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Commodity derivative valuation under a factor model with time-varying market prices of risk

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Abstract It is well known that market prices of risk play an important role in commodity derivative valuation. There is an extensive literature showing that market prices of risk vary through time. Based on these results, a factor model, with two long- and short-term factors, with market prices of risk depending on these underlying asset factors is proposed and estimated, using data from crude oil, heating oil, unleaded gasoline and natural gas futures prices traded at NYMEX. The valuation results obtained with an extensive sample of commodity American options traded at NYMEX show that this model with time-varying market prices of risk outperforms standard models with constant market prices of risk.

Keywords Market price of risk · Commodity prices · Commodity derivatives · Stochastic processes · Kalman filter

JEL Classification C32 · C51 · C60 · G13

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1 Introduction

In equity markets, the market price of risk is the excess return over the risk-free rate per unit standard deviation ($(\mu - r)/\sigma$) that investors want as compensation for taking risk, which is also called the Sharpe ratio. This ratio plays an important role in derivatives valuation. If the underlying asset is a traded asset, it is possible to build a risk-free portfolio by buying the derivative and selling the underlying asset or vice versa. Consequently, the market price of risk does not appear in the derivatives valuation model.

However, if the underlying asset is not a traded asset, there is no way of building a riskless portfolio by buying the derivative and selling the underlying asset or vice versa; therefore, we must know how much return is needed to compensate the unhedgeable risk. This is why the market price of risk must be estimated to obtain a theoretical value for the derivative asset.

In commodity markets, the market price of risk has a slightly different definition. As noted by Kolos and Ronn (2008), equities require a costly investment and, consequently, return the risk-free rate under the risk-neutral measure. In the case of commodities, it should be noted that sometimes there is a storage cost associated with storing the commodity and also a convenience yield associated with holding the commodity rather than the derivative asset. Nevertheless, futures contracts are costless to enter into; therefore, their risk-neutral drift is zero. Thus, the market price of risk in commodity markets is defined as the ratio of the asset return to its standard deviation (μ/σ). Additionally, whereas the market price of risk must be positive in equity markets, it can be negative in commodity markets.

There have been several papers that have analyzed the properties of market prices of risk in commodity markets and their relation with other variables. Fama and French (1987, 1988) note the importance of allowing for time-varying market prices of risk as negative correlations between spot prices and market prices of risk can generate mean reversion in spot prices. Bessembinder (1992) shows that market prices of risk in financial and commodity markets are related to the covariance of the market portfolio and the futures returns. Routledge et al. (2001) and Bessembinder and Lemmon (2002) relate market prices of risk to several measures of uncertainty, such as price volatility, spikes and uncertainty in demand. Moosa and Al-Loughani (1994), Sardosky (2002) and Jalali-Naini and Kazemi-Manesh (2006) find evidence of variable market prices of risk in oil markets using GARCH models.

More recently, Kolos and Ronn (2008) estimate the market prices of risk for energy commodities, finding positive long-term and negative short-term market prices of risk. Lucia and Torro (2011) find that market prices of risk in the Nordic Power Exchange (Nord Pool) vary seasonally over the year and are related to unexpected low reservoir levels.

There are also several recent papers that find determinants of commodity market prices of risk, such as Hong and Yogo (2012) or Archarya et al. (2013) among others. Moreover, Trolle and Schwartz (2010) and Prokopczuk and Simen (2013) find that the variance of market prices of risk varies through time. Baker and Routledge (2012) analyze the dynamics of risk premium in oil markets by solving a Pareto risk-sharing problem. Basu and Miffre (2013) find that systematic hedging pressure is a significant determinant of commodity futures risk premia. Le and Zhu (2013) find that risk premia in gold lease rates are time-varying and increasing in the level and slope of gold lease rates and volatility.

There have also been several papers that have analyzed the importance of allowing for time-varying market prices of risk from the point of view of asset valuation. Following the ideas in Fama (1984) and Fama and Bliss (1987), Duffee (2002) and Dai and Singleton (2002) propose interest rate models where market prices of risk are linear functions of the state variables. Casassus and Collin-Dufresne (2005) propose and estimate a three-factor model for commodity spot prices, convenience yields and interest rates where convenience yields depend on spot prices and interest rates, and time-varying (state depending) market prices of risk using a maximum likelihood method. They also test the importance of time-varying market prices of risk and the dependence of convenience yields on spot prices and interest rates on the valuation of a set of theoretical commodity European call options.

More recently, Trolle and Schwartz (2009) and Casassus et al. (2013) propose models with time-varying market prices of risk to value commodity derivatives within an affine framework. Trolle and Schwartz (2009) allow for stochastic volatility within a factor model for commodity prices, where market prices of risk depend only on the volatility factors. Casassus et al. (2013) propose a multi-commodity factor model for commodity prices, where market prices of risk depend on the model factors. They apply their model to the valuation of crack-spread options. However, they do not compare the effects of time-varying and constant market prices of risk from the point of view of option valuation. Bhar and Lee (2011) propose a three-factor model for crude oil prices with time-varying market prices of risk. They apply their model to the valuation of crude oil futures prices, but they do not test the effect of time-varying market prices of risk on option pricing.

In this paper, we extend these ideas by proposing and estimating a commodity derivative valuation model with time-varying market prices of risk, which is applied to the valuation of an extensive database of exchange-traded commodity American options. Thus, there are three key questions that we try to address in this work: (i) Are market prices of risk constant or depend on the state variables? (ii) Can a two-factor model with time-varying market prices of risk help to understand the stochastic behavior of commodity futures prices? and (iii) Can this model allowing for time-varying market prices of risk help to reduce estimation errors when valuing commodity American options compared to standard constant market prices of risk models?

Specifically, we propose an extended Schwartz and Smith (2000) two factor model allowing for market prices of risk to depend on the long- and short-term model factors. The model is estimated using crude oil, heating oil, gasoline and natural gas futures prices traded on the NYMEX, through the Kalman filter method.

The valuation results obtained with an extensive sample of commodity American options, traded on the NYMEX, show that the proposed model with time-varying market prices of risk outperforms standard models with constant market prices of risk. These results confirm the previous findings shown in the literature of non-constant market prices of risk. It is important to note that in the papers by Trolle and Schwartz (2009) and Casassus et al. (2013) models allowing for time-varying market prices of risk are presented and are applied to the valuation of exchange-traded options. However, they do not analyze the effect of time-varying market prices of risk from the point of view of option pricing, given that their objective is to incorporate stochastic volatility effects in the first case (Trolle and Schwartz 2009) or to present a multi-commodity factor model in the second one (Casassus et al. 2013). Therefore, the main contribution of this paper is the analysis of the effect of time-varying market prices of risk, compared to the constant case, on option pricing using exchange-traded data.

The remainder of this paper is organized as follows. Section 2 presents the data sets used in the paper. The factor model with time-varying market prices of risk is proposed and estimated in Sect. 3. Section 4 presents the option valuation results obtained with the models with time-varying and constant market prices of risk. Finally, Sect. 5 concludes with a summary and discussion.

2 Data

The data set used in this paper consists of weekly observations of WTI (light sweet) crude oil, heating oil, unleaded gasoline (RBOB) and natural gas (Henry Hub) futures prices traded on the NYMEX.

Currently, there are futures being traded on NYMEX for WTI crude oil with maturities of 1 month to 7 years, for heating oil from 1 to 18 months, for gasoline from 1 to 12 months and for Henry Hub natural gas from 1 month to 6 years. However, there is not enough liquidity for the futures with longer maturities, especially in the case of gasoline. Therefore, in the cases of WTI crude oil and heating oil, our data set is comprised of futures prices from 1 to 18 months (1,338 weekly observations) between 1/1/1985 and 8/16/2010. In the case of RBOB gasoline, the data set is comprised of futures prices from 1 to 9 months (1,338 weekly observations) between 1/1/1985 and 8/16/2010. Finally, in the case of Henry Hub natural gas, the data set is comprised of futures prices from 1 to 18 months (1,064 weekly observations) between 4/2/1990 and 8/16/2010. The main descriptive statistics of these variables are contained in Table 1.

To assess the robustness of the results, two different data sets have been employed for each commodity. The first set contains more data (weeks) but fewer futures contracts, while the second set contains fewer weeks but more futures contracts.

In the case of WTI crude oil, the first data set is comprised of contracts F1, F3, F5, F7 and F9 from 1/1/1985 to 8/16/2010, yielding 180 quotations for each contract. F1 is the contract for the month closest to maturity, F2 is the contract for the second-closest month to maturity, and so on. The second data set for WTI crude oil is comprised of contracts F1, F4, F7, F11, F15 and F18 from 9/9/1996 to 8/16/2010, yielding 82 quotations for each contract.

In the case of heating oil, the first data set is comprised of contracts F1, F3, F6, F8 and F10 from 10/14/1985 to 8/16/2010, yielding 177 quotations for each contract. The second data set for heating oil is comprised of contracts F1, F4, F8, F11, F15 and F18 from 9/9/1996 to 8/16/2010, yielding 82 quotations for each contract.

In the case of RBOB gasoline, the first data set is comprised of contracts F1, F3, F4, F5 and F7 from 4/29/1985 to 8/16/2010, yielding 181 quotations for each contract.

Table 1 Descriptive statistics

	WTI crude oil		Heating oil		RBOB gasoline		Henry Hub natural gas	
	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.
F1	33.39	23.56	38.91	27.65	39.76	26.01	4.04	2.60
F2	33.4	23.79	38.94	27.94	39.53	26.01	4.13	2.66
F3	33.37	23.97	38.98	28.22	39.34	25.97	4.19	2.71
F4	33.32	24.11	38.99	28.46	39.16	25.91	4.22	2.73
F5	33.26	24.23	38.97	28.65	39.03	25.93	4.25	2.75
F6	33.2	24.33	38.94	28.81	38.92	25.97	4.27	2.76
F7	33.14	24.41	39	29.06	38.95	26.17	4.29	2.77
F8	33.08	24.48	38.98	29.17	39.56	26.71	4.3	2.78
F9	33.03	24.54	38.94	29.21	40.52	27.28	4.3	2.78
F10	33.15	24.67	38.94	29.27			4.29	2.76
F11	33.6	24.96	39.39	29.59			4.29	2.74
F12	34.24	25.27	40.44	30.17			4.33	2.73
F13	35.1	25.63	42.17	30.92			4.53	2.73
F14	35.35	25.75	42.78	31.25			4.52	2.73
F15	35.6	25.96	43.79	31.77			4.52	2.73
F16	35.7	26.09	44.62	32.29			4.52	2.73
F17	35.8	26.17	46.5	33.03			4.53	2.73
F18	36.2	26.43	49.27	33.84			4.56	2.73

The table shows the mean and standard deviation (S.D.) of the four commodity series prices. F1 is the futures contract closest to maturity, F2 is the contract second-closest to maturity and so on. In the cases of WTI crude oil and heating oil the data set is comprised of futures prices from 1 to 18 months (1,338 weekly observations) from 1/1/1985 to 8/16/2010. In the case of RBOB gasoline, the data set is comprised of futures prices from 1 to 9 months (1,338 weekly observations) from 1/1/1985 to 8/16/2010. In the case of Henry Hub natural gas, the data set is comprised of futures prices from 1 to 18 months (1,064 weekly observations) from 4/2/1990 to 8/16/2010

The second data set for heating oil is comprised of contracts F1, F3, F5, F7 and F9 from 7/17/1995 to 8/16/2010, yielding 92 quotations for each contract.

Finally, in the case of Henry Hub natural gas, the first data set is comprised of contracts F1, F4, F6, F9 and F11 from 4/16/1990 to 8/16/2010, yielding 135 quotations for each contract. The second data set for Henry Hub natural gas prices is comprised of contracts F1, F4, F8, F12, F15, F18, F22, F26, F29, F31 and F35 from 5/28/1997 to 8/16/2010, yielding 76 quotations for each contract.

3 A factor model with time-varying market prices of risk

In this section, a factor model with time-varying market prices of risk depending on the model factors is proposed and estimated. The model will be an extension of the Schwartz and Smith (2000) two-factor model, which is one of the most popular models in the literature to capture the stochastic behavior of commodity prices.

In the Schwartz and Smith (2000) model, the log-spot price (X_t) is assumed to be the sum of two stochastic factors, a short-term deviation (χ_t) and a long-term equilibrium price level (ξ_t).[1] Thus,

$$X_t = \xi_t + \chi_t \tag{1}$$

The stochastic differential equations (SDEs) for these factors are as follows:

$$\begin{cases} d\xi_t = \mu_\xi dt + \sigma_\xi dW_{\xi t} \\ d\chi_t = -\kappa \chi_t dt + \sigma_\chi dW_{\chi t} \end{cases} \tag{2}$$

where $dW_{\xi t}$ and $dW_{\chi t}$ can be correlated ($dW_{\xi t} dW_{\chi t} = \rho_{\xi\chi} dt$) and with $\rho_{\xi\chi}$ representing the coefficient of correlation between long- and short-term factors.

To value derivative contracts, we must rely on the "risk-neutral" version of the model. The SDEs for the factors under the equivalent martingale measure can be expressed as:

$$\begin{cases} d\xi_t = (\mu_\xi - \lambda_\xi)dt + \sigma_\xi dW^*_{\xi t} \\ d\chi_t = (-\kappa \chi_t - \lambda_\chi)dt + \sigma_\chi dW^*_{\chi t} \end{cases} \tag{3}$$

where λ_ξ and λ_χ are the market prices of risk for the long- and short-term factors, respectively, and $W^*_{\xi t}$ and $W^*_{\chi t}$ are the factor Brownian motions under the equivalent martingale measure.

Moreover, in the cases of commodities, such as natural gas, heating oil and gasoline, a deterministic seasonal component is added, as suggested by Sorensen (2002).[2] Therefore, the log spot price for heating oil, gasoline and natural gas (X_t) is assumed to be the sum of two stochastic factors (χ_t and ξ_t) and a deterministic seasonal trigonometric component (α_t), $X_t = \xi_t + \chi_t + \alpha_t$. The SDEs for ξ_t and χ_t are given by expressions (2) and:

$$\begin{aligned} d\alpha_t &= 2\pi\varphi\alpha^*_t \, dt \\ d\alpha^*_t &= -2\pi\varphi\alpha_t \, dt \end{aligned} \tag{4}$$

where α^*_t is the other seasonal factor, which complements α_t, and φ is the seasonal period.

The model with time-varying market prices of risk will be an extension of the two-factor model described above, where the log spot price for heating oil, gasoline and natural gas (X_t) is assumed to be the sum of two stochastic factors (χ_t and ξ_t) and a deterministic seasonal trigonometric component (α_t), $X_t = \xi_t + \chi_t + \alpha_t$ ($X_t = \xi_t + \chi_t$ for crude oil). The SDEs for the long- and short- term factors under the equivalent martingale measure, with time-varying market prices of risk, can be expressed as:

[1] In this model large moves affecting the long-term run are captured through the long-term factor, whereas small, short-term deviations from the long-term run are captured through the short-term factor.

[2] Sorensen (2002) suggests introducing into the model a deterministic seasonal component for agricultural commodities. Here, we use Sorensen's proposal for heating oil, gasoline and natural gas, which present a strong seasonal behavior (see, for example, Mirantes et al. 2012a).

$$\begin{cases} d\xi_t = (\mu_\xi - \lambda_{\xi_t})dt + \sigma_\xi dW^*_{\xi t} \\ d\chi_t = (-\kappa\chi_t - \lambda_{\chi_t})dt + \sigma_\chi dW^*_{\chi t} \end{cases} \tag{5}$$

where, as before, $W^*_{\xi t}$ and $W^*_{\chi t}$ are the factor Brownian motions under the equivalent martingale measure, and $\lambda_{\xi t}$ and $\lambda_{\chi t}$ are time-varying market prices of risk for the long- and short-term factors, respectively.[3,4]

Following Duffee (2002), Dai and Singleton (2002) and Casassus and Collin-Dufresne (2005), the market prices of risk are expressed as linear functions of the underlying long- and short-term factors:

$$\begin{aligned} \lambda_{\xi t} &= \lambda_{\xi 0} + \lambda_{\xi 1} \cdot \xi_t + \lambda_{\xi 2} \cdot \chi_t \\ \lambda_{\chi t} &= \lambda_{\chi 0} + \lambda_{\chi 1} \cdot \xi_t + \lambda_{\chi 2} \cdot \chi_t \end{aligned} \tag{6}$$

As stated in previous studies, one of the main difficulties in estimating the parameters of the two-factor model is that the short- and long-term factors (or state variables) are not directly observable. Instead, they must be estimated from spot and/or futures prices.[5]

The formal method to estimate the model is to use the Kalman filter methodology, which is briefly described in the "Appendix". The results of the estimation of this factor model with time-varying market prices of risk, together with the results of the standard two-factor Schwartz and Smith (2000) model with constant market prices of risk for the four commodity series using both the first and the second data sets described in Sect. 2 are shown in Table 2 (WTI crude oil), Table 3 (heating oil), Table 4 (RBOB gasoline) and Table 5 (Henry Hub natural gas).

The results in Tables 2, 3, 4 and 5 confirm what we can find in other papers like Schwartz (1997) or Schwartz-Smith (2000), that is, the presence of the mean reversion effect, typically observed in commodity markets (parameter κ is significant in all cases). Moreover, as expected, both long- and short-term factors are found to be stochastic (their corresponding standard deviations, σ_ξ and σ_χ, respectively, are significant), although the short-term standard deviation is found to be higher than the corresponding long-term standard deviation, suggesting that short-term effects have

[3] As shown in Schwartz and Smith (2000), their short-term/long-term model is equivalent to the stochastic convenience yield model by Gibson and Schwartz (1990), in which the convenience yield is assumed to follow an Ornstein–Uhlenbeck process. Therefore, although not explicitly considered in our model with time-varying market prices of risk, convenience yields are assumed to follow a mean-reverting process. Recently, Bakshi et al. (2013) point out that a third factor, the commodity momentum, is needed to describe the cross-sectional and time-series variation of commodity returns.

[4] Here we assume homoskedasticity in the error terms. Trolle and Schwartz (2009) present a model allowing for stochastic volatility for crude oil prices, using daily data. More recently, Christoffersen et al. (2013) present a discrete-time GARCH-type model allowing for both time-varying volatility and jumps. However, in this paper we have confined ourselves to the constant volatility case with no jumps for several reasons. Firstly, here we are using weekly data. Secondly, a stochastic volatility model with jumps is probably more realistic, but also more complex so much the Kalman filter formulae cannot be computed explicitly in an exact way and it is necessary the use of approximations, such as the extended Kalman filter, whereas all the formulae in this article are exact.

[5] The exact expression for the futures price under the Schwartz and Smith (2000) two-factor model with seasonal factors can be found in Mirantes et al. (2012a).

Table 2 Estimation results of the factor models with time-varying and constant market prices of risk WTI crude oil

	First data set		Second data set	
	WTI Constant MPR	WTI Variable MPR	WTI Constant MPR	WTI Variable MPR
μ_ξ	0.0452*	0.0404	0.1128***	0.0868**
	(0.0270)	(0.0268)	(0.0334)	(0.0410)
κ	1.9748***	1.1859***	1.1254***	1.3257***
	(0.0234)	(0.2318)	(0.0103)	(0.2478)
σ_ξ	0.1936***	0.1919***	0.1761***	0.2160***
	(0.0030)	(0.0170)	(0.0037)	(0.0283)
σ_χ	0.2467***	0.1799***	0.2763***	0.1393***
	(0.0043)	(0.0266)	(0.0065)	(0.0384)
$\lambda_{\xi 0}$	0.0907***	0.3821*	0.1669***	0.4945***
	(0.0271)	(0.1991)	(0.0334)	(0.1505)
$\lambda_{\xi 1}$	–	−0.0856	–	−0.1009***
		(0.0569)		(0.0387)
$\lambda_{\xi 2}$	–	0.8030*	–	1.2291*
		(0.4184)		(0.6864)
λ	0.0453	1.1308***	−0.0333	0.3670**
	(0.0346)	(0.3736)	(0.0524)	(0.1758)
$\lambda_{\chi 1}$	–	−0.3251***	–	−0.1022**
		(0.1084)		(0.0473)
$\lambda_{\chi 2}$	–	0.8121***	–	−0.1048
		(0.2647)		(0.2702)
$\rho_{\xi\chi}$	0.1494***	0.5775***	0.0445	0.7349***
	(0.0239)	(0.0949)	(0.0320)	(0.0791)
σ_η	0.0079***	0.0078***	0.0093***	0.0093***
	(0.0001)	(0.0001)	(0.0001)	(0.0001)
Log-L	50,007.70	50,160.36	32,146.72	32,163.08
AIC	49,991.70	50,136.36	32,130.72	32,139.08
SIC	49,950.11	50,073.97	32,093.99	32,084.00

The table shows the estimation results of the model with time-varying market prices of risk (MPR), depending on the model long- and short-term factors, together with those obtained with the standard Schwartz and Smith (2000) two-factor model with constant market prices of risk. The table shows the results obtained with both the first and the second data sets described in Sect. 2. Standard errors are in parentheses. The estimated values are reported with * denoting significance at 10 %, ** denoting significance at 5 %, and *** denoting significance at 1 %

a higher impact on spot prices than long-term effects.[6] However, as explained above, it must be kept in mind that short-term effects tend to disappear with time (the short-

[6] This fact is also found in Schwartz and Smith (2000) and Mirantes et al. (2012b), among others.

Table 3 Estimation results of the factor models with time-varying and constant market prices of risk heating oil

	First data set		Second data set	
	Heating oil Constant MPR	Heating oil Variable MPR	Heating oil Constant MPR	Heating oil Variable MPR
μ_ξ	0.1659***	0.3994***	0.2248***	0.0635
	(0.0310)	(0.1237)	(0.0327)	(0.0283)
κ	1.8522***	1.1312***	1.7080***	0.4026***
	(0.0911)	(0.0205)	(0.0158)	(0.0000)
σ_ξ	0.1770***	0.3691***	0.1696***	0.1905***
	(0.0035)	(0.0031)	(0.0037)	(0.0000)
σ_χ	0.3343***	0.6105***	0.2441***	0.3758***
	(0.0158)	(0.0206)	(0.0072)	(0.0000)
φ	0.9976***	0.9974***	0.9972***	0.9974***
	(0.0000)	(0.0000)	(0.0002)	(0.0002)
$\lambda_{\xi 0}$	0.2239***	0.3642	0.2322***	0.2883
	(0.0327)	(0.2214)	(0.0329)	(0.0000)
$\lambda_{\xi 1}$	–	−0.2846***	–	0.0216***
		(0.0000)		(0.0000)
λ_2	–	−0.8459***	–	−0.5506***
		(0.0225)		(0.0198)
$\lambda_{\chi 0}$	0.6999***	0.5173**	0.3546***	−0.9568***
	(0.1079)	(0.2067)	(0.0496)	(0.0000)
$\lambda_{\chi 1}$	–	0.5407***	–	0.0462***
		(0.0198)		(0.0000)
$\lambda_{\chi 2}$	–	0.6409***	–	1.1536***
		(0.0378)		(0.0000)
$\rho_{\xi\chi}$	−0.1229***	−0.5889***	0.4488***	−0.4248***
	(0.0431)	(0.0340)	(0.0307)	(0.0000)
σ_η	0.0209***	0.0143***	0.0190***	0.0187***
	(0.0003)	(0.0001)	(0.0001)	(0.0001)
Log-L	47,709.23	51,511.79	42,417.57	42,573.40
AIC	47,691.23	51,485.79	42,399.57	42,547.40
SIC	47,644.72	51,418.61	42,358.26	42,487.73

The table shows the estimation results of the model with time-varying market prices of risk (MPR), depending on the model long- and short-term factors, together with those obtained with the standard Schwartz and Smith (2000) two-factor model with constant market prices of risk. The table shows the results obtained with both the first and the second data sets described in Sect. 2. Standard errors are in parentheses. The estimated values are reported with * denoting significance at 10 %, ** denoting significance at 5 %, and *** denoting significance at 1 %

term process is stationary), whereas long-term effects do not disappear with time (the long-term process is integrated).

However, the most important issue in Tables 2, 3, 4 and 5 from the point of view of the goal of this paper, is that the parameters associated with the market prices of

Table 4 Estimation results of the factor models with time-varying and constant market prices of risk rbob gasoline

	First data set		Second data set	
	RBOB Constant MPR	RBOB Variable MPR	RBOB Constant MPR	RBOB Variable MPR
μ_ξ	−0.4000***	0.2855***	0.2621***	0.0909
	(0000)	(0.0173)	(0.0315)	(0.0200)
κ	3.1144***	0.4002***	2.0500***	2.1285***
	(0.0916)	(0.0206)	(0.0558)	(0.1590)
σ_ξ	0.2093***	0.3023***	0.1877***	0.2458***
	(0.0034)	(0.0000)	(0.0045)	(0.0000)
σ_χ	0.3770***	0.3212***	0.3084***	0.5067***
	(0.0088)	(0.0000)	(0.0084)	(0.0000)
φ	0.9947***	0.9940***	1.0028***	1.0002***
	(0.0001)	(0.0000)	(0.0004)	(0.0003)
$\lambda_{\xi 0}$	−0.3919***	0.6893***	0.3439***	0.0298
	(0.0041)	(0.0000)	(0.0323)	(0.1130)
$\lambda_{\xi 1}$	–	0.0576***	–	−0.0045
		(0.0140)		(0.0396)
$\lambda_{\xi 2}$	–	0.3876***	–	−0.8974***
		(0.0000)		(0.0000)
$\lambda_{\chi 0}$	−0.3849***	0.6412*	0.3791***	0.9855***
	(0.0288)	(0.0000)	(0.0548)	(0.2980)
$\lambda_{\chi 1}$	–	0.4053***	–	−0.1642*
		(0.0000)		(0.0901)
$\lambda_{\chi 2}$	–	1.1309***	–	−0.0410
		(0.0482)		(0.1778)
$\rho_{\xi\chi}$	0.0764**	−0.2500***	0.1072***	−0.7064***
	(0.0322)	(0.0000)	(0.0404)	(0.0000)
σ_η	0.0162***	0.0151***	0.0162***	0.0159***
	(0.0001)	(0.0001)	(0.0002)	(0.0002)
Log-L	42,227.28	42,409.56	25,273.54	25,374.57
AIC	42,209.28	42,383.56	25,255.54	25,348.57
SIC	42,162.60	42,316.14	25,213.51	25,287.86

The table shows the estimation results of the model with time-varying market prices of risk (MPR), depending on the model long- and short-term factors, together with those obtained with the standard Schwartz and Smith (2000) two-factor model with constant market prices of risk. The table shows the results obtained with both the first and the second data sets described in Sect. 2. Standard errors are in parentheses. The estimated values are reported with * denoting significance at 10 %, ** denoting significance at 5 %, and *** denoting significance at 1 %

risk ($\lambda_{\xi 0}$, $\lambda_{\xi 1}$, $\lambda_{\xi 2}$, $\lambda_{\chi 0}$, $\lambda_{\chi 1}$ and $\lambda_{\chi 2}$) are significant in most of the cases, confirming that market prices of risk vary through time depending on the economic conditions (proxied in this paper by the model long- and short-term factors).

Table 5 Estimation results of the factor models with time-varying and constant market prices of risk Henry Hub natural gas

	First data set		Second data set	
	Henry Hub Constant MPR	Henry Hub Variable MPR	Henry Hub Constant MPR	Henry Hub Variable MPR
μ_ξ	−0.3996***	−0.2996***	0.0719***	0.0284***
	(0.0307)	(0.0621)	(0.0252)	(0.0000)
κ	1.8158***	0.8416***	1.1163***	1.0458***
	(0.0000)	(0.0607)	(0.0138)	(0.0000)
σ_ξ	0.2515***	0.2334***	0.1297***	0.3325***
	(0.0000)	(0.0295)	(0.0040)	(0.0000)
σ_χ	0.5547***	0.5475***	0.4779***	0.1714***
	(0.0087)	(0.0285)	(0.0155)	(0.0000)
φ	0.9957***	0.9997***	0.9999***	0.9992***
	(0.0001)	(0.0002)	(0.0001)	(0.0001)
$\lambda_{\xi 0}$	−0.1397***	0.4892***	0.1236***	−0.0587***
	(0.0488)	(0.0284)	(0.0253)	(0.0000)
$\lambda_{\xi 1}$	–	−0.3987***	–	−0.0029
		(0.0539)		(0.0506)
$\lambda_{\xi 2}$	–	−0.3539***	–	1.9929***
		(0.0768)		(0.0000)
$\lambda_{\chi 0}$	0.0008	0.4166*	−0.2177**	−0.0252***
	(0.0994)	(0.1922)	(0.0928)	(0.0000)
$\lambda_{\chi 1}$	–	0.1618	–	−0.0358
		(0.1466)		(0.0212)
$\lambda_{\chi 2}$	–	0.8608***	–	0.0452***
		(0.0000)		(0.0000)
$\rho_{\xi\chi}$	−0.5678***	−0.7205***	−0.0222	0.9166***
	(0.0000)	(0.0677)	(0.0471)	(0.0000)
σ_η	0.0916***	0.0914***	0.0399***	0.0383***
	(0.0006)	(0.0006)	(0.0002)	(0.0002)
Log-L	22,625.01	22,758.45	39,438.12	40,032.74
AIC	22,607.01	22,732.45	39,420.12	40,006.74
SIC	22,562.30	22,667.87	39,379.28	39,947.75

The table shows the estimation results of the model with time-varying market prices of risk (MPR), depending on the model long- and short-term factors, together with those obtained with the standard Schwartz and Smith (2000) two-factor model with constant market prices of risk. The table shows the results obtained with both the first and the second data sets described in Sect. 2. Standard errors are in parentheses. The estimated values are reported with * denoting significance at 10 %, ** denoting significance at 5 %, and *** denoting significance at 1 %

If we define the Schwartz information criterion (SIC) as $\ln(L_{ML}) - q\ln(T)$, where q is the number of estimated parameters, T is the number of observations and L_{ML} is the value of the likelihood function using the q estimated parameters, then the fit

Table 6 Likelihood ratio tests

Statistic	WTI First data set	WTI Second data set	Heating oil First data set	Heating oil Second data set
Log-L Const. MPR	50,007.70	32,146.72	47,709.23	42,417.57
Log-L Var. MPR	50,160.36	32,163.08	51,511.79	42,573.40
LR (*p* value)	305.32 (0.00)	32.72 (0.00)	7,605.12 (0.00)	311.66 (0.00)
Statistic	RBOB First data set	RBOB Second data set	Henry Hub First data set	Henry Hub Second data set
Log-L Const. MPR	42,227.28	25,273.54	22,625.01	39,438.12
Log-L Var. MPR	42,409.56	25,374.57	22,758.45	40,032.74
LR (*p* value)	364.56 (0.00)	202.06 (0.00)	266.88 (0.00)	1189.24 (0.00)

The Table shows the values of the maximized log-likelihood function (Log-L) for the models with constant and time-varying market prices of risk (MPR), the likelihood ratio (LR) and asymptotic *p* values for the four commodities under study and for the two data sets employed in the estimation procedure

is better when the SIC is higher. The same conclusions are obtained with the Akaike information criterion (AIC), which is defined as $\ln(L_{ML}) - 2q$. It is worth noting that in Tables 2, 3, 4 and 5, the values of the SIC and the AIC are higher in the model with time-varying market prices of risk. This finding confirms the results obtained by Casassus and Collin-Dufresne (2005), in that allowing for time-varying market prices of risk improves the estimation results.[7]

A similar way of comparing the models is to compute a likelihood ratio test. The model with time-varying market prices of risk nests the constant market prices of risk one when $\lambda_{\xi 1} = \lambda_{\xi 2} = \lambda_{\chi 1} = \lambda_{\chi 2} = 0$ in Eq. (6). Therefore, the restrictions imposed by the constant market prices of risk model can be tested by means of a likelihood ratio test. The results are contained in Table 6. The value of the LR statistic is quite large in all cases, indicating the rejection of the null hypothesis that the true model is the restricted one, i.e., the constant market prices of risk one.

In the next section, we use these results for commodity option valuation purposes. Specifically, we show the importance of allowing for time-varying market prices of risk in valuing a set of market traded commodity options. As stated in the Introduction, there have been several papers that have estimated factor models allowing for time-varying market prices of risk (Casassus and Collin-Dufresne 2005; Trolle and Schwartz 2009; Bhar and Lee 2011; 2013). However, they do not analyze the effect of time-varying market prices of risk, compared to the constant case, from the point of view of the valuation of exchange-traded options.

4 Option valuation with time-varying market prices of risk

As stated above, in this section, we apply our model with time-varying market prices of risk to the valuation of an extensive set of commodity market traded options.

[7] It should be noted that in the paper by Casassus and Collin-Dufresne (2005) the estimation is carried out using the the maximum likelihood method, whereas in the present paper we use the Kalman filter method.

4.1 Option data

The data set used in the estimation procedure consists of daily observations of WTI, heating oil, RBOB gasoline and Henry Hub natural gas American call and put options quoted at the NYMEX and corresponding to the years from 2006 until 2010. The number of series is 1,293 call and 2,153 put (223,272 and 118,316 observations, respectively) in the case of WTI crude oil; 1,567 call and 302 put (177,927 and 45,725 observations, respectively) in the case of heating oil; 1,633 call and 938 put (145,354 and 59,576 observations, respectively) in the case of RBOB gasoline; and 681 call and 758 put (79,957 and 99,828 observations, respectively) in the case of Henry Hub natural gas.

In the NYMEX, WTI option contracts mature each month for the current year and for the next 5 years. Additionally, the June and December months are listed beyond the sixth year. Strike prices are the one at-the-money strike price, twenty strike prices in increments of \$0.50 per barrel above and below the at-the-money strike price, and the next 10 strike prices in increments of \$2.50 above the highest and below the lowest existing strike prices for a total of at least 61 strike prices.

In the case of heating oil and RBOB gasoline options, there are listed contracts for the next 36 consecutive months, and available strike prices are the at-the-money, twenty strike prices in \$0.01 per gallon increments above and below the at-the-money strike price, and the next 10 strike prices in \$0.05 increments above the highest and below the lowest existing strike prices for a total of at least 61 strike prices.

Finally, in the case of Henry Hub natural gas options, there are listed contracts for the consecutive months for the balance of the current year plus 5 additional years. Strike prices are the one at-the-money strike prices, twenty strike prices in increments of \$0.05 per mmBtu above and below the at-the-money strike price in all months, plus an additional 20 strike prices in increments of \$0.05 per mmBtu above the at-the-money price will be offered in the first three nearby months, and the next 10 strike prices in increments of \$0.25 per mmBtu above the highest and below the lowest existing strike prices in all months, for a total of at least 81 strike prices in the first three nearby months and a total of at least 61 strike prices for 4 months and beyond.[8]

In all cases, the underlying asset is the corresponding WTI, heating oil, RBOB gasoline or Henry Hub natural gas futures contract.

4.2 Option valuation methodology

The computation of American option prices is a challenging problem which implies solving an optimal stopping problem. The problem can be simplified employing Monte Carlo techniques. The starting point of these methods is to replace the time interval of exercise dates by a finite subset. The solution of the corresponding discrete optimal stopping problem reduces to an effective implementation of the dynamic programming principle. However, the conditional expectations involved in the iterations of the dynamic programming cause the main difficulty for the development of the

[8] Additional details about the contracts can be found on the CME Group web page.

Monte Carlo techniques. One way of treating this problem is the method presented in Longstaff and Schwartz (2001), which is one of the most popular American option valuation methods and will be the method used in this section to value commodity American options.

Specifically, the method proposed by Longstaff and Schwartz (2001) consists of estimating the conditional expected pay-off to the holder of the option from continuation using least squares regression techniques.

For the purpose of option valuation, we need a full description of the model. In matrix form, the state dynamics can be described as follows:

$$dZ_t = (\mu + AZ_t)\,dt + dW_t. \tag{7}$$

To clarify, let us take U_t to be a unit of Brownian motion (i.e., $dU_t dU_t^T = Idt$) and rewrite (7) as:

$$dZ_t = (\mu + AZ_t)\,dt + RdU_t. \tag{8}$$

For parameter estimation purposes, we use Kalman filter equations to estimate $Z_{t|t-1} = E\left[Z_t/Z_1, \ldots, Z_{t-1}\right]$, and as an intermediate result, $Z_{t-1|t-1} = E\left[Z_{t-1}/Z_1, \ldots, Z_{t-1}\right]$. This process (estimating using current or even future information) is termed "aliasing" in the Kalman filter literature. The series $Z_{t|t}$ is used as initial states for option valuation.

4.3 Option valuation results

Table 7 presents several metrics to analyze the predictive power ability of the models for the data set of WTI, heating oil, RBOB gasoline and Henry Hub natural gas American options. The models considered are the time-varying market prices of risk and the standard constant (two-factor) market prices of risk. Moreover, the results shown in the table are based on the estimation results obtained from both the first and the second data sets described in Sect. 2.

The statistics presented in Table 7 are the root mean squared error (RMSE), the percentage root mean squared error (PRMSE) and the mean absolute error (MAE), which are defined as:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(f_{i,m} - f_{i,t}\right)^2}$$

$$PRMSE = \frac{\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(f_{i,m} - f_{i,t}\right)^2}}{\sqrt{\frac{1}{n}\sum_{i=1}^{n} f_{i,m}^2}}$$

$$MAE = \frac{1}{n}\sum_{i=1}^{n}\left|f_{i,m} - f_{i,t}\right|$$

Table 7 American option valuation results error descriptive statistics

	Constant MPR			Time-varying MPR		
	RMSE	PRMSE	MAE	RMSE	PRMSE	MAE
Panel A: WTI American options						
First data set	0.9727	27.4746	0.6853	0.8974	25.20429	0.6416
Second data set	0.9675	28.49709	0.6835	0.9313	26.39755	0.6672
Panel B: Heating oil American options						
First data set	3.1407	43.2489	2.7943	1.3379	14.0521	1.0127
Second data set	1.3484	16.3243	1.1052	1.3807	17.0274	1.0706
Panel C: RBOB gasoline American options						
First data set	5.5027	164.1529	4.6894	1.4065	39.7270	1.0744
Second data set	1.4952	37.8901	1.1821	0.9294	28.1595	0.7342
Panel D: Henry Hub natural gas American options						
First data set	0.1192	64.2655	0.0913	0.1124	54.8146	0.0878
Second data set	0.1055	71.8211	0.0846	0.0864	14.0521	0.0678

The table presents several metrics, root mean squared error (RMSE), percentage root mean squared error (PRMSE) and mean absolute error (MAE), to analyze the predictive power ability of the models under study: the time-varying market prices of risk (MPR) model and the standard (two-factor) model with constant market prices of risk. The data set is comprised of daily observations of WTI American call and put options quoted at NYMEX during the years 2006–2010. For each series, we have calculated the corresponding statistic. These results correspond to the median value of these multiple means. The total number of observations is 341,588, 223,652, 204,930 and 179,785 for WTI crude oil, heating oil, RBOB gasoline and Henry Hub natural gas respectively

where $f_{i,m}$ and $f_{i,t}$ are the market and the theoretical prices, respectively, of option i.

The values shown in the table are the median of the different means for each option series. It can be observed that we achieve better results with the time-varying market prices of risk model for all commodities under study with all three statistics (except in the case of heating oil using the RMSE with the second data set). It is also worth noting that, in general, we achieve better results using the first data set (at least in the case of WTI crude oil, heating oil and RBOB gasoline). Furthermore, it can be appreciated that the best results of the time-varying model are achieved with RBOB gasoline, followed by heating oil.

These results confirm that the constant market price of risk assumption in standard option valuation models has an important effect in terms of valuation errors. Therefore, the fact that market prices of risk vary over time according to the market conditions (proxied by the model long- and short-term factors) must be taken into account in option valuation models. In this paper, we have seen that, in fact, by allowing for time-varying (state-depending) market prices of risk option valuation, errors can be reduced compared to those obtained with standard (constant market prices of risk) models.

Finally, as stated above, there have been several papers that have estimated factor models allowing for time-varying market prices of risk (Casassus and Collin-Dufresne 2005; Trolle and Schwartz 2009; Bhar and Lee 2011; Casassus et al. 2013). However, these papers do not test the importance of time-varying market prices o f risk on the valuation of exchange-traded options, compared to the constant market prices of risk case.

5 Conclusions

In this paper, we note the importance of allowing for time-varying market prices of risk in a commodity derivative model.

Based on previous research showing that market prices of risk vary over time, a factor model with market prices of risk depending on the market conditions and proxied by long- and short-term price factors is proposed and estimated. The valuation results obtained with a sample of futures contracts on crude oil, heating oil, gasoline and natural gas show that the proposed model with time-varying market prices of risk depending on the model factors outperforms the standard two-factor model with constant market prices of risk. This finding confirms the results obtained by Casassus and Collin-Dufresne (2005) or Casassus et al. (2013) among others, in that allowing for time-varying market prices of risk improves the estimation results.

However, the most important contribution of this paper is the application of the model with time-varying market prices of risk to the valuation of an extensive sample of exchange-traded commodity derivatives, and the analysis of the importance of allowing for time-varying market prices of risk, compared to the constant case, from the exchange-traded option valuation point of view. Specifically, the data base is comprised of American options on WTI, heating oil, RBOB gasoline and Henry Hub natural gas futures contracts, traded at NYMEX. The results indicate that by allowing for time-varying (state-depending) market prices of risk option valuation, errors can be reduced compared to those obtained with standard (constant market prices of risk) models. Consequently, it is important to take into account the dependence of market prices of risk on the economic conditions in valuing derivative contracts.

Appendix: Kalman filtering[9]

Let $Z_t = (\xi_t \chi_t \alpha_t \alpha_t^*)'$ be the vector of all factors.[10] The "risk-neutral" SDE of Z_t can be expressed as $dZ_t = \left(b^\diamond + AZ_t\right) dt + \Omega dW_{Zt}^\diamond$, where $dW_{Zt}^\diamond$ is a vector of independent Brownian motions, and therefore $\text{Var}(dZ_t) = R = \Omega\Omega^T$ (Ω^T is the transpose matrix of Ω), with the restriction explained above: $b^\diamond = (\,\mu_t - \lambda_{\xi 0} \; -\lambda_{\chi 0} \; 0 \; 0\,)$ and:

[9] Detailed accounts for Kalman filtering are given in Harvey (1989) and also in Bakshi and Wu (2010) among others.

[10] It is worth noting that α_t and α_t^* are deterministic factors and therefore their volatility is zero. Moreover, $\alpha_t = \alpha_t^* = 0$ in the case of crude oil.

$$A = \begin{pmatrix} -\lambda_{\xi_1} & -\lambda_{\xi_2} & 0 & 0 \\ -\lambda_{\chi 1} & -\kappa - \lambda_{\chi 2} & 0 & 0 \\ 0 & 0 & 0 & \varphi \\ 0 & 0 & -\varphi & 0 \end{pmatrix}$$

Under this notation $X_t = cZ_t$, where $c = (\,1\ 1\ 1\ 0\,)$.

It is easy to prove that the (unique) solution of that problem is (Oksendal 1992):

$$Z_t = e^{At}\left[Z_0 + \int_0^t e^{-As} b^{\diamond} ds + \int_0^t e^{-As} \Omega dW_{Zs}^{\diamond}\right] \tag{9}$$

It is clear that, under the risk-neutral measure, given Z_0, Z_t is Gaussian, with mean and variance[11]

$$E^*[Z_t] = e^{At}\left[Z_0 + \int_0^t e^{-As} b^{\diamond} ds\right] \tag{10}$$

$$Var^*[Z_t] = e^{At}\left[\int_0^t e^{-As} R(e^{-As})^T ds\right](e^{At})^T \tag{11}$$

As $X_t = cZ_t = \xi_t + \chi_t + \alpha_t$, then under the risk-neutral measure, X_t is also Gaussian with mean and variance:

$$E^*(X_t) = cE^*(Z_t)$$
$$Var^*(X_t) = cVar^*(Z_t)c^T$$

This provides a valuation scheme for all sorts of commodity contingent claims as financial derivatives on commodity prices, real options, investment decisions and other more. In particular, the price of a futures contract traded at time "t" with maturity at time "$t+T$" is: $F_{t,T} = E^*\left[S_{t+T} \,\|\, I_t\right] = \exp\left\{E^*\left[X_{t+T} \,\|\, I_t\right] + \frac{1}{2}Var^*\left[X_{t+T} \,\|\, I_t\right]\right\}$, where I_t is the information available at time "t". It can be expressed as:

$$F_{t,T} = \exp\left[ce^{AT} Z_t + g(T)\right] \tag{12}$$

where $g(T) = ce^{AT} \int_t^{t+T} e^{-As} b^{\diamond} ds + \frac{1}{2} ce^{AT}\left[\int_t^{t+T} e^{-As} R(e^{-AT})^T ds\right](e^{AT})^T c^T$, which is a deterministic function.

The Kalman filter technique is a recursive methodology that estimates the unobservable time series and the state variables or factors (Z_t) based on an observable time series (Y_t), which depends on these state variables.

If the difference between the current period and the initial period is one period time, Z_t follows the discrete process:

$$Z_t = c_t + TZ_{t-1} + \psi_t \qquad t = 1, \ldots, N_t \tag{13}$$

[11] E*[] and Var*[] are the mean and variance under the risk neutral measure.

where $c_t = e^{At} \int_{t-1}^{t} e^{-As} bds \in \Re^h, T = e^A \in \Re^{hxh} and \psi_t \in \Re^h$ is a vector of serially uncorrelated Gaussian disturbances with zero mean and covariance matrix $Q = (e^A)\left[\int_{t-1}^{t} e^{-As} R(e^{-As})^T ds\right](e^A)^T$. This equation will be called, following standard conventions in the literature, the *transition equation*. It is worth noting that expression (13) can be derived from expression (9) and thus c_t, T and Q can be computed from expression (9).

The measurement equation is just the expression of the log-futures prices (Y_t) in terms of the factors (Z_t) by adding serially uncorrelated disturbances with zero mean (η_t) to take into account measurement errors derived from bid-ask spreads, price limits, non-simultaneity of observations, errors in data, etc. To avoid dealing with a great amount of parameters, the covariance matrix H_t will be assumed diagonal with main diagonal entries equal to σ_η. This simple structure for the measurement errors is imposed so that the serial correlation and cross correlation in the log-prices is attributed to the variation of the unobservable state variables. The measurement equation (which can be derived from expression (12))will be expressed as:

$$Y_t = d_t + M_t Z_t + \eta_t \qquad t = 1, \ldots, N_t \tag{14}$$

where $Y_t, d_t \in \Re^n$, $M_t \in \Re^{nxh}$, $Z_t \in \Re^h$, h is the number of state variables, or factors, in the model, and $\eta_t \in \Re^n$ is a vector of serially uncorrelated Gaussian disturbances with zero mean and covariance matrix H_t.

Let $Y_{t|t-1}$ be the conditional expectation of Y_t and let Ξ_t be the covariance matrix of Y_t conditional on all information available at time $t-1$. Then, after omitting unessential constants, the log-likelihood function can be expressed as:

$$l = -\sum_t \ln |\Xi_t| - \sum_t (Y_t - Y_{t|t-1})' \Xi_t^{-1} (Y_t - Y_{t|t-1}) \tag{15}$$

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