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Gender Bias in Parental Investments in Children's Education: A Theoretical Analysis

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Abstract. Human capital accumulation is one of the main engines of economic growth. Thus, many LDCs have introduced laws over the past 30 years for compulsory education and have increased their investment in public schooling. Nevertheless, the level of education in most poor countries is still very low, particularly for girls. The goal of this article is to develop a model of household decision-making in order to better understand what variables affect parents' decision to educate girls less than boys. In the first part of the paper, a unitary model, a non-cooperative household model, and a bargaining model are developed and compared to explain factors that might produce gender bias in investment in education. As a result, the number of years of education for male and female children depends on the different costs and returns of educating girls and boys and, in the non-consensus models, on each parent's preferences and decision power. The second part of the paper contains a simulation of the models assuming different policies for increasing women's education using figures from the Living Standard Measurement Studies of Cote d'Ivoire.

Keywords: education, gender bias, household bargaining model

JEL Classfication: D1, C78, J16

Introduction

Many studies have underlined the importance of increasing female education in LDCs to reduce fertility rate (Susan H. Cochrane, 1979; Dina Abu-Ghaida and Stephen Klasen, 2004), to ameliorate child health conditions (Duncan Thomas, 1990, 1994; Lawrence Haddad and John Hoddinott, 1994; Kambhampati Subbarao and Laura Raney, 1995), and to change the patterns of household consumption leading to a reduction in the income share spent on adult goods (Nancy R. Folbre, 1984; Mark Rosenzweig and Kennet Wolpin, 1988; Lawrence Haddad and Ravi Kanbur, 1990; Mark Pitt, Rosenzweig and Hassan, 1990; John Hodinott and Lawrence Haddad, 1995).

The literature on investment in child education makes use mainly of the unitary model.[1] Gary S. Becker and Nigel Tomes (1976) assume altruistic parents choosing the 'quantity' and the 'quality' of their children, where the quality depends on both endowed ability and education received. This model can be used to analyze gender bias in education: if returns to female education are lower and parents do not prefer boys, girls receive more education in order to reach the same income level as their brothers. However, when the cost of educating a girl is greater than the cost of educating a boy, parents might decide to invest more in boys' human capital and, possibly, to compensate the girls with a cash transfer. Jere Behrman, Robert Pollak and Paul Taubman (1982) integrated the Becker and Tomes' unitary model by considering inequality-adverse parents and parents who care only about the efficiency of their investment. Only in the 'efficiency' case do parents tend to invest more in high-returns children.[2]

The unitary model has recently been criticized because of its weak theoretical foundations (Pierre A. Chiappori, 1992), its inability to be used to perform intra-household welfare analysis (Patricia Apps and Ray Rees, 1988), and its empirical failures in both developed and developing countries (see Jere R. Berhman, 1997, for a review). Furthermore, in the context of developing countries, Lawrence Haddad, John Hoddinot, and Harold Aldeman (1997) argue that using a unitary model as a guideline for policy prescriptions may lead to serious policy failures.

In order to avoid the theoretical and empirical problems posed by the traditional unitary model, a new approach (Marilyn Manser and Murray Brown, 1980; Marjorie McElroy and Mary Jean Horney, 1981) has been developed starting from the John Nash bargaining model (1950, 1953): household decisions are considered the result of a bargaining process among family members who differ in their preference orderings and decision powers. Another important approach to the household decision process is given by the *collective approach* model developed by Pierre A. Chiappori (1988, 1992) in which decisions are assumed to be always Pareto-efficient and no restrictions are imposed on the decision process. The literature has pointed out to an *observational equivalence* (Jere R. Behrman, 1997; Lawrence Haddad et al., 1997): there are cases where the unitary models and the collective models yield similar predictions.

The aim of the paper is to show how a two-period household Nash-bargaining model can be developed and used to analyze how in poor countries market factors interact with traditional values in affecting parents' decision to invest in their children's education. The Nash-bargaining model has been preferred to the collective model *à la Chiappori* because it also allows us to consider non-cooperative outcomes that are especially relevant in poor countries (Christopher Udry, 1996). In order to take observational equivalence into consideration, a unitary model has also been developed.

The paper is organized as follows: in Sections 1, 2 and 3, respectively, a unitary, a non-cooperative, and a bargaining model are presented; Section 4 shows how different mother's and father's preferences, decision powers, and different costs and returns to female and male schooling can interact to produce under-investment in girls' education; in Section 5 the models are simulated using Cote d'Ivoire data while in Section 6 calibration is used to show the possible effects of alternative policies. Conclusions follow.

1. The unitary model: a starting point to analyze household decisions

The different household decision models proposed in this paper take into account some of the possible causes of gender bias in education (parents' preferences, costs and returns on female education, traditional values) in order to analyze which are the most relevant factors that affect parents' decisions about children's education and how gender bias in education can result. The first step is to develop a unitary model that will be used in the following sections to study possible cases of observational equivalence.

Let's consider a household composed of a father-husband (m = male), a mother-wife (f = female), and two children, a boy (b) and a girl (g)[3,4]. Individuals live three periods: in the period 0 they are children and they live with their parents who invest in their education. Without loss of generality, children's consumption is set equal to zero. In period 1, people get married and have children; in this period they decide their consumption, labor supply, and investment in children's education. In the period 2, individuals are old and they do not work, but they survive with their savings and with the transfers they receive from their children (since in most poor countries there are no pensions).

There are two types of goods:

1. private goods, consumed by each individual in the second and third period of his/her life;
2. public goods for the couple, consumed only within the family.

In the unitary model parents' preferences are represented by a unique utility function (U^{HH}), assumed additive separable in the two periods of adulthood,[5] twice continuously differentiable, and strictly quasi-concave. Therefore:

$$U^{HH} = U^{HH}(c_1^m, c_1^f, c_2^m, c_2^f, x_1^b, x_1^g)$$

where c_t^i is the consumption of market goods for parent i (= m, f) in period t (= 1, 2) and x_1^j is the investment in child j's education ($j = b,g$) expressed in years.[6] In the unitary model preferences are altruistic and all goods can be considered as public.[7]

The labor supply is assumed inelastic and equal to one.[8] Parent's income is given by the sum of non-labor income I^i (given by social transfers, allowances and aid) and labor income $\bar{w}^i(1 + \eta^i \bar{x}_0^i)$, where $\bar{w}^i$ is the wage of the non-educated worker, $\bar{x}_0^i$ is the level of i's education acquired in period 0 and η^i is the rate of return to education. If there is no discrimination in the labor market, $\bar{w}^i$ and η^i are equal for men and women.

Parents decide on how much to invest in their children's education given that:

1. the minimum cost of providing one year of education (as in Jere R. Behrman et al., 1999), p_1^j, may be different for boys and girls, given that it includes both direct and indirect costs that are usually different. In particular, the opportunity cost of girls' education is often higher since young girls, especially in South Asia and in Africa, help their mothers in subsistence agriculture, in looking after their younger siblings, or in gathering wood and water.[9] In contrast, boys usually

start to work later in household productive activities (Pitt, Rosenzweig, and Hassan, 1990; Simon Appelton et al., 1995) and therefore they can study at least for a few years prior to their work in household production.[10] Indirect costs thus also include foregone income from child labor;[11]

2. returns to education, $\eta^j \in [0,1]$, are generally lower for girls than for boys because of discrimination in the labor market,[12] and the gap is particularly large in poor countries;[13]
3. there exists a 'social norm' according to which adults give a fraction $\lambda^j \in [0,1]$ of their income to their old parents (as in Alessandro Cigno, 1993)[14]. In fact, in poor countries with no pension scheme, old people rely on the resources received from their adult children. Also the value of λ^j is, in general, different for men and women.[15] Assuming that this contribution is independent of spouse's income, we can ignore the potential effects of education via the marriage market.

When parents decide how much to invest in their children's education,[16] they do not consider the positive effects of girls' education on fertility rates and on their children's well-being. If social returns to women's education are greater than private returns, the resulting level of investment in female human capital will tend to be sub-optimal, leaving room for policy action.

Since household decisions concern present consumption, future consumption and investment, it is necessary to make some assumptions about expectations. Here static expectations are assumed and therefore λ^i, η^j, $\bar{w}^i$ e I^i are constant over time and across generations and hence we have $\lambda^i = \lambda^j$, $\eta^i = \eta^j$ and $\bar{w}^i = \bar{w}^j$ for $i = m$ and $j = b$, and for $i = f$ and $j = g$, respectively.

The household's maximization problem can be written as:

$$\max_{c_1^m, c_1^f, c_2^m, c_2^f, x_1^b, x_1^g} \quad U^{HH} = U^{HH}(c_1^m, c_1^f, c_2^m, c_2^f, x_1^b, x_1^g)$$

$$\begin{aligned}\text{subject to:} \quad & p_1^m c_1^m + p_1^f c_1^f + p_2^m (1+r)^{-1} c_2^m + p_2^f (1+r)^{-1} c_2^f + p_1^b x_1^b + p_1^g x_1^g \\ &= (1-\lambda^b)\{[\bar{w}^m(1+\eta^b \bar{x}_0^m)] + I^m\} + (1-\lambda^g)\{[\bar{w}^f(1+\eta^g \bar{x}_0^f)] + I^f\} \\ &\quad + (1+r)^{-1}\{\lambda^b[\bar{w}^m(1+\eta^b x_1^b) + I^b] + \lambda^g[\bar{w}^f(1+\eta^g x_1^g) + I^g]\}\end{aligned}$$

where r is the real interest rate used to actualize future values, p_t^i is the price of private goods in period t[17] and p_1^j is the cost of one year of schooling. The actualized sum of parents' consumption plus the total expenses for children's education must be equal to the present value of the household's resources. These are given by the parents' incomes in period 1 plus the sum of the transfers that the parents receive in period 2 from their adult son and daughter.

If there are no liquidity constraints and $r = 0$ (for simplicity), the first order conditions for x_1^b and x_1^g are given by:

$$\frac{\partial U^{HH}}{\partial x_1^b} - \upsilon[p_1^b - \lambda^b \bar{w}^m \eta^b] = 0 \qquad \text{and} \qquad \frac{\partial U^{HH}}{\partial x_1^g} - \upsilon[p_1^g - \lambda^g \bar{w}^f \eta^g] = 0,$$

where υ is the Lagrangian multiplier. These conditions are both sufficient and necessary since we assumed strictly quasi-concave utility functions.

Let's assume a log-linear utility function such that:

$$U^{HH} = \alpha \ln c_1^m + a \ln c_1^f + \beta \ln c_2^m + b_2^f + \delta \ln x_1^b + \varepsilon \ln x_1^g,$$

where α, a, β, b, δ, and ε are all positive parameters. The first order conditions become:

$$\frac{\delta}{x_1^b} - \upsilon[p_1^b - \lambda^b \bar{w}^m \eta^b] = 0 \qquad \text{and} \qquad \frac{\varepsilon}{x_1^g} - \upsilon[p_1^g - \lambda^g \bar{w}^f \eta^g] = 0$$

and therefore:

$$\frac{x_1^g}{x_1^b} = \frac{p_1^b - \lambda^b \bar{w}^m \eta^b}{p_1^g - \lambda^g \bar{w}^f \eta^g} \frac{\varepsilon}{\delta}$$

The first term on the right-hand side is the ratio between the parents' net benefits of investing in one year of education for the son and the daughter, while the second term shows the effect of parents' preferences. In Section 4 the possible causes of gender bias in investment of education will be discussed and it will be shown when this result is *observationally equivalent* to the result of a bargaining model.

2. A non-cooperative model for household decisions

In this section the more realistic assumption that the father and the mother have different preferences is introduced. In the non-cooperative setting, each parent, with different preferences and separate economic spheres, decides how much to consume and how much to invest in his/her son's and daughter's education. As in Shelly Lundberg and Robert A. Pollak (1993), the non-cooperative outcome or, more precisely, the level of indirect utility that each parent can reach in a non-cooperative setting, will be used as a threat point for the cooperative Nash-bargaining model (in the next section).

Parents have egoistic preferences (see note 8) that depend on private and public goods. In fact, in the non-unitary models c_t^i becomes a private good, while x_1^j is public. *m*'s and *f*'s intertemporal utility functions have the same characteristics as before (additive separable in the two periods, twice continuously differentiable, and strictly quasi-concave), and can be written as:

$$U^m = U^m(c_1^m, c_2^m, x_1^b, x_1^g) \qquad \text{and} \qquad U^f = U^f(c_1^f, c_2^f, x_1^b, x_1^g)$$

In the non-cooperative solution each parent chooses the level of his/her own contribution to each child's education given the spouse's contributions (Zhiqi Chen and Frances Woolley, 2001), i.e. parent *i* pays $x_{1,i}^b$ years of the boy's schooling and $x_{1,i}^g$ years of the girl's schooling. The choice of these contributions is made simultaneously with the choice of c_1^i and c_2^i in order to maximize *i* 's utility function subject to the budget constraint and to the spouse's (*k*'s) contributions to the children's education $\bar{x}^b_{1,k}$ and $\bar{x}^g_{1,k}$, that i considers as given. The Cournot–Nash equilibrium gives the two pairs of parents' contributions to their children's education ($x^{b^*}_{1,i}, x^{g^*}_{1,i}$), where $i = m, f$.

Let's consider m's maximization problem, that can be written as:

$$\max_{c_1^m, c_2^m, x^b_{1,m}, x^g_{1,m}} \quad U^m = U^m[c_1^m, c_2^m, (x^b_{1,m} + \bar{x}^b_{1,f}), (x^g_{1,m} + \bar{x}^g_{1,f})]$$

$$\text{subject to:} \quad p_1^m c_1^m + p_2^m (1+r)^{-1} c_2^m + p_1^b x^b_{1,m} + p_1^g x^g_{1,m} = (1-\lambda^b)[\bar{w}^m(1+\eta^b \bar{x}_0^m) + I^m]$$
$$+ (1+r)^{-1} \frac{1}{2} \{\lambda^b [\bar{w}^m (1+\eta^b x_1^b) + I^b] + \lambda^g [\bar{w}^f (1+\eta^g x_1^g) + I^g]\}$$

where $x_1^j = x^j_{1,m} + \bar{x}^j_{1,f}$.

The sum of m's private consumption in the two periods plus the amounts devoted to the children's education must be equal to the sum of m's income (minus the transfer m makes to his old parents) and the actualized value of the transfer that m will receive in his old age from his adult children, assuming that the parents divide the resources received from their children equally and that period 2 has the same length for both parents.[18] If there are no liquidity constraints the present value of consumption is equal to the present value of incomes.

The demand functions for $x^b_{1,m}$ and $x^g_{1,m}$ are conditional upon the values of $\bar{x}^b_{1,f}$ and $\bar{x}^g_{1,f}$ chosen by f , and therefore they must be considered as reaction functions. Since $x^b_{1,m}$ and $x^g_{1,m}$ can not be negative, m's best reaction functions are given by:

$$B(x^b_{1,m}) = \max\left\{ \hat{x}^b_{1,m}(\bar{x}^b_{1,f}, \bar{x}^g_{1,f}, p_t^m, p_t^j, \lambda^j, \bar{w}^j, \eta^j, \bar{x}_0^m, I^m, I^j, r), 0 \right\}$$
$$B(x^g_{1,m}) = \max\left\{ \hat{x}^g_{1,m}(\bar{x}^b_{1,f}, \bar{x}^g_{1,f}, p_t^m, p_t^j, \lambda^j, \bar{w}^j, \eta^j, \bar{x}_0^m, I^m, I^j, r), 0 \right\},$$

where $j = b$ and g and $t = 1, 2$.

If f chooses a level of contribution to j's education that guarantees what m considers 'sufficient', m's contribution will be zero in the Nash equilibrium with common consistent beliefs.

In the same way, we can analyze f's maximization problem in order to find f's best reaction functions to m's choice.

The outcome of the non-cooperative equilibrium is generally not Pareto-efficient and, as we will see in the simulation of the model (Section 5), parents can reach higher levels of utility by cooperating. However, as Luisa Smith (1994) and Christopher Udry (1996) have pointed out, non-cooperation occurs quite often, especially in African countries where spouses have separate economic spheres, and therefore the Cournot–Nash equilibrium can be relevant in explaining parents' decisions.

Now let's assume log-linear utility functions:

$$U^m = \alpha \ln c_1^m + \beta \ln c_2^m + \delta \ln(x^b_{1,m} + x^b_{1,f}) + \varepsilon \ln(x^g_{1,m} + x^g_{1,f})$$

$$U^f = a \ln c_1^f + b \ln c_2^f + d \ln(x^b_{1,m} + x^b_{1,f}) + e \ln(x^g_{1,m} + x^g_{1,f})$$

For realistic values of the parameters and of the exogenous variables (such as those used in the simulation of the model in Section 5), the best reaction functions have the following slopes:

$$\frac{\partial x^b_{1,i}}{\partial x^b_{1,k}} 0 \quad \frac{\partial x^g_{1,i}}{\partial x^g_{1,k}} 0 \quad \text{where } k \text{ is } i\text{'s spouse}$$

When a parent increases his/her contribution to one child's schooling, the other parent augments the contribution to the other child's education. The log-linearity of the utility functions leads to optimal solutions in which both children receive a contribution to their education from at least one parent. Since preferences are concave and the budget constraint is linear, i's best reaction functions have a unique value for each pair of k's contributions to public goods. If both parents pay for both public goods we have an internal solution. Otherwise, there is a corner solution.

The Cournot–Nash equilibrium is given by the simultaneous solution of the four best reaction functions. The equilibrium values for the private consumption in the two periods also depend on $\bar{x}^b_{1,k}$ and $\bar{x}^g_{1,k}$ and can be calculated after we have obtained the equilibrium level of m's and f's contributions to each of the public goods. The demand functions for consumption and children's schooling do not depend only on the individual's income, prices, and preference parameters, but also on the spouse's income that conditions his/her contribution to public goods.

Ted Bergstrom, Larry Blume, and Hal Varian (1986) showed that in a bargaining model with public goods, best reaction functions are continuous in a compact and convex set and, as a consequence, we can use the Brower Fixed Point Theorem to show that a Nash equilibrium exists. This equilibrium is unique if the public good is a normal good.

Moreover, Peter Warr (1983) showed that in a voluntary Nash equilibrium with more than one public good, when players have egoistic preferences and payment for the public goods is voluntary, income redistribution between contributors that leave total income unchanged can modify, in the event of corner solutions, both the level of each individual contributions to each public good and the total amount of contributions (and therefore the total amount spent on public goods).[19] In fact, after redistribution, some of the contributors may decide not to pay anymore for one of the public goods (e.g. the education of one of the two children).[20]

The equilibrium values for c^i_1, c^i_2, x^b_1 and x^g_1 that are obtained are then substituted into i's utility function and we find the value of the indirect utility V^i for the non-cooperative solution. This value is used in the next section to define the threat point for i in the cooperative bargaining model.

3. A cooperative bargaining model for gender bias in investment in education

In the cooperative model spouses bargain in order to decide how to allocate resources between the consumption of private and public goods. Spouses' cooperation is based on the threat of non-cooperation and the indirect utilities corresponding to these threat points are given by V^m and V^f obtained in the previous section.[21]

The Nash bargaining problem consists of the maximization of the product of each player's gain from cooperation, measured by the difference between the level of utility reached by cooperating and the level of utility obtained in case of non-cooperation (threat point), and it can be written as:

$$\max \mathrm{N} = [U^m(c_1^m, c_2^m, x_1^b, x_1^g) - V^m]\,[U^f(c_1^f, c_2^f, x_1^b, x_1^g) - V^f]$$

subject to the household's budget constraint[22]

$$\begin{aligned} &p_1^m c_1^m + p_1^f c_1^f + p_2^m (1+r)^{-1} c_2^m + p_2^f (1+r)^{-1} c_2^f + p_1^b x_1^b + p_1^g x_1^g = \\ &(1-\lambda^b)\{[\bar{w}^m(1+\eta^b \bar{x}_0^m)] + I^m\} + (1-\lambda^g)\{[\bar{w}^f(1+\eta^g \bar{x}_0^f)] + I^f\} \\ &\quad + (1+r)^{-1}\{\lambda^b[\bar{w}^m(1+\eta^b x_1^b) + I^b] + \lambda^g[\bar{w}^f(1+\eta^g x_1^g) + I^g]\} \end{aligned}$$

If there are no liquidity constraints and $r = 0$ (as before), the first order conditions for x_1^b and x_1^g are given by:

$$\frac{\partial U^m}{\partial x_1^b}(U^f - V^f) + \frac{\partial U^f}{\partial x_1^b}(U^m - V^m) - \upsilon[p_1^b - \lambda^b \bar{w}^m \eta^b] = 0 \qquad (1)$$

$$\frac{\partial U^m}{\partial x_1^g}(U^f - V^f) + \frac{\partial U^f}{\partial x_1^g}(U^m - V^m) - \upsilon[p_1^g - \lambda^g \bar{w}^f \eta^g] = 0 \qquad (2)$$

where υ is the Lagrangian multiplier. These conditions are again both sufficient and necessary since we assumed strictly quasi-concave utility functions.

When we solve the full set of first order conditions, we obtain the optimal level of consumption for both private and public goods. As in the non-cooperative equilibrium, the father's and the mother's incomes enter separately into the demand functions, and therefore the effects of changes in the income of one of the parents might differ from the effects of changes in the income of the other parent.

If we assume log-linear utility functions, conditions (1) and (2) become:

$$\frac{\delta}{x_1^b}(U^f - V^f) + \frac{d}{x_1^b}(U^m - V^m) + \upsilon[p_1^b - \lambda^b \bar{w}^m \eta^b] = 0$$

$$\frac{\varepsilon}{x_1^g}(U^f - V^f) + \frac{e}{x_1^g}(U^m - V^m) + \upsilon[p_1^g - \lambda^g \bar{w}^f \eta^g] = 0,$$

$$\text{and therefore:} \quad \frac{x_1^g}{x_1^b} = \frac{p_1^b - \lambda^b \bar{w}^m \eta^b}{p_1^g - \lambda^g \bar{w}^f \eta^g}\,\frac{\varepsilon(U^f - V^f) + e(U^m - V^m)}{\delta(U^f - V^f) + d(U^m - V^m)}. \qquad (3)$$

As in the unitary model, the first fraction on the right-hand side is the ratio between the parents' net benefits of investing in one year of education for a son and a daughter. In contrast, the second ratio, shows how preference parameters interact with the values of the indirect utility associated with the threat points in affecting the parents' decision concerning their children's education. In fact, $(U^i - V^i)$ represents the gain from cooperation that, *ceteris paribus*, decreases as the utility of the threat point increases. Therefore, V^i can be considered as a measure of i's decision power within the family. ε, e, δ and d are the parameters representing the weights of a boy's and a girl's education in parental utility functions.

Equation (3) allows us to consider different hypotheses about the parents' preferences. The next section analyzes four hypotheses that seem to be most realistic in the context of poor countries.

4. Decision powers, preferences and gender bias in education

Hypothesis 1. When m and f have the same preferences about their children's education and there is no gender bias, we have $\delta = d = \varepsilon = e$ and (3) becomes:

$$\frac{x_1^g}{x_1^b} = \frac{p_1^b - \lambda^b \bar{w}^m \eta^b}{p_1^g - \lambda^g \bar{w}^f \eta^g} \tag{4}$$

This is exactly the result we find in the unitary model under the assumption of equal weight in the household's utility function a boy's and a girl's education ($\delta = \varepsilon$).[23]

We find gender bias in education ($x_1^g < x_1^b$) only when $p_1^b - \lambda^b \bar{w}^m \eta^b < p_1^g - \lambda^g \bar{w}^f \eta^g$. In poor countries this is likely to happen because of three different issues. First, the cost of education is higher for girls than for boys (see Section 1) because both direct costs and indirect costs are higher. As a consequence, $p_1^b < p_1^g$.

As we have seen in Section 1, private returns to education are often lower for women because of discrimination in the labor market. Cristina Echevarria and Antonio Merlo (1999) attribute this to women's reproductive role since they usually have to stop working when they have children, especially where social protection of working mothers is absent. Moreover, relative to men, women are less likely to work in the formal sector and parents under-evaluate the positive effects of education on household activities and child rearing.[24] In terms of the model, this means that the wage of the unskilled worker and the rate of return to education are higher for men than for women ($\bar{w}^m > \bar{w}^f$ and $\eta^b > \eta^g$). Finally, the contribution to old parents' consumption (λ^j) is usually greater for males, because when a woman gets married, she often leaves her family and becomes a member of the husband's household. Therefore, she contributes less than her brothers to her old parents' consumption (Susan Horton, 1988; Simon Appleton, 1996) and we have $\lambda^b > \lambda^g$.[25]

The joint effect of these circumstances produces gender bias in education in many LDCs. This model can also be applied to rich countries where, in some cases, women's education is still lower than men's. While in developed countries the cost of educating a girl is equal to the cost of educating a boy, discrimination still exists in the labor market, causing returns to female education to be lower than returns to male education.

Hypothesis 2. There is no gender bias in parents' preferences, but the mother cares more than the father about the children's education, and therefore $d > \delta$, $e > \varepsilon$, but $d = e$ and $\delta = \varepsilon$. In fact, Duncan Thomas (1990) and Jessica Patton (1993) showed that mothers tend to spend a greater income share in goods benefiting their children. As in the previous case, (3) becomes equal to (4) and we obtain the same results as before in terms of the ratio between x_1^g and x_1^b, but not in terms of levels.

Hypothesis 3. Here the mother not only gives more importance to the children's education than the father, but she also has a preference for the daughter, while the father prefers the son. This means that $\delta > \varepsilon$ and $d < e$, but $\delta < d$ and ε. The assumption that mothers prefer girls seems to be confirmed by Aysit Tansel (1997) for Cote d'Ivoire and Ghana, and by Duncan Thomas (1994), who analyzed the impact of mother's and father's education on children's health in the United States, Brazil and Ghana. In this case, equation (3) cannot be simplified and we have to analyze the effects of changes in the values of the threat points. For LDCs, under the assumptions regarding the values of the parameters that were mentioned above, we have (see proofs in the Appendix):

$$\frac{\partial\left(\frac{x_1^g}{x_1^b}\right)}{\partial V^f} > 0 \quad \text{and} \quad \frac{\partial\left(\frac{x_1^g}{x_1^b}\right)}{\partial V^m} \tag{5}$$

Improvements in the value of the mother's threat point decreases gender bias in education, while an increase in the value of the father's threat point increases the difference between x_1^g and x_1^b. The relative magnitude of the two derivatives depends on the relative gains from cooperation.

Hypothesis 4. Both parents may have a preference for the son, especially where traditional values are dominant. Gender bias in parents' preferences (Geeda G. Kingdon, 1998) reinforces the effects of the market. In fact, if both the mother and the father want to spend more on the boy's education we have $\delta > \varepsilon$ and $d > e$, and if they give the same weight in their utility function to the children of the same gender we also have $\delta = d$ and $\varepsilon = e$. Therefore, equation (3) becomes:

$$\frac{x_1^g}{x_1^b} = \frac{p_1^b - \lambda^b \bar{w}^m \eta^b}{p_1^g - \lambda^g \bar{w}^f \eta^g} \frac{\varepsilon}{\delta}$$

and, as a consequence of the assumption made on costs and returns to boys' and girls' education, we obtain a greater difference between x_1^g and x_1^b. In contrast to Hypothesis 3, under Hypothesis 4 parents' individual threat points do not affect the ratio between x_1^g and x_1^b. In fact, threat points affect this ratio only when one parent has a stronger preference for the boy while the other has a stronger preference for the girl. Here again, we have a case of observational equivalence (both in terms of levels and in terms of the ratio) with a unitary model in which $\delta > \varepsilon$.

In the model presented, the sources of gender bias in education are related not only to the different costs and returns of schooling for boys and girls, but also to the different parental preferences and decision powers.[26]

The entire model has been developed under the assumption of one boy and one girl in the household. This assumption does not affect the result obtained (see note 5), either in the theoretical model, or in the simulation. However, what matters is the total number of children in the household: when the number of children increases, if parents want to invest a constant share of their income in child education, the average number of years of schooling for each boy and each girl decreases.

In all models presented (unitary, non-cooperative and cooperative), no liquidity constraints are assumed. When we consider binding liquidity constraints, we have two different effects: investments in the children's education and the parents' consumption in period 1 are reduced with respect to their optimal value, but here the parents' consumption in period 2 is also reduced, since it depends on the level of the children's education that is now lower. Therefore, liquidity constraints can have a large negative impact both on parents' and on children's well-being. Jere R. Behrman (1997) shows that in a unitary model with no gender bias in parents' preferences, differences in costs and returns to education may induce parents to invest differently in their children's education when liquidity constraints are binding.

5. Simulations

In this section the model is simulated in order to obtain an explicit solution that allows us to study how parents' preferences interact with market characteristics and with traditional households' attitudes in order to produce gender bias in education.

Calibration has often been used for economic policy purposes (Finn E. Kydland and Edward C. Prescott, 1982; Finn E. Kydland, 1992; Fabio Canova 1993; Charlotte Wojcik, 2000). The results obviously depend on the choice of the values for the parameters and for the exogenous variables that is usually made in order to obtain the observed values of the endogenous variables or, alternatively, by using estimates obtained in other studies (Fabio Canova, 1994; Thomas F. Cooley, 1997). The crucial point is, therefore, to select a set of coherent values for the parameters and to evaluate the results in the limited area defined by the set of chosen parameters.

For the purpose at hand, the values for the exogenous variables are taken from the World Bank Living Standard Measuring Study (LSMS) dataset for Cote d'Ivoire for the years 1985–1986. This dataset contains information about household socio-economic characteristics, earnings, and consumption of almost 1600 families and 15,000 individuals in different areas of the country. Around 70% of the sample households live in rural areas, while only 12% live in Abidjan (the capital).

Cote d'Ivoire experienced quite high rates of economic growth during the 70s, mainly due to export of crops and public investments. In the 80s, due to a severe recession, Cote d'Ivoire had to reduce public investments, particularly in education.

As a consequence, like most Sub-Saharan countries, Cote d'Ivoire has very low levels of education, especially for women. According to the survey, girls between 12 and 16 receive on average 3.31 years of schooling, while boys receive 4.75; girls between 17 and 19 have on average 3.53 years of education while boys have 5.7 (see also Aysit Tansel, 1997). This situation did not improve much in the last few years, as confirmed by the most recent World Bank data reported in Dina Abu-Ghaida and Stephan Klasen (2004) that show a female enrolment rate in primary schools of 66% compared to 88% of males, and a female enrolment rate in secondary education of 15% compared to the male rate of 28%.

In choosing the values for the simulation of the model, the female non-skilled worker wage is assumed to be 96.5% of the male non-skilled worker wage. It is also assumed that the mother has 0.47 years of schooling while the father has 1.55.[27] The rates of return to education are those estimated by Paul T. Schultz (1993) for Cote d'Ivoire: 10.9% for women and 14% for men.[28] Using these values, we can obtain a female labor income rate equal to 83.4% of the male labor income rate at average levels of education. This is consistent with the results obtained by Simon Appleton et al. (1995) and Aysit Tansel (1997) for Cote d'Ivoire. Moreover, individuals' non-labor income is assumed to be 10% of the total income. The cost of educating a girl is assumed to be 20% higher than the cost of educating a boy, and this reflects mainly the difference in indirect costs. Finally, the assumption that men transfer 20% of their income to their old parents, while women transfer only 10%, is considered. However, we know that households in Cote d'Ivoire can be either patri- or matri-lineal, and when the latter is the case $\lambda^g > \lambda^b$. As shown in the sensitivity analysis, results will not change much when we consider the case of matri-lineal households. The values of the parameters in the parents' utility functions are chosen in order to obtain an income share of close to 5% spent on children's education. In fact, according to Christiaan Grootaert and Ravi Kanbur (1995), the average expenditure on education for parents' of school-age children in Cote d'Ivoire is 5.75% of their total expenses in Abidjan, 5.4% in other urban areas, 4.5% in rural areas, and only 2.5% in the poorest areas of the country.

By substituting these values for the exogenous variables and for the various parameters it is possible to simulate the model to obtain a numerical solution under the four different hypotheses. The results are presented both for cooperative and non-cooperative behavior. In Africa, in fact, household members often do not pool their incomes and spouses have separate economic spheres (Luisa Smith, 1994), and therefore, both models can be relevant in explaining household decisions on investment in children's education.

Hypothesis 1. Here we assume the following values of the preference parameters: $\alpha = a = 0.52$, $\beta = b = 0.42$ and $\delta = d = \varepsilon = e = 0.03$. The results of the simulation are shown in Table 1: in the non-cooperative case the father pays equally for both the girl's and the boy's education, while the mother, who has a lower income, pays only for the boy. Here, gender bias in education is the result of an efficient investment strategy that favors less costly and more productive children (i.e. boys).

Table 1. Simulation—Hypothesis 1.

	Non-cooperative	Cooperative
Father's consumption in $t=1$	18.74	17.97
Father's consumption in $t=2$	15.13	14.52
Mother's consumption in $t=1$	18.74	17.97
Mother's consumption in $t=2$	15.13	14.52
Boy's years of education	1.56 [0.96 (*m*)/0.6 (*f*)]	7.20
Girl's years of education	0.96 (*m*)	2.13
Father's indirect utility	2.677	2.708
Mother's indirect utility	2.677	2.708

In the cooperative case the demand for both public goods increases. However, the gender gap in education also grows. Again, this is the result of the efficient parents' strategy. We can finally observe that, under the cooperative behavior, both parents reach a higher level of utility (V^m) and (V^m), and this means that it is advantageous for them to cooperate.

As already mentioned, this cooperative model in which parents have the same utility function gives exactly the same results as a unitary model where the unique parents' utility function is such that $\alpha = a = 0.26$, $\beta = b = 0.21$ and $\delta = \varepsilon = 0.03$.

Hypothesis 2. Here the values for the parameters in the utility functions are set as follows: $\alpha = 0.52, \beta = 0.42, \delta = 0.03, \varepsilon = 0.03, a = 0.50, b = 0.40, d = 0.05$ and $e = 0.05$. Table 2 shows the results obtained under this assumption.

In both cases we obtain higher levels of schooling for both children compared with the results of Hypothesis 1. In the non-cooperative case, the mother pays for the son's education while the father pays a lower amount for the daughter's education. In the cooperative case, we have a greater difference in absolute value in the level of education between children.

Hypothesis 3. Here the values for the parameters of the utility functions are set as follows: $\alpha = 0.54, \beta = 0.42, \delta = 0.022, \varepsilon = 0.018, a = 0.50, b = 0.397, d = 0.026,$ and $e = 0.077$. In Table 3 we can observe how the non-cooperative case produces a lower

Table 2. Simulation—Hypothesis 2.

	Non-cooperative	Cooperative
Father's consumption in $t = 1$	19.84	19.16
Father's consumption in $t = 2$	16.02	15.47
Mother's consumption in $t = 1$	17.53	16.14
Mother's consumption in $t = 2$	14.02	12.91
Boy's years of education	2.53 (*f*)	9.44
Girl's years of education	1.02 (*m*)	2.79
Father's indirect utility	2.747	2.784
Mother's indirect utility	2.535	2.577

Table 3. Simulation—Hypothesis 3.

	Non-cooperative	Cooperative
Father's consumption in $t=1$	19.87	18.61
Father's consumption in $t=2$	15.49	14.54
Mother's consumption in $t=1$	17.25	17.61
Mother's consumption in $t=2$	13.70	13.89
Boy's years of education	1.29 (*f*)	5.65
Girl's years of education	0.59 (*m*)	3.41
Father's indirect utility	2.763	2.764
Mother's indirect utility	2.429	2.621

gender bias in education but also less schooling for both children. The mother pays completely for the boy's schooling, while the father pays only for the girl (as under Hypothesis 2). In the cooperative case, we obtain a decrease in boy's schooling, but an increase in girl's education with respect to all the previous cases. The preference of the mother for the girl's schooling is so strong that it compensates for her lower bargaining power.

As discussed in Section 5 and in the Appendix, an equal increase in the value of the indirect utilities of the father and the mother have different effects on the level of boys' and girls' education. In particular, if we increase the father's indirect utility by 0.5, the girl's education decreases by 0.45% while the boy's education remains practically unchanged (and therefore the ratio x_1^g/x_1^b decreases). On the contrary, when we increase the mother's indirect utility by 0.5, the girl's education increases by 0.47% while the boy's education remains practically unchanged (and therefore the ratio x_1^g/x_1^b increases).

Hypothesis 4. For this last hypothesis, the values of the parameters are set at: $\alpha = a = 0.52, \beta = b = 0.42, \delta = d = 0.04,$ and $\varepsilon = e = 0.02$. In Table 4 we can see that in the non-cooperative case the father (the richest parent) pays for the boy's education while the mother pays for the girl's schooling. When we consider the

Table 4. Simulation–Hypothesis 4.

	Non-cooperative	Cooperative
Father's consumption in $t=1$	18.82	18.00
Father's consumption in $t=2$	15.20	14.54
Mother's consumption in $t=1$	18.74	17.93
Mother's consumption in $t=2$	15.14	14.49
Boy's years of education	2.09 (*m*)	9.60
Girl's years of education	0.64 (*f*)	1.42
Father's indirect utility	2.690	2.725
Mother's indirect utility	2.686	2.721

cooperative case, the difference between the girl's and boy's education reaches its maximum both in absolute and in relative values because preference bias and market factors work in the same direction.

Under this hypothesis, the results for the boy's and the girl's education [29] in the cooperative case are equal to the results obtained with a unitary model in which the values of the parameters in the unique utility function are such that $\alpha > a$, $\beta > b$ and $\delta = 0.04 > \varepsilon = 0.03$. Therefore, again we have observational equivalence between the cooperative and the unitary model.

As we have seen, under all the hypotheses but Hypothesis 3 the cooperative case produces a larger gender gap in education than the non-cooperative one. In the cooperative case the outcome of the bargaining is always more efficient and, therefore, the parents tend to invest more in the less expensive and more productive child. The cooperative outcome of Hypothesis 3 seems to better fit actual average values of boys' and girls' education in Cote d'Ivoire, confirming that mothers have lower bargaining power but care more about their children than the 'more powerful' fathers, and they try to favor their daughters in the household's resources allocation process. All the simulations have been performed under the assumption of no liquidity constraints. If liquidity constraints are binding, under all hypotheses we observe a decrease in the boy's and the girl's years of education and in the father's and mother's consumption in both period 1 and 2.[30] In the simulations performed here, however, liquidity constraints are never binding.

5.1. *Sensitivity analysis* [31]

The objective of the sensitivity analysis is to evaluate how robust the results are to reasonable perturbation of the parameters around the calibrated values (Fabio Canova, 1995). In particular, a change in the parents' preference parameters is assumed such that the parents' want to spend a lower (higher) percentage of their income on their children's education and a higher (lower) percentage for their own consumption.[32] Under all hypotheses the results of the simulation change in the expected direction and are of reasonable magnitude. This seems to indicate that the model is not too sensitive to the choice of the values of the parameters.

Robust results have also been obtained when assuming changes in the values of the exogenous variables (for example, in the average years of education of the mother and father, or in the level of the women's and men's non-skilled labor income).

The assumption of the non-cooperative model that parents equally share the resources they receive from their children has also been relaxed by assuming different types of division.[33] The simulations of the non-cooperative model show that x_1^b and x_1^g change in the expected direction, and these changes are always relatively small. In the cooperative model, instead, changes in the assumption on how parents divide the resources received from their children produce only small changes in the values of the indirect utility of the threat points with consequent small changes in the values of x_1^b and x_1^g. These changes are all in the expected direction with respect to the changes in V^m and V^f.

Another assumption that has been relaxed is the assumption that period 2 has the same length for both women and men. In fact, this is not the case when women get married to older men, and therefore tend to die later than their husbands. Relaxing this assumption means a decrease in the value of β and an increase in the value of b.[34] These changes have no effects on the level of the boy's and the girl's education: the values of δ, ε, d, and e do not change.

Finally, the models have been simulated under different assumptions regarding the values of λ^g and λ^b. Since we know that households in Cote d'Ivoire can be both patri- and matri-lineal, the model has also been simulated under the two different assumption that $\lambda^g = \lambda^b$ and that $\lambda^g > \lambda^b$. The results of the simulations show that changes in the relative values λ^is only have a marginal effect on the level of boys' and girls' education and on the ratio between these two levels. It seems that the main sources of gender bias in investment in education are the different costs and returns to education: even a λ^g greater than λ^b is not able to overcome the effect of such bias.

6. Which policy to reduce gender bias in education?

In order to reduce gender bias in education in poor countries two different types of policies are considered:

- *Cost Reduction*: Policies that reduce costs (direct and/or indirect) of educating a girl, such as public investment in those services that reduce the importance of girls' work for their families (like waterworks), building more schools even in small villages, and giving grants to parents who decide to educate their daughters. In the model, this means to set $p_1^b = p_1^g$;
- *Increase in Returns*: Policies that increase returns to women's schooling, by eliminating gender discrimination in the labor market. In the model, this means to set both $\bar{w}^m = \bar{w}^f$ and $\eta^b = \eta^g$.

The simulation permits evaluation of which policy is more effective in the context identified by the assumptions made. If parents behave non-cooperatively, we observe that policies to reduce gender bias in education increase girls' education without reducing boys' education under all assumptions regarding parental preferences. In particular, in all hypotheses but in hypothesis 1, it seems that policies reducing the cost of educating a girl are more effective than policies increasing returns on female education. When parents behave cooperatively, we observe that both the absolute values of years of education are always higher and also that the increases produced by policy interventions are greater. As in the non-cooperative case, cost reduction is more effective than an increase in returns, and policies reducing gender bias in education have no negative effects on boys' education.

If we consider the model more consistent with the actual data (the cooperative model under hypothesis 3), even the introduction of extreme equalitarian policies that eliminate some of the sources of gender bias in education does not seem to be sufficient for solving the problem of girls' education: we need to reduce the cost of girls' schooling by 30.3% in order to eliminate gender bias in education! This obviously entails an enormous amount of public resources that a poor country would have to pay out of tax revenue, implying a decrease in net returns from education for both men and women. The results of these simulations can be found in Silvia Pasqua (1997).

Another relevant policy that can be considered is the introduction of pension schemes that reduce the importance of children for parents' consumption in their old age. In terms of the model, this means assuming that adult children do not transfer resources to their old parents (and therefore $\lambda^b = \lambda^g = 0$). This policy has a very strong effect on gender bias in education in all the hypotheses and especially in the cooperative case (under hypothesis 3 girls are even more educated than boys), but we also observe a relevant decrease in the level of both children's education because parents do not get part of the returns to their children's education and therefore they are less interested in investing in them (Table 5).

Once again, the introduction of pension schemes is a very costly policy, especially for countries with a vast informal sector, and their introduction can reduce returns to education for those employed in the formal sector.

All the models have been based on the assumption of 'one boy-one girl' households. However, when there are more than two children in the household and parents want to (can) invest only a constant share of their income in children's education, if the cost of schooling is given, a decrease in the number of children may increase the average level of education of both girls and boys. But, as is well known, the most effective policies for reducing fertility are policies that increase female education and bargaining power within the household, which brings us back to the discussion above.

Table 5. Ratio between girl's and boy's education $(x_1^g/x_1^b)^*$.

		Without pension schemes/perfect capital markets	With pension schemes/perfect capital markets
Hyp1	Non-cooperative	0.61	0.73
	Cooperative	0.29	0.86
Hyp2	Non-cooperative	0.40	0.86
	Cooperative	0.30	0.86
Hyp3	Non-cooperative	0.46	0.70
	Cooperative	0.60	1.75
Hyp4	Non-cooperative	0.30	0.43
	Cooperative	0.14	0.43

*The value observed in the Cote d'Ivoire is 0.62.

7. Conclusions

In almost all poor countries children go to school only for a few years because of their important role in helping parents both in domestic and commercial activities. Moreover, the *Human Development Reports* show that gender bias in education persists in many poor countries, especially in South Asia and in Sub-Saharan Africa.

What policy can be put in place to increase investment in girls' education?

In order to answer this question we need to consider those factors affecting parents' decisions about children education. In Sections 1, 2 and 3 different models of household decision-making were developed. In these models children's education depends not only on costs and returns to schooling, but also on parents' preferences and decision powers, which are different for mothers and fathers.

Therefore, it becomes important to analyze how different hypotheses regarding parents' behavior and preferences interact with market factors (costs and returns to education) in causing gender bias in education. Market factors determine both direct and indirect costs of children's schooling and returns to education, and these are not always equal for men and women. Hence, gender bias in education is the result of a household investment strategy that tends to devote more resources to the less costly and more productive child (the boy). Moreover, the traditional women's role and parents' preferences for sons may contribute to an increase in the difference in the level of education between boys and girls.

According to the model simulation presented here, introducing policies for reducing the costs of girls' education is a more effective means of increasing female education than policies that augment returns to female schooling. However, very little can be done in the short run when traditional social values are strong and parents have a preference for boys. The introduction of pension schemes may reduce gender bias in education by diminishing the importance of children for parents' consumption in their old age, but such pension schemes may have the adverse effect of decreasing the amount that parents want to spend on their children's education. Furthermore, eliminating liquidity constraints can also have positive effects. In particular, it would be possible to use microcredit programs to allow poor parents, unable to procure a loan because of lack of collateral, to invest in their children's education with positive effects on their own consumption in old age as well.

The importance of eliminating gender bias in education in poor countries has been recognized by the UN Beijing Conference in 1995: better educated women have more access to economic resources, can earn higher wages, are more productive in domestic activities and child care, and tend to have fewer children. As a consequence, investing in women's education has high social returns in terms of fertility reduction, better child health conditions, and a more equal distribution of resources within the family. All policies analyzed are rather expensive and the lack of public resources in many LDCs do not allow governments to sufficiently invest in education in order to reach the dual objectives of universal primary education and gender equality in schooling.

8. Appendix

In this appendix the proof of result (5) is presented. Let's consider equation (3)

$$\frac{x_1^g}{x_1^b} = \frac{p_1^b - \lambda^b \bar{w}^m \eta^b}{p_1^g - \lambda^g \bar{w}^f \eta^g} \frac{\varepsilon(U^f - V^f) + e(U^m - V^m)}{\delta(U^f - V^f) + d(U^m - V^m)}$$

If we want to calculate the effect of a change in the mother's threat point on the ratio between years of the girl's and the boy's education, we must calculate the partial derivative of x_1^g/x_1^b with respect to V^f that is equal to:

$$\frac{\partial\left(\frac{x_1^g}{x_1^b}\right)}{\partial V^f} = \frac{p_1^b - \lambda^b \bar{w}^m \eta^b}{p_1^g - \lambda^g \bar{w}^f \eta^g} \frac{(\delta e - \varepsilon d)(U^m - V^m)}{[\delta(U^f - V^f) + d(U^m - V^m)]^2}$$

The first fraction is constant and its value is between 0 and 1. The second fraction always has a positive denominator. Also $(U^m - V^m)$ is positive, because it is m's gain from cooperating with f. The sign of the partial derivative therefore depends on the sign of $(\delta e - \varepsilon d)$ which, in Hypothesis 3, is positive and therefore:

$$\frac{\partial\left(\frac{x_1^g}{x_1^b}\right)}{\partial V^f} > 0$$

In the same way, we have:

$$\frac{\partial\left(\frac{x_1^g}{x_1^b}\right)}{\partial V^m} = \frac{p_1^b - \lambda^b \bar{w}^m \eta^b}{p_1^g - \lambda^g \bar{w}^f \eta^g} \frac{(\varepsilon d - \delta e)(U^f - V^f)}{[\delta(U^f - V^f) + d(U^m - V^m)]^2}$$

As before the sign of the partial derivative depends only on the sign of $(\varepsilon d - \delta e)$. Given the assumption about parents' preferences, this expression has a negative value and therefore:

$$\frac{\partial\left(\frac{x_1^g}{x_1^b}\right)}{\partial V^m} < 0$$

When the mother gives more weight than the father to the children's education and both parents have a preference for the boy, the sign of the two partial derivatives depends on the value of the parameters.

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Notes

1. In the unitary model parents' preferences are represented by a single utility function that is maximized subject to a single budget constraint.
2. Jere R. Behrman (1988) applied the model to gender bias in food distribution in an rural Indian village and he found that preference for males is the result of an efficiency strategy of investment, especially during periods of famine.
3. The case of polygamy is not considered as in Luisa Smith (1994) and Simon Appleton (1996).
4. As it is possible to show, the assumption of one boy and one girl is not crucial for the model. Since returns to education are assumed constant and parents do not know which of their children will survive until adulthood to help them in their old age, any strategy to invest more in one specific son's or in one specific daughter's education is not optimal. Therefore, instead of analyzing the level of education of the single boy and of the single girl, we can consider as endogenous variables the years of education parents want to give to each of their sons and daughters, respectively.
5. In fact in period zero, childhood, individuals do not consume.
6. Instead of considering only formal education, it is possible to assume that parents are interested in their children's level of human capital. The notion of human capital is wider and includes both formal and 'informal' education that children can acquire, for example, by spending time with their parents. Considering human capital in the model would however complicate the analysis without many gains in terms of results. In fact, it would be possible to assume a 'human capital' production function where the level of human capital of each child depends on the number of years of education and on the time parents' devote to their children (therefore also time constraints for both parents should be introduced). However, to avoid zero investment in child education it would also be necessary to assume that formal education is more 'productive' than parents' time and this would produce results similar to those obtained in the model here presented.
7. A good is defined 'private' if the consumption by one individual excludes consumption by other individuals; the opposite is true for a public good. Given two private goods c_1 and c_2, consumed exclusively by individual *1* and by individual *2* respectively, and a public good x, we can have: egoistic preferences when the individual's utility function depends on the quantity of private and public good consumed and therefore $U_i(c_i, x)$ for $i = 1, 2$; caring preferences when the individual's utility function also depends on the level of utility reached by the other individual and therefore $W_i = (U_i(c_i, x), U_j(c_j, x))$; altruistic preferences when all goods are public and therefore $U_i(c_i, c_j, x)$.
8. If we assume that labor supply is endogenous, we need to add a time constraint for each parent. The introduction of time constraints into the model do not change the results in terms of the effects of possible policies that can be adopted to increase investment and to reduce gender bias in education (Silvia Pasqua, 1997) and therefore here the simplifying assumption of exogenous labor supply is considered.
9. See Paul J. Gertler and Paul Glewwe (1992) for rural Peru and Mark M. Pitt and Mark Rosenzweig (1990) for Indonesia.
10. Melissa Blinder (1988) found that in Mexico in 1990 older boys were more likely than girls to participate in the labor force: between 12 and 14 years 11% of boys and only 3% of girls worked; between 15 and 19 years the percentages are 47% and 18% respectively. Ranjan Ray (2000) found that in Peru and Pakistan girls work especially in domestic activities, while boys have higher participation rates. Emmanuel Skoufias (1994) showed that in rural India girls devote more time to domestic activities than boys and the opposite is true for time at school.
11. The introduction of income from child labor into the indirect costs of education is again a simplifying assumption that makes it possible not to consider explicitly a time constraint for each child and the consequence that in the non-consensus models both parents should agree on the time each child has to devote to work.
12. Geeda G. Kingdon (1998) found that in Uttar Pradesh returns to women's education are 45% lower than returns to men's education. Rosenzweig and Schultz (1982) explained male–female differences in child survival rates in rural India as a consequence of the different expected returns to labor that affect parental investment in sons and daughters.

13. Moreover, most women's work is unpaid work within the family (substance agriculture and domestic activities).
14. The value of λ^j can be made endogenous if we assume altruism toward parents.
15. Rosenzweig and Schultz (1982) and Ashish Garg and Jonathan Morduch (1998).
16. In this model parents transfer resources to their children only in the form of investment in education. No compensative transfers are assumed since these are 'unproductive' from the point of view of the parents unless we assume a marriage market where more educated children or better endowed with money (in the form of a dowry, for example) can get married with a 'better' spouse. Since the introduction of the marriage market would complicate the model, no money transfers from parents to children are assumed.
17. The prices of consumption goods for *m* and *f* are assumed different because spouses can consume different private goods. In fact, Jessica Patton (1993) found that in Cote d'Ivoire women spend less money than men on 'adult goods' such as alcohol, cigarettes and food outside the home, and more money on food, domestic goods and goods for children.
18. As we will see in the simulation, these assumptions are not crucial for the results.
19. When there is a single public good, on the contrary, income redistribution among contributors does not modify the total amount of the public good purchased.
20. Under the caring preferences assumption this is true only for particular functional forms of the utility functions (see Ted Bergstrom et al., 1986).
21. Most of the bargaining models (Marilyn Manser and Murray Brown, 1980; Marjorie McElroy and Mary Jean Horney, 1981; Marjorie McElroy, 1990) use the indirect utility that each spouse can obtain outside the marriage as threat points. This seems to be too 'strong' a threat in day-to-day marriage and therefore Shelly Lundberg and Robert A. Pollak (1993) proposed to use the non-cooperative outcomes as threat points.
22. The budget constraint is the same as in the unitary model.
23. Note that the result is the same not only in terms of the ratio between the boy's and the girls' education, but also in terms of levels.
24. To make labor force participation of men and women endogenous, we need to introduce time constraints (see note 9).
25. It is possible that daughters may transfer a greater proportion of their income to their old parents than sons. This, for example, occurs in countries like the Philippines where many young girls emigrate to work overseas or in matrilineal countries/households. In this case $\lambda^b < \lambda^g$. If the difference between the two values is big enough, parents may want to invest more on girls' than on boys' education.
26. The model can be extended to include a household domestic production function in which the home produced goods are made using inputs bought on the market and time devoted by *m* and *f* to domestic activities. In this way it is also possible to introduce a time constraint for parents and to obtain a general equilibrium model for household decision-making that also considers labor supply decisions, assumed to be fixed in the model presented (Silvia Pasqua, 1997).
27. This is the average education received by the parents of children between 17 and 19 years of age in the survey (see also Aysit Tansel, 1997).
28. The difference between women's and men's returns to primary education are similar to those of secondary education.
29. But not for the level of father's and mother's consumption.
30. Jere R. Behrman (1997) shows how in a unitary model with no gender bias in preferences, a lower investment in girl's education can be due to liquidity constraints.
31. For reasons of space the result of the sensitivity analysis are only summarized and not fully presented.
32. First an increase of 0.01 in the values of d, δ, e and ε and a decrease of 0.01 in the values of a, α, b and β has been assumed, then the model has been simulated assuming a decrease of 0.01 in the values of d, δ, e and ε and an increase of 0.01 in the values of a, α, b and β.
33. In particular, first it has been assumed that the father receives 70% of total resources transferred from the children, while the mother receives 30%; then that the father receives 40% while the mother receives 60%; and finally that the father receives 30% of total resources, while the mother receives 70%.

34. Another possibility, that gives however the same results, is to assume different intertemporal discount rates for the father and for the mother.

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