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Mathematics-writing profiles for students with mathematics difficulty

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Abstract
Increasingly, students must demonstrate knowledge in mathematics through mathematics writing, yet research lags in understanding how students engage in mathematics-writing tasks. Most available research on mathematics writing focuses on typically achieving students without considering students with mathematics difficulty (MD). In this study, we explored how students with MD who participated in a word-problem intervention randomized-control trial performed on a mathematics-writing task. We sampled 144 third-grade students with MD and evaluated student performance on an explanatory mathematics-writing measure. Overall, students with MD, on a mathematics-writing rubric with five categories, scored between 1 and 2 points out of a possible 5 points for each category. For one of the five rubric categories, Mathematics Content, the students in the word-problem interventions marginally outperformed the students in the business-as-usual condition. On average, students wrote 33.3 words, numbers, and symbols in response to the mathematics-writing prompt with an average of 8.7 mathematics vocabulary words. Of the mathematics vocabulary words used, students most frequently used formal mathematics vocabulary and names for symbolic numbers, with symbolic symbols and general vocabulary used to a lesser extent. The trends in this study will support future research to enhance mathematics-writing instruction.

Keywords Mathematics · Mathematics writing · Explanatory writing · Mathematics difficulty

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Mathematics writing provides students with the opportunity to reason and communicate ideas (Casa et al., 2016). Mathematics writing meets the requirements set by Common Core State Standards to understand and create arguments with assumptions, definitions, and evidence from previously established results (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). When students engage in mathematics writing, students report that mathematics writing supports their mathematical content knowledge (Tan & Garces-Bacsal, 2016). Yet, teachers frequently fail to include mathematics writing in their curriculum (Powell et al., 2021a). This is problematic as half of states in the U.S. require students to use writing in response to mathematics prompts (Powell & Hebert, 2022).

Students must use mathematics writing to demonstrate mathematics competency. Therefore, to prepare students for such tasks, improvements may be needed to the mathematics-writing supports within content area instruction and intervention for students performing across mathematical levels. To achieve this goal, it is necessary to first understand how students perform on mathematics-writing tasks when they have participated in classroom mathematics instruction and when they have participated in a content area intervention. We are interested in determining what students include in their mathematics-writing samples and if it differs based on participating in a content area intervention, especially for students who experience difficulty in mathematics. In this introduction, we review mathematics writing and the factors that impact mathematics writing. Then, we address the importance of understanding how students with mathematics difficulty (MD) perform on mathematics-writing tasks. Next, we review the challenges of word-problem solving, a content area frequently paired with mathematics writing. Finally, we present the purpose and research questions of the study.

Mathematics writing

As stated, mathematics writing supports student reasoning and communication within mathematics, and mathematics writing may allow for equitable opportunities for students to participate in mathematics reasoning and discourse (Casa et al., 2016). Such reasoning and communication in mathematics writing falls into four categories: exploratory, explanatory, argumentative, and mathematically creative (Casa et al., 2016). In exploratory mathematics writing, students act as their own audience to make sense of their mathematical ideas. In explanatory mathematics writing, students provide information, through descriptions and explanations, about a mathematical concept. In argumentative writing, students construct arguments or critique the reasoning of others. In mathematically creative writing, students think creatively about mathematical ideas to communicate fluently and flexibly in mathematics (Casa et al., 2016). We focus on explanatory mathematics writing in the present study because it is the most common category of mathematics writing in the elementary grades (Powell et al., 2017).

Students communicate mathematically in novel ways within these four mathematics-writing categories (Kostos & Shin, 2010). Students may participate in mathematics writing using a variety of methods, include journal writing (Baxter et al., 2005; Glogger et al., 2012; Lim & Pugalee, 2004), letter writing (Norton & Rutledge, 2010; Shield & Galbraith, 1998), or by responding to prompts (Cohen et al., 2015; Hughes et al., 2019; Kiuhara et al., 2020; Moran et al., 2014). Not only does mathematics writing allow students to express reasoning and mathematical ideas, but students can also learn mathematics through writing (Graham et al., 2020). Students may write about mathematical content related to computation (Powell & Hebert, 2016), fractions (Kiuhara et al., 2020), measurement and geometry (Cohen et al., 2015), algebra (Kasmer & Kim, 2012), calculus (Idris, 2009), or problem solving (Hughes & Lee, 2020). To communicate using mathematics writing, students use skills related to both general writing (e.g., grammar, writing sentences, writing paragraphs) and mathematics knowledge (e.g., computation, mathematical representations, mathematics vocabulary; Hebert & Powell, 2016; Powell & Hebert, 2016). In the following sections, we highlight the importance of these skills for mathematics writing.

General writing skills

General writing skills support communication across academic domains for communicating arguments, information, and narratives (National Governors Association Center for Best Practice & Council of Chief State School Officers, 2010). Writing across domains, a multi-faceted task, connects transcription, text generation, and executive functioning as they relate to working memory (Berninger, 2009). This includes the use of handwriting, spelling, language, planning, organization, and self-regulation (Berninger, 2009; Graham et al., 2017).

General writing skills moderately correlate with mathematics-writing performance (Powell & Hebert, 2016). In particular, the use of organization plays an especially valuable role in mathematics writing (Powell & Hebert, 2016; Stonewater, 2002). Similar to responding to a general writing prompt (e.g., Describe your favorite game and three reasons you like the game), students must introduce their writing, provide details with their reasoning, and include a conclusion. Depending on the type of mathematics-writing prompt, students may need to plan and organize additional components of their written response (Hughes et al., 2020). For example, explanatory mathematics writing requires an introduction, a description of the mathematical concepts or procedures, reasoning, and a conclusion (Hughes et al., 2020).

Frequently, students experience difficulty with such organization when responding to mathematics-writing prompts (Hughes et al., 2020). In fact, students are 2.6 times less likely to include an introduction and 1.8 times less likely to include a conclusion when responding to a mathematics-writing prompt than a general essay writing prompt (Hebert & Powell, 2016). In explanatory mathematics writing, students typically only describe their mathematical procedures with a conclusion sentence. Students tend to omit an introduction sentence and rarely provide a rationale for their mathematical procedures (Hughes et al., 2020). Yet, a limited list of ideas arranged in an unconnected way within mathematics writing is not enough to perform well on mathematics-writing tasks (Stonewater, 2002).

Computation

To accurately respond to mathematics prompts, students must negotiate the mathematics content. In explanatory mathematics writing, students may be asked to complete a mathematics problem or review a pseudo student's work before explaining their reasoning (Hebert & Powell, 2016; Hughes & Lee, 2020; Powell & Hebert, 2016). For example, when presented with a word-problem mathematics-writing prompt, students must solve the word problem and then explain how they solved the problem (Hughes & Lee, 2020). Alternatively, in situations when students must explain a pseudo student's work, students must first accurately perform one or more computation problems to check the student's work before they can accurately respond to the prompt (Hebert & Powell, 2016; Powell & Hebert, 2016).

Hughes et al. (2020) documented computation errors made by middle school students with MD when responding to an explanatory fraction word-problem prompt. Out of 51 students, 10 students made computation errors while responding to the prompt. Although these data provided insight into the frequency of such errors, there is still limited understanding of student use of computation within mathematics writing. More research should be completed to better understand how students use calculations when responding to mathematics-writing prompts.

Mathematical representation

Mathematical representations support students' ability to build a connection between conceptual and procedural content in mathematics (Agrawal & Morin, 2016). The use of representations through the Concrete-Semi-concrete-Abstract framework improves student performance across mathematical concepts including place value, computation, word problems, fractions, and algebra (Agrawal & Morin, 2016; Mancl et al., 2012). Considering the prevalence and efficacy of the Concrete-Semi-concrete-Abstract framework in mathematics (Agrawal & Morin, 2016; Mancl et al., 2012), it has been hypothesized that students are more likely to include Semi-concrete or abstract representations in their mathematics writing because they frequently see and understand mathematical representations (Hebert & Powell, 2016).

Recent research investigated student use of Semi-concrete and abstract representations in mathematics writing to support communication (Hebert & Powell, 2016; Hughes et al., 2020). This research indicated that a correlation exists between the use of Semi-concrete representations in mathematics writing with correct responses to mathematics-writing prompts (Hughes et al., 2020). Yet even though the use of Semi-concrete representations supports mathematics writing, a limited number of students include such representations in their writing. Hebert and Powell (2016) reported that even when mathematics word-problem and fraction writing prompts included a Semi-concrete representation, only 10% of fourth-grade students drew a picture in their mathematics-writing responses. Students more frequently included equations in their mathematics-writing responses with 34% of students including equations. Similarly, Hughes et al. (2020) measured performance of fourth- and fifth-grade students with learning disabilities in mathematics fraction and decimal word-problem writing prompts. When responding to fraction word-problem prompts, half of the students included pictures or equations in their written responses. Alternatively, when responding to the decimal word-problem prompts, only 10% of students included pictures or equations in their written responses. These results indicate that student use of pictures or equations in their written responses may be connected to performance in mathematics writing and the mathematics content addressed in the prompt.

Mathematics vocabulary

Monroe and Panchyshyn (1995) classified mathematics vocabulary into four categories: technical, subtechnical, general, and symbolic. Technical vocabulary in mathematics includes terms specific to mathematics, not typically used in everyday language (i.e., *denominator*, *multiplication*). Subtechnical vocabulary in mathematics typically includes terms with multiple meanings, a meaning in mathematics context and in everyday language (i.e., *total*, *difference*; Monroe & Panchyshyn, 1995). General vocabulary in mathematics includes vocabulary terms used in mathematics but not specific to mathematics (i.e., *wrong*, *answer*). Last, symbolic vocabulary includes alphabetic numbers (i.e., *twenty*, *five*) and nonalphabetic symbols (i.e., *20*, +; Hebert & Powell, 2016; Monroe & Panchyshyn, 1995). Mathematics vocabulary plays a role across mathematics domains, including word-problem solving, computation, measurement, geometry, and mathematics writing (Forsyth & Powell, 2017; Hebert & Powell, 2016; Peng & Ling, 2019).

Mathematics vocabulary especially impacts mathematics content areas involving high language use such as word-problem solving (Peng & Lin, 2019). Although little research has been conducted on mathematics vocabulary in mathematics writing, evidence suggests that mathematics vocabulary influences mathematics writing (Hebert & Powell, 2016; Hughes et al., 2020; Stonewater, 2002). In mathematics-writing tasks, students need mathematics vocabulary to communicate ideas (Hebert & Powell, 2016). Students who score higher on mathematics-writing measures use specific and clear mathematics vocabulary (Stonewater, 2002). Hebert and Powell (2016) reported in mathematics writing, fourth-grade students use a term with mathematics connotation every three to four words. Additionally, students incorporated symbolic mathematics-vocabulary terms more frequently than other mathematics-vocabulary terms. They included general terms slightly more often than technical terms and rarely included subtechnical terms in mathematics writing (Hebert & Powell, 2016).

Hughes et al. (2020) described slightly different trends for argumentative writing. In argumentative writing, students incorporated general vocabulary most frequently, symbolic vocabulary second, and rarely used technical and subtechnical vocabulary. Considering the variation in mathematics-vocabulary use within mathematics writing (Hebert & Powell, 2016; Hughes et al., 2020), it is necessary to continue to investigate how students include mathematics vocabulary within mathematics writing. Prior research has typically investigated the four mathematics-vocabulary categories in mathematics writing, without identifying if vocabulary differs based on problem-specific content (Hebert & Powell, 2016; Hughes et al., 2020). Therefore, it also remains unknown how the problem content included in the prompt influences the vocabulary included by students in their written output.

Students with MD

The complex task of mathematics writing requires students to access both mathematics and writing prerequisite skills (Powell & Hebert, 2016). For example, typically achieving (TA) students tend to write fewer words when responding to mathematics-writing prompts than to general essay composition prompts (Hebert & Powell, 2016). Students with MD may be especially challenged by mathematics-writing prompts. Students with MD typically perform below TA students across mathematics skills (Andersson, 2010; Cirino et al., 2015; Nelson & Powell, 2018b). Specifically, they make more errors than their TA peers in areas such as counting fact retrieval, mathematics fact fluency, multi-digit computation tasks, place value, understanding operational symbols, mathematics language, and word-problem solving (Andersson, 2010; Arsenault & Powell, 2022; Cirino et al., 2015).

To date, little research has been conducted about how students with MD perform on mathematics-writing prompts. In one study, Hughes et al. (2020) sampled the mathematics writing of students with learning disabilities and reported that students experienced difficulty with accurately responding to the prompts. When students responded accurately, students with learning disabilities tended to have trouble explaining their mathematical reasoning. While testing a mathematics-writing intervention with a focus on fractions, Kiuhara et al. (2020) reported that, at pretest, students with MD had difficulty writing about fraction content, the number of words written, and the quality of their written mathematics reasoning. These studies represent some of the only investigations on how students with MD perform on mathematics-writing tasks. Therefore, to support students with MD in mathematics writing, it is necessary to build a knowledge base on how students with MD perform on mathematics-writing tasks.

Word problems

Before conducting mathematics-writing instruction, it is important to first understand how the content area instruction and intervention impacts student performance on mathematics-writing tasks. Understanding how content area impacts student performance on mathematics-writing tasks can inform implementation of mathematics-writing instruction. Therefore, while mathematics writing is the focus of this study, we provide background information about word-problem solving because we conducted this study within a randomized-controlled trial focused on a word-problem intervention. Word problems consist of mathematical problems embedded within text which must be interpreted and solved for an unknown. To accurately solve word problems, students apply both language and computation skills by interpreting the language of the story, planning to solve the problem, and finally completing the necessary computation(s) to solve for the unknown (Björn et al., 2016; Fuchs et al., 2015).

Word-problem solving especially challenges students with MD compared to their TA peers (Arsenault & Powell, 2022). Students with MD are taught schema types to reduce the strain on reasoning and working memory by building mental representations and identifying problem types (Fuchs et al., 2019). By learning schema types, students learn to classify problems into problem types (Riley & Greeno, 1988). Teaching students with MD using schema instruction leads to growth in word-problem solving accuracy (Peltier & Vannest, 2017). Such schema instruction may also impact student performance on mathematics-writing prompts focused on word-problem solving due to the increased understanding of the content area mathematics. Therefore, it is important to understand how students who participated in schema instruction versus without perform on mathematics-writing prompts.

Purpose and research questions

Mathematics writing supports students' ability to reason and communicate in mathematics (Casa et al., 2016). The ability to communicate using mathematics writing is a required standard as stated in the Common Core Standards and is frequently measured on normative assessments (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010). Yet, while current research indicates that mathematics-writing tasks challenge all students (Hebert & Powell, 2016), limited research measures student performance on mathematics-writing tasks and what students can do in mathematics writing (Hebert & Powell, 2016; Hughes et al, 2020; Powell et al., 2017). Additionally, little is known on how to support students with mathematics difficulty, including if content area instruction and intervention supports mathematics writing (Powell et al., 2017). The purpose of this study was to determine how students with MD performed on a mathematics-writing task, what features they included in their mathematics writing, and if such student performance is impacted by content area (i.e., word-problem) intervention. Understanding how students with MD wrote in mathematics after either participating in classroom instruction or a word-problem intervention will lead to a better understanding of how to support students with MD in mathematics writing.

In this study, we asked the following research questions:

1. How do students with MD perform on a mathematics-writing prompt about a word problem? Is performance on a mathematics-writing prompt influenced by participation in a word-problem intervention?
2. What features (i.e., words, symbols, equations, pictures, and mathematics vocabulary terms) do students with MD include in their responses to a mathematics-writing prompt? Do the features students include vary based on student participation in a word-problem intervention?

Method

Context and setting

We collected the data for this analysis from a cohort of Grade 3 students participating in a study related to word-problem intervention (Powell et al., 2021b). All students attended 51 classrooms from 13 elementary schools in a large, urban public school district in the Southwest of the United States. This school district served over 80,000 students. On average, the district reported 55.5% of students as Hispanic, 29.6% as White, 7.1% as Black, and 7.7% as belonging to another race or ethnic category. Overall, 27.1% of students identified as emergent bilinguals, 52.4% qualified as economically disadvantaged, and 12.1% received special education services. The district's graduation rate was 90.7%.

Participants

In the fall of the school year, our research team screened 818 Grade 3 students using *Single-Digit Word Problems* (Jordan & Hanich, 2000). Students scoring below the 25th percentile qualified as experiencing MD with word-problem solving ($n=236$), with the 25th percentile being a common cut-off score in MD-related research (Geary et al., 2012; Hecht & Vagi, 2010; Nelson & Powell, 2018a). This measure of *Single-Digit Word Problems* has been used as a word-problem screener for MD in other studies, see Powell et al. (2021b) as an example.

We only had the capacity to include approximately 3 students with MD from each of the 51 classrooms. Some classrooms only had 1 student with MD, whereas others had more than 10. For this reason, we randomly selected 3–4 students with MD from each classroom for a total of 159 students with MD who could participate in the intervention portion of the study. Of these 159 students, we randomly assigned them to one of three conditions: word-problem intervention with a focus on equation solving and the equal sign ($n=60$), word-problem intervention without a focus on equation solving and the equal sign ($n=38$), and business-as-usual comparison (BaU; $n=61$). At posttest, after implementation of the interventions, we retained 50 students in the first word-problem condition, 34 students in the second word-problem condition, and 61 students in the BaU comparison. This permitted a sample size of 145 students with MD who completed the mathematics-writing measure in the spring of Grade 3. Table 1 displays demographics of the sample.

Intervention

The Grade 3 students in both word-problem interventions participated in one-on-one intervention sessions three times per week for 30 min per session (i.e., students completed between 45 to 48 completed session). The core content of the two word-problem intervention conditions focused on schema instruction with an attack strategy. For the attack strategy, students learned to RUN through a problem. They

Table 1 Student demographics

	All students (*N* = 145)	Word-problem conditions (*n* = 84)	Business-as-usual (*n* = 61)
Sex			
Female	85 (58.6%)	50 (59.5%)	35 (57.4%)
Male	59 (40.7%)	34 (40.5%)	25 (41.0%)
Not reported	1 (0.7%)	0 (0.0%)	1 (1.6%)
Race/ethnicity			
Black	20 (13.8%)	12 (14.3%)	8 (13.1%)
Asian	3 (2.1%)	1 (1.2%)	2 (3.3%)
Hispanic	99 (68.3%)	57 (67.8%)	42 (68.8%)
White	4 (2.7%)	3 (3.6%)	1 (1.6%)
Multi-racial	14 (9.7%)	9 (10.7%)	5 (8.2%)
Other	4 (2.8%)	2 (2.4%)	2 (3.4%)
Not reported	1 (0.7%)	0 (0.0%)	1 (1.6%)
Special education			
Not in special education	125 (86.2%)	69 (82.1%)	56 (91.8%)
Receiving special education	19 (13.1%)	15 (0.7%)	4 (6.6%)
Not reported	1 (0.7%)	0 (0.0%)	1 (1.6%)
Emergent bilinguals			
Emergent bilinguals	88 (60.7%)	51 (60.7%)	37 (60.7%)
Not emergent bilinguals	56 (38.6%)	33 (39.3%)	23 (37.7%)
Not reported	1 (0.7%)	0 (0.0%)	1 (1.6%)

would *R*ead the problem, *U*nderline the label and cross off irrelevant information, and *N*ame the problem type (i.e., choose the correct schema for the problem; Powell et al., 2021b). Students also learned three additive schemas: Total, Difference, and

Change. In Total problems, parts are put together for a total. In Difference problems, two amounts are compared for a difference, and for Change problems, a starting amount increases or decreases for a new amount (Powell et al., 2021b).

The 30-min sessions consisted of five activities for each session: (1) Math Fact Flashcards, (2) Equation Quest or Pirate Crunch, (2) Buccaneer Problems, (4) Shipshape Sorting, and (5) Jolly Roger Review. During Math Fact Flashcards, interventionists displayed addition and subtraction math-fact flashcards (addends 0–9; minuends 0–18; and subtrahends 0–9) to students during two, 1-min timings. Students in the first word-problem intervention then completed Equation Quest, working with the interventionists to solve equations and interpreting the equal sign as a relational symbol. As the second activity, students in the second word-problem intervention completed Pirate Crunch—mathematics review activities including time, money, geometry, place value, and fractions. In the third activity, Buccaneer Problems, interventionists guided all students through three word-problems using schema instruction with the RUN attack strategy. For the fourth activity, Shipshape Sorting, all students were given 1 min to sort word problems by schema type. For the last activity, Jolly Roger Review, all students completed a timed review of the concepts covered in the session with 1 min to complete mathematics facts, computation problems, or write equations for the word-problem schemas and 2 min to solve a word problem using the schema and RUN strategies (Powell et al., 2021b). During these sessions, students did not receive any incorrect worked-example practice, mathematics-writing practice, or specific mathematics-vocabulary instruction.

General education instruction

All students with MD participated in regular mathematics instruction provided by their general education teacher. In the district, teachers primarily used the *GO Math!*, *Investigations in Number, Data, and Space for the Common Core*, or *Motivation Math* curricula to guide mathematics instruction. Students in the word-problem interventions received supplemental, individual intervention about word-problem solving from our research team; BaU students did not receive supplemental intervention from our team. The interventionists did not provide intervention during the students' regular mathematics instruction to ensure students continued to fully participate in the district's mathematics curriculum.

Measures

To identify students with MD, we used the *Single-Digit Word Problems* (Jordan & Hanich, 2000). *Single-Digit Word Problems* included 14 one-step word problems involving sums or minuends of 9 or less categorized into the Total, Difference, and Change schemas. Examiners read each word problem aloud and could re-read each problem up to one time upon student request. We scored *Single-Digit Word Problems* as the number of correct responses (maximum = 14). We calculated Cronbach's α for the full sample at 0.88 (Powell et al., 2021b).

Mathematics-writing measure

We used a *Mathematics-Writing Measure* to assess mathematics writing for the students with MD (see Fig. 1), a measure adapted from a measure used in prior work with students who experienced reading and mathematics difficulty (Hebert et al., 2019). We administered this measure at posttest only, in the spring of Grade 3. An examiner read the prompt aloud to students in a small group situation (2–3 students). The examiner then provided the students 10 min to answer the mathematics-writing prompt: “Sam made several mistakes. Write about Sam’s mistakes

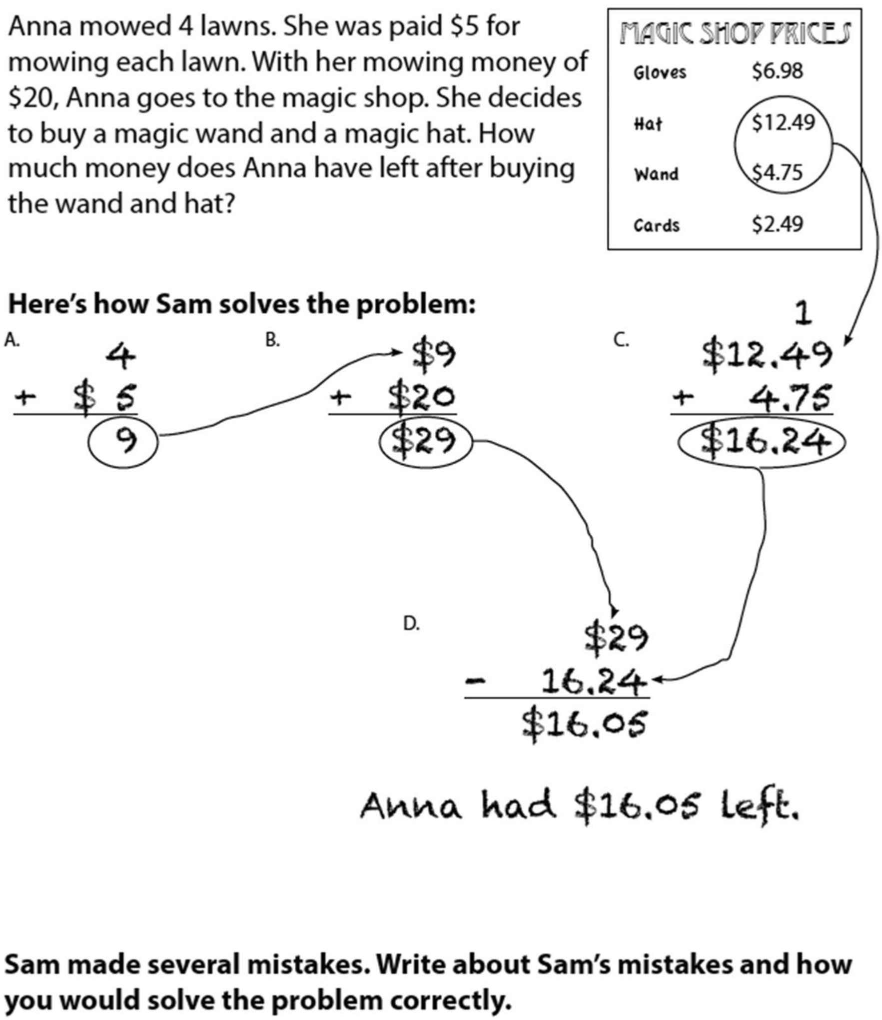

Fig. 1 Mathematics-writing prompt

and how you would solve the problem correctly.” Examiners provided students with one page with the prompt and one page with horizontal lines for writing.

In the prompt, a student, “Sam,” solves a multi-step word problem about how much money “Anna” spent at a magic shop after earning $20 mowing lawns. Sam makes several mistakes. In Step A, Sam adds irrelevant information, an unnecessary step to complete the problem. In Step B, Sam adds the sum from irrelevant information in Step A to the total amount of money Anna earned mowing lawns, a second unnecessary step to accurately solving the problem. In Step C, Sam correctly adds the amount of money Anna spent on a magic wand, $12.49, and a magic hat, $4.75, but makes an error while regrouping. Sam writes the sum of $12.49 and $4.75 as $16.24 instead of $17.24. Last, in Step D, instead of subtracting the amount of money Anna spent, $17.24, from the total amount of money she earned mowing lawns, $20, Sam subtracts the incorrect amount of money Anna spent, $16.24 from Step C, from the incorrect amount of money Anna earned, $29.00 from Step B. In addition to using the incorrect numbers in Step 4, Sam also lines up the numbers incorrectly, leading to an incorrect difference.

We scored student writing samples based on a five-category rubric (see Fig. 2), adapted from a previously used mathematics-writing rubric (Hebert & Powell, 2016). Maximum score for the mathematics-writing prompt was 25, and Cronbach’s α was 0.81.

		5	4	3	2	1
Math content	**Compare to an ideal response and consider all math that should be discussed**	Complete understanding of all math concepts *and* procedures	Adequate understanding (for all parts of the problem) OR Addresses most math concepts *and* procedures (may be missing one part of the problem OR addresses procedures but not all concepts)	Partial understanding (for all parts of the problem) OR Addresses some math concepts *and* procedures (may be missing some parts of the problem)	Limited understanding of math concepts and procedures (may address only one or two parts of the problem) OR focused mostly on procedures OR mixes correct and incorrect explanations	No understanding of math concepts and procedures (may mention a concept with no understanding of content)
Math vocabulary		Accurate use of formal math vocabulary AND shows sophisticated understanding of the meaning of each vocabulary term	Adequate use of formal math vocabulary with some informal vocabulary BUT shows sophisticated understanding of the meaning of each vocabulary term	Use of formal and informal math vocabulary BUT does not always show sophisticated understanding of the meaning of each vocabulary term	Use of informal math vocabulary only OR use of formal vocabulary with limited understanding of the meaning of each vocabulary term; May include at least one example of incorrect use of vocabulary	No use of (or incorrect use of) math vocabulary
Organization of math ideas		Clear progression of ideas with fully-developed explanations, logical sequencing with effective transitions	Adequate progression of ideas with mostly-developed explanations, logical sequencing with adequate transitions	Weak progression; May not include strong explanations and/or may have difficulty with sequencing and lack of transitions	Limited progression of ideas with limited explanations; No organization or flow with limited transitions or connectors OR may have addressed only one part of the problem with a strong explanation	No progression of ideas, limited use of transitions and connectors OR may have addressed only one part of the problem with weak or no explanation
Writing grammar (considering math terms)	**Only judge student's sample**	Free of grammatical errors and use of fluent sentences; Includes appropriate and consistent use of numbers and symbols	1 or 2 grammatical errors and use of fluent sentences; Includes appropriate and consistent use of numbers and symbols	Several errors, but they do not interfere with comprehension; Mostly fluent sentences; Includes minor inconsistencies with use of numbers and symbols	Many errors interfere with comprehension; Limited fluent sentences; Includes inconsistencies with use of numbers and symbols	Grammatical errors make comprehension impossible
Clarity and precision	**Compare to an ideal response**	Explains procedures and concepts in a clear and precise way (considering all parts of the problem); Provides enough detail, elaboration, or examples while discussing the concepts and procedures so that there is no ambiguity; Covers all aspects of the problem	Explains most concepts and procedures in a clear and precise way but may not address 1 to 2 elements (considering all parts of the problem) OR discusses all parts of the problem and provides enough detail or elaboration of some of the statements but not others	Explains some procedures and concepts in a clear and precise way but does not address all parts of the problem (considering all parts of the problem) OR discusses all parts of the problem but their explanation is not clear or precisee but does not provide enough detail or elaboration of their statements	Explains maybe only 1 or 2 concepts and procedures in a clear and precise way (consdering all parts of the problem) OR discusses some or many parts but the explanation is not clear or precise and not enough detail or elaboration of their statements	Explains maybe only 1 or 2 concepts and procedures with minimal clariety

Fig. 2 Mathematics-writing rubric

Coding

We coded each mathematics-writing sample for word-problem content. We counted the total number of words and numbers used in the written response (e.g., *12.49*, *4*, and *20*) and the number of symbols students used in their writing (e.g., *$*, +, or =). We also recorded whether students calculated the problem independently from their written answer, included a picture, or included any expressions or equation in their written response (e.g., *4+5*). Last, we counted the vocabulary terms students wrote, such as word-problem schema vocabulary (e.g., *difference*, *total*), formal mathematics vocabulary (e.g., *multiply*, *decimal*), problem-specific vocabulary (e.g., *hat*, *buy*), number words (e.g., *four*, *nine*), numerals (e.g., *29*, *5*), mathematical symbols (e.g., *$*, +), general vocabulary (e.g., *solve*, *mistake*), and informal mathematics vocabulary (e.g., *answer*, *carried*). We coded the words as written if that word or a variation of the word was used (i.e., *add* and *addition* both coded as *add*). We also gave students credit for the word if it was misspelled (i.e., *ad* coded as *add*). Table 2 lists the word-problem content we coded, including the number of words, numbers, and symbols. Table 3 provides the vocabulary terms coded from the student work samples.

Interrater reliability

Two raters (i.e., the fifth and sixth authors) scored the writing samples against the mathematics-writing rubric. Reliability of coding was 94.7% between the two coders. Then, two additional raters (i.e., the first and fourth authors), coded for the word-problem content used in the writing samples. For training purposes, these raters scored two writing samples together, agreeing on 100% of the content. Then, the raters coded two writing samples and initially scored 92.0% reliability, but then reviewed resolved discrepancies and came to as resolution, this increased to 100%. The raters then scored seven writing samples independently, with initial 89.1%

Table 2 Scoring of mathematics writing by number of words and vocabulary terms

Variables	*M*	*SD*
Number of words and numbers	31.7	27.1
Number of symbols	1.7	2.6
Number of words, numbers, and symbols	33.3	28.0
Mathematics vocabulary	8.7	7.9
Schema vocabulary	0.2	0.5
Formal mathematics vocabulary	2.0	2.2
Problem-specific vocabulary	0.9	1.7
Symbolic number words	0.3	1.1
Symbolic numbers	2.2	3.3
Symbolic symbols	1.6	2.5
General vocabulary	1.2	1.5
Informal mathematics vocabulary	0.3	0.6

Table 3 Mathematics vocabulary occurrences

Mathematics Vocabulary Category	Term	Instances	Occurrences per student			
			Once	Twice	Three	Four or more
Schema vocabulary	*Total/Whole*	10	6	2		
	Change	1	1			
	Difference	0				
	Start/Finish	11	9	1		
	End	1	1			
	Part	0				
	Even	0				
Formal mathematics vocabulary	*Regroup*	4	2	1		
	Add	88	31	14	7	2
	Subtract	55	33	8	2	
	Multiply	7	5	1		
	Divide	4	4			
	Dollar	43	15	4	2	3
	Money	24	11	2	3	
	Equal	14	2	1		2
	Number	54	14	9		4
	Decimal	0				
Problem-specific vocabulary	*Gloves*	2	2			
	Hat	24	13	2	1	1
	Shop	3	3			
	Wand	22	11	2	1	1
	Cards	2	2			
	Buy	11	3	4		
	Sam	35	23	3	2	
	Anna	30	20	5		
Symbolic number words	*Four*	10	8	1		
	Five	9	6	1		
	Nine	13	4	3	1	
	Twenty	9	4			1
Symbolic numbers	*4*	60	24	7	3	3
	5	41	24	4	3	
	9	33	9	9	2	
	20	34	24	3	1	
	29	32	11	8		1
	12.49	31	25	3		
	4.75	31	24	2	1	
	16.24	30	20	5		
	17.24	11	5	3		
	2.76	0				
	16.05	20	11	3	1	
Symbolic symbols	*$*	68	9	8	6	5

Table 3 (continued)

Mathematics Vocabulary Category	Term	Instances	Occurrences per student			
			Once	Twice	Three	Four or more
	+	81	23	9	5	5
	−	24	18	3		
	=	56	13	7	7	2
	×	5	1			1
	÷	0				
General vocabulary	*Wrong*	36	19	7	1	
	Right	29	11	6	2	
	Correct	9	7	1		
	Chart	0				
	Mistake	60	33	5	4	1
	Solve	19	16		1	
	Check	8	3			1
	Same	6	4	1		
Informal mathematics vocabulary	*Carry*	4				
	Answer	34				
	Point	0				

reliability that increased to 100% after resolving discrepancies. Next, they randomly sampled 30 more writing samples to independently double code. The raters initially scored 91.5% for reliability which increased to 100% after resolving discrepancies.

Procedure

Examiners employed as graduate research assistants administered all posttest sessions after the completion of a word-problem intervention in March and April of the school year. Examiners participated in a training on administration of the tests during one, 2-h session conducted in February of the school year. Before administering the measures to students, examiners practiced with one another and conducted a practice testing session with the project manager. Examiners administered the posttest measures in small groups (2–3 students). We conducted five, 30-min posttest sessions for each student, and all testing sessions occurred within a 2-week window. We administered the mathematics-writing measure in the fifth posttest session immediately following administration of a distal word-problem measure.

Fidelity test implementation

Examiners recorded all testing sessions at posttest. We randomly selected 20% of audio recordings for analysis, evenly distributed across examiners, and measured fidelity to testing procedures against detailed fidelity checklists. We measured posttest fidelity at 97.9% ($SD = 0.02$).

Data analysis

As noted, this data was collected as part of a larger randomized-controlled trial (Powell et al., 2021b). In that trial, we compared two variants of word-problem intervention, and we noted no significant difference on the word-problem outcome between the two word-problem interventions with students in both word-problem interventions outperforming students in the BaU comparison. Therefore, for this analysis, we combined the students for each of the word-problem interventions to represent a singular word-problem intervention. We used SPSS and Excel to complete the analysis for each research question.

To investigate our first research question, we measured student performance on the mathematics-writing rubric. We reported the mean performance in each rubric category for the word-problem intervention and the BaU groups. Then, we conducted a one-way ANOVA to compare the student performance between conditions (i.e., word-problem intervention versus BaU) for each of the five categories on the mathematics-writing rubric. We then calculated Cohen's *d* to determine the effect sizes for each category between the word-problem intervention and the BaU groups (Cohen, 1992).

To investigate our second research question, we reported means, standard deviations, and frequencies based on word-problem features. First, we reported the mean and standard deviation for number of words and numbers used; number of symbols used; and number of words, numbers, and symbols used. Then, we compared the word-problem intervention and the BaU groups for each category using a one-way ANOVA and calculated Cohen's *d* for effect sizes. Next, we reported the mean and standard deviation number of mathematics-vocabulary terms used across all categories and for each mathematics-vocabulary category (i.e., schema vocabulary, formal mathematics vocabulary, problem-specific vocabulary, symbolic number words, symbolic numbers, symbolic symbols, general vocabulary, and informal mathematics vocabulary). We compared the word-problem intervention and the BaU groups for each category using one-way ANOVA. Next, we measured the percentage of students who drew pictures, computed a problem separate from their writing sample, and included equations in their writing sample. Last, we calculated the total frequency of each mathematics-vocabulary word used across all student writing samples.

Results

Mathematics-writing performance

We used the five-category rubric (see Fig. 2) to understand the mathematics-writing performance of students with MD. In this section, we compare the performance of the 84 students with MD who participated in the word-problem interventions to the 61 students with MD who participated in the BaU classroom instruction (see Table 4). Across both groups and all five categories, the students with MD scored between 1 and 2 points out of a possible 5 points.

Table 4 Scoring of mathematics writing by rubric category

	Word-problem conditions ($n=84$)		Business-as-usual ($n=61$)		Word-problem conditions versus Business-as-usual		
	M	*SD*	*M*	*SD*	*F*	*p*-value	*d*
Mathematics content	1.33	0.57	1.16	0.45	3.50	0.063	0.33
Mathematics vocabulary	1.26	0.47	1.16	0.49	1.31	0.254	0.21
Organization of mathematics ideas	1.29	0.51	1.23	0.59	0.47	0.494	0.11
Writing grammar	1.62	0.76	1.46	0.67	1.76	0.187	0.22
Clarity and precision	1.10	0.30	1.08	0.42	0.67	0.796	0.06

The students with MD in the word-problem interventions ($M=1.33$) performed marginally higher than the BaU students ($M=1.16$) on the Mathematics Content category of the mathematics-writing rubric $F(1,143)=3.50$, $p=0.063$. The two groups did not perform significantly different on the remaining four categories of the mathematics-writing rubric, although the average scores for students who participated in the word-problem interventions were higher in all four categories of Mathematics Vocabulary, Organization of Mathematics Ideas, Writing Grammar, and Clarity and Precision.

Mathematics-writing features

In our second research question, we asked about which features students with MD included in their responses to a mathematics-writing prompt and do the features students include vary based on student participation in a word-problem intervention. We separated the features into two categories: (a) mathematics representations and (b) mathematics vocabulary.

Mathematics representations

In this study, mathematics representations included pictures, computations separate from the mathematics writing, and equations embedded within the mathematics writing. None of the students included pictures in or separate from their written response. A total of 38.9% of students attempted or completed computation to solve the word problem separately from their written response (see Fig. 3). Additionally, 33.3% of students included equations within their written response (see Fig. 4). When examined by group, the students in the word-problem intervention and the BaU groups did not perform significantly different on inclusion of pictures, computation separate from the mathematics writing, and equations embedded within the mathematics writing.

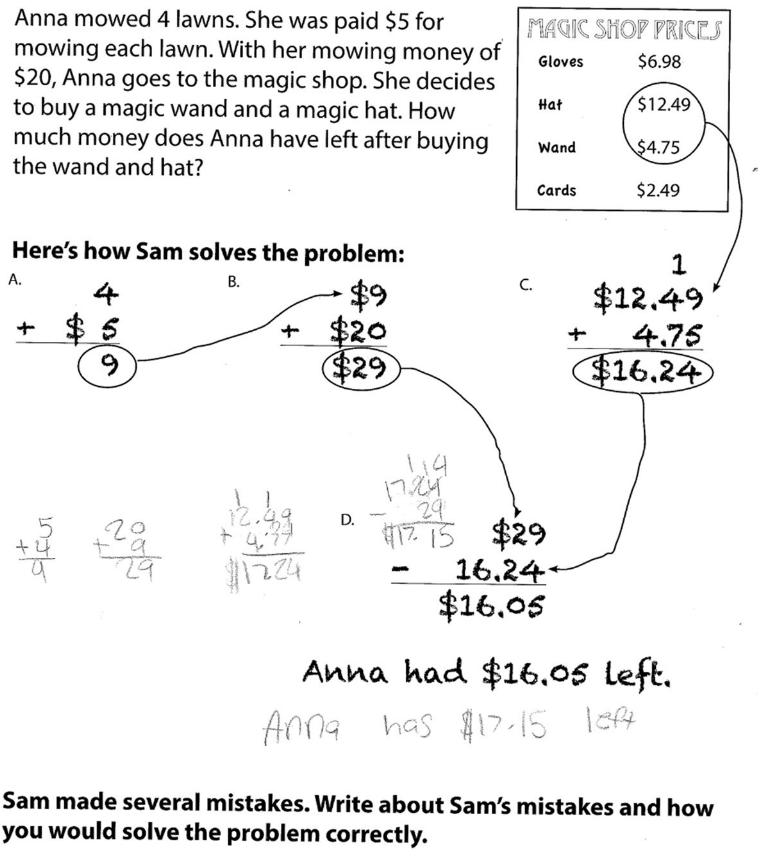

Fig. 3 Computation completed separate from written answer

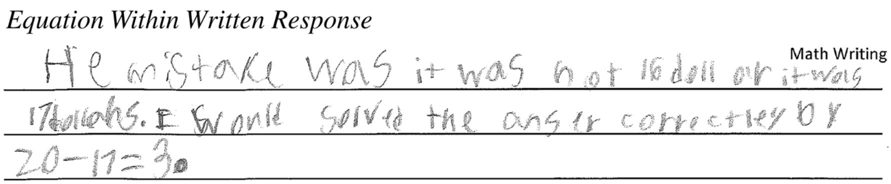

Fig. 4 Equation within written response

Mathematics vocabulary

For mathematics vocabulary, we examined the average use and frequency of words, numbers, and symbols for students with MD. Table 2 includes the average use of words, numbers, and symbols. Students with MD wrote on average 31.7 words and numbers in response to the mathematics-writing prompt. They also wrote an average 1.7 symbols in their writing samples. Together, students wrote an average of 33.3 words, numbers, and symbols. When examined by group, students in the word-problem intervention and the BaU groups did not perform significantly different on their average use of words, numbers, and symbols; words and numbers; or symbols included in their responses to the mathematics-writing prompt.

For mathematics vocabulary, we coded eight vocabulary categories: schema vocabulary, formal mathematics vocabulary, problem-specific vocabulary, symbolic number words, symbolic numbers, symbolic symbols, general vocabulary, and informal mathematics vocabulary. On average, across all the words, numbers, and symbols we coded, students used 8.7 mathematics vocabulary terms per writing sample (27% of the average writing sample). Within the eight categories, students most frequently used symbolic numbers (i.e., *4*, *5*) and formal mathematics vocabulary (i.e., *regroup*, *add*). Students also frequently used symbolic symbols (i.e., +, =), general vocabulary (i.e., *wrong*, *right*), and problem-specific vocabulary (i.e., *hat*, *shop*). Students were less likely to use symbolic number words (i.e., *four*, *five*), informal mathematics vocabulary (i.e., *carry*, *answer*), and schema vocabulary (i.e., *change*, *part*). When examined by group, students in the word-problem intervention and the BaU groups did not perform significantly different on their average use of mathematics vocabulary terms per writing sample, schema vocabulary, formal mathematics vocabulary, symbolic number words, symbolic numbers, symbolic symbols, general vocabulary, or informal mathematics vocabulary. For the inclusion of problem-specific vocabulary, students in the word-problem interventions ($M = 1.11$, $SD = 1.79$) did perform marginally significantly higher than the students in the BaU group ($M = 0.61$, $SD = 1.60$) on their inclusion of problem-specific vocabulary $F(1,143) = 3.03$, $p = 0.084$ with a small effect size ($d = 0.30$).

Table 3 lists how frequently students wrote each term within the eight mathematics vocabulary categories. Within symbolic numbers, students wrote the numbers *4* (60 times) and *5* (41 times) most often. In the writing prompt, the numbers *4* and *5* appear in the word problem and in the first step as well as the first error completed by the pseudo student. Students wrote the other symbolic numbers from the pseudo student's work less frequently (between 20 to 34 times) We coded for two symbolic numbers not written by the pseudo student (Sam), but needed for the correct computation of the problem, *17.24* and *2.76*. The symbolic number *17.24* was written 11 times, but no students wrote the correct answer of *2.76*.

Within the formal mathematics vocabulary, frequencies of words ranged between 0 and 88 occurrences. Students frequently wrote *add*, *subtract*, and *dollar*; all words describing symbols or operations used in the pseudo student's work. They also used *number* in 54 instances, a term describing the numbers in the prompt. Students used *money* and *equal* with some regularity, both which would support addressing the prompt. The students used terms such as *regroup*, *multiply*, *divide*, and *decimal* the least.

For symbolic symbols, students most frequently used the symbols related to the problem. Students wrote the plus sign (+) 81 times, similar to the number of times they wrote *add*. They also wrote the dollar sign (*$*) 68 times. Similarly, students wrote the formal word *dollar* more often than most formal mathematics words. In contrast to the formal mathematics vocabulary word *equal* (14 instances), students wrote the equal sign (=) 56 times. Unlike the formal term *subtract*, students wrote the minus sign (−) only a moderate number of times (24 times). Students rarely wrote the multiplication symbol (×) and never wrote a division symbol (÷), similar to their formal terms.

For general vocabulary, students most commonly used terms which could address the pseudo student's errors. Students wrote *mistake* 60 times, a word used in the prompt asking about the *mistakes* the pseudo student made. Students also used words such as *wrong* (36 times) and *right* (29 times). Alternatively, the students rarely wrote the word *correct* (9 times). Students wrote *solve* (19 times) a moderate amount, but rarely or never wrote *chart*, *check*, or *same*.

For problem-specific words, students regularly used the names of the pseudo student, *Sam* (35 times), and the person in the word problem, *Anna* (30 times). Students used the words *hat* (24 times) and *wand* (22 times) a moderate amount. Both words were written in the question sentence of the word problem. Alternatively, even though *buying* was also in the question sentence, they only wrote *buy* 11 times. The students wrote *gloves*, *shop*, and *cards* the least.

Students infrequently wrote symbolic number words, informal mathematics vocabulary, and schema vocabulary. For symbolic number words, students wrote *nine* a total of 13 times and *four* 10 times. They also wrote both *five* and *twenty* 9 times. For informal mathematics vocabulary, we only coded three words. The students wrote *answer* 34 times, *carry* 4 times, and no students wrote *point*. This aligns with how frequently the students wrote the formal counterparts of *regroup* and *decimal*. Last, students also infrequently wrote schema vocabulary terms. The terms *total* or *whole* (10 times) and *start* or *finish* (11 times) were written the most frequently. Students wrote *change* and *end* once each and never wrote *difference*, *part*, and *even*.

Discussion

In this study, we examined the written responses of students with MD to an explanatory mathematics-writing prompt. We focused on two research questions. First, we asked how students with MD performed on the mathematics-writing measure and if their performance was impacted by participation in a word-problem intervention. Second, we asked which features students with MD included in their responses to the mathematics-writing prompt and if the features varied based on student participation in a word-problem intervention. These questions extend previous research investigations about how both students with and without MD perform on mathematics-writing prompts for explanatory writing (Hebert & Powell, 2016; Hughes et al., 2020). By expanding the understanding of how students with MD perform on a mathematics-writing measure after participation in a word-problem intervention, more can be known about how to support students with MD through the development of mathematics-writing assessments and interventions.

First, we examined how students with MD performed on the mathematics-writing measure. Overall, students with MD scored low on the mathematics-writing measure. Students often scored “no” (1) or “limited” (2) across all five categories of the rubric which shows most students with MD have an opportunity for growth with their mathematics writing. Our results align with previous research, specifically the research of Hughes et al. (2020). They asked students in Grades 4 and 5 with learning disabilities to complete two mathematics-writing measures and noted that few students provided strong explanations in their mathematics writing.

Although our students with MD performed low on all categories of the mathematics-writing rubric, the students in the word-problem interventions did perform marginally higher than the students in the BaU group on Mathematics Content, indicating that intervention in word-problem solving may impact performance on a mathematics writing focused on word-problem solving. During the word-problem interventions, students did not receive any experiences with mathematics writing. Therefore, any impact on mathematics-writing occurred implicitly. The implicit learning may have been impacted by the explicit instruction and practice in solving word problems. By increasing their ability to solve word problems, students may have had an increased ability to interpret and communicate about the word-problem content.

For our second research question, we examined the mathematics-writing features students with MD included in their writing and if the features vary based on student participation in a word-problem intervention. In our analysis of mathematics representations, approximately one-third of students attempted to calculate the problem themselves, as evidenced by the students’ mathematics-writing paper. In our *Mathematics Writing Measure* (Fig. 1), students learn that Sam made several mistakes and students are prompted to write about how they would solve the problem correctly. Because the prompt specifically asks students to explain how they would solve the problem correctly, it is unsurprising that students would show computation on the page or include an equation in their writing (see Fig. 3 for an example of computation by a student). What is surprising, however, is that less than half of students did this. To solve the problem correctly, students must add (or check Sam’s addition for) $12.49 and $4.75, which equals $17.24. Then, students subtract (or find the difference between) $20.00 minus $17.24 for a difference of $2.76. To do this mathematics without written computation would be difficult for many adults as well as Grade 3 students with MD.

Similarly, one-third of students included equations within their written response to explain Sam’s errors or to describe how to solve the problem correctly. In research in Grades 4 and 5, both Hebert and Powell (2016), with TA students, and Hughes et al. (2020), with students with MD, noted 30 to 50% of students used equations in their mathematics writing, and this data aligns with that we collected from our Grade 3 students. Using computation to solve a problem and equations to explain the mathematics are likely important strategies for successful mathematics writing and more students may want to rely on these strategies. Educators can support students with MD to use computation to solve a problem and equations to explain mathematics using structured metacognitive steps guiding students through the mathematics-writing process (Hebert et al., 2019; Hughes & Lee, 2020; Kiuhara et al., 2020). In these steps, students can be prompted to solve the problem and to use clear and concise language and symbols when composing their written response (Kiuhara et al., 2020).

In our present study, no students included pictures in or separate from their written responses. This deviates from previous examinations of student mathematics-writing samples in which students sometimes used pictures to supplement their writing (Hebert & Powell, 2016; Hughes et al., 2020). The contrast with previous research may be due to the numbers in the present study's prompt (i.e., whole numbers vs. fractions; see Fig. 1) or to student understanding of the prompt. When students drew pictures in Hebert and Powell (2016), the mathematics-writing prompt included visuals of fraction models that likely encouraged students to do their own drawings in addition to the provided visuals. Alternatively, our prompt included no visuals. This differentiation in prompts may indicate that students may be more likely to respond to a prompt with a visual with a picture than to a prompt without a visual. A second factor which may have impacted the lack of pictures could be due to student understanding of the prompt. The students in our sample on average scored between the "no" (1) or "limited" (2) levels across all five categories of the rubric, while inclusion of pictures by students in previous research correlated with accuracy in response to mathematics-writing prompts (Hughes et al., 2020).

We also measured mathematics vocabulary used in the mathematics writing of students with MD. First, we counted the total words, numbers, and symbols written in each student's mathematics writing. As described, students with MD wrote an average of 33.3 words, numbers, and symbols. This number of words, numbers, and symbols is drastically lower than that counted by Hebert and Powell (2016) with a representative sample of Grade 4 students. With a similar (but not exact) prompt about Sam and the magic shop, Hebert and Powell (2016) noted students wrote an average of 59.0 words, numbers, and symbols. While not a direct comparison, our results indicated that students with MD wrote fewer words, numbers, and symbols in their mathematics writing, and this may mean under-developed explanations of mathematical ideas. The use of fewer words, numbers, and symbols of students with MD may be due to the limited use of computations to check the pseudo students work and mathematics vocabulary within their writing. The limited use of computation and mathematics vocabulary aligns with previous research indicating that students with MD frequently experience difficulty within the areas of computation and mathematics vocabulary (Hughes et al., 2020; Peng & Lin, 2019).

We then analyzed the frequency of mathematics vocabulary used by students. Students most frequently used formal mathematics vocabulary (e.g., *add*, *dollar*), symbolic numbers (e.g., *4*, *12.49*), symbolic symbols (e.g., *$*, +), and general vocabulary (e.g., *mistake*, *wrong*), similar to the Grade 4 students of Hebert and Powell (2016). As noted in Table 3, many students used these terms only once, but some students used the same term two, three, or four times in their writing sample. Therefore, some students used this mathematics vocabulary, and used it frequently, whereas others did not use any of these mathematics-vocabulary terms. Compared to symbolic numbers, students demonstrated a trend of using a limited number of symbolic number words (e.g., *four;* Hebert & Powell, 2016). Students favored using a numeral (e.g., *5*) instead of its number word counterpart (e.g., *five*). Interestingly, students wrote *17.24*, the first step to solving the problem correct, 11 times. But no students wrote *2.76*, the correct response to the word-problem prompt. As students with MD often have difficulty with multi-step word-problem solving (Jitendra et al., 2013), this result pointed to some students understanding the first step of solving the problem but minimal work on the second step of the problem.

In this study, we extended previous research by investigating student use of schema vocabulary, problem-specific vocabulary, and informal mathematics vocabulary. Of these, we noted limited use of schema vocabulary. For the 84 students who participated in the word-problem interventions, they participated in 45 to 48, 30-min sessions focused on developing schema knowledge for Total, Difference, and Change word problems. We included the examination on schema vocabulary because we hypothesized that students might write using terms they had heard and said dozens, if not hundreds, of times in their intervention sessions. For example, to identify Total problem, students asked themselves, "Are parts put together for a total?" One step to solving the *Mathematics-Writing Measure* was to identify two parts (i.e., *hat* and *wand*) and put those parts together for a total (*$17.24*). Yet very few (if any) students used this language in their explanation of how to solve the problem correctly. We did note greater instances of use of problem-specific vocabulary (e.g., *Sam*, *hat*, *wand*), with a marginally significant advantage for students who participated in the word-problem intervention, and informal mathematics vocabulary (e.g., *answer*) over schema vocabulary. These patterns in the use of mathematics vocabulary should be explored in future iterations of this research, ideas which we describe in the following section.

Implications for research

Although our study was limited in scope, by only analyzing the mathematics writing of Grade 3 students identified with MD through one word-problem measure, we can provide several directions for future research that would expand the literature in mathematics writing. First, we administered our measure to a sample of students who we identified as experiencing MD based on their performance on one word-problem measure. In future iterations of mathematics-writing research, researchers and educators should administer the same mathematics-writing measure to students with and without MD to understand whether students with MD perform differentially on mathematics-writing measures. Furthermore, researchers should examine mathematics writing across grade levels to understand whether performance patterns persist at different grade levels.

Second, we observed a slight advantage for students who participated in the word-problem interventions on one category (i.e., Mathematics Content) of the five-category rubric. Future research should investigate whether participation in intensive mathematics intervention can lead to improved mathematics writing in the mathematics content (e.g., word problems, fractions, geometry) of the intervention. If researchers learned that the content of intervention transferred to mathematics writing in a similar content, this would be important information for intervention design and assessment.

Third, future research should investigate the efficacy of focused vocabulary instruction within mathematics writing. We noted that students used different mathematics vocabulary terms with varying levels of frequency. Perhaps students do not use mathematics vocabulary in their mathematics writing because of unfamiliarity with terms; therefore, researchers should explore the connection between mathematics vocabulary instruction and mathematics writing. Researchers may also want to explore the connection between mathematics-vocabulary terms used in intervention and the rate of transfer of those terms to mathematics writing.

Implications for practice

In the present study, students with MD demonstrated limited mathematics-writing performance; therefore, educators must realize the value in supporting mathematics writing. All students must be able to solve mathematics problems and communicate about mathematics problems (National Council of Teachers of Mathematics, 2000). An important first step for supporting mathematics writing may be to support mathematics content. Based on our results, students with MD in a word-problem intervention group performed marginally higher than students in a BaU group on Mathematics Content. By modeling and practicing mathematics content, students may transfer this knowledge to mathematics writing. A second step for educators may be through providing students with more opportunities to write in mathematics. Recent research indicates the usefulness of mathematics-writing interventions for students with MD (Hughes & Lee, 2020; Kiuhara et al., 2020), but more research is needed to develop an evidence base of mathematics-writing practices. A third suggestion may be to help students use effective mathematical strategies in their own mathematics writing. For example, using representations (e.g., drawings) has been shown as an important component for mathematics learning (Jitendra et al., 2016), yet no students used pictures in their mathematics writing. Educators should help students learn how to use mathematics strategies in their mathematics writing.

Limitations

Before concluding, we describe several limitations to the present study. Primarily, we focused solely on students with MD. We categorized students as experiencing MD based on performance on a single word-problem screener. We did not examine the mathematics-writing of a representative sample of Grade 3 students to examine how students with MD performed compared to their TA peers. Another limitation to the study was that students responded to only one mathematics-writing measure. While this single measure may provide an approximate view of a student's mathematics-writing performance, multiple measures would have provided a more complete understanding of the mathematics writing of a student. Last, we focused on the different mathematics-vocabulary terms students used in their mathematics writing, but the prompt, with a focus on word-problem solving, only sampled a limited number of mathematics-vocabulary terms. With more mathematics-writing measures focused on different content, we could collect a more in-depth understanding of how students use mathematics vocabulary.

Conclusion

This study expands on the current research on mathematics writing by examining the performance of students with MD on a mathematics-writing measure. Overall, students with MD experienced difficulty with mathematics writing. Interestingly, participation in an intensive intervention led to marginally higher scores in Mathematics Content of the mathematics writing. Additionally, students with MD often included formal mathematics vocabulary, symbolic numbers, symbolic symbols, and general vocabulary in their written responses. Although this study provides detail about how students with MD perform in mathematics writing and what they include in their mathematics-writing samples, more research is needed to understand how to assess and support students with MD in mathematics writing.

References

Andersson, U. (2010). Skill development in different components of arithmetic and basic cognitive functions: Findings from a 3-year longitudinal study of children with different types of learning difficulties. *Journal of Educational Psychology, 102*(1), 115–134. https://doi.org/10.1037/a0016838

Agrawal, J., & Morin, L. L. (2016). Evidence-based practices: Applications of concrete representational abstract framework across math concepts for students with mathematics disabilities. *Learning Disabilities Research & Practice, 31*(1), 34–44. https://doi.org/10.1111/ldrp.12093

Arsenault, T. L., & Powell, S. R. (2022). Word-problem performance differences by schema: A comparison of students with and without mathematics difficulty. *Learning Disabilities Research & Practice, 37*(1), 37–50. https://doi.org/10.1111/ldrp.12273

Baxter, J. A., Woodward, J., & Olson, D. (2005). Writing in mathematics: An alternative form of communication for academically low-achieving students. *Learning Disabilities Research & Practice, 20*(2), 119–135. https://doi.org/10.1111/j.1540-5826.2005.00127.x

Berninger, V. W. (2009). Highlights of programmatic, interdisciplinary research on writing. *Learning Disabilities Research & Practice, 24*(2), 69–80. https://doi.org/10.1111/j.1540-5826.2009.00281.x

Björn, P. M., Aunola, K., & Nurmi, J.-E. (2016). Primary school text comprehension predicts mathematical word problem-solving skills in secondary school. *Educational Psychology, 36*(2), 362–377. https://doi.org/10.1080/01443410.2014.992392

Casa, T. M., Firmender, J. M., Cahill, J., Cardetti, F., Choppin, J. M., Cohen, J., Cole, S., Colonnese, M. W., Copley, J., DiCicco, M., Dieckmann, J., Dorl, J., Gavin, M. K., Hebert, M. A., Karp, K. S., LaBella, E., Moschkovich, J. N., Moylan, K., Olinghouse, N. G., Powell, S. R., Price, E., Pugalee, D. K., Rupp Fulwiler, B., Sheffield, L. J., & Zawodniak, R. (2016). *Types of and purposes for elementary mathematical writing: Task force recommendations.* https://mathwriting.education.uconn.edu/wp-content/uploads/sites/1454/2016/04/Types_of_and_Purposes_for_Elementary_Mathematical_Writing_for_Web-2.pdf

Cirino, P. T., Fuchs, L. S., Elias, J. T., Powell, S. R., & Schumacher, R. F. (2015). Cognitive and mathematical profiles for different forms of learning difficulties. *Journal of Learning Disabilities, 48*(2), 156–175. https://doi.org/10.1177/0022219413494239

Cohen, J. (1992). A power primer. *Psychological Bulletin, 112*(1), 155. https://doi.org/10.1037/0033-2909.112.1.155

Cohen, J. A., Miller, H. C., Casa, T. M., & Firmender, J. M. (2015). Characteristics of second graders' mathematical writing. *School Science and Mathematics, 115*, 344–355. https://doi.org/10.1111/ssm.12138

Forsyth, S. R., & Powell, S. R. (2017). Differences in the mathematics-vocabulary knowledge of fifth-grade students with and without learning difficulties. *Learning Disabilities Research & Practice, 32*(4), 231–245. https://doi.org/10.1111/ldrp.12144

Fuchs, L. S., Fuchs, D., Compton, D. L., Hamlett, C. L., & Wang, A. Y. (2015). Is word-problem solving a form of text comprehension? *Scientific Studies of Reading, 19*, 204–223. https://doi.org/10.1080/10888438.20151005745

Fuchs, L. S., Fuchs, D., Seethaler, P. M., Cutting, L. E., & Mancilla-Martinez, J. (2019). Connections between reading comprehension and word-problem solving via oral language comprehension: Implications for comorbid learning disabilities. *New Directions for Child and Adolescent Development, 2019*(165), 73–90. https://doi.org/10.1002/cad.20288

Geary, D. C., Hoard, M. K., Nugent, L., & Bailey, D. H. (2012). Mathematical cognition deficits in children with learning disabilities and persistent low achievement: A five-year prospective study. *Journal of Educational Psychology, 104*(1), 206–223. https://doi.org/10.1037/a0025398

Glogger, I., Schwonke, R., Holzäpfel, L., Nückles, M., & Renkl, A. (2012). Learning strategies assessed by journal writing: Prediction of learning outcomes by quantity, quality, and combinations of learning strategies. *Journal of Educational Psychology, 104*(2), 452–468. https://doi.org/10.1037/a0026683

Graham, S., Collins, A. A., & Rigby-Wills, H. (2017). Writing characteristics of students with learning disabilities and typically achieving peers: A meta-analysis. *Exceptional Children, 83*(2), 199–218. https://doi.org/10.1177/0014402916664070

Graham, S., Kiuhara, S. A., & MacKay, M. (2020). The effects of writing on learning in science, social studies, and mathematics: A meta-analysis. *Review of Educational Research, 90*(2), 179–226. https://doi.org/10.3102/0034654320914744

Hebert, M. A., & Powell, S. R. (2016). Examining fourth-grade mathematics writing: Features of organization, mathematics vocabulary, and mathematical representations. *Reading and Writing, 29*(7), 1511–1537. https://doi.org/10.1007/s11145-016-9649-5

Hebert, M. A., Powell, S. R., Bohaty, J. J., & Roehling, J. (2019). Piloting a mathematics writing intervention with late elementary students at-risk for learning difficulties. *Learning Disabilities Research and Practice, 34*(3), 144–157. https://doi.org/10.1111/ldrp.12202

Hecht, S. A., & Vagi, K. J. (2010). Sources of group and individual differences in emerging fraction skills. *Journal of Educational Psychology, 102*(4), 843–859. https://doi.org/10.1037/a0019824

Hughes, E. M., & Lee, J.-Y. (2020). Effects of a mathematical writing intervention on middle school students' performance. *Reading & Writing Quarterly, 36*(2), 1–17. https://doi.org/10.1080/10573569.2019.1677537

Hughes, E. M., Lee, J.-Y., Cook, M. J., & Riccomini, P. J. (2019). Exploratory study of a self-regulation mathematical writing strategy: Proof-of-concept. *Learning Disabilities: A Contemporary Journal, 17*(2), 185–203.

Hughes, E. M., Riccomini, P. J., & Lee, J.-Y. (2020). Investigating written expressions of mathematical reasoning for students with learning disabilities. *The Journal of Mathematical Behavior, 58*, 100775. https://doi.org/10.1016/j.jmathb.2020.100775

Idris, N. (2009). Enhancing students' understanding in calculus trough writing. *International Electronic Journal of Mathematics Education, 4*(1), 36–55. https://doi.org/10.29333/iejme/229

Jitendra, A. K., Nelson, G., Pulles, S. M., Kiss, A. J., & Houseworth, J. (2016). Is mathematical representation of problems an evidence-based strategy for students with mathematics difficulties? *Exceptional Children, 83*(1), 8–25. https://doi.org/10.1177/0014402915625062

Jitendra, A. K., Rodriguez, M., Kanive, R., Huang, J.-P., Church, C., Corroy, K. A., & Zaslofsky, A. (2013). Impact of small-group tutoring interventions on the mathematical problem solving and achievement of third-grade students with mathematics difficulties. *Learning Disability Quarterly, 36*(1), 21–35. https://doi.org/10.1177/0731948712457561

Jordan, N. C., & Hanich, L. B. (2000). Mathematical thinking in second grade children with different forms of LD. *Journal of Learning Disabilities, 33*, 567–578. https://doi.org/10.1177/002221940003300605

Kasmer, L. A., & Kim, O. K. (2012). The nature of student predictions and learning opportunities in middle school algebra. *Educational Studies in Mathematics, 79*(2), 175–191. https://doi.org/10.1007/s10649-011-9336-z

Kostos, K., & Shin, E. (2010). Using math journals to enhance second graders' communication of mathematical thinking. *Day Care and Early Education, 38*(3), 223–231. https://doi.org/10.1007/s10643-010-0390-4

Kiuhara, S. A., Rouse, A. G., Dai, T., Witzel, B. S., Morphy, P., & Unker, B. (2020). Constructing written arguments to develop fraction knowledge. *Journal of Educational Psychology, 112*(3), 584–607. https://doi.org/10.1037/edu0000391

Lim, L., & Pugalee, D. K. (2004). Using journal writing to explore they communicate to learn mathematics and they learn to communicate mathematically. *Ontario Action Researcher, 7*(2), 17–24.

Mancl, D. B., Miller, S. P., & Kennedy, M. (2012). Using the concrete-representational-abstract sequence with integrated strategy instruction to teach subtraction with regrouping to students with learning disabilities. *Learning Disabilities Research & Practice, 27*(4), 152–166. https://doi.org/10.1111/j.1540-5826.2012.00363.x

Monroe, E. E., & Panchyshyn, R. (1995). Vocabulary considerations for teaching mathematics. *Childhood Education, 72*(2), 80–83. https://doi.org/10.1080/00094056.1996.10521849

Moran, A. S., Swanson, H. L., Gerber, M. M., & Fung, W. (2014). The effects of paraphrasing interventions on problem-solving accuracy for children at risk for math disabilities. *Learning Disabilities Research and Practice, 29*(3), 97–105. https://doi.org/10.1111/ldrp.12035

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Author.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common core state standards mathematics*. http://corestandards.org/

Nelson, G., & Powell, S. R. (2018a). A systematic review of longitudinal studies of mathematics difficulty. *Journal of Learning Disabilities, 51*(6), 523–539. https://doi.org/10.1177/0022219417714773

Nelson, G., & Powell, S. R. (2018b). Computation error analysis: Students with mathematics difficulty compared to typically achieving students. *Assessment for Effective Intervention, 43*(3), 144–156. https://doi.org/10.1177/153450841774567

Norton, A., & Rutledge, Z. (2010). Measuring task posing cycles: Mathematical letter writing between algebra students and preservice teachers. *The Mathematics Educator, 19*(2), 32–45.

Peltier, C., & Vannest, K. J. (2017). A meta-analysis of schema instruction on the problem-solving performance of elementary school students. *Review of Educational Research, 87*(5), 899–920. https://doi.org/10.3102/0034654317720163

Peng, P., & Lin, X. (2019). The relation between mathematics vocabulary and mathematics performance among fourth graders. *Learning and Individual Differences, 69*, 11–21. https://doi.org/10.1016/j.lindif.2018.11.006

Powell, S. R., & Hebert, M. A. (2016). Influence of writing ability and computation skill on mathematics writing. *The Elementary School Journal, 117*(2), 310–335. https://doi.org/10.1086/688887

Powell, S. R., Hebert, M. A., Cohen, J. A., Casa, T. M., & Firmender, J. M. (2017). A synthesis of mathematics writing: Assessments, interventions, and surveys. *Journal of Writing Research, 8*(3), 493–526. https://doi.org/10.17239/jowr-2017.08.03.04

Powell, S. R., Hebert, M. A., & Hughes, E. M. (2021a). How educators use mathematics writing in the classroom: A national survey of mathematics educators. *Reading and Writing: An Interdisciplinary Journal, 34*(2), 417–447. https://doi.org/10.1007/s11145-020-10076-8

Powell, S. R., Berry, K. A., Fall, A-M., Roberts, G., Fuchs, L. S., & Barnes, M. A. (2021b). Alternative paths to improved word-problem performance: An advantage for embedding pre algebraic reasoning instruction within word-problem intervention. *Journal of Educational Psychology, 113*(5), 898–910. https://doi.org/10.1037/edu0000513

Powell, S. R., & Hebert, M. A. (2022). Thinking beyond symbols: Writing and reading in mathematics. In Z. A. Philippakos & S. Graham (Eds.), *Writing and reading connections: Bridging research and classroom practice* (pp. 246–264). Guilford

Riley, M. S., & Greeno, J. G. (1988). Developmental analysis of understanding language about quantities and of solving problems. *Cognition and Instruction, 5*(1), 49–101. https://doi.org/10.1207/s1532690xci0501_2

Shield, M., & Galbraith, P. (1998). The analysis of student expository writing in mathematics. *Educational Studies in Mathematics, 36*, 29–52. https://doi.org/10.1023/a:1003109819256

Stonewater, J. K. (2002). The mathematics writer's checklist: The development of a preliminary assessment tool for writing in mathematics. *School Science and Mathematics, 102*(7), 324–334. https://doi.org/10.1111/j.1949-8594.2002.tb18216.x

Tan, T., & Garces-Bacsal, R. M. (2016). The effect of journal writing on mathematics achievement among high-ability students in Singapore. *Gifted and Talented International, 28*(1–2), 173–184. https://doi.org/10.1080/15332276.2013.11678412

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