

ORIGINAL ARTICLE

Regulation of non-marketed outputs and substitutable inputs

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Abstract We study the regulation of a monopolistic firm that provides a non-marketed output based on multiple substitutable inputs. The regulator is able to observe the effectiveness of the provision, but faces information asymmetries with respect to the efficiency of the firm's activities. Specifically, we consider a setting where one input and the output are observable, while another input and related costs are not. Multi-dimensional information asymmetries are introduced by discrete distributions for the functional form of the marginal rate of substitution between the inputs as well as for the input costs. For this novel setting, we investigate the theoretically optimal Bayesian regulation mechanism. We find that the first-best solution cannot be obtained in case of shadow costs of public funding. The second-best solution implies separation of the most efficient type with first-best input levels, and upwards distorted (potentially bunched) observable input levels for all other types. Moreover, we compare these results to a simpler non-Bayesian approach, i.e., a single pooling contract, and hence, bridge the gap between the academic discussion and regulatory practice. In a numerical simulation, we identify certain conditions in which a single contract non-Bayesian

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regulation can indeed get close to the second-best solution of the Bayesian menu of contracts regulation.

Keywords Regulation · Asymmetric information · Mechanism design · Non-marketed goods · Substitutable inputs

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1 Introduction

Numerous goods and services are provided by regulated firms with a monopolistic status. For instance, a single firm is usually responsible for grid infrastructures in the electricity or telecommunication sector. The service to be delivered by these firms is typically well-defined and often fixed ex-ante, such that the regulator will be well aware whether or not it has been provided *effectively*. For instance, it is straightforward to verify the number of blackouts in electricity grids or the speed of the internet in telecommunication networks. In contrast, it is often difficult for the regulator to judge the *efficiency* of the firm's measures to provide the output. Technical systems are often highly complex and characterized by a production function with multiple interdependent inputs that are hard to assess. Hence, firms may have an informational advantage on their internal activities, which clearly complicates efficient regulation.

In practice, production functions often involve two types of activities or inputs, respectively, that are substitutable to a certain extent: on the one hand, it needs phys*ical assets*, such as electricity lines or data cables, whose level of deployment is relatively easy to observe. On the other hand, operational measures are required for the efficient usage of physical infrastructures, such as data routing or line switching, which are more difficult to asses (for instance, the number of newly built lines can simply be counted, while measuring the productivity in using a operational software can be quite ambiguous). From this setting, different sources of information asymmetry may arise: First, caused by substantial changes on the demand side or new technological options, the regulator must assess the necessary level of activity by the firm to provide the observable output. As an example, consider the rapidly increasing deployment of renewable energies in the electricity sector or the use of broadband internet in the telecommunication industry. For instance, in Germany the regulator approves a detailed electricity grid development plan for the transmission system operators (Netzentwicklungsplan 2013), which is elaborated by the operators themselves. There is control by public consultation and model-based studies; however, the complexity of expanding and operating the electricity grid precludes elimination of all information asymmetry about the necessary level of actions. Second, a source of asymmetric information exists if the regulator may be unable to verify the unit costs of one or multiple inputs. As previously stated, this is the case if the effects of operational measures and hence, related costs are difficult to assess. In the given example of the electricity or telecommunication grid, the regulator can hardly assess how physical assets

can be (partially) substituted by using efficient operational measures, e.g., intelligent management of redundancies in the grid.

In theory as well as in practice, such problems of information asymmetry between the regulator and the firm have been tackled by different forms of regulation. Typical approaches in regulatory practice range from cost-based regulation to widely applied incentive regulation (discussed, e.g., in Joskow 2014), or a linear combination of those two extremes (e.g., Schmalensee 1989). From an academic viewpoint, theoretical approaches suggest that the best theoretical solution consists of the regulator offering the firm a *menu* of contracts, such that the firm reveals her private information (e.g., Laffont and Tirole 1993). Even though the dichotomy between such Bayesian models of regulation (which tend to dominate the academic discussion) and simpler non-Bayesian models (which are closer to regulatory practice) is well perceived, corresponding explanations are rather vague. For instance, as Armstrong and Sappington (2007) note, "[...] *regulatory plans that encompass options are 'complicated', and therefore prohibitively costly to implement*".

Against the above background, the goal of this paper is twofold: First, to identify and investigate the optimal Bayesian regulation for the multi-dimensional problem at hand, and second, to bridge the gap between the theoretically optimal solution and simpler regimes applied in regulatory practice. For the latter, we provide a theoretical as well as computational analysis to identify circumstances under which a simple regime comes near the optimal solution.

To derive an optimal regulation strategy, we build on the theory of incentives and contract menus. It is well known that in a simple setting with two types of the firm, the efficient type is incentivized via a contract with first-best (price) levels along with some positive rent, while the inefficient type's contract includes prices above the first-best and no rent (e.g., Laffont and Tirole 1993). The same logic applies for the case of one-dimensional continuous type spaces (ibid.).

By analyzing asymmetric information with respect to total factor productivity in a two-input production function, Besanko (1985) extended this approach and presented a paper with noticeable similarities to ours. Specifically, he presented a result which is congruent with the one we obtain in a reduced version of our model.¹ However, there remain several important differences: First, Besanko assumed a model with distributional welfare preferences while we consider shadow costs of public funding. Second, the optimal menu in Besanko (1985) consists of the observable input along with a regulated price for the output, while we build on the observable input along with a transfer payment. And third, we study a discrete multi-dimensional type space instead of a univariate continuous distribution.

Multi-dimensional problems of adverse selection have been studied, e.g., by Lewis and Sappington (1988b), Dana (1993), Armstrong (1999) or Aguirre and Beitia (2004). While Dana (1993) analyzes a multi-product environment, Lewis and Sappington (1988b), Armstrong (1999) and Aguirre and Beitia (2004) consider two-dimensional adverse selection with only one screening variable. Specifically, the latter three derive

¹ We thank an anonymous reviewer for drawing our attention towards this paper and the results presented therein.

factor costs.³

optimal regulation strategies in a marketed-good environment (in the sense of Caillaud et al. 1988) with unknown cost and demand functions. In our paper, unlike Lewis and Sappington (1988b) and Armstrong (1999), we consider shadow costs of public funding instead of distributional welfare preferences. Despite technical differences, this is largely in line with the analysis of Aguirre and Beitia (2004).² However, in contrast to all these papers, we solve the two-dimensional adverse selection problem for a non-marketed good environment and a production process that involves two substitutable inputs with an uncertain rate of substitution (i.e., isoquant) and input

With this novel setting of multi-dimensional inputs and a non-marketed output, we contribute to the general insights from the above literature. We find that expected social welfare necessarily includes positive rents for some types of the firm, such that the first-best solution cannot be achieved. While the efficient type is always set to first-best input levels, the other contracts' (observable) input levels are distorted upwards.⁴ Separation of at least three types is always possible, while bunching of two types may be unavoidable in case of a very asymmetric distribution of costs or very flat isoquants.

We then compare the obtained optimal Bayesian regulation to the results of a non-Bayesian regulation that we obtain by restricting our regulation problem to one single *pooling* contract.⁵ We find that despite the general inferiority, a non-Bayesian costbased regulatory regime may indeed be close to the optimal Bayesian solution for specific circumstances. This especially holds true if the overall input level is likely to be high, or if the substitutability in the observable input is low (such that its power as contract variable is weak).

Meanwhile, in practical applications the specific results will naturally depend on the specification of the functions and parameters. For instance, in our exemplary calculations based on a CES production function, we find an exponential increase in the second-best observable quantities when moving the probability of low isoquants upwards. With respect to welfare, the simple contract performs close to the optimal contract if realizations of high input level requirements as well as costs are likely, if the elasticity of substitution is high, or if the share parameter favors the unobservable input factor. Hence—depending on the prevailing conditions—a simple contract might indeed be a suitable solution in regulatory practice to avoid overly complicated menus



 $^{^2}$ Aguirre and Beitia (2004) show the difference between shadow costs of public funding and distributional welfare preferences based on a model with continuous probability distribution, while we assume a discrete distribution.

³ Noticeably, with the (discrete) two-dimensional adverse selection problem, our problem setting is technically closest to the model discussed by Armstrong (1999).

⁴ Upwards distorted observable input levels coincide with upwards distorted prices for the inefficient type as shown in Laffont and Tirole (1993). They also agree with the results in a setting with unknown cost and demand functions as long as shadow costs of public funding are considered (Aguirre and Beitia 2004). Noticeably, the case of prices below marginal costs, as found in Lewis and Sappington (1988b) and Armstrong (1999), is mainly triggered by using a distributive social welfare function instead of shadow costs of public funding.

⁵ We will argue that in our context, this type of regulation is more suitable than other "simple" mechanisms like rate of return or revenue cap regimes.

of contracts which might even turn out to be inefficient overall (e.g., when adding costly planning and coordination processes to the analysis). For instance, in the above example of electricity networks in Germany, the regulator imposes a cost-based regulation on the responsible firms—and might eventually be close to an efficient regulatory outcome when considering large budgets and ongoing public discussions about grid expansion as an indication for the aforementioned circumstances.

The paper is organized as follows: Sect. 2 introduces the model; Sect. 3 presents the optimal regulation strategy; Sect. 4 compares the optimal regulation to simpler regimes; Sect. 5 provides additional insights based on our computational analysis; and Sect. 6 concludes.

2 The model

Consider a single firm that is controlled by a regulator. The firm uses two inputs to provide an output in terms of a good or service level q that is requested by the regulator. The regulator's choice of q could, for instance, result from counterbalancing the economic value of the provided with the related social costs. For simplicity, however, we assume q to be invariant throughout the paper.⁶

In our model, probability μ (respectively $1 - \mu$) leads to a low (high) aggregated input that is necessary to reach the same requested output q. This could, e.g., be an exogenous shock induced by the deployment of additional infrastructure, e.g., due to additional grid necessary for renewable energies or high speed internet etc. From the firm's perspective, an output level q can be provided by means of two different inputs, one of which is observable (x) and one non-observable (y) by the regulator. The observable input x might be thought as actually physical infrastructure, which is easily observable. The non-observable input might be something more hidden, e.g., more sophisticated operation procedures, optimization of the existing infrastructure etc. The tradeoff between those two inputs needed to reach output q is commonly described by a production function q = f(x, y) which can be illustrated by means of isoquants. We assume smooth and decreasing marginal returns of both inputs, such that the isoquants are downward sloping, convex and differentiable. Noticeably, two different isoquants can never cross. An example fulfilling these requirements is a Cobb–Douglas-type production function. The inverse production function g(q, x)reflects the necessary level of the non-observable input y needed to reach output q, given a level of x. We will mostly use this inverse function hereafter. Due to the exogenous shock leading to a low (l) or high (h) aggregated input necessary for the envisaged output level q, the inverse function takes one of two possible functional forms, i.e. $g_i(q, x)$, with $i \in [l, h]$ and $g_l(q, x) < g_h(q, x)$.

⁶ Although this assumption might seem restrictive at first sight, it may indeed fit many relevant cases very well. For instance, due to the very high societal value of uninterrupted electricity transmission, changes in costs will hardly affect the desired level of the transmission service quality q. In the same vein, access to infrastructure in rural areas, e.g., high-speed internet or telecommunication services, may not be cost-efficient but considered a public service.



Fig. 1 a Cost variation small compared to isoquant variation. b Cost variation large compared to isoquant variation. Problem setting with double adverse selection

The optimal rate of substitution between the two inputs minimizing total costs for reaching the requested output depends on the cost functions of the inputs. We consider the cost function $c^x(x)$ of the the observable input to be fixed and common knowledge, while the cost function of the non-observable input $c_j^y(y)$ is subject to a nature draw, which leads with probability v (respectively 1 - v) to a low (high) cost function (i.e., $j \in [l, h]$). For simplicity, we assume constant factor costs of both inputs, i.e., $c^x(x) = c^x$ and $c_j^y(y) = c_j^y$. The realization of c_j^y influences the isocost line of the two inputs and hence, the optimal rate of substitution.⁷ Hence, depending on the two random draws for the isoquant and the costs of the non-observable input, there are four possible first-best bundles of inputs, which we denote by $\{x_{ll}^{fb}, y_{ll}^{fb}\}, \{x_{lh}^{fb}, y_{lh}^{fb}\}, \{x_{hl}^{fb}, y_{hl}^{fb}\}$ and $\{x_{hh}^{fb}, y_{hh}^{fb}\}$. As a last precondition, we require the expansion path, i.e. the curve connecting the optimal input combinations of the different isoquants, to be pointing rightwards as the necessary aggregated input increases.⁸ In terms of the first-best input levels, this requires $x_{ll}^{fb} > x_{hl}^{fb}$ and $x_{lh}^{fb} > x_{hh}^{fb}$, which again holds true for a wide range of possible production function specifications, including the above mentioned Cobb–Douglas type.

Figure 1 provides an intuition of the problem setting with double adverse selection. Two different cases are depicted: on the left hand side, cost variation is small compared to isoquant variation, while on the right hand side the opposite is true. Distinguishing these cases will become relevant for regulatory purposes in Sect. 3.

Under optimal Bayesian regulation, the goal of the regulator is to incentivize the firm via a suitable contract framework to choose the welfare-optimizing bundle of inputs, which we will derive based on classic mechanism design entailing truthful direct revelation. Contrary to the firm, the regulator cannot observe the realizations

⁷ As it is well known from production theory, the optimal rate of substitution is determined by equating the marginal rate of technical substitution between the factors (i.e., the slope of the isoquant) with the relative factor costs (i.e., the slope of the isocost line).

⁸ For an analysis involving continuous variables, this would require the expansion path to behave like a function with a unique function value y for each x, or, in other words, an expansion path that is not bending backwards.



Fig. 2 Timing

of the two random draws, although the possible realizations as well as the occurrence probabilities are common knowledge. She knows the cost function of the observable input and can observe the corresponding input level. The output is also observable and verifiable.⁹ For an optimal regulation, the regulator offers the firm a menu of four contracts, each with a level of the observable input x_{ij} and a corresponding transfer T_{ij} . Naturally, the contracts can be conditioned on observable parameters only, i.e., the output as well as the amount of the observable input used. Both are enforceable by means of suitably high penalties in case the firm deviates from the requested/contracted level.

The timing—as shown in Fig. 2—is as follows. First, the random draws are realized and the cost function of the non-observable input and the necessary aggregated input relation (isoquant) are observed by the firm. The firm then chooses between several (in our case, four) contracts offered by the regulator. She then realizes the input levels to produce the requested output. The regulator observes one input level (x) and whether the output is as requested; if those are as agreed upon, the contract is executed and the transfer realized.

The rent of the firm R_{ij} given a realization $i \in [l, h]$ and $j \in [l, h]$, results from the transfer T_{ij} minus the private cost of the firm's activities:¹⁰

$$R_{ij} = T_{ij} - c^{x} x_{ij} - c^{y}_{j} g_{i}(q, x_{ij})$$
(1)

The regulator maximizes expected social welfare, defined as the sum of expected social utility and firm surplus, by adjusting the observables, i.e.:

$$\max_{x_{ij}, T_{ij}} W = \mathbb{E}\left[\underbrace{S_q - (1+\lambda)T_{ij}}_{\text{Net social utility}} + \underbrace{(T_{ij} - c^x x_{ij} - c^y_j g_i(q, x_{ij}))}_{\text{Firm's rent}(R_{ij})}\right]$$
(2)

where S_q is the gross social utility from reaching output q, and λ denotes the shadow costs of public funding, i.e., the costs due to raising and transferring finances through public channels (for a discussion, see, e.g., Laffont and Tirole 1993). As discussed previously, we assume q—and hence also gross social utility S_q —to be invariant and

⁹ Stochastic deviations due to force majeure are supposed to be detectable and excludable from the contract framework.

¹⁰ It goes without saying here that the firm is characterized such that she tries to maximize her rent.

independent of the random draws, yielding.11

$$\max_{x_{ij}, T_{ij}} W = S_q - \mathbb{E}\left[\underbrace{(1+\lambda)T_{ij}}_{\text{Transfer costs}} - \underbrace{(T_{ij} - c^x x_{ij} - c^y_j g_i(q, x_{ij}))}_{\text{Firm's rent}(R_{ij})}\right]$$
(3)

As an important consequence of Eq. (3), we see that the optimization problem of the regulator can be reformulated in terms of a cost-minimization problem, essentially stating that the desired output shall be reached at minimal expected social costs:

$$\min_{x_{ij}, T_{ij}} C = \mathbb{E} \left[C_{ij} \right] = \mathbb{E} \left[\lambda \underbrace{R_{ij}}_{\text{Firm's rent}} + (1+\lambda) \left(\underbrace{c^x x_{ij}}_{\text{Costs of}} + \underbrace{c^y_j g_i(q, x_{ij})}_{\text{Costs of}} \right) \right]$$
(4)

While choosing x_{ij} and T_{ij} such that social costs are minimized, the regulator is restricted by several participation and incentive constraints for the firm's rent:

$$R_{ij} \ge 0 \qquad \qquad \forall i, j \qquad (5)$$

$$R_{ij} \ge R_{i'j'} + c_{j'}^{y} g_{i'}(q, x_{i'j'}) - c_{j}^{y} g_{i}(q, x_{i'j'}) \qquad \forall \text{ pairs } i, j \text{ and } i', j'$$
(6)

Equation (5) ensures that all types of firms have a non-negative profit and therefore participate. We allow no shutdown of a firm due to the essential service the firm offers to society (e.g., electricity or telecommunication infrastructure). In line with the revelation principle, Eq. (6) provides the firm with the incentive to truthfully report the realized isoquant and non-observable input costs.

3 Optimal regulation

3.1 Preparatory analysis

As a first preparatory step in the analysis we shall check whether the contract variable *x* is actually suitable to provide incentives to the firm to reveal her true type. To this

¹¹ This is the reason why q appears as a subscript here. In case of a more complex model with an endogenous demand, q would be a variable for the regulator, and S_q be replaced by S(q) (with $\frac{\partial S}{\partial q} > 0$), indicating that social utility is increasing as the regulator chooses higher output q. The additional first order condition would write as $\frac{\partial W}{\partial q} = 0 \rightarrow \frac{\partial S(q)}{\partial q} = c_j^y \frac{\partial g_i(q, x)_i j}{\partial q}$, expressing the optimal balance of the social value of the provided output and the related social costs. In our model, this would involve the complexity of one additional choice variable and the associated emergence of a continuum of isoquants. Qualitatively, the efficiency of all results obtained in our model would need to be checked against their social value, and adjusted in case of a mismatch. Starting from the requested output in the first-best case, a second-best solution would thus entail the need to reduce the requested output.

end, we investigate whether the incentive to choose another type's contract regarding one of the two random draws is impacted by an adjustment of x, as reflected in the firm's rent $R_{ij}(x)$. This is often referred to as "single crossing" conditions. For the incentive to choose another type's contract regarding the realized input cost, we find that¹²

$$\frac{\partial}{\partial x}(R_{ih}(x) - R_{il}(x)) = \left(c_l^y - c_h^y\right)g_i'(q, x) \quad \text{for} \quad i = l, h, \tag{7}$$

which is clearly greater than zero due to $c_h > c_l$ and $g'_i(q, x) < 0$. Hence, by an upwards distortion of x, we are able to reduce the incentive for the firm to choose the contract of a high cost type instead of truly revealing the realized low cost type.

Similarly, for the incentive to choose a contract for an isoquant different from the realized one, we find that

$$\frac{\partial}{\partial x}(R_{hj}(x) - R_{lj}(x)) = c_j^y(g_l'(q, x) - g_h'(q, x)) \quad \text{for} \quad j = l, h$$
(8)

which is greater than zero as long as $g'_h(q, x) < g'_l(q, x)$. Recalling from Sect. 2 that we have assumed rightwards pointing expansion paths, this condition will always hold true. Hence, upwards distorting *x* will provide a possibility to reduce the incentive for the firm to choose the contract with a high isoquant instead of truly revealing the realized low isoquant.¹³

The effect of changing incentives following a distortion of x helps us to derive a first characterization of the optimal solution of our regulatory problem. In fact, in order to comply with the incentive constraints (6) (which need to be fulfilled for the optimal solution anyway), input levels x_{ij} need to follow a certain ordering. Note that for each pair of types there are two relevant incentive constraints. Adding those and using the above single crossing conditions, the necessary ordering can be obtained as follows:

$$x_{ll} \le x_{lh} \le x_{hh} \tag{9}$$

$$x_{ll} \le x_{hl} \le x_{hh} \tag{10}$$

Moreover, from the incentive constraints follows that only the participation constraints of the high cost types (i.e., *lh* and the *hh*-type) may remain relevant for further analyses. In contrast, the other two participation constraints are implicitly fulfilled if these two incentive constraints hold.

So far unclear from the above analysis, however, is the ordering of the intermediate cases x_{lh} and x_{hl} , which depends on whether the term $R_{hl}(x) - R_{lh}(x)$ is increasing or decreasing in x within the relevant range (i.e., the range covered by x in the optimal

¹² Here and in the following, a prime denotes derivation with respect to x.

¹³ Note that if Eq. (8) does not hold, the effect is reversed: incentives for the lj-types to claim higher isoquants could then be reduced by a downwards distortion of x. For the sake of conciseness, we omit further discussions of this particular case.

contract). Differentiating with respect to x yields

$$\frac{\partial}{\partial x}(R_{hl}(x) - R_{lh}(x)) = (c_h^y g_l'(q, x) - c_l^y g_h'(q, x)) \tag{11}$$

which is increasing in x as long as

$$\frac{c_h^y}{c_l^y} < \frac{g_h(q, x)}{g_l(q, x)},\tag{12}$$

and decreasing in x otherwise.¹⁴ Together with the two constraints relating the types lh and hl we infer that if the cost variation is small compared to the isoquant variation, then $x_{lh} \le x_{hl}$. If the aggregated input level variation is small compared to the cost variation, then $x_{lh} \ge x_{hl}$. For an intuition, recall Fig. 1. If the aggregated input level variation and hence the distance between the isoquants is large, x_{hl}^{fb} is larger than x_{lh}^{fb} . If the cost variation, and hence, the vertical distance between the corresponding first-best solutions is large, x_{lh}^{fb} is larger than x_{hl}^{fb} .

The results of our preparatory analysis are summarized in the following two Lemmas.

Lemma 1 Participation is only an issue for the high-cost types. Hence, the participation constraints for the high cost types are relevant, whereas the ones for the low cost types are implicitly fulfilled.

Lemma 2 In order to reach incentive compatibility, input levels x_{ij} must be ordered as follows:

(A) If the cost variation is small compared to the isoquant variation, then $R_{hl}(x) - R_{lh}(x)$ is increasing in x and requires

$$x_{ll} \le x_{lh} \le x_{hl} \le x_{hh}. \tag{13}$$

(B) If the cost variation is large compared to the isoquant variation, then $R_{hl}(x) - R_{lh}(x)$ is decreasing in x and requires

$$x_{ll} \le x_{hl} \le x_{lh} \le x_{hh}. \tag{14}$$

3.2 Full information benchmark

If the regulator had no information deficit, she would observe the realized isoquant as well as the realized isocost line. Differentiating all possible realizations of the social cost function C_{ij} with respect to the observable input levels x_{ij} shows that

¹⁴ Note that Condition (12) (or its inverse) is a precondition for the subsequent analysis. We assume it holds unambiguously within the entire relevant range covered by the contract variable x. In more practical applications, this will depend on the (parametric or non-parametric) specification of the functions $g_l(q, x)$ and $g_h(q, x)$.

all of them are single-peaked with a unique minimum at $g'_i(q, x_{ij}) = -\frac{c^x}{c^y_j}$, which is necessarily realized at $x_{ij} = x_{ij}^{fb}$. The regulator would easily derive the first-best levels of inputs to supply the requested output at minimal social costs, i.e., $\{x_{ij}^{fb}, y_{ij}^{fb}\}$, by equating the known realized marginal rate of technical substitution of the inputs with the realized isocost line. Moreover, she would be able to enforce the implementation of the first-best due to the full observability. The corresponding optimal transfers would be $T_{ij}^{fb} = c^x x_{ij}^{fb} + c^y_j y_{ij}^{fb}$, leaving all types of the firm with zero rent. In the case of full information, social costs amount to $C_{ij}^{fb} = (1 + \lambda)T_{ij}^{fb} = (1 + \lambda)(c^x x_{ij}^{fb} + c^y_j y_{ij}^{fb})$, corresponding to the welfare-optimizing first-best solution that could thus be obtained.

3.3 Asymmetric information

In the case of asymmetric information, the only two observables for the regulator are the output q and the observable input x. In addition, she can choose an appropriate level of transfer payment T. As q is invariable and observable, its implementation can be enforced by means of suitably high penalties in case the firm deviates. Hence, xand T are the two variables the regulator will condition her contracts on. The general idea for the regulator's optimal regulation strategy is to offer a menu of contracts with optimized variables { x_{ij}^* , T_{ij}^* }, such that expected social costs are minimized (as stated in Eq. (4)), and participation (Eq. (5)) and incentive constraints (Eq. 6) fulfilled. Hence, we restrict our attention to incentive compatible contracts ensuring that the firm always reveals her true type. Under these conditions, the revelation principle requires that the solution found (if any) is a Bayesian-Nash equilibrium (Myerson 1979; Laffont and Martimort 2002).

Note that the contractual combinations of inputs and transfers are closely interlinked. Therefore, they may be interpreted as minimum quantity constraints, rewarding the firm conditional on the realized input. For instance, the regulator offers the firm a revenue of T_{hl}^* conditional on the utilization of at least a certain input quantity x_{hl}^* , which in turn translates into a minimum output level. In our example of electricity grid infrastructure, this could correspond to some minimum reliability standard (such as the n-1 criterion).¹⁵

3.3.1 One-dimensional asymmetric information

We shall first investigate a simplified problem with one-dimensional asymmetric information only, i.e., isoquant *or* cost uncertainty. Eliminating the isoquant uncertainty (by setting $\mu = 0$, $\mu = 1$ or $g_{lj} = g_{hj}$), we are left with two constraints binding: the participation constraint of the high cost type and the incentive constraint from the low to the high cost type. This leads to the simplified cost function:

$$C = \nu \left[\lambda \left(g_i(x_{ih}) (c_h^y - c_l^y) \right) + (1 + \lambda) \left(c^x x_{il} + c_l^y g_i(x_{il}) \right) \right]$$

¹⁵ We thank an anonymous reviewer for pointing out this interpretation.

$$+ (1-\nu)\left[(1+\lambda)\left(c^{x}x_{ih}+c^{y}_{h}g_{i}(x_{ih})\right)\right]$$
(15)

Derivating with respect to x_{ij} , $j \in l$, h yields the following first order conditions:

$$\frac{\partial C}{\partial x_{il}} = 0 \Leftrightarrow g'_i(x^*_{il}) = -\frac{c^x}{c^y_l},\tag{16}$$

$$\frac{\partial C}{\partial x_{ih}} = 0 \Leftrightarrow \underbrace{\nu\lambda(c_{h}^{y} - c_{l}^{y})g_{i}'(x_{ih}^{*})}_{<0} + \underbrace{(1 - \nu)(1 + \lambda)(c^{x} + c_{h}^{y}g_{i}'(x_{ih}^{*}))}_{=0 \text{ for } x_{ih} = x_{ih}^{fb}}_{<0 \text{ for } x_{ih} < x_{ih}^{fb}}_{>0 \text{ for } x_{ih} > x_{ih}^{fb}}$$
(17)

Equation (16) shows that in case of no isoquant uncertainty, the observable input levels of the low cost type are first-best. In contrast, Eq. (17) depicts an upwards distortion of the high cost type: as the first part is strictly smaller than zero, the second part needs to compensate to achieve the first order condition which in turn can only be true when the high cost type is distorted upwards.

Similarly, in case of no cost uncertainty, the observable input levels of the low isoquant type are first-best, whereas the high isoquant type observable input level is distorted upwards:¹⁶

Lemma 3 In case of asymmetric information about either costs or isoquants, the observable input level of the respective *l*-type is set to first-best, while it is distorted upwards compared to its first-best for the *h*-type.

Note that the result of an adverse selection problem with one-dimensional information asymmetry on costs is well-known from the literature (e.g., Baron and Myerson 1982 or Sappington 1983). For the more specific case of one-dimensional asymmetric information regarding total factor productivity in a two-input production function, it was derived by Besanko (1985) (based upon a model with continuous distribution of types). Also note that the results concerning isoquant uncertainty are strikingly different compared to the one-dimensional demand uncertainty with non-decreasing marginal costs which is studied by Lewis and Sappington (1988a) and shows similarities with the isoquant uncertainty in our setting. They find that the first-best can be achieved in the one-dimensional case of demand uncertainty, due to the fact that rent extraction from the high-demand type by means of reduced transfer payments is both feasible and optimal. The low-demand type is therefore never attracted by the contract for the high-demand type. In contrast, the fact that we study a non-marketed good with fixed input factor costs excludes the regulator from being able to reduce the transfer in the high-demand contract without letting the high-demand type firm run at a loss. Therefore, the only possibility to reduce the attractiveness of the high-demand contract is to distort its quantity in an upwards direction.

¹⁶ Due to the symmetry of the problem, we omit the detailed calculation here.



3.3.2 Two-dimensional asymmetric information

Solving the full optimal regulation problem requires minimization of social costs, subject to all imposed four participation and twelve incentive constraints. Due to the large number of constraints, we approach the optimization by solving a relaxed problem where only a subset of the constraints is considered. To this end, we need to come up with an educated guess about the binding constraints in the optimum. If we can later show that the remaining constraints are fulfilled at the solution of the relaxed problem, we will have obtained the solution of the full problem.

We already know from Lemma 1 that the participation constraints of the high-cost types are the only relevant ones. Furthermore, it generally seems to be a good approach to assume the "upwards" incentive constraints, i.e., from low to high isoquant, and from low to high costs, to be binding. Moreover, it seems plausible to assume binding incentive constraints from the most efficient to an intermediate type (i.e., *lh* or *hl*), and from an intermediate type to the least efficient type. If we consider the isoquant variation more relevant than the cost variation, assuming the incentive constraints according to the ordering shown in Lemma 2, Case (*A*), to be binding appears to be the most educated guess we can come up with.¹⁷ Hence, we assume that incentive constraints $ll \rightarrow lh, lh \rightarrow hl$, and $hl \rightarrow hh$ are fulfilled with equality. In addition, we assume the participation constraint of the *hh*-type to bind since this is the only type remaining that is not attracted by any other type. Figure 3 illustrates with arrows the binding incentive constraints, such that the former type is not attracted by the latter type-contract. Diamonds mark the binding participation constraints.

Under the constraints considered binding for Case (A) the social cost function (4) becomes

$$C = \mu \nu \left[\lambda \left(g_h(x_{hh}) \left(c_h^y - c_l^y \right) + c_l^y g_h(x_{hl}) - c_h^y g_l(x_{hl}) + g_l(x_{lh}) \left(c_h^y - c_l^y \right) \right) + (1 + \lambda) \left(c^x x_{ll} + c_l^y g_l(x_{ll}) \right) \right]$$

¹⁷ The ordering and solution of Case (B) is reversed, but similar. The corresponding discussion can be found in the "Appendix".

$$+ \mu(1-\nu) \left[\lambda \left(g_h(x_{hh}) \left(c_h^y - c_l^y \right) + c_l^y g_h(x_{hl}) - c_h^y g_l(x_{hl}) \right) \right. \\ \left. + (1+\lambda) \left(c^x x_{lh} + c_h^y g_l(x_{lh}) \right) \right] \\ \left. + (1-\mu)\nu \left[\lambda \left(g_h(x_{hh}) \left(c_h^y - c_l^y \right) \right) + (1+\lambda) \left(c^x x_{hl} + c_l^y g_h(x_{hl}) \right) \right] \\ \left. + (1-\mu)(1-\nu) \left[(1+\lambda) \left(c^x x_{hh} + c_h^y g_h(x_{hh}) \right) \right].$$
(18)

To derive the optimal observable input levels, we need to derive the above equation with respect to each of the four possible x_{ij} . Minimizing *C* with respect to x_{ll} yields

$$g_l'(x_{ll}^*) = -\frac{c^x}{c_l^y},$$
(19)

which implies that $x_{ll}^* = x_{ll}^{fb}$. Derivations of *C* with respect to x_{lh} , x_{hl} and x_{hh} take the following forms:

$$\frac{\partial C}{\partial x_{lh}} = \underbrace{\mu \nu \lambda(c_{h}^{y} - c_{l}^{y})g_{l}'(x_{lh})}_{<0} + \underbrace{\mu(1 - \nu)(1 + \lambda)(c^{x} + c_{h}^{y}g_{l}'(x_{lh}))}_{=0 \text{ for } x_{lh} = x_{lh}^{fb}}_{<0 \text{ for } x_{lh} < x_{lh}^{fb}}$$

$$\frac{\partial C}{\partial x_{hl}} = \underbrace{\mu \lambda(c_{l}^{y}g_{h}'(x_{hl}) - c_{h}^{y}g_{l}'(x_{hl}))}_{<0} + \underbrace{(1 - \mu)\nu(1 + \lambda)(c^{x} + c_{l}^{y}g_{h}'(x_{hl}))}_{=0 \text{ for } x_{hl} = x_{hl}^{fb}}_{<0 \text{ for } x_{hl} < x_{hl}^{fb}}$$

$$\frac{\partial C}{\partial x_{hh}} = \underbrace{(\mu + (1 - \mu)\nu)\lambda g_{h}'(x_{hh})(c_{h}^{y} - c_{l}^{y})}_{<0} + \underbrace{(1 - \mu)(1 - \nu)(1 + \lambda)(c^{x} + c_{h}^{y}g_{h}'(x_{hh}))}_{=0 \text{ for } x_{hl} < x_{hl}^{fb}}$$

$$= 0 \text{ for } x_{hh} = x_{hh}^{fb}}_{<0 \text{ for } x_{hh} < x_{hh}^{fb}}}$$

$$= 0 \text{ for } x_{hh} = x_{hh}^{fb}}_{<0 \text{ for } x_{hh} < x_{hh}^{fb}}}$$

We find that this set of assumptions does indeed lead us to the optimal regulation strategy. The results are summarized in the following Proposition 1.

Proposition 1 For Case (A),

(i) Optimal regulation is achieved under the following set of observable input levels:

$$x_{ll}^* = x_{ll}^{fb} \tag{23}$$

$$x_{lh}^* \ge x_{lh}^{fb} \tag{24}$$

$$x_{hl}^* \ge x_{hl}^{fb} \tag{25}$$

$$x_{hh}^* \ge x_{hh}^{fb},\tag{26}$$

while respecting $x_{ll}^* < x_{lh}^* \le x_{hl}^* \le x_{hh}^*$.

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(22)

(ii) The most efficient (ll) type can always be separated. Moreover, separation of at least three types is always possible, while bunching of the lh and hl types is unavoidable in case of $v \to 1$. The hl and hh types may need to be bunched in case of $g'_1(q, x) \to 0$ together with c^y_1 being large.

Proof See "Appendix".

Corollary 1 For $\lambda = 0$, the optimal solution is first-best. All input levels amount to $x_{ii}^* = x_{ii}^{fb}$, and expected social costs to $C = C^{fb}$.

Proof Follows immediately from the solution of Case (*A*) when setting $\lambda = 0$.

According to Corollary 1, with no shadow costs of public funding, all input levels x_{ij}^* are first-best. The regulator optimizes overall welfare, but has no preference regarding the distribution of social surplus. Hence, she can give the firm an arbitrarily high budget at no social costs, and the firm maximizes her rent by setting efficient input levels. In this case, the maximization of the firm and the maximization of social welfare coincide, i.e., there is no problem of aligning the activities of the firm with social interests. Of course, larger parts of the welfare are then given to the firm.

For the general case of $\lambda > 0$, observable input levels of all types besides the *ll*-one are distorted above first-best levels, leading to a second-best solution only. Naturally, the overall level of inefficiency increases in λ , but also for decreasing μ and ν (i.e., when there is a high probability for "costly" outcomes of the random draws) as well as for $c_h^y - c_l^y$ and $g_h(q, x) - g_l(q, x)$ getting large. In contrast, however, the less significant the cost variation becomes compared to the isoquant variation, the more efficient the solution will be.

Due to keeping the most efficient (*ll*) type at first-best level combined with the ordering according to Lemma 2, the type can always be separated in the contract framework. Moreover, we find that at least three types can always be separated, while bunching of two types may be unavoidable in case of vanishing isoquant or cost uncertainties, or if the isoquant variation becomes extremely large. As a last remark, it is worth mentioning that the ordering of rents is (and must be) as depicted in Fig. 3, i.e. $0 = R_{hh}^* < R_{lh}^* < R_{lh}^*$.

The results for Case (*B*) are symmetric but structurally identical to Case (*A*), i.e., the *ll*-type is incentivized to first-best input levels while the other types show upwards distortions of x_{ij} . However, roles of isoquants and costs are interchanged, reflected in the inverse occurrence of the terms $g_i \leftrightarrow c_j^y$ and $\mu \leftrightarrow \nu$. At the same time, as imposed by Lemma 2, Case (*B*), the sequence of the "intermediate" types is now $hl \rightarrow lh$. Hence, the ordering of observable input levels x_{lh} and x_{hl} as well as rents R_{lh} and R_{hl} need to be reversed to obtain an optimal regulatory contract framework.¹⁸

Considering a distributional preference of the regulator (e.g., as in Besanko 1985) instead of shadow costs of public funds, we find that our general findings remain valid. The optimal solution with distributional preference still sets first best levels for the *ll*-type's observable input levels. In contrast, the upwards distortion of the other types will increase because the regulator considers social costs related to the firm's rent more

¹⁸ See the "Appendix" for a detailed discussion and the corresponding proposition and proof.

important than social costs stemming from input levels. Or—in other words—for these types the regulator will need to readjust the basic tradeoff between rent extraction and input efficiency.

4 Comparing the optimal regulation to simpler regimes

In contrast to the optimal Bayesian menu of contracts studied in the previous section, regulatory authorities often apply alternative, simpler approaches, i.e., non-Bayesian or *single* contracts instead of applying Bayesian contracts in terms of *menus* of contracts. In electricity networks, England, Wales and the UK are the only examples for menus of contracts being applied in regulatory practice (Joskow 2014). In contrast, examples of simpler regimes are manifold. For instance, German system operators get reimbursed for their costs related to self-formulated grid expansion plans.

In practice, regulators (and the firm) may prefer simple mechanisms for the sake of easier implementation and execution, and to avoid the need to calibrate the different parameters of our stylized model. In fact, estimating the shape of the isoquants, costs as well as the corresponding probabilities to build a suitable menu of contracts might be a challenging and resource-consuming task. Therefore, extended planning and coordination processes might even turn the provision of a menu of contracts into a third-best solution. In contrast, pooling contract regimes, revenue caps or rate-of-return regulation—specifically addressing only a simplified set of contract variables while being aware of distorting effects—might indeed be advantageous overall.

In case of a **rate-of-return** regulation conditioned on the observable input, the regulator would simply pay a rate on the claimed observable input level. This would incentivize the firm to always opt for the hh-type, as this ensures the highest level of observable input and thus, the highest profit. The regulator would have no other choice than to accept this claim regarding the unobservable high input level, and to pay the related unobservable input costs. In our model, this leads to an extreme Averch-Johnson-effect, and thus renders the mechanisms rather unfavorable.

With a **revenue cap**, the regulator allows the firm the budget to cope with the most adverse situation, i.e., to cover the costs of the most inefficient type (hh), and leaves the firm with the discretion to realize the (efficient) input levels. Meanwhile—depending on its type—the rent of the firm can be very high. This is socially undesirable especially if shadow costs of public funding are substantial. Moreover, the regime also requires the regulator not to take away the realized rent ex-post.

Yet another alternative is constituted by a **single pooling contract**, where the regulator defines one observable input level which the firm shall realize. Optimization ensures that the best possible single-input level is chosen. Moreover, to ensure participation, a transfer needs to be provided sufficient to cover the high isoquant case given the fixed level of observable input.

In addition to the above, a comparison between the revenue cap and a pooling contract reveals benefits for the latter in the following relevant aspects: First, the regulator knows and commits to a specific level of observable input. In the context of electricity grid infrastructure, investment projects (i.e., the observable input) often face noticeable public opposition, such that enhanced control and planning reliability

is highly desirable. Moreover, compared to a revenue cap, the pooling mechanism allows the regulator to restrict the range of rents that can be realized by the firm. In fact, the only uncertainty left for the regulator (and the public) is whether the firm faces a high or low isoquant. If the isoquant is low, the low unobservable quantity will be realized, thus leaving the firm with a rent. For the high isoquant the firm is forced to implement the plan and gets no rent. In contrast to the possible outcomes under a revenue cap regime, this is not only more efficient in case of large shadow costs of public funding, but might also be aligned with distributional preferences of the society.

In the following, we will implement a simple non-Bayesian pooling mechanism into our modeling approach to explore and compare its features (we will take our analysis further in our computational analysis in Sect. 5). To do so, we need to limit the set of regulatory choice variables to one single contract with contract variables \bar{x} and \bar{T} , such that the objective function of the regulator (in contrast to Eq.(4) as in the case of optimal regulation) becomes:

$$\min_{\bar{x},\bar{T}} \bar{C} = \mathbb{E} \left[\lambda \underbrace{\bar{R}_{ij}}_{\text{Firm's rent}} + (1+\lambda) \left(\underbrace{c^x \bar{x}}_{\text{Costs of}} + \underbrace{c^y_j g_i(q,\bar{x})}_{\text{Costs of non-observable input}} \right) \right]$$
(27)

In contrast to the solution of the optimal regulation, this minimization is only subject to the participation constraints (5). With a sole contract and hence, only one observable input \bar{x} for all types, the regulator has no possibility to separate types, which makes the incentive constraints obsolete. As before, the only participation constraint holding with equality is the one of the *hh*-type. Considering that this type gets full cost reimbursement but cannot be distinguished from the other types, it becomes clear that all other types must then necessarily receive a positive rent. The following proposition summarizes the solution of this non-Bayesian regulatory approach.¹⁹

Proposition 2 Under a single contract cost-based regulation with quantity restriction, the optimal input level \bar{x}^* represents the expected value of the first-best solutions of the four possible types,²⁰ adjusted by some upwards distortion in case of $\lambda > 0$. As an expected average, it lies between the extreme types' first-best input levels, i.e. $x_{II}^{fb} < \bar{x}^* < x_{Ih}^{fb}$.

Proof See "Appendix".

¹⁹ Note that the solution for a pure cost-based regulation without quantity restriction would simply reimburse the costs of the observable input. This would incentivize the firm to choose infinitely high values of x (known as the gold-plating effect). Assuming that the regulator restricts her set of choices by an upper level of $\bar{x} = x_{hh}^{fb}$ in order to limit excessive (socially costly) rents, all types would then choose this level. In contrast to this *very* simple approach, the regulatory regime considered in this section makes use of being able to use the observable input x as a contracting variable.

²⁰ I.e., the point where the ratio of the expected costs equals the expected slope of the isoquants.

Table 1 Fixed parameters	c ^x	c_l^y	c_h^y	θ_l	θ_h	q
	20	10	11	2	1	10

Note that it is easy to show that the optimal solution of the single contract is a feasible solution of the menu of contracts problem. Due to the fact that the solution for the menu of contracts, as stated in Proposition 1, is both optimal and different from the one in Proposition 2, it must necessarily be superior.

Further characteristics of the simple regulatory regime as well as a detailed comparison with the optimal menu of contracts will be presented in the next section, based on our computational analysis.

5 Computational analysis

In order to gain further insights into the previously developed theoretical insights, we shall now present a computational analysis around an exemplary constant elasticity of substitution (CES) production function which is defined as follows:

$$q = \theta_i [ax^{\rho} + (1-a)y^{\rho}]^{\frac{1}{\rho}},$$
(28)

where θ_i is the factor productivity, ρ the elasticity of substitution, and *a* the share parameter. Note that the CES production function complies with all the necessary assumptions discussed in Sect. 2 as long as $\rho < 1.^{21}$

The inverse function can easily be derived as

$$y = g_i(q, x) = \left(\frac{1}{1-a}\right)^{\frac{1}{\rho}} \left[\left(\frac{q}{\theta_i}\right)^{\rho} - ax^{\rho} \right]^{\frac{1}{\rho}},$$
(29)

just as its derivative

$$\frac{\partial g_i(q,x)}{\partial x} = -a \left(\frac{1}{1-a}\right)^{\frac{1}{\rho}} x^{\rho-1} \left[\left(\frac{q}{\theta_i}\right)^{\rho} - ax^{\rho} \right]^{\frac{1-\rho}{\rho}}.$$
 (30)

For the sake of traceability, we will assume certain parameters to be fixed throughout the subsequent numerical analysis, as listed in the following Table 1.

Note that these assumptions ensure that Condition (12) is fulfilled in the relevant range of the contract variable *x*, thus implying $x_{lh} \le x_{hl}$, or, in other words, that Case (*A*) holds.²²

To exemplify the schematic figure given in Sect. 2, Fig. 4 illustrates exemplary inverse production functions g(q, x) for the above assumptions and the case of a = 0.5

²¹ Otherwise, the isoquants are no longer strictly convex.

²² For the sake of brevity, we leave out a computational analysis of Case B.





and $\rho = 0.1$, e.g., to verify their compliance with the necessary characteristics (i.e., downward sloping, convex and differentiable behavior).

In the following, we will proceed in showing the results of our numerical results for the following parametric ranges: $\lambda \in \{0, 0.5, 1\}, a \in [0.1, 0.9], \rho \in [0.1, 0.9], \mu \in [0.1, 0.9]$ and $\nu \in [0.1, 0.9]$. Specifically, we will first focus on the impact of the exact numerical specification of the production function (i.e., a variation of *a* and ρ), and then turn our attention to the impact of the random draw probabilities (i.e., a variation of μ and ν).

This will allow to illustrate our theoretical findings and to develop a better understanding of the orders of magnitude, and thus, to gain further insights.

5.1 Impact of the production function specification

Figure 5 shows the numerical results of the optimal solution for the observable input level x_{ij}^* in comparison to their first-best levels x_{ij}^{fb} , given $\lambda = 0.5$ and a range of parameters defining the (inverse) production function, i.e., different *a* and ρ . The full set of results with three different levels of λ is shown in the "Appendix", Fig. 10. In addition, the optimal level of the observable input level in the simple contract, \bar{x}^* , is



Fig. 5 a *x* versus *a*. b *x* versus ρ . Numerical results of *x*, for different *a* and ρ , and $\lambda = 0.5$



Fig. 6 a *C* versus *a*. b *C* versus ρ . Numerical results of the expected costs *C*, for different *a* and ρ , and $\lambda = 0.5$

also shown. For reading the following figures without color printout, recall that it must hold that $x_{ll} \le x_{lh} \le x_{hh} \le x_{hh}$.

As expected, there is no upwards distortions of any x_{ij} for the case of $\lambda = 0$. In the same vein, the *ll*-type is never distorted, independent of λ . Furthermore, it can be verified that the simple contract variable \bar{x}^* is an adjusted expected average of the different types, and thus lies within the range opened up by x_{ij}^* , as found in Proposition 2.

For increasing *a*, i.e., a relative shift of productivity from the non-observable (*y*) to the observable input (*x*), all *x* show a clear tendency to increase. However, while \bar{x}^* is increasing monotonically, x_{ij}^* as well as x_{ij}^{fb} show a peak at high *a* and a decrease after, due to the fact that the functions $\frac{\partial g}{\partial x}$ for different *a* can indeed cross. In contrast, an increasing elasticity of substitution ρ (right hand side in Fig. 5) implies monotonically decreasing *x*, due to the fact that input factor *x* is more costly than *y*.

Bunching of two types occurs for a large range of parameters *a* and ρ . While all types must be clearly separated for $\lambda = 0$, the *hl* and *hh*-types are bunched for $\lambda = 0.5$ and $\lambda = 1$ across the entire range of *a* (given $\rho = 0.1$), as well as for the most part of ρ (given a = 0.5).

Furthermore, note that the upwards distortion of x_{lh} is always small, whereas it can be significant for x_{hl} and x_{hh} , driven by the fact that we are dealing with the case here where cost uncertainty is relatively small compared to the isoquant uncertainty.

Based on these insights, we can shift our focus to the related cost efficiency of the numerical analysis. Figure 6 presents expected costs for the different regulatory regimes and parameter constellations (again, the full set of results with three different levels of λ is shown in the "Appendix", Fig. 11).

As found in Corollary 1, the optimal regulatory regimes achieves first-best results for $\lambda = 0$. Interestingly, a maximum occurs in the first-best expected costs for *a* being approximately at the ratio $c^x : c^y$, due to the fact that even though input factor costs increase for an increasing usage of the (then more productive) observable input factor *x* (compare Fig. 5)²³, overall expected costs can be decreased at after the cost ratio of the two input factors has been reached because more rents can then be extracted.

²³ Recall that input factor x was assumed to be more expensive than y.



Fig. 7 **a** x_{lh} . **b** x_{hl} . **c** x_{hh} . Numerical results of x, for different μ and ν , and $\lambda = 0.5$

In line with Proposition 1, $\lambda > 0$ implies upwards distortions and rents of the firm, and thus higher expected costs than in the first-best case. Interestingly, however, optimal regulation may still approach first-best efficiency when *a* becomes large, i.e., when productivity in the observable input is high and the unobservability of *y* thus not so relevant. In contrast, the simple mechanism can never approach first-best results, and is always worse than the second-best optimal regulation, however, can be identical with the latter for small *a* (i.e., when productivity in the observable input *x* becomes irrelevant, such that its usability as contract variable is very weak), as well as for high ρ (essentially following the same logic as for *a*).²⁴

5.2 Impact of the random draw probabilities

We now discuss the impact of the random draw probabilities on our numerical results. Figure 7 contains the regulated observable input quantities x for different μ and v when $\lambda = 0.5$ (results for all λ are in the "Appendix", Fig. 12). Recall that for $\lambda = 0$, all second-best quantities are first-best, independent of μ and v. As there is no variation,

²⁴ Technically, both specifications result in low values of $\frac{\partial g}{\partial x}$.



the figures for $\lambda = 0$ are not included. Meanwhile, the different first-best quantities are included as a reference in Figs. 7 and 12.

For all x_{ij} it holds true that the upwards distortion is increasing in λ , while the general pattern remains identical. x_{lh}^* is independent of the isoquant probability μ as the relevant binding incentive constraint prevents the *ll*-type from claiming the *lh*-type contract which is on the same isoquant. In contrast, lower probabilities for high costs (i.e., high ν) result in higher upwards distortions of the *lh*-type, as the expected costs of the upwards distortion then get a lower weight.

In a similar vein, the upwards distortions of the *hl* and *hh*-types increase at an increasing rate for increasing μ , and can be substantial compared to the one of the *lh*-type (due to the fact that we are in Case A). Meanwhile, increases in ν now lead to decreasing upwards distortions. The results for the *hh*-type are largely driven by the often occurring need to bunch it with the *hl*-type. In fact, they are bunched for the entire range of μ for $\nu = 0.1$, as well as for $\mu < 0.3$ ($\mu < 0.7$) when $\nu = 0.5$ ($\nu = 0.9$) which may be recognized in the actual numbers as well as in a slight kink in the curves for x_{hh}^* . Note that this also causes the curves for x_{hh}^* to cross for different ν .

As a last step in our computational analysis, we investigate and compare the expected costs for the different regulatory approaches for varying random draw probabilities, as presented in Fig. 8 (and 13 for all three levels of λ).

Naturally, the costs monotonically decrease in the probability for low input factor needs (i.e., high μ), just as in the probability for low input factor costs c_l^y (i.e., high ν). As in Fig. 6, expected costs of the optimal regulation are first-best for $\lambda = 0$, while the simple mechanism must always perform worse. The difference between the regulatory regimes becomes more relevant for increasing λ and μ . Meanwhile, in line with our finding in Sect. 4, the performance of the simple mechanism is similar to the optimal regulatory regime for λ large and μ small.

Furthermore, Figs. 7 and 8 reveal a remarkable dependency of the results on μ and ν individually, but also on their specific combination. This may be seen as a strong argument for our two-dimensional analysis and provides insights complementary to Besanko (1985). Specifically, while the effect of ν on distorted quantities is negligible for small μ , values nearly double for the considered range of ν when μ is high. The effect is lower for total costs, but still clearly visible. Consequently, the specific conditions of the two-dimensional uncertainty may have a significant impact, and

should thus be evaluated carefully by the regulator before deciding on the suitability of such mechanisms.

Transferring the above discussion to regulatory practice, one may indeed come to the conclusion that practically applied non-Bayesian regulatory approaches could be close to the optimal second-best strategy. For instance, it is likely that the large-scale penetration of renewable energies requires strong reinforcements in the electricity infrastructure. Meanwhile, renewable energies also elevate the costs for electricity generation, such that consumers might be increasingly skeptical vis-a-vis a redistribution of welfare towards the firm—which in turns yields higher costs of the financing process. In the end, however, reasons for the chosen regulation are probably manifold, and might also include preferences of the regulator, commitment problems²⁵ or the prohibitively high costs of implementing a 'complicated' regulatory regime (Armstrong and Sappington 2007).

6 Conclusion

We considered a regulated firm providing a non-marketed output with substitutable inputs. We presented the optimal Bayesian regulation in terms of a menu of contracts when the regulator faces information asymmetries regarding the aggregated input level needed to provide the output as well as the realized optimal marginal rate of substitution between the inputs. Finally, the optimal Bayesian regulation was compared to a simpler non-Bayesian approach, i.e., a single pooling contract, which appears to be closer to regulatory practice.

We found that in the optimal Bayesian regulation, the first-best solution cannot be achieved under the considered information asymmetries and shadow costs of public funding. This implies a strictly positive rent for the firm. The second-best solution that we then characterized depends on the relative importance of the information asymmetries. However, the most efficient type is always set to first-best, while the levels of the observable input are distorted upwards for all other types. At least three types can always be separated, while bunching of two types may be unavoidable in case of a very asymmetric distribution of costs or very flat isoquants. These results are structurally similar to the solutions for multi-dimensional adverse selection problems in the literature (e.g. Lewis and Sappington 1988b; Armstrong 1999 or Aguirre and Beitia 2004). However, in contrast to existing results, our model explains upwards distortions of input levels rather than prices. Hence, we obtained important insights regarding the optimal mechanism design in the context of a regulated monopolistic firm producing a non-marketed good with multi-dimensional inputs.

²⁵ Noticeably, a commitment problem of the regulator might impede the implementation of an incentivebased approach, which would be welfare-superior compared to a cost-based regulation. If the firm gets an unconditional payment representing the pay-off of the *hh*-type, i.e., $\tilde{T} = c^x x_{hh}^{fb} + c_h^y g_h(q, x_{hh}^{fb})$, she will realize first-best input quantities $\{x_{ij}^{fb}, y_{ij}^{fb}\}$. In this case, the realized rent of the firm becomes $R_{ij} = c^x x_{hh}^{fb} + c_h^y g_h(q, x_{hh}^{fb}) - c^x x_{ij}^{fb} - c_j^y g_i(q, x_{ij}^{fb})$. However, due to the (observable) separation of types via the realized input *x*, the regulator might be tempted to adjust the regulatory contract ex-post, and hence, jeopardize the regulatory success if the firm anticipates this behavior.

Our theoretical and computational comparison to a single contract cost-based approach, as it is often applied in regulatory practice, showed that the menu of contracts is welfare superior. However, there are situations in which the performance of the approaches converge. This is the case if the overall input level is likely to be high, or if the substitutability in the observable input is low, such that its power as contract variable is weak. Moreover, the pooling contract avoids the possibly complicated and costly design of menus of contracts, and allows the regulator to commit to a specific observable input level and to restrict the range of rents that can be realized by the firm. These circumstances and features may indeed prevail in some cases,²⁶ possibly explaining the gap between the theoretically optimal Bayesian approach and the simpler non-Bayesian regulation applied in practice. However, as demonstrated in our computational analysis, prevailing conditions should first be revised carefully to investigate the their impact on the optimality of regulatory regimes.

Lastly, we note that our general approach as well as our insights might also be applicable to other industries that show similar characteristics, such as public works or administrative services. Besides investigating such areas of application, future research could relax the participation assumption and hence, allow for a shut down of firms. Another expansion could allow the good to be marketed, which would trigger a demand reaction of the regulator (or consumers) and possibly lead to interesting variations of the conclusions derived in this paper.

Appendix

Proof of Proposition 1

- *Proof* (i) From Eq. (20), we see that $\frac{\partial C}{\partial x_{lh}}$ is strictly smaller than 0 for $x_{lh} = x_{lh}^{fb}$ and monotonically increasing in x_{lh} , which implies that $x_{lh}^* > x_{lh}^{fb}$ must always hold. The same logic applies for x_{hl}^* and x_{hh}^* .
- The same logic applies for x_{hl}^{*} and x_{hh}^{*} . (ii) From the fact that $x_{ll}^{fb} < x_{lh}^{fb}$ and the strict upwards distortion of all other types, it follows that the *ll*-type can always be separated. In order to investigate whether the types *lh*, *hl* and *hh* can be separated or need to be bunched, we proceed as follows: For each of the possible pairs lh - hl, hl - hh and lh - hh, we check the derivative of *C* with respect to the former type at the optimal level of x^* of the latter type (derived from the first order condition). If the change in *C* is greater than 0 we can conclude that we have already surpassed the optimal level of the former type, which then must be smaller than the optimal level of the latter type. In other words, we check the level of upwards distortion for the *lh*, *hl* and *hh* types while considering the necessary ordering of the types according to Lemma 2. For the pair *lh-hl*, we find that x_{lh}^* may surpass x_{hl}^* in case of $\nu \to 1$, while they are otherwise clearly separated from each. For the pair *hl-hh*, bunching may occur for $g'_l(q, x) \to 0$ together with c_l^{y} being large. Furthermore, we find that

 $^{^{26}}$ E.g. the cost-based regulation of German TSOs, where the necessary is suspected to be high, but the German public heavily discusses the related costs for consumers.

lh-hh can always be separated, implying that at most two types (i.e., either *lh-hl* or *hl-hh*) may be bunched under certain parameter constellations.

Lastly, it is straightforward to check that the remaining constraints are satisfied under the obtained solution of the relaxed problem. Hence, we have indeed obtained to optimal solution for the full regulatory problem we are facing in Case (A).

Proof of Proposition 2

Proof Written explicitly, Eq. (27) becomes

$$\bar{C} = \mu \nu \left[\lambda \left(c_h^y g_h(\bar{x}) - c_l^y g_l(\bar{x}) \right) + (1 + \lambda) \left(c^x \bar{x} + c_l^y g_l(\bar{x}) \right) \right]
+ \mu (1 - \nu) \left[\lambda \left(c_h^y g_h(\bar{x}) - c_h^y g_l(\bar{x}) \right) + (1 + \lambda) \left(c^x \bar{x} + c_h^y g_l(\bar{x}) \right) \right]
+ (1 - \mu) \nu \left[\lambda \left(c_h^y g_h(\bar{x}) - c_l^y g_h(\bar{x}) \right) + (1 + \lambda) \left(c^x \bar{x} + c_l^y g_h(\bar{x}) \right) \right]
+ (1 - \mu) (1 - \nu) \left[(1 + \lambda) \left(c^x \bar{x} + c_h^y g_h(\bar{x}) \right) \right].$$
(31)

Deriving the above with respect to \bar{x} yields, after a few calculations, $\mathbb{E}(g'_i(\bar{x}^*))\mathbb{E}(c^y_j) + c^x + \lambda(c^y_h g'_h(\bar{x}^*) + c^x) = 0$. Hence, for $\lambda = 0$, $\mathbb{E}(g'_h(\bar{x}^*)) = -\frac{c^x}{\mathbb{E}(c^y_j)}$.

Two-dimensional asymmetric information, Case (B): cost variation large compared to isoquant variation

To solve the second case following from Lemma 2, we need to apply a different educated guess with respect to the binding constraints. However, we apply a similar reasoning as in Case (*A*), but take account of the fact that now, cost variation is more relevant than isoquant variation. Hence, we choose a symmetric setting and imply incentive constraints $ll \rightarrow hl$, $hl \rightarrow lh$ and $lh \rightarrow hh$ to be binding. Again, we assume the participation constraint of the *hh*-type to be binding. Figure 9 illustrates this setting.



After having determined the results and checked all remaining constraints, we find the setting of binding constraints as in Fig. 9 indeed to be optimal for Case (B). Results are summarized in the following Proposition 3.

Proposition 3 For case (B),

(i) Optimal regulation is achieved under the following set of observable input levels:

$$x_{ll}^* = x_{ll}^{fb} \tag{32}$$

$$x_{lh}^* \ge x_{lh}^{fb} \tag{33}$$

$$x_{hl}^* \ge x_{hl}^{fb} \tag{34}$$

$$x_{hh}^* \ge x_{hh}^{fb},\tag{35}$$

while respecting $x_{ll}^* < x_{hl}^* \le x_{lh}^* \le x_{hh}^*$.

- (ii) The most efficient (ll) type can always be separated. Moreover, separation of at least three types is always possible, while bunching of the hl and lh types is unavoidable in case of $\mu \rightarrow 1$. The lh and hh types may be bunched in case of c_l^y being small and $g'_h(q, x)$ large.
- *Proof* (i) Under the constraints considered binding for Case (B)—as discussed and shown in Fig. 9—the social cost function (4) becomes

$$C = \mu \nu \left[\lambda \left(c_{h}^{y}(g_{h}(x_{hh}) - g_{l}(x_{hh})) + c_{h}^{y}g_{l}(x_{lh}) - c_{l}^{y}g_{h}(x_{lh}) + c_{l}^{y}(g_{h}(x_{hl}) - g_{l}(x_{hl})) \right) + (1 + \lambda) \left(c^{x}x_{ll} + c_{l}^{y}g_{l}(x_{ll}) \right) \right]$$

+ $\mu (1 - \nu) \left[\lambda \left(c_{h}^{y}(g_{h}(x_{hh}) - g_{l}(x_{hh})) \right) + (1 + \lambda) \left(c^{x}x_{lh} + c_{h}^{y}g_{l}(x_{lh}) \right) \right]$
+ $(1 - \mu)\nu \left[\lambda \left(c_{h}^{y}(g_{h}(x_{hh}) - g_{l}(x_{hh})) \right) + c_{h}^{y}g_{l}(x_{lh}) - c_{l}^{y}g_{h}(x_{lh}) + (1 + \lambda) \left(c^{x}x_{hl} + c_{l}^{y}g_{h}(x_{hl}) \right) \right]$
+ $(1 - \mu)(1 - \nu) \left[(1 + \lambda) \left(c^{x}x_{hh} + c_{h}^{y}g_{h}(x_{hh}) \right) \right].$ (36)

Minimizing C with respect to x_{ll} yields

~ ~

$$g_l'(x_{ll}^*) = -\frac{c^x}{c_l^y},$$
(37)

which implies that $x_{ll}^* = x_{ll}^{fb}$. Derivation of *C* with respect to x_{lh} , x_{hl} and x_{hh} yields:

$$\frac{\partial C}{\partial x_{lh}} = \underbrace{\mu\lambda(c_h^y g_l'(x_{lh}) - c_l^y g_h'(x_{lh})}_{<0} + \underbrace{\mu(1 - \nu)(1 + \lambda)(c^x + c_h^y g_l'(x_{lh}))}_{=0 \text{ for } x_{lh} = x_{lh}^{fb}}_{<0 \text{ for } x_{lh} < x_{lh}^{fb}}$$
(38)

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$$\frac{\partial C}{\partial x_{hl}} = \underbrace{\mu \nu \lambda(c_l^y g'_h(x_{hl}) - c_l^y g'_l(x_{hl}))}_{<0} + \underbrace{(1 - \mu)\nu(1 + \lambda)(c^x + c_l^y g'_h(x_{hl}))}_{<0} (39)$$

$$= 0 \text{ for } x_{hl} = x_{hl}^{fb}$$

$$> 0 \text{ for } x_{hl} < x_{hl}^{fb}$$

$$> 0 \text{ for } x_{hl} > x_{hl}^{fb}$$

$$+ \underbrace{(1 - \mu)\nu(1 - \nu)(1 + \lambda)(c^x + c_h^y g'_h(x_{hh}))}_{<0} + \underbrace{(1 - \mu)(1 - \nu)(1 + \lambda)(c^x + c_h^y g'_h(x_{hh}))}_{>0 \text{ for } x_{hh} < x_{hh}^{fb}}$$

$$(40)$$

From Eq. (38), we see that $\frac{\partial C}{\partial x_{lh}}$ is strictly smaller than 0 for $x_{lh} = x_{lh}^{fb}$ and monotonically increasing in x_{lh} , which implies that $x_{lh}^* > x_{lh}^{fb}$ must always hold. The same logic applies for x_{hl}^* and x_{hh}^* .

(ii) From $x_{ll}^{fb} < x_{lh}^{fb}$ and the strict upwards distortion of all other types, it follows that the *ll*-type can always be separated. x_{hl}^* may surpass x_{lh}^* in case of $\mu \to 1$. If the low costs c_l^y are small and $g'_h(q, x)$ becomes large, *lh* and *hh* types may need to be bunched, without impacting the separation of the other types. The remaining constraints are satisfied under the obtained solution.

As in Case (A), the first-best solution can be obtained for $\lambda = 0$, while the solution is second-best and incurring an increasing level of inefficiency for increasing levels of λ . Also again, the most efficient type can always be separated, while bunching of the *hl* and *lh* types (*lh* and *hh* types) may occur for very high occurrence probability of low isoquants, or if $g_h(q, x)$ is very steep and c_l^{γ} small.

Impact of the production function specification: numerical results for different levels of λ

See Figs. 10 and 11.

x vs. *a*

0.5 a 0.6 0.7

0.3 0.4

14

12

10

ε

6

4

2

0

14

0.1 0.2

×

 $\lambda = 0$

x^{*}_{hh} x^{fb}_{hh} x^{*}_{hl}

x^{fb}

X_{lh} X^{ft}

ׄ

xfi





Fig. 10 Numerical results of *x*, for different λ , *a* and ρ



Fig. 11 Numerical results of the expected costs *C*, for different λ , *a* and ρ

Impact of the random draw probabilities: numerical results for different levels of $\boldsymbol{\lambda}$

See Figs. 12 and 13.



Fig. 13 Numerical results of *C*, for different λ , μ and ν



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