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Minimum quality standard regulation under imperfect quality observability

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Abstract Minimum quality standards (MQS) constitute an important regulatory tool that can be used to raise product qualities, to benefit consumers and to increase market participation. One of the main assumptions in the existing literature is that firms must comply with standards. Nevertheless, in many industries, and in particular the service industry, quality observability and enforceability are not perfect. Some low quality firms do not comply with standards. What are the welfare implications of an MQS regulation in such an environment? We develop a price competition model of vertical differentiation that accounts for these empirical observations. Contrary to well-established results in the literature, MQS can increase quality disparity between firms and raise hedonic prices. Some consumers get hurt and market participation decreases.

Keywords Vertical differentiation · Regulation · Minimum quality standards · Imperfect quality observability · Imperfect enforceability

JEL Classification D43 · L13 · L51 · L88

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1 Introduction

Minimum quality standards (MQS) constitute an important regulatory tool that can be used to raise product qualities, to benefit consumers and to increase market participation.[1] Economic analyses of the effects of MQS regulation have maintained the assumption, implicitly or explicitly, that firms fully comply with such laws.[2] This seems to be a reasonable assumption when quality is about a "technical" component and the error involved in quality assessment is low, e.g., miles-per-gallon performance, emissions. Nevertheless, the situation can be quite different when quality is about a service and the human factor is involved. In such cases it is difficult to measure and monitor quality perfectly. Recent empirical literature has documented evidence of firms' imperfect compliance of minimum quality standards in various service industries such as child care centers, hospitals and nursing homes, which casts serious doubt on the perfect observability/enforceability assumption.[3] For example, one third of the nursing homes in California failed to comply with the regulation 3 years after the minimum nurse staffing standards took effect (Chen 2009). An important reason for the imperfect enforcement in the nursing home industry is that regulators observe quality with noise. The enforcement agency monitors nursing homes largely through yearly on-site surveys, which only captures a snapshot of the true quality and is subject to randomness. Moreover, the quality measures used in the inspection process, such as deficiencies, have been widely criticized as subjective and inconsistent across survey teams.

To the best of our knowledge, our paper is the first one that relaxes the perfect observability/enforceability assumption that has been made by the theoretical contributions in the literature. In particular, we ask: what are the welfare implications of an MQS regulation when compliance with the regulation is not observed perfectly?[4]

In a vertically differentiated duopoly, when quality is perfectly observable by a regulator and there are no enforcement issues, the low quality firm complies fully with the new standard. The high quality firm in response raises its quality in order to preserve some of the vertical differentiation, but, because quality is costly, the increase

[1] See Sappington (2005) and Armstrong and Sappington (2007) for recent surveys on the developments in the theory of regulation.

[2] In Leland (1979) and Shapiro (1983) firms are price takers. In Ronnen (1991) firms compete in prices and marginal cost is independent of the quality level. Crampes and Hollander (1995) assume price competition with marginal cost being a function of quality, while Valletti (2000) assumes quantity competition. Häckner (1994), Ecchia and Lambertini (1997) and Napel and Oldehaver (2011) examine the impact of an MQS on the stability of collusion, under various assumptions about the mode of competition and the costs. Scarpa (1998) relaxes the duopoly assumption and shows that with three firms an MQS can lower the high quality. Constantatos and Perrakis (1998) allow the timing of the quality choices to vary. Garella and Petrakis (2008) assume that consumers are imperfectly informed regarding product qualities. All papers make the perfect compliance assumption.

[3] About 20% of child care centers are found to be out of compliance with regulations (such as maximum group size), often by a substantial margin, Blau and Mocan (2002). Using cost report data from the California Office of Statewide Health Planning, Harrington and O'Meara (2006) estimate that 27% of nursing homes failed to comply with the minimum staffing standards by 2003.

[4] Dai (2010) examines the effect of imperfect verification and a subsequent appeals process in regulatory settings where there is an upper bound on penalties that can be imposed.

in quality is less than the increase of the low quality firm. The end result is less vertical differentiation and lower hedonic prices, as it has been eloquently described in Ronnen (1991). Not only did the MQS succeed in raising qualities, but it also lowered prices, making all consumers better off.

We depart from the existing literature in that we assume that a regulator observes quality with some noise. If the signal the regulator receives falls short of the imposed MQS, then the firm pays a fine, which is a function of how far the signal is below the standard. In this environment, the low quality firm will "optimally" increase its quality, in response to an MQS, but it 'usually' has no incentives to fully comply with the standard. In turn, the high quality firm raises its quality for *two* reasons: (i) to preserve the vertical differentiation and (ii) to optimally reduce the probability of sending a signal to the regulator that falls short of the MQS. The first effect is standard, we now provide some intuition for the second effect.

The low quality firm may optimally choose to provide a level of quality that falls below the standard. Hence, it may be the high quality firm whose quality level is closer to the standard. The closer a firm's quality level is to the standard the higher is the effect of a marginal increase of quality in reducing the probability of being found in noncompliance. The combination of these two effects implies that the high quality firm may have stronger incentives for quality improvements than the low quality firm, resulting in *higher* vertical differentiation and *higher* hedonic prices for both firms. In sharp contrast with the perfect observability model, low-valuation consumers among those who purchase the low quality product become worse off when an MQS is imposed. Market participation can decrease and fewer consumers consume the high quality product.

Our predictions match two key empirical observations from the nursing home industry that we use as a motivating example. First, some facilities choose to provide a quality that is below the standard. Second, it is the facilities that are positioned closer to the standard (and these need not be the lowest quality facilities) that have the strongest incentive to improve quality (Chen 2009). These predictions are not only for the nursing home industry, and a standard perfect observability/enforceability model is unable to deliver these predictions.

Our findings have interesting policy implications. The conventional wisdom was that an MQS will raise the price of the low quality firm, forcing some consumers to drop out of the market, e.g., Leland (1979) and Shapiro (1983).[5] Ronnen (1991), as we mentioned above, showed that this prediction can be overturned. We show that the 'older' conventional wisdom can be accurate, but for different reasons than those presented in the earlier papers.

The rest of the paper is organized as follows. The model is presented in Sect. 2. In Sect. 3, we examine the effects of MQS. In Sect. 4, we analyze the consequences of a higher penalty and a more precise inspection process. We offer a numerical example in Sect. 5 and a discussion on Sect. 6. We conclude in Sect. 7. Most of the proofs can be found in the "Appendix".

[5] Or, some consumers will be worse off because their favorite qualities are no longer available when some firms exit the market.

2 The description of the model

Two firms—low quality L and high quality H, indexed by i—sell competing brands to a continuum of consumers. Consumers differ in their tastes (or income), described by the parameter θ which is uniformly distributed on the interval $[a, b]$ with density 1 (Gabszewich and Thisse 1979; Shaked and Sutton 1982). The quality of firm H's product is denoted by q_H and that of firm L by q_L, with $q_H \geq q_L \geq 0$. Let p_H and p_L denote the prices that firm H and L charge respectively. Each consumer buys at most one unit of the good. Consumers have the same indirect utility function which is described as follows

$$U = \begin{cases} \theta q_i - p_i, & \text{if a consumer purchases firm } i\text{'s product} \\ 0, & \text{otherwise.} \end{cases}$$

We denote the difference in product qualities, $q_H - q_L$, by Δ. The consumer who is located at point $\theta_1 = (p_H - p_L)/\Delta$ is indifferent between the high and the low quality product and the consumer who is located at point $\theta_0 = p_L/q_L$ is indifferent between the low quality and the outside (no purchase) option. We assume that the firm incurs a fixed up-front (design) cost, $C_i = q_i^2/2$, to develop a product of quality q_i, but after that every unit that is produced costs zero.

A regulator imposes a minimum quality standard (MQS), denoted by $q_{\min}$. Moreover, the regulator inspects the firms to determine whether they meet the MQS. Quality, however, is not perfectly observable. Rather, the regulator observes $\tilde{q}_i = q_i + \varepsilon_i$, where ε_i is independently distributed on $\left[\underline{\varepsilon}, \bar{\varepsilon}\right]$ according to F (and density $f > 0$, for $\varepsilon \in \left(\underline{\varepsilon}, \bar{\varepsilon}\right)$ and $f(\varepsilon) = 0$ at the two endpoints of the support) with mean zero and $\underline{\varepsilon} < 0$. We assume that the density is symmetric about zero and single-peaked.[6] The regulator relies on the signal he receives to assess quality and if $\tilde{q}_i < q_{\min}$, then firm i pays a penalty $P = k(q_{\min} - \tilde{q}_i)$ which is proportional to how far its quality is from the MQS.[7] We assume that consumers observe the quality perfectly. The difference with the regulator is that the regulator needs 'hard evidence' in order to impose a fine and the evidence collection process is not perfect.

For instance, we can assume that observed quality depends on actual quality plus some daily randomness. Consumers consume the good/services over a long period of time and hence the noise averages out. Moreover, consumers focus on a relatively small set of choices and can spend more time on researching each candidate. The regulator, on the other hand, due to resource constraints, obtains a smaller sample than the consumers and hence his observation is noisier. For simplicity, we assume that the quality consumers consume is subject to zero noise. Alternatively, the regulator cannot rely on word-of-mouth information in order to impose fines. Consumers can

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6 Note that for some low realizations of ε the quality signal of a firm $\tilde{q}_i$ can become negative. This is not so crucial because we can always assume that the qualities chosen here represent improvements over a positive base quality.

7 The linearity assumption is made for simplicity and tractability. In Sect. 6 we offer a discussion on how our results are affected if, instead, we assume non-linear penalty functions.

base their purchase decisions on such information, which, as we assume, is reliable (on average).

The game unfolds as follows. We take the penalty k and the MQS, $q_{\min}$, as exogenously given and we perform comparative statics with respect to these two parameters.[8] We examine the effects of a higher penalty k in Sect. 4.1. In stage 1, firms choose their qualities and in stage 2 they choose prices. Consumers, after observing prices and qualities, make their purchases.

3 Analysis

We look for a subgame perfect Nash equilibrium. We solve the game backwards.

The regulator inspects both firms. If $\tilde{q}_i < q_{\min}$, then firm i pays a fine $P = k\,(q_{\min} - \tilde{q}_i)$. Consumers observe qualities perfectly. Those with taste parameter θ above θ_1 buy from the high quality firm, those with θ between θ_0 and θ_1 buy from the low quality firm and the rest do not buy at all.

3.1 Stage 2: firms set prices

Firms' demand functions are

$$d_H = b - \frac{(p_H - p_L)}{\Delta} \quad \text{and} \quad d_L = \frac{p_H - p_L}{\Delta} - \frac{p_L}{q_L}.$$

The equilibrium prices are

$$p_H = \frac{2\Delta b q_H}{4q_H - q_L} \quad \text{and} \quad p_L = \frac{\Delta b q_L}{4q_H - q_L}. \tag{1}$$

3.2 Stage 1: firms choose qualities

The revenue functions, when prices are given by (1), are

$$R_H\,(q_L, q_H) \equiv p_H d_H = \frac{4b^2 q_H^2\,(q_H - q_L)}{(4q_H - q_L)^2} \quad \text{and}$$
$$R_L\,(q_L, q_H) \equiv p_L d_L = \frac{b^2 q_H q_L\,(q_H - q_L)}{(4q_H - q_L)^2}. \tag{2}$$

[8] In the nursing home industry, for example, competition among nursing facilities is local, but the penalty is the same across many local markets. Hence, regulators do not customize the penalty to the specific characteristics of a local market. In this respect, the penalty in any given local market is not the optimal one.

The marginal revenue functions of the two firms are given by

$$\frac{\partial R_H}{\partial q_H} \equiv MR_H(q_L, q_H) = \frac{4b^2 q_H \left(4q_H^2 - 3q_H q_L + 2q_L^2\right)}{(4q_H - q_L)^3} \tag{3}$$

and

$$\frac{\partial R_L}{\partial q_L} \equiv MR_L(q_L, q_H) = \frac{b^2 q_H (4q_H - 7q_L)}{(4q_H - q_L)^3}. \tag{4}$$

The expected profit function of the high quality firm is[9]

$$\pi_H = R_H - \int_{\underline{\varepsilon}}^{q_{\min} - q_H} k(q_{\min} - q_H - \varepsilon) f(\varepsilon)\, d\varepsilon - \frac{q_H^2}{2} \tag{5}$$

where the second term on the right hand side represents the expected fine for the high quality firm.

The first order condition with respect to q_H, using Leibniz's rule to differentiate the integral, is

$$\frac{\partial \pi_H}{\partial q_H} = MR_H + kF(q_{\min} - q_H) - q_H = 0. \tag{6}$$

Similarly, the expected profit function of the low quality firm is

$$\pi_L = R_L - \int_{\underline{\varepsilon}}^{q_{\min} - q_L} k(q_{\min} - q_L - \varepsilon) f(\varepsilon)\, d\varepsilon - \frac{q_L^2}{2}. \tag{7}$$

The first order condition with respect to q_L is

$$\frac{\partial \pi_L}{\partial q_L} = MR_L + kF(q_{\min} - q_L) - q_L = 0. \tag{8}$$

The second terms in the right hand side of (6) and (8) represent the expected savings in the noncompliance penalty from a marginal increase of quality. It is the per-unit penalty k times the probability of the firm being found in noncompliance with the MQS.[10]

[9] We suppress the dependence of the revenue and marginal revenue curves on the qualities.

[10] The second order conditions are satisfied

$$\frac{\partial^2 \pi_i}{\partial q_i^2} = \frac{\partial MR_i}{\partial q_i} - kf(q_{\min} - q_i) - 1 < 0,$$

Let $r \equiv q_H/q_L$ denote the quality ratio. Using (1), the hedonic prices for the low and high quality goods are

$$x_L \equiv \frac{p_L}{q_L} = \theta_0 = \frac{r-1}{4r-1} \quad \text{and} \quad x_H \equiv \frac{p_H}{q_H} = \frac{2(r-1)}{4r-1}. \tag{9}$$

When the quality ratio increases hedonic prices increase (and vice versa) $dx_L/dr > 0$ and $dx_H/dr > 0$. Moreover,

$$\theta_1 = \frac{2r-1}{4r-1}$$

with $d\theta_1/dr > 0$.

We will look for conditions under which a more stringent MQS increases the quality ratio and leads to higher hedonic prices. First, we present the unregulated equilibrium qualities in the following Lemma.

Lemma 1 *When there is no MQS regulation, or when the fine k is zero, then the equilibrium qualities are*

$$q_H^{ur} = 0.2533b^2 \quad \textit{and} \quad q_L^{ur} = 0.0482b^2. \tag{10}$$

Proof If there is no MQS, or if $k = 0$, then the second terms in the right hand side of (6) and (8) disappear. The rest follows from Proposition 1 in Motta (1993). □

The unregulated equilibrium (10) is valid (that is, the market is indeed uncovered) if $a < 0.2125b$, which implies $\theta_0 > a$. We will maintain the assumption of an uncovered market. Next, we examine how an MQS affects the quality levels of the two firms.[11]

Proposition 2 *Both firms increase their qualities in response to an MQS, i.e., $\partial q_H/\partial q_{\min} \geq 0$ and $\partial q_L/\partial q_{\min} \geq 0$. Moreover, $\partial q_H/\partial q_{\min} < 1$ and $\partial q_L/\partial q_{\min} < 1$.*

Proposition 2 suggests that firms' quality choices do not move one-to-one with the MQS. Thus, eventually, the low quality firm (and possibly the high quality if the standard becomes too stringent) falls out of compliance.

Footnote 10 continued
since the marginal revenue is decreasing in own-quality (see Lemma 6). What remains to be verified is that the low quality firm does not find it profitable to 'leapfrog' the high quality firm. When we set the penalty $k = 0$ our model reduces to the vertical differentiation price competition model of Motta (1993). Motta showed that the low quality firm i obtains negative profits if it leapfrogs the high quality firm, that is, $\pi_i\left(q_i, q_H^*\right) < 0$, for any $q_i \geq q_H^*$. By an application of the envelope theorem, profits are monotonically decreasing in k, that is $d\pi_i^*/dk = -\int_{\underline{\varepsilon}}^{q_{\min}-q_i^*}\left(q_{\min} - q_i^* - \varepsilon\right) f(\varepsilon)\, d\varepsilon \leq 0$. Therefore, if the low quality firm does not want to become the high quality firm when $k = 0$, it must not find such a deviation profitable when $k > 0$, as in our model. Deviation profits, in this case, will be even more negative.

[11] In what follows, we assume that the penalty is 'reasonable', in the sense that it does not drive the firms, and in particular the low quality firm, out of the market. More specifically, profits are strictly positive when $k = 0$ and are monotonically decreasing in k (by an application of the envelope theorem). Hence, there must exist a unique k, denoted by $\bar{k}$, such that profits are negative for $k > \bar{k}$ and positive for $k \in [0, k]$.

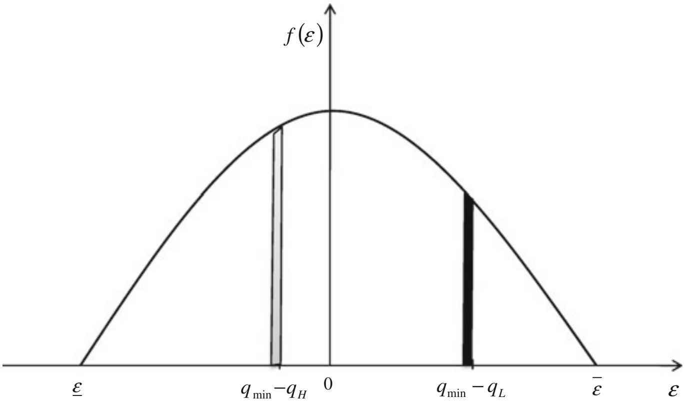

Fig. 1 High quality firm is above the MQS, $q_{\min} - q_H < 0$, while the low quality firm is below the MQS, $q_{\min} - q_L > 0$. Moreover, high quality firm is closer to the standard, in absolute, than the low quality firm

There are two marginal benefits, with respect to the expected penalty of a firm, when a firm increases its quality by one unit: (i) given the probability of getting caught, the penalty is reduced by k and (ii) the probability of getting caught is also reduced. The second marginal effect is zero and hence the marginal expected savings of a quality increase is $kF\,(q_{\min} - q_i)$, as given by the second term in (6) and (8). How a firm responds to a change in $q_{\min}$ depends on the distance between $q_{\min}$ and q_i, which in turn affects the marginal expected savings. This is given by the derivative of the expected savings which is $-kf\,(q_{\min} - q_i)$. This effect is, in general, different for the two firms due to their differential distance from the standard, see Fig. 1.

We have established that both firms will increase quality as a response to an MQS and that the quality of the low quality firm may fall below the standard (this can be true even for the high quality firm). What remains to be shown is which firm increases its quality more. This will determine the degree of vertical differentiation, the hedonic prices and ultimately consumer welfare. This is summarized in the next Proposition.

Proposition 3 *We can distinguish between the following two cases:*

(i) There exists a $\underline{q}_{\min}$, such that for any $q_{\min} < \underline{q}_{\min}$ an increase in the standard $dq_{\min}$ reduces vertical differentiation, $dr/dq_{\min} < 0$, and lowers hedonic prices $dx_L/dq_{\min} < 0$ and $dx_H/dq_{\min} < 0$, making all consumers better off.

(ii) There exists a $\overline{q}_{\min} \geq \underline{q}_{\min}$, such that for all $q_{\min} > \overline{q}_{\min}$ an increase in the standard $dq_{\min}$ increases vertical differentiation, $dr/dq_{\min} > 0$, and the hedonic prices of both firms also increase $dx_L/dq_{\min} > 0$ and $dx_H/dq_{\min} > 0$. Therefore, there exists a group of consumers who are worse off as a result of the MQS policy. Market participation shrinks.

Our main result is driven by uncertainty in the accuracy of measurement of a firm's compliance with the MQS. When the perfect observability/enforceability assumption is relaxed the low quality firm chooses quality that falls short of the MQS. This implies that it can be the high quality firm that is closer to the standard (in absolute), as Fig. 1 depicts. The high quality firm improves quality, in response to a higher standard, for two reasons: (i) to preserve the vertical differentiation and (ii) to lower the probability of being found in noncompliance. The second effect is stronger for the high quality firm if it is closer to the standard. Figure 1 illustrates this. The marginal savings of the high quality firm if it increases quality by one unit in response to an increase of the MQS (represented by the grey area) is higher than the corresponding marginal savings of the low quality firm (represented by the black area). This is because 'average' error realizations are more likely, due to the single-peak assumption for the error density $f(\varepsilon)$, than 'extreme' ones.

The combination of these two effects implies that it can be the high quality firm that enhances quality more. Vertical differentiation and hedonic prices increase.[12] Low valuation consumers among those who purchase the low quality product become worse off. Some consumers among those who consume the high quality product switch to the low quality product, because θ_1 increases when r increases. Finally, market participation shrinks.[13]

We examine a policy that strengthens a standard that may already be in existence. This is a common practice in many markets where standards increase gradually. The predictions of Proposition 3 are consistent with the empirical findings from the nursing home industry, where the highest quality improvements come not from the lowest quality facilities, but rather from those that are closer to the standard, as it is documented in Chen (2009). This has important policy implications.

The conventional wisdom prior to Ronnen (1991) was that an MQS can be harmful because it will increase the price of the low quality product. Ronnen (1991) showed that the opposite can be true. In his setup an MQS can be harmful only when it forces the low quality firm to exit. If exit does not happen, then vertical differentiation decreases and hedonic prices decrease when an MQS increases, *regardless* of where $q_{\min}$ is.[14]

In contrast, we argue that *even* if low quality firms do not exit, as it is the implicit assumption in Proposition 3, MQS can be harmful if the standard is already too strict. Our model therefore suggests that regulators should be cautious when they raise MQS, but for different reasons. It has to do with the lack of perfect quality observability and enforceability instead of reasons stated by Ronnen (1991) and other prior literature. Furthermore, we show that even the hedonic price of the high quality firm can increase.

[12] When firms compete in quantities, Valletti (2000) demonstrated that an MQS can raise the hedonic price of the low quality firm, thereby hurting some consumers. The hedonic price of the high quality firm, however, decreases.

[13] Even an MQS that is below the lowest quality can have an effect on the equilibrium qualities, due to the error with which the regulator observes qualities. This is also true in Garella and Petrakis (2008), but it is the consumers who have imperfect information about product qualities.

[14] This follows from (6) if we set the fine k equal to zero and assume full compliance $dq_L = dq_{\min}$. In this case

$$\frac{d\left(q_H/q_L\right)}{dq_L} < 0 \Leftrightarrow r > \frac{\partial MR_H/\partial q_L}{-\partial MR_H/\partial q_H + 1}.$$

The last inequality holds because the marginal revenue functions are homogeneous of degree zero, see Lemma 6, and from Euler's theorem we have $q_H\left(\partial MR_H/\partial q_H\right) + q_L\left(\partial MR_H/\partial q_L\right) = 0$. This suggests that under perfect compliance an increase of the standard $dq_{\min}$ will always reduce the quality disparity regardless of where $q_{\min}$ is, as long as both firms remain in the market.

So, it is not only the low valuation consumers that may be hurt from an MQS policy. If, on the other hand, the standard is not too strict, then the first part of Proposition 3 predicts that a marginal increase of the standard will benefit all consumers.

3.3 The effect of MQS on social surplus

Social surplus is given by

$$SS = q_L \int_{\theta_0}^{\theta_1} \theta d\theta + q_H \int_{\theta_1}^{b} \theta d\theta - \frac{q_H^2}{2} - \frac{q_L^2}{2}.$$

We differentiate SS with respect to $q_{\min}$

$$\frac{dSS}{dq_{\min}} = -\frac{dr}{dq_{\min}}\left[\frac{\partial\theta_0}{\partial r} q_L \theta_0 + \frac{\partial\theta_1}{\partial r}\theta_1 (q_H - q_L)\right] + \left[\int_{\theta_0}^{\theta_1} \theta d\theta - q_L\right] + \left[\int_{\theta_1}^{b} \theta d\theta - q_H\right].$$

Using the first order conditions, (6) and (8), the above derivative can be re-expressed as follows

$$\frac{dSS}{dq_{\min}} = -\frac{dr}{dq_{\min}}\left[\frac{\partial\theta_0}{\partial r} q_L \theta_0 + \frac{\partial\theta_1}{\partial r}\theta_1 (q_H - q_L)\right] + \left[\int_{\theta_0}^{\theta_1} \theta d\theta - MR_L - kF(q_{\min} - q_L)\right] + \left[\int_{\theta_1}^{b} \theta d\theta - MR_H - kF(q_{\min} - q_H)\right]. \quad (11)$$

When the standard is mild, i.e., $q_{\min} = q_L + \underline{\varepsilon}$ which implies that $F(q_{\min} - q_L) = F(q_{\min} - q_H) = 0$, then from the first part of Proposition 3 we know that $dr/dq_{\min} < 0$, suggesting that the first term in (11) is positive.[15] Moreover, marginal social benefit in the low $\int_{\theta_0}^{\theta_1} \theta d\theta$ and high $\int_{\theta_1}^{b} \theta d\theta$ consumer segments exceeds the marginal private benefit, MR_L and MR_H, implying that the second and third terms in (11)

[15] Recall that

$$\frac{\partial\theta_0}{\partial r} > 0 \quad \text{and} \quad \frac{\partial\theta_1}{\partial r} > 0.$$

are also positive. This confirms the finding in Ronnen (1991) where a mild standard improves social welfare. There are two sources of welfare improvements. First, the standard reduces the excessive vertical differentiation, resulting in higher market participation and in more consumers purchasing the high quality product. Second, the MQS mitigates the problem of underprovision of quality in both market segments.

But when the standard becomes more stringent, then, as we showed in the second part of Proposition 3, $dr/dq_{\min} > 0$. The first term in (11) becomes negative. Market participation declines and fewer consumers consume the high quality product. Furthermore, because $F(q_{\min} - q_L) > 0$ and $F(q_{\min} - q_H) > 0$, it may very well be the case that the marginal private benefit exceeds the social marginal benefit, resulting in quality overprovision.[16] Hence, a restrictive MQS can harm social welfare for reasons that have gone unnoticed by the literature. In a perfect observability/enforceability model a non-mild MQS, that does not induce exit, will make the second and perhaps third term in (11) negative, suggesting the existence of an endogenously determined standard, e.g., Ecchia and Lambertini (1997). However, in our model the mechanism and the underlying components of welfare changes are different.

3.4 The effect of MQS on profits

We now examine how an MQS affects the profitability of the firms. We differentiate (7) and (5) with respect to $q_{\min}$

$$\begin{aligned}\frac{d\pi_L}{dq_{\min}} &= \frac{\partial R_L}{\partial q_L}\frac{\partial q_L}{\partial q_{\min}} + \frac{\partial R_L}{\partial q_H}\frac{\partial q_H}{\partial q_{\min}} \\ &\quad + \frac{\partial q_L}{\partial q_{\min}} kF(q_{\min} - q_L) - kF(q_{\min} - q_L) - q_L\frac{\partial q_L}{\partial q_{\min}} \\ &= \frac{\partial R_L}{\partial q_H}\frac{\partial q_H}{\partial q_{\min}} - kF(q_{\min} - q_L)\end{aligned}$$

where the second equality follows from (8) and

$$\begin{aligned}\frac{d\pi_H}{dq_{\min}} &= \frac{\partial R_H}{\partial q_L}\frac{\partial q_L}{\partial q_{\min}} + \frac{\partial R_H}{\partial q_H}\frac{\partial q_H}{\partial q_{\min}} \\ &\quad + \frac{\partial q_H}{\partial q_{\min}} kF(q_{\min} - q_H) - kF(q_{\min} - q_H) - q_H\frac{\partial q_H}{\partial q_{\min}} \\ &= \frac{\partial R_H}{\partial q_L}\frac{\partial q_L}{\partial q_{\min}} - kF(q_{\min} - q_H)\end{aligned}$$

where the second equality follows from (6).

[16] This can happen if k is high, because firms want to reduce the penalty they pay to the regulator. In making this argument we have ignored the nonnegativity constraint on profits, which may bind before overprovision of quality.

Because $\partial R_L/\partial q_H > 0$ and $\partial R_H/\partial q_L < 0$, the high quality firm's profit is decreasing, but the impact of an MQS on the low quality firm is ambiguous.[17] If the standard is mild, i.e., $q_{\min} = q_L + \underline{\varepsilon}$, then $q_L = q_L^{ur}$ and $F(q_{\min} - q_L) = 0$, implying that the profit of the low quality firm increases, $d\pi_L/dq_{\min} > 0$, as in Ronnen (1991). But as the MQS becomes more stringent $F(q_{\min} - q_L) > 0$ and the low quality firm's profit can decrease. The next Proposition summarizes the findings.

Proposition 4 *The effect of an MQS on firm profits is described as follows: The profits of the high quality firm decrease. There are two opposing forces on the profits of the low quality firm. Initially, that is for a mild standard, profits increase, but for a high MQS profits can decrease.*

In the numerical example of Sect. 5, π_L is decreasing in the MQS (for the values we considered), which implies that, overall, profits are inverse U-shaped.

4 Extensions

We examine the following two cases. First, we allow the regulator to increase the penalty k and second we make the inspection process more precise.

4.1 Increasing the penalty k

We perform some comparative statics with respect to k. In Proposition 3, we showed that after a certain threshold a more stringent MQS increases the degree of vertical differentiation and raises both hedonic prices. Here, we demonstrate that in this situation a higher penalty k can reduce quality disparity, lower both hedonic prices and benefit all consumers.

Proposition 5 *A higher penalty k can decrease the quality disparity and lower both hedonic prices, whereas a higher MQS produces the exact opposite results.*

This reveals that the MQS and the penalty should be complementary regulatory tools. The numerical example in Sect. 5 verifies this.

The intuition is as follows. As (6) and (8) show, the expected marginal savings on penalty from quality improvement is $kF(q_{\min} - q_i)$. Therefore, how each of these two policies affects quality choices simply depends on how it affects $kF(q_{\min} - q_i)$ differently. A more stringent MQS increases the probability of the firm getting caught. On the other hand, an increase in k affects the per-unit penalty but not the probability of noncompliance. This affects the firms' incentives to build quality differentially. For instance, if the low quality firm is far below from the standard and the high quality firm faces a very low probability of being found in non-compliance, then the high

[17] It can be easily shown that

$$\frac{\partial R_H}{\partial q_L} = -\frac{4b^2 q_H^2 (2q_H + q_L)}{(4q_H - q_L)^3} < 0 \quad \text{and} \quad \frac{\partial R_L}{\partial q_H} = \frac{b^2 q_L^2 (2q_H + q_L)}{(4q_H - q_L)^3} > 0.$$

quality firm has stronger incentives to improve quality than the low quality firm when the standard increases as opposed to when k increases.

4.2 Second-order stochastic dominance

We now assume that the regulator makes an investment that yields a new distribution $\tilde{F}(\varepsilon)$ which second order stochastically dominates $F(\varepsilon)$ (both distributions have mean zero). To put differently, $F(\varepsilon)$ is a mean preserving spread (mps) of $\tilde{F}(\varepsilon)$. More specifically, the mps we consider is a one under which the two distributions cross once at the mean (zero) and the less dispersed distribution $\tilde{F}(\varepsilon)$ crosses $F(\varepsilon)$ from below, see Mas Colell et al. (1995). What is the impact of such a (local) change in the error distribution on the equilibrium qualities?[18]

If $q_{\min} < q_L < q_H$, then both firms decrease their qualities. This can be seen by inspecting the first order conditions (6) and (8) and noting that the errors $\varepsilon \equiv q_{\min} - q_i$ in this case are negative. Since the probability of being found in noncompliance decreases for both firms (when we go from $F(\varepsilon)$ to $\tilde{F}(\varepsilon)$), both firms respond by lowering their qualities. Therefore, when the standard is weak, making the inspection process more precise will actually lead to lower qualities being produced by both firms.

If $q_L < q_{\min} < q_H$, then, following the logic we outlined above, the high quality firm will lower its quality, while the low quality firm will raise its quality, since $\varepsilon_L > 0$ and $\varepsilon_H < 0$. The quality of the high quality firm decreases.

Finally, if $q_L < q_H < q_{\min}$, then both firms will raise their qualities.

The first two cases we consider above highlight the interesting possibility that a more precise inspection process can have a detrimental effect on quality. The intuition is as follows. Suppose that both firms comply with the standard $q_{\min} < q_L < q_H$ and the inspection noise is reduced. This implies that it is less likely that the regulator will make a mistake and penalize one of the firms. In this case firms can 'safely' lower their qualities. But if a firm is below the standard, as in $q_L < q_{\min} < q_H$, then the low quality firm, knowing that if the inspection noise decreases the chances of getting caught will increase, supplies higher quality.

5 A numerical example

Consider the following triangular distribution of the error ε on $[-1/m, 1/m]$

$$F(\varepsilon) = \begin{cases} \frac{1}{2}(1 + m\varepsilon)^2, & \text{if } \varepsilon \leq 0 \\ \frac{1}{2} + \frac{(2-m\varepsilon)m\varepsilon}{2}, & \text{if } \varepsilon \geq 0. \end{cases}$$

[18] In the nursing home industry, for example, to increase the precision of information the regulatory agencies receive, they can increase inspection frequency. They can also add inspection dimensions and have more highly trained and experienced inspectors. Finally, the regulatory agencies can have nursing homes fill out and submit more report cards and provide greater details on different elements of qualities.

We assign the following values to the parameters: $k = 0.002$, $m = 5$ and $b = 1$.[19] This implies that the error ε has support $[-0.2, 0.2]$. The unregulated qualities are given by (10). When $q_{\min} = 0.10$, then the following qualities satisfy the system of first order conditions (6) and (8)

$$q_H = 0.25352 \quad \text{and} \quad q_L = 0.04932 \tag{12}$$

implying a quality ratio $r = 5.14067$. The expected profit of the low quality firm is $\pi_L = 0.0014 > 0$. The expected profits of the high quality firm will certainly be strictly positive. Now suppose that the MQS increases to $q_{\min} = 0.11$. The following qualities satisfy the system of first order conditions

$$q_H = 0.25355 \quad \text{and} \quad q_L = 0.04937$$

implying a quality ratio $r = 5.13564$. The expected profit of the low quality firm is $\pi_L = 0.00138 > 0$. The degree of vertical differentiation has decreased. Hedonic prices decrease. This is the first part of Proposition 3. The low quality firm is closer to the standard than the high quality firm $\varepsilon_L \equiv q_L - q_{\min} = 0.11 - 0.04937 = 0.06063$ and $\varepsilon_H \equiv q_H - q_{\min} = 0.11 - 0.25355 = -0.14355$, which imply $f(\varepsilon_L) > f(\varepsilon_H)$.

As the second part of Proposition 3 shows, the above prediction changes when we consider a higher MQS as a starting point. For example, when $q_{\min} = 0.22$ then the following qualities satisfy the system of first order conditions

$$q_H = 0.2542 \quad \text{and} \quad q_L = 0.049747 \tag{13}$$

implying a quality ratio $r = 5.10984$. The expected profit of the low quality firm is $\pi_L = 0.00119 > 0$. Now suppose that the MQS increases to $q_{\min} = 0.23$. The following qualities satisfy the system of first order conditions

$$q_H = 0.25428 \quad \text{and} \quad q_L = 0.04976$$

implying a quality ratio $r = 5.11015$. The degree of vertical differentiation and the hedonic prices have increased. The expected profit of the low quality firm is $\pi_L = 0.00117 > 0$. The high quality firm is relatively closer now than before to the MQS and hence it has stronger incentives to invest in quality improvements than the low quality firm. In particular, $\varepsilon_L \equiv q_L - q_{\min} = 0.23 - 0.04976 = 0.18024$ and $\varepsilon_H \equiv q_H - q_{\min} = 0.11 - 0.25355 = -0.02355$, which imply $f(\varepsilon_L) < f(\varepsilon_H)$.[20]

[19] Due to the complexity of the model, analytic solutions are not possible.

[20] Quality disparity is minimized when $q_{\min} = 0.218$.

5.1 Higher penalty k

Let's now examine the effect of a higher penalty k. Suppose $q_{\min} = 0.22$ and $k = 0.0021$ (instead of 0.002). Then,

$$q_H = 0.25424 \quad \text{and} \quad q_L = 0.04982$$

implying higher qualities relative to (13), and a quality ratio $r = 5.10300$, which is less than $r = 5.10984$, the quality ratio when $k = 0.002$. This verifies Proposition 5, where it is shown that a higher penalty can reduce vertical differentiation, lower both hedonic prices and be beneficial to all consumers when, in the same situation, a more stringent MQS hurts some consumers by raising both hedonic prices.

5.2 Second-order stochastic dominance

Fix $k = .002$, $m = 4$ and $q_{\min} = 0.045$. The following qualities satisfy the system of first order conditions (6) and (8)

$$q_H = 0.25344 \quad \text{and} \quad q_L = 0.048961.$$

Now allow m to increase from 4 to 4.2. This shrinks the support of the error distribution from $[-0.25, 0.25]$ to $[-0.238, 0.238]$ and generates a counterclockwise rotation of the distribution function. In this case, $q_{\min}$ is below q_L and q_H. The following qualities satisfy the system of first order conditions (6) and (8)

$$q_H = 0.25343 \quad \text{and} \quad q_L = 0.048960.$$

The product quality of both firms, as predicted in Sect. 4.2, decreases.

Now let's consider the case where $q_{\min}$ is greater than q_L and below q_H. Fix $k = .002$, $q_{\min} = 0.10$ and allow m to increase from 5 to 5.1. The following qualities satisfy the system of first order conditions (6) and (8)

$$q_H = 0.25351 \quad \text{and} \quad q_L = 0.04932.$$

Relative to (12), the product quality of the high quality firm, as predicted in Sect. 4.2 , decreases, while that of the low quality firm increases.

6 Discussion

Below, we offer a discussion about how our main results depend on the various modeling assumptions, such as the linearity of the penalty function and the monitoring technology.

6.1 On the linear penalty function

Our main result depends on the curvature of the penalty function as follows. With a linear penalty function, the marginal expected savings for a firm when it increases its quality is constant with respect to how far the firm's quality level is from the standard. With a convex function, on the other hand, the low quality firm–who is usually below the standard and hence higher on the penalty function–has a higher marginal expected saving than the high quality firm. This gives the low quality firm stronger incentives to increase its quality and introduces an *additional* effect that is muted under the linear penalty assumption. Therefore, our main result weakens quantitatively. Nevertheless, we were able to construct a numerical example with a quadratic penalty function where our main results continue to hold.[21] Conversely, with a concave penalty function, our results become stronger. The evidence on this issue in the literature is mixed. Some studies on environment regulations assume that the penalty function is linear (e.g. Stranlund et al. 2009). Others contend that emission standards will be effective only when the return from noncompliance increases at a decreasing rate, and assume the penalty function to be strictly convex in violation size (e.g. Hardford 1978; Shaffer 1990).[22] We believe that a penalty function cannot be everywhere convex. Firms will find ways to avoid paying increasingly higher penalties and eventually limited liability constraints will become binding.[23]

6.2 On the monitoring technology

Previous literature has studied the choice of policy instruments to control pollution when incomplete enforcement is an issue (Montero 2002; Rousseau and Proost 2005; Macho-Stadler 2008), and the interaction between abatement technology adoption and imperfect compliance (Villegas-Palacio and Coria 2010). The common assumption about incomplete enforcement is that firms are required to self-report their compliance status, and the regulator only monitors stochastically. However, once the regulator monitors a firm, it is able to perfectly determine the firm's compliance status. Suppose we make a similar assumption, that is if firm i is in non-compliance with the MQS, $q_{\min} - q_i > 0$, then it gets detected with a probability that depends on how far the firm is from the standard $F(q_{\min} - q_i)$ and pays a penalty k. In this case, the high quality firm, who most likely is in compliance, will never respond to a higher standard. Only the low quality firm would increase its quality. Vertical differentiation would

21 More specifically, we use the triangular distribution of Sect. 5. We assume a quadratic penalty $P = k(q_{\min} - \tilde{q}_i)^2$ and we set $k = 0.0002$, $m = 8$ and $b = 1$. When we go from $q_{\min} = 0.20$ to $q_{\min} = 0.21$ the ratio of qualities r increases, consistent with Proposition 3.

22 Stranlund et al. (2009) mention that neither of these assumptions is justified by actual enforcement strategies. "Our assumption of a constant expected marginal penalty for tax evasion is not common in the theoretical literature on compliance with incentive-based policies. Most authors assume expected penalties that are some combination of strictly convex penalty functions, and probabilities of detecting violations that may depend on firms' emissions reports, on the regulator's expectation of their emissions, or on their actual emissions. These assumptions are not justified by actual enforcement strategies."

23 For instance, firms can appeal to court, delaying the collection of the penalty considerably.

decrease and a MQS in this case would benefit all consumers, as in Ronnen (1991). Perfect quality inspection is a reasonable assumption for pollutant emissions, which can be observed and measured accurately using cutting edge devices. However, in many real-world settings, in particular service industries and health care, the regulator observes quality with noise, either due to randomness in certain dimensions of product quality, or subjectivity, or high inspection costs and limited resources. Hence, our paper complements the existing literature by taking this practically and theoretically important factor into account. Our main purpose is to illustrate how imperfect quality observability may lead to excessive quality differentiation and become another source of policy failure under minimum quality standards regulation.

7 Conclusion

We examine the implications of minimum quality standards in a market characterized by vertical differentiation and where regulators observe quality imperfectly. The existing literature assumes that firms have no choice but to comply with standards. In many markets, we do not observe full compliance, the service industry in particular. Our paper provides an explanation for the existence of noncompliance and firms' strategic quality choice, as observed in reality. Uncertainty in the accuracy of measurement of a firm's compliance with the MQS drives our main result. There are many products, for example, medical care, educational services, research activities and financial services whose quality is complex, multi-dimensional, subject to randomness, and therefore difficult to measure. Combined with the limited resources regulators can utilize to inspect firms, the uncertainty leads to random error in the quality measurement process. Thus, both low quality and high quality firms face risks of being found in noncompliance. The risks rather than absolute certainty of being identified as non-compliant fundamentally change firms' incentives for quality improvements and generate results that contrast with the conventional wisdom.

Assuming the MQS is always perfectly enforced, the existing literature argues that low-quality firms will be forced to meet the standards or exit, and high-quality firms will stay at their initial position (e.g. Leland 1979; Shapiro 1983) or raise quality in an effort to vertically differentiate themselves from their improved low quality rivals (e.g. Ronnen 1991; Crampes and Hollander 1995). We relax the standard assumption that quality has been observed and enforced perfectly, and allow low-quality firms the choice of whether to comply with the MQS or not. We argue that besides the incentive to vertically differentiate and soften price competition, firms also improve quality in order to reduce their chances of being "labeled" as low quality and not in compliance. Hence, firms closer to the standard have stronger incentives to raise quality, because the marginal effect of a quality improvement on the probability of noncompliance is higher. We also show that MQS can be harmful even if low quality firms do not exit. A non-mild standard leads to a higher degree of vertical differentiation, weaker price competition and higher hedonic prices. Some consumers are definitely worse off due to an MQS policy and market participation decreases. Social welfare can also be reduced.

The mechanism we identify in this paper is novel and it has important policy implications. Many markets fit our modeling assumptions where MQS increase gradually and uncertainty in the accuracy measurement of quality exists. An important question in these markets is whether a further increase of a standard will improve welfare, relative to the status quo. We show that when the standard is low, raising the standard will increase vertical differentiation, lower hedonic prices and benefit all consumers. However, when the standard is already set at a high level, then a further increase of the standard will generate the adverse effects for (some) of the consumers that we describe.

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A Appendix

A.1 Lemma 6

The results in the next Lemma are used extensively in the proofs. The proof of the Lemma is straightforward.

Lemma 6 *The marginal revenue curves of the two firms, MR_H and MR_L, given by (3) and (4), are downward sloping with respect to a firm's own quality, with $MR_H > MR_L$, for any $q_H \geq q_L$. Moreover,*

$$\frac{\partial MR_H}{\partial q_H} = -\frac{8b^2 q_L^2 (5q_H + q_L)}{(4q_H - q_L)^4} \quad \text{and} \quad \frac{\partial MR_L}{\partial q_L} = -\frac{2b^2 q_H^2 (8q_H + 7q_L)}{(4q_H - q_L)^4}$$

and

$$\frac{\partial MR_H}{\partial q_L} = \frac{8b^2 q_H q_L (5q_H + q_L)}{(4q_H - q_L)^4} > \frac{\partial MR_L}{\partial q_H} = \frac{2b^2 q_H q_L (8q_H + 7q_L)}{(4q_H - q_L)^4}.$$

Moreover,

$$q_H \frac{\partial MR_H}{\partial q_H} + q_L \frac{\partial MR_H}{\partial q_L} = 0 \quad \text{and} \quad q_L \frac{\partial MR_L}{\partial q_L} + q_H \frac{\partial MR_L}{\partial q_H} = 0.$$

Finally,

$$\frac{\partial MR_H}{\partial q_H} \frac{\partial MR_L}{\partial q_L} = \frac{\partial MR_H}{\partial q_L} \frac{\partial MR_L}{\partial q_H}.$$

A.2 Proof of Proposition 2

First, we find how the equilibrium qualities are affected by a marginal increase of the MQS. To this end, we invoke the Implicit Function Theorem, using the first order conditions (6) and (8)

$$\begin{pmatrix} \frac{\partial q_H}{\partial q_{\min}} \\ \frac{\partial q_L}{\partial q_{\min}} \end{pmatrix} = -(J)^{-1} \begin{pmatrix} kf(q_{\min} - q_H) \\ kf(q_{\min} - q_L) \end{pmatrix}$$

where

$$J = \begin{pmatrix} \frac{\partial MR_H}{\partial q_H} - kf(q_{\min} - q_H) - 1 & \frac{\partial MR_H}{\partial q_L} \\ \frac{\partial MR_L}{\partial q_H} & \frac{\partial MR_L}{\partial q_L} - kf(q_{\min} - q_L) - 1 \end{pmatrix}$$

is the Jacobian of the system of first order conditions as it is given by (6) and (8) with

$$(J)^{-1} = \frac{1}{|J|} \begin{pmatrix} \frac{\partial MR_L}{\partial q_L} - kf(q_{\min} - q_L) - 1 & -\frac{\partial MR_H}{\partial q_L} \\ -\frac{\partial MR_L}{\partial q_H} & \frac{\partial MR_H}{\partial q_H} - kf(q_{\min} - q_H) - 1 \end{pmatrix}$$

where $|J|$ is the determinant of the Jacobian. Then, we have

$$\begin{pmatrix} \frac{\partial q_H}{\partial q_{\min}} \\ \frac{\partial q_L}{\partial q_{\min}} \end{pmatrix} = -\frac{1}{|J|} \times \begin{pmatrix} \left(\frac{\partial MR_L}{\partial q_L} - kf(q_{\min} - q_L) - 1\right) kf(q_{\min} - q_H) - \frac{\partial MR_H}{\partial q_L} kf(q_{\min} - q_L) \\ \left(\frac{\partial MR_H}{\partial q_H} - kf(q_{\min} - q_H) - 1\right) kf(q_{\min} - q_L) - \frac{\partial MR_L}{\partial q_H} kf(q_{\min} - q_H) \end{pmatrix}. \quad (14)$$

Note that

$$|J| = \left(\frac{\partial MR_H}{\partial q_H} - kf(q_{\min} - q_H) - 1\right)\left(\frac{\partial MR_L}{\partial q_L} - kf(q_{\min} - q_L) - 1\right) - \frac{\partial MR_H}{\partial q_L}\frac{\partial MR_L}{\partial q_H} > 0$$

because from Lemma 6

$$\frac{\partial MR_H}{\partial q_H} < 0, \frac{\partial MR_L}{\partial q_L} < 0 \quad \text{and} \quad \frac{\partial MR_H}{\partial q_H}\frac{\partial MR_L}{\partial q_L} = \frac{\partial MR_H}{\partial q_L}\frac{\partial MR_L}{\partial q_H}.$$

We can then conclude that $\partial q_H/\partial q_{\min} \geq 0$ and $\partial q_L/\partial q_{\min} \geq 0$. Moreover, $\partial q_H/\partial q_{\min} = 0$ and $\partial q_L^r/\partial q_{\min} = 0$, when $k = 0$ and, using L'Hopital's rule, $\lim_{k\to\infty} \partial q_H/\partial q_{\min} = 1$ and $\lim_{k\to\infty} \partial q_L/\partial q_{\min} = 1$. Both firms comply fully with the MQS only when the penalty is infinitely high (ignoring the nonnegativitiy profit constraint).

So far we have proved than when the penalty k is zero, $\partial q_i^r/\partial q_{\min} = 0$, and when $k \to \infty$, $\partial q_i^r/\partial q_{\min} \to 1$. Is there a k such that $\partial q_i^r/\partial q_{\min} > 1$? We solve

$\partial q_i^r/\partial q_{\min} - 1 = 0, i = L, H$, with respect to k. This yields (after a simplification using Lemma 6)

$$k_L = \frac{-\frac{\partial MR_H}{\partial q_H} - \frac{\partial MR_L}{\partial q_L} + 1}{f\left(q_{\min} - q_H\right)\left(\frac{\partial MR_L}{\partial q_L} + \frac{\partial MR_L}{\partial q_H} - 1\right)} \quad \text{and}$$

$$k_H = \frac{-\frac{\partial MR_H}{\partial q_H} - \frac{\partial MR_L}{\partial q_L} + 1}{f\left(q_{\min} - q_L\right)\left(\frac{\partial MR_H}{\partial q_H} + \frac{\partial MR_H}{\partial q_L} - 1\right)}.$$

The numerators are positive (see Lemma 6). The denominator of k_L is negative for any $q_H > q_L$. This can be seen as follows. The marginal revenue functions are homogeneous of degree zero and from Euler's theorem we have $q_L\left(\partial MR_L/\partial q_L\right) + q_H\left(\partial MR_L/\partial q_H\right) = 0$, see also Lemma 6. This implies

$$\frac{q_H}{q_L} = \frac{-\frac{\partial MR_L}{\partial q_L}}{\frac{\partial MR_L}{\partial q_H}} \Rightarrow 1 < \frac{q_H}{q_L} < \frac{-\frac{\partial MR_L}{\partial q_L} + 1}{\frac{\partial MR_L}{\partial q_H}} \Rightarrow \frac{\partial MR_L}{\partial q_L} + \frac{\partial MR_L}{\partial q_H} - 1 < 0.$$

Hence, there does not exist a positive k such that $\partial q_L^r/\partial q_{\min} = 1$, suggesting that $\partial q_L^r/\partial q_{\min} < 1$.

The same is also true for the high quality firm. The denominator of k_H is negative. Following a similar argument, we have $q_H\left(\partial MR_H/\partial q_H\right) + q_L\left(\partial MR_H/\partial q_L\right) = 0$, leading to

$$\frac{q_H}{q_L} = \frac{-\frac{\partial MR_H}{\partial q_L}}{\frac{\partial MR_H}{\partial q_H}} \Rightarrow 1 < \frac{q_H}{q_L} < \frac{-\frac{\partial MR_H}{\partial q_L} + 1}{\frac{\partial MR_H}{\partial q_H}} \Rightarrow \frac{\partial MR_H}{\partial q_H} + \frac{\partial MR_H}{\partial q_L} - 1 < 0.$$

Therefore, we also have $\partial q_H/\partial q_{\min} < 1$.

A.3 Proof of Proposition 3

First, we would like to show that a more stringent MQS increases the quality ratio $r \equiv q_H/q_L$, i.e., $dr/dq_{\min} > 0$. By setting $f\left(q_{\min} - q_L\right) = 0$ and assuming that $f\left(q_{\min} - q_H\right) > 0$ from (14) we have

$$\frac{\partial\left(q_H/q_L\right)}{\partial q_{\min}} > 0 \Leftrightarrow \frac{\frac{\partial q_H}{\partial q_{\min}}}{q_H} > \frac{\frac{\partial q_L}{\partial q_{\min}}}{q_L} \Leftrightarrow \frac{-\frac{\partial MR_L}{\partial q_L} + 1}{q_H} > \frac{\frac{\partial MR_L}{\partial q_H}}{q_L} \Leftrightarrow r < \frac{-\frac{\partial MR_L}{\partial q_L} + 1}{\frac{\partial MR_L}{\partial q_H}}.$$

The last inequality follows because the marginal revenue functions are homogeneous of degree zero and from Euler's theorem we have $q_L\left(\partial MR_L/\partial q_L\right) + q_H\left(\partial MR_L/\partial q_H\right) = 0$, see also Lemma 6. By continuity this will also hold in the neighborhood of $f\left(q_{\min} - q_L\right) = 0$. This case can arise when the quality of the low quality firm is way below the standard so that the density is very low. From (8) it follows easily that if the fine k is not very high, then the low quality firm may optimally set a quality level so that the probability of being found in noncompliance $F(q_{\min} - q_L)$ is 100% and at this point $f(q_{\min} - q_L) = 0$.

Now we would like to show the opposite, i.e., $dr/dq_{\min} < 0$. By setting $f\big(q_{\min} - q_H\big) = 0$ and assuming that $f(q_{\min} - q_L) > 0$ from (14) we have

$$\frac{\partial (q_H/q_L)}{\partial q_{\min}} < 0 \Leftrightarrow \frac{\frac{\partial q_H}{\partial q_{\min}}}{q_H} < \frac{\frac{\partial q_L}{\partial q_{\min}}}{q_L} \Leftrightarrow \frac{\frac{\partial MR_H}{\partial q_L}}{q_H} < \frac{-\frac{\partial MR_H}{\partial q_H} + 1}{q_L} \Leftrightarrow r > \frac{\frac{\partial MR_H}{\partial q_L}}{-\frac{\partial MR_H}{\partial q_H} + 1}.$$

The last inequality follows because the marginal revenue functions are homogeneous of degree zero and from Euler's theorem we have $q_H(\partial MR_H/\partial q_H) + q_L(\partial MR_H/\partial q_L) = 0$, see also Lemma 6. By continuity this will also hold in the neighborhood of $f(q_{\min} - q_H) = 0$. This case can arise when the quality of the high quality firm is way above the standard so that the density is very low.

When $q_{\min} = q_L^{ur} + \underline{\varepsilon}$, the standard is not binding and the equilibrium qualities are given by (10). As $q_{\min}$ increases the low quality firm will increase its quality and despite the fact that $f(q_{\min} - q_H) = 0$, for $q_{\min}$ in the neighborhood of the $q_L^{ur} + \underline{\varepsilon}$, the high quality firm will also increase its quality, see (14). This holds in the region $\big(q_L^{ur} + \underline{\varepsilon}, \underline{q}_{\min}\big)$. As we proved above, $dr/dq_{\min} < 0$ in this region. As $q_{\min}$ continues to increase after a threshold $\overline{q}_{\min}$, the low quality firm is so below the standard so that $f(q_{\min} - q_L) = 0$. In this region we have $dr/dq_{\min} > 0$.

A.4 Proof of Proposition 5

First, we find how the equilibrium qualities are affected by a marginal increase of k. To this end, we invoke the Implicit Function Theorem, using the first order conditions (6) and (8)

$$\begin{pmatrix} \frac{\partial q_H}{\partial k} \\ \frac{\partial q_L}{\partial k} \end{pmatrix} = -(J)^{-1} \begin{pmatrix} F(q_{\min} - q_H) \\ F(q_{\min} - q_L) \end{pmatrix}$$

where

$$J = \begin{pmatrix} \frac{\partial MR_H}{\partial q_H} - kf(q_{\min} - q_H) - 1 & \frac{\partial MR_H}{\partial q_L} \\ \frac{\partial MR_L}{\partial q_H} & \frac{\partial MR_L}{\partial q_L} - kf(q_{\min} - q_L) - 1 \end{pmatrix}$$

is the Jacobian of the system of first order conditions as it is given by (6) and (8) with

$$(J)^{-1} = \frac{1}{|J|} \begin{pmatrix} \frac{\partial MR_L}{\partial q_L} - kf(q_{\min} - q_L) - 1 & -\frac{\partial MR_H}{\partial q_L} \\ -\frac{\partial MR_L}{\partial q_H} & \frac{\partial MR_H}{\partial q_H} - kf(q_{\min} - q_H) - 1 \end{pmatrix}$$

where $|J|$ is the determinant of the Jacobian. Then, we have

$$\begin{pmatrix} \frac{\partial q_H}{\partial k} \\ \frac{\partial q_L}{\partial k} \end{pmatrix} = -\frac{1}{|J|} \times \begin{pmatrix} \left(\frac{\partial MR_L}{\partial q_L} - kf(q_{\min} - q_L) - 1\right) F(q_{\min} - q_H) - \frac{\partial MR_H}{\partial q_L} F(q_{\min} - q_L) \\ \left(\frac{\partial MR_H}{\partial q_H} - kf(q_{\min} - q_H) - 1\right) F(q_{\min} - q_L) - \frac{\partial MR_L}{\partial q_H} F(q_{\min} - q_H) \end{pmatrix}.$$

First note that because $q_H > q_L$, we always have $F(q_{\min} - q_L) > F(q_{\min} - q_H)$. In the proof of Proposition 3, we showed that when $f(q_{\min} - q_L) = 0$ then $dr/dq_{\min} > 0$. A natural question that arises is: Can, at that point, all consumers benefit if instead of increasing the MQS the regulator increases the penalty k? To answer this question, we assume that $F(q_{\min} - q_H) = 0$. Then, we have

$$\begin{pmatrix} \frac{\partial q_H}{\partial k} \\ \frac{\partial q_L}{\partial k} \end{pmatrix} = -\frac{1}{|J|} \begin{pmatrix} -\frac{\partial MR_H}{\partial q_L} F(q_{\min} - q_L) \\ \left(\frac{\partial MR_H}{\partial q_H} - kf(q_{\min} - q_H) - 1\right) F(q_{\min} - q_L) \end{pmatrix}.$$

$$\frac{\partial (q_H/q_L)}{\partial k} < 0 \Leftrightarrow \frac{\frac{\partial q_H}{\partial k}}{q_H} < \frac{\frac{\partial q_L}{\partial k}}{q_L} \Leftrightarrow$$

$$\frac{\frac{\partial MR_H}{\partial q_L} F(q_{\min} - q_L)}{q_H} < \frac{-\left(\frac{\partial MR_H}{\partial q_H} - kf(q_{\min} - q_H) - 1\right) F(q_{\min} - q_L)}{q_L} \Leftrightarrow$$

$$r > \frac{\frac{\partial MR_H}{\partial q_L} F(q_{\min} - q_L)}{-\left(\frac{\partial MR_H}{\partial q_H} - kf(q_{\min} - q_H) - 1\right) F(q_{\min} - q_L)}.$$

Then we have

$$\frac{\partial (q_H/q_L)}{\partial k} < 0 \Leftrightarrow r > \frac{\frac{\partial MR_H}{\partial q_L}}{-\left(\frac{\partial MR_H}{\partial q_H} - kf(q_{\min} - q_H) - 1\right)}.$$

From the proof of Proposition 3 we have

$$r > \frac{\frac{\partial MR_H}{\partial q_L}}{-\frac{\partial MR_H}{\partial q_H} + 1} > \frac{\frac{\partial MR_H}{\partial q_L}}{-\left(\frac{\partial MR_H}{\partial q_H} - kf(q_{\min} - q_H) - 1\right)}.$$

Hence, $dr/dk < 0$ also holds in the neighborhoods of $F(q_{\min} - q_H) = 0$. This demonstrates that a low $F(q_{\min} - q_H)$ guarantees that a higher penalty reduces the quality disparity.

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