



# C–H activation of ethane on palladium clusters: a computational study at the dual levels of density functional theory and coupled-cluster theory

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## Abstract

We studied the C–H activation of ethane on the palladium (Pd) atomic clusters at the CCSD(T)/TZ//B3LYP/DZ level of theory, where TZ and DZ denote the LANL2 valence triple- $\zeta$  and double- $\zeta$  basis sets that include a relativistic pseudopotential for the Pd core electrons. In this study, we globally optimized first the Pd<sub>n</sub> clusters ( $n = 1–8$ ) and then the transition state (TS) structures of Pd<sub>n</sub> + C<sub>2</sub>H<sub>6</sub> → H–Pd<sub>n</sub>–C<sub>2</sub>H<sub>5</sub>. For each cluster size, we studied four spin states ( $S = 0, 1, 2, 3$ ). The Pd atom is in the singlet electronic state ([Kr]4d<sup>10</sup>) at the CCSD(T) level. The CCSD(T) global minima of the Pd<sub>3</sub>, Pd<sub>7</sub>, and Pd<sub>8</sub> clusters are also in the singlet electronic state, whereas the CCSD(T) global minima of Pd<sub>2</sub>, Pd<sub>4</sub>, Pd<sub>5</sub>, and Pd<sub>6</sub> are in the triplet electronic state. The atomization energy of Pd<sub>n</sub> increases monotonically with the cluster size. Pd<sub>4</sub> and Pd<sub>6</sub> are particularly stable relative to their neighboring sizes. Among all sizes, Pd<sub>4</sub> is the least active toward the C–H bonds in ethane, followed by sizes 5, 7, 3, and 1, whereas Pd<sub>2</sub>, Pd<sub>6</sub>, and Pd<sub>8</sub> are the most active: the enthalpy of activation of the Pd<sub>n</sub> + C<sub>2</sub>H<sub>6</sub> → H–Pd<sub>n</sub>–C<sub>2</sub>H<sub>5</sub> reaction at room temperature are –29, –21, and 8 kJ/mol at these three sizes, respectively, indicating their strong ability to activate ethane. Among these three sizes, Pd<sub>2</sub> is highly unstable and thus less ideal. Pd<sub>6</sub> and Pd<sub>8</sub> are both energetically stable and active toward the C–H bonds of ethane.

**Keywords** Palladium · Ethane · Ethene · Dehydrogenation · B3LYP · CCSD(T)

## Introduction

Every year more than 100 million tons of ethylene are produced from ethane [1]. The rate-limiting step of the conversion from ethane to ethene involves the C–H bond breaking in ethane. The energy barrier of this rate-limiting step can

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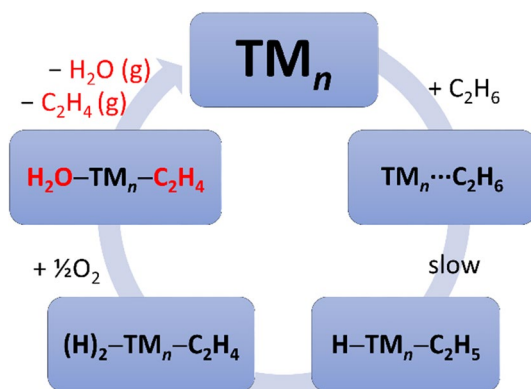
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be reduced by using catalysts that effectively weaken or break a C–H bond in ethane. Considering the magnitude at which this reaction occurs, it is significant to investigate new catalysts to further bring the energy barrier down. Lowering the energy barrier for breaking a C–H bond in ethane, even by only a few kJ/mol, could potentially be valuable. Therefore, numerous efforts have been made to find novel catalysts to break the C–H bond more easily [2–36]. We have also been studying the atomic clusters of various transition metals (TM) as potential catalysts for the production of ethene via the oxidative dehydrogenation (ODH) of ethane (the catalytic cycle is depicted in Fig. 1) [15, 22, 23].

In our previous density functional theory (DFT) studies on the abilities of the Pt, Ir, and Rh atomic clusters to break a C–H bond in ethane (and propane) [15, 22, 23], several common features were observed. In all three cases, the C–H bond is weakened and eventually broken by the transition metal cluster, not because the d electrons of the TM cluster flow into the  $\sigma^*$  bonding orbital of ethane, but because the  $\sigma$  bonding electrons in ethane flow into an empty d orbital of the transition metal cluster. The direction of the electron flow is evidenced directly by the Mulliken charge analysis; this is also evidenced indirectly by the correlation between the catalytic ability of a TM cluster and the diffusiveness of its LUMO that can accept electrons from ethane. In addition, we compared the TM cluster-mediated bond breaking in ethane and propane: the additional electron-pushing methyl group in propane lowers the energy barrier of the C–H bond cleavage. This substituent effect supports our conjecture that the C–H bond is weakened or broken by the Pt/Ir/Rh clusters because these TM clusters have empty d orbitals that readily accept the  $\sigma$  bonding electrons of ethane. In addition, we found that the natural bond orbital (NBO) charges [37, 38] and Wiberg bond indexes (WBIs) [39–41] may serve as reliable tools for prescreening of the various catalytic sites on the Ir and Rh clusters, and potentially other similar transition metal (e.g., Pt) clusters that activate the C–H bonds of ethane and other light alkane [23].

After the Pt/Ir/Rh clusters were studied, Pd, the remaining element in the  $2 \times 2$  square of  $^{45}\text{Rh}$ ,  $^{46}\text{Pd}$ ,  $^{77}\text{Ir}$ , and  $^{78}\text{Pt}$  in the periodic table is studied with a focus on its atomic clusters' thermal stability and activity toward ethane. The ground state electron configurations of Rh, Pd, Ir, and Pt are  $[\text{Kr}]4d^85s^1$ ,  $[\text{Kr}]4d^{10}$ ,  $[\text{Xe}]4f^{14}5d^76s^2$ ,

**Fig. 1** The catalytic cycle of the ODH of ethane. The step labeled “slow” is the presumably rate-limiting step of the catalytic cycle. Molecular formulas in red font are either vibrationally energetic in the TM complex or translationally energetic in the gas phase



and  $[\text{Xe}]4f^{14}5d^96s^1$ , respectively. Pd is special because, among these four elements, the Pd atom is the only one with completely filled d orbitals and thus may lack the ability to accommodate the  $\sigma$  bonding electrons of ethane. Since every subshell of the Pd atom is either completely filled or completely empty, it would be interesting to see how Pd atoms form metallic bonds between each other and how the Pd clusters interact with ethane. While numerous computational studies of the Pd atomic clusters or Pd-containing bimetal alloys and their interactions with hydrocarbons have been performed and published, most of these studies, if not all, have their accuracy levels limited by the computational methods, typically DFT methods that may not describe dispersion accurately, in their studies [42–50]. Therefore in this study, we used the B3LYP method coupled with Grimme's empirical dispersion to obtain the optimized structures of the Pd clusters and the transition states (TS) for the Pd cluster-mediated C–H bond-breaking reactions. For an enhanced level of accuracy, we further used the “gold standard” of computational chemistry, the CCSD(T) method, to study the energetics of the Pd clusters and their reactions with ethane. The ideal sizes of the Pd atomic clusters were then proposed based on a balance between their energetic stability and activity toward ethane.

## Computational methods

The B3LYP hybrid GGA method [51–53] was used in this study to carry out the optimization and frequency calculations of the Pd atomic clusters and the transition states (TS) of the  $\text{Pd}_n + \text{C}_2\text{H}_6 \rightarrow \text{H-Pd}_n\text{-C}_2\text{H}_5$  reactions. The D3 empirical dispersion formulated by Grimme with Becke–Johnson damping [54] (D3BJ) was included in the B3LYP calculations. Hereafter, this computing scheme is denoted by B3LYP-D3BJ. The vibrational frequency calculations ensure that the  $\text{Pd}_n$  local minima have zero imaginary frequencies, whereas every TS structure has exactly one imaginary frequency that involves the interaction between the Pd cluster and a C–H bond of ethane. Some highly symmetric  $\text{Pd}_n$  structures such as the linear  $\text{Pd}_3$  and the square planar  $\text{Pd}_4$  were found to have one or even more imaginary frequencies. The intrinsic reaction coordinate (IRC) calculations, a.k.a. the minimum energy path (MEP) calculations, were carried out to ensure that each identified transition state connects the reactants ( $\text{Pd}_n + \text{C}_2\text{H}_6$ ) and the desired product ( $\text{H-Pd}_n\text{-C}_2\text{H}_5$ ).

In the B3LYP-D3BJ optimization and frequency calculations, polarized valence double- $\zeta$  basis sets were used. The LANL2DZ basis set and the LANL2 effective core potential (ECP) [55], augmented with f-type polarization functions [56], were employed on the Pd atoms. The polarized valence double- $\zeta$  6-31G(d) basis sets were employed on the C, H, and O atoms [57, 58]. Hereafter, this combination of the valence double- $\zeta$  basis sets is denoted by DZ. Overall this B3LYP-D3BJ/DZ computing scheme used in the optimization, frequency, and IRC calculations is the same as or very similar to those used in our previous studies on the C–H activation by the Pt, Ir, and Rh atomic clusters [15, 22, 23]; it is also similar to Liu et al.'s calculations on the functionalization of C–H bonds [59]. The natural bond orbital (NBO) [37, 38] analysis was carried out using version 3.1 of the NBO program [60].

The CCSD(T) single-point energy calculations were carried out on the B3LYP-D3BJ/DZ structures to achieve a higher level of accuracy. In the CCSD(T) calculations, The valence triple- $\zeta$  LANL2TZ basis set and the LANL2 effective core potential (ECP) [55], augmented with f-type polarization functions [56], were employed on the Pd atoms. The polarized valence triple- $\zeta$  6-311G(d) basis sets were employed on the C, H, and O atoms [57, 58]. Hereafter, this well-balanced combination of valence triple- $\zeta$  basis sets is denoted by TZ. The CCSD(T) computing scheme is denoted by CCSD(T)/TZ//B3LYP-D3BJ/DZ or simply CCSD(T)//B3LYP in short for visual clarity. The 298 K internal energy, 298 K enthalpy (H), and 298 K Gibbs free energy (G) presented in this study are the sums of the CCSD(T)/TZ single-point electronic energy and the corresponding thermal corrections calculated at the B3LYP-D3BJ/DZ level of theory.

For the smallest Pd<sub>2</sub> and Pd<sub>3</sub> clusters, we were able to carry out the geometry optimization calculations at the CCSD(T)/TZ level to compare the CCSD(T)/TZ and B3LYP-D3BJ/DZ optimized structures. The CCSD(T) optimization calculations were completed numerically due to the lack of analytical gradients at the CCSD(T) level of theory. We also used the double-differentiation numerical method to obtain the CCSD(T) vibrational frequencies of the Pd<sub>2</sub> clusters in the singlet and triplet electronic states.

The B3LYP and other DFT calculations were carried out with the Gaussian 09 program (Windows version) [61]. The CCSD(T) calculations were carried out with the Gaussian 16 program (Linux version) [62].

## Results and discussion

### CCSD(T)/TZ vs B3LYP-D3BJ/DZ

In this study, we compared the performance of the hybrid and pure DFT methods against the CCSD(T) benchmark and also against the experimental data (0.81 eV) [63] of the singlet–triplet (S-T) gap of the Pd atom. The S-T gap is simply the energy of the triplet electronic state of the Pd atom minus its energy in the singlet electronic state. Table 1 shows the comparison of the S-T gap of the Pd atom using the hybrid B3LYP and the pure BLYP DFT methods, along with the CCSD(T) method. Two different basis sets were used in the calculations: a smaller double- $\zeta$  LANL2DZ(f) basis set and a larger triple- $\zeta$  LANL2TZ(f) basis set. The table shows that as the basis set size increases, the calculated singlet–triplet gap decreases. The difference between the BLYP and B3LYP results is negligible (0.83 vs. 0.83 eV) when the double- $\zeta$  basis set is used but becomes more appreciable (0.72 eV vs. 0.69 eV) when the larger triple- $\zeta$  basis set is used. Among all these computing schemes, the B3LYP and BLYP calculations with the smaller DZ basis set result in the smallest discrepancies (only 0.02 eV) from the experimental value. Overall, the B3LYP values are even slightly more accurate than the CCSD(T) calculations with either the smaller DZ or the larger TZ basis set. In theory, the higher-level CCSD(T) method should be more accurate than the DFT methods when an infinitely large basis set is used. Therefore, the better performance of the B3LYP/DZ method is likely due

**Table 1** CCSD(T) and DFT calculated energies of the singlet and triplet Pd atom and the singlet–triplet (S-T) gap

Basis set	CCSD(T)		B3LYP		BLYP	
	DZ	TZ	DZ	TZ	DZ	TZ
<sup>1</sup> Pd (a.u.) <sup>a</sup>	−126.0780	−126.1200	−126.7068	−126.7075	−126.6138	−126.6155
<sup>3</sup> Pd (a.u.)	−126.0470	−126.0941	−126.6761	−126.6811	−126.5831	−126.5900
S-T gap (eV)	0.84	0.71	0.83	0.72	0.83	0.69
Error <sup>b</sup> (eV)	0.03	−0.10	0.02	−0.09	0.02	−0.12

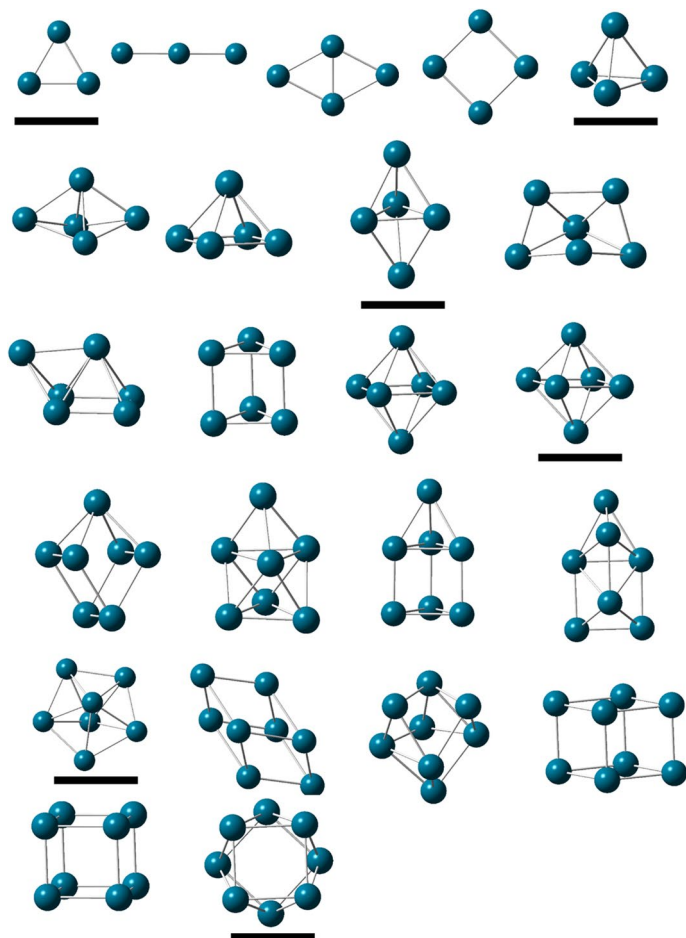
<sup>a</sup>1 a.u. = 27.2114 eV = 2625.50 kJ/mol

<sup>b</sup>The errors were calculated against the experimental value (0.81 eV)

to a fortuitous cancelation of errors for the studied system. Nonetheless, based on its agreement with experimental values of the S-T gap of the Pd atom as well as its success in the previous studies of the Ir and Rh systems [15, 23], we are sufficiently confident in using the B3LYP-D3BJ/DZ method for the geometry optimization and vibrational frequency calculations of the Pd clusters in this study.

Fig. 2 shows the 23 atomic structures of Pd<sub>*n*</sub> (*n* = 3–8) we optimized at the B3LYP-D3BJ/DZ level of theory. The structures of the Pd atom and the Pd<sub>2</sub> dimer were not included in the figure because of their simplicity. For Pd<sub>3</sub>, the triangular and linear structures were locally optimized. For sizes larger than 3, linear structures are no longer studied due to their much higher energies [64]. For Pd<sub>4</sub>, the rhombus, square planar, and tetrahedral structures were optimized. Because the planar structures of Pd<sub>4</sub> are much less stable than the nonplanar structures [64], planar structures were no longer considered beyond size 4. For Pd<sub>5</sub>, the square pyramidal and trigonal bipyramidal structures were optimized. For Pd<sub>6</sub>, the fused bi-tetrahedron, capped square pyramidal, trigonal prism, elongated/compressed octahedral, and octahedral structures were optimized. For Pd<sub>7</sub>, the capped trigonal prism, capped octahedral structures, and pentagon bipyramidal structures were optimized. For Pd<sub>8</sub>, the bicapped octahedral structure, the cubic structure, and various distorted cubic structures were optimized. In general, the choices of the initial structures of the Pd clusters were based on several previous studies [48, 64, 65].

For each atomic cluster structure, four different electronic structures with *S* = 0, 1, 2, 3 were studied, where *S* is the electron spin quantum number. The *S* = 3 spin states with six unpaired electrons were found to be highly unstable and therefore higher than *S* = 3 spin states were not examined in this study. The ground electronic state of the Pd atom is a singlet at the B3LYP and CCSD(T) levels. In partial agreement with some previous DFT studies [46, 48, 50, 64, 65], we found the CCSD(T) global minima of the Pd clusters are in a singlet electronic state at sizes 3, 7, and 8, whereas the global minima of Pd<sub>2</sub>, Pd<sub>4</sub>, Pd<sub>5</sub>, and Pd<sub>6</sub> are in the triplet electronic state. It was experimentally verified that the global minimum of Pd<sub>2</sub> is indeed in the triplet state [66, 67]. Our CCSD(T) predicted spin states of the Pd<sub>*n*</sub> global minima agree with the BLYP study [48] but differ from the PW91 DFT calculations by Zhang et al. [64], which suggest that the Pd<sub>*n*</sub> global minima are all in the triplet electronic states



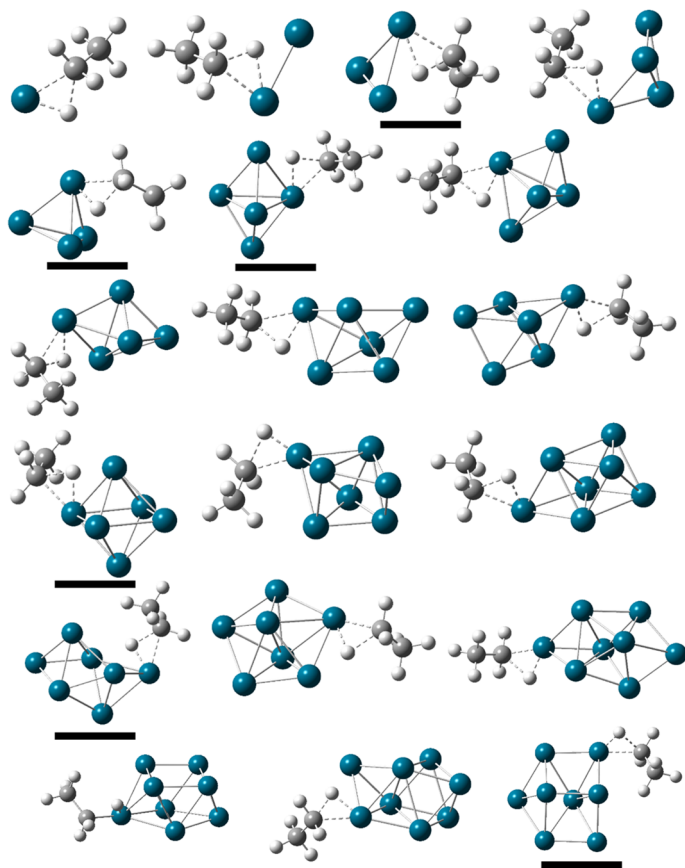
**Fig. 2** Optimized structures of  $\text{Pd}_n$  clusters ( $n=3-8$ ). The CCSD(T) global minimum structures are underscored. The spin quantum number  $S$  is 1 for sizes 2, 4, 5, and 6;  $S$  is 0 for all other sizes

except that  $\text{Pd}_8$  is in the singlet state [64]. Interestingly, another PW91 study conducted by Luo et al. [50] suggests that  $\text{Pd}_3$  and  $\text{Pd}_6$  global minima are in the singlet electronic state whereas sizes 2, 4, 5, 7, and 8 are in the triplet electronic state. There are two reasons why these two PW91 studies made different predictions regarding the spin states of the  $\text{Pd}_n$  global minima. One reason is that they used different basis sets: Zhang et al. used a plane wave basis set [64], whereas Luo et al. used a double numerical basis set [50]. Another reason is that the energy spacings between the  $\text{Pd}_n$  global and local minima are typically so small that slightly different computing schemes (e.g., different basis sets) may result in different energy rankings. Nonetheless, the atomic structures of the  $\text{Pd}_n$  global minima at the CCSD(T) level are the same as those proposed by Zhang et al. [64] and very close to those proposed by Luo et al. [50]. From size 3 to size 8, the global minimum structures at both the

CCSD(T) and PW91 [64] levels of theory are triangular, tetrahedral, trigonal bipyramidal, nearly octahedral, pentagonal bipyramidal, and twisted cubic (in which the bottom square of the cube is rotated 45 degrees about the vertical axis). The PW91 study by Luo et. al. proposed the same global minimum structures for all sizes except for size 8: they suggest that the  $\text{Pd}_8$  lowest-energy structure is a bicapped octahedron instead of a twisted cube. The CCSD(T) predicted  $\text{Pd}_n$  global minimum structures also agree with the BLYP [48] and BP86 [46] calculations in general. In these DFT studies, the global minimum structures of  $\text{Pd}_n$  ( $n=3-7$ ) are also triangular, tetrahedral, trigonal bipyramidal, nearly octahedral, and pentagonal bipyramidal. Size 8 was not examined in the BLYP study [48] whereas the BP86 [46] calculations suggest that the  $\text{Pd}_8$  global minimum is a bicapped octahedron. Overall, the PW91, BP86, and BLYP DFT methods predicted the same lowest-energy structures as our CCSD(T)//B3LYP-D3BJ method with very few exceptions [46, 48, 50, 64]. While the prior DFT calculations and our CCSD(T) calculations do not completely agree on the spin states of the  $\text{Pd}_n$  global minima, we are more confident with our CCSD(T) predictions for two reasons: one is that we included Grimme's empirical dispersion in the DFT calculations to obtain more accurate optimized structures; the other is that the CCSD(T) method we used in this study is widely accepted as the gold standard of computational chemistry and is typically more accurate and reliable than the DFT methods both in theory and in practice.

We then studied these Pd atomic clusters' abilities to activate a C–H bond in ethane. The 19 studied TS atomic structures of the  $\text{Pd}_n + \text{C}_2\text{H}_6 \rightarrow \text{H}-\text{Pd}_n-\text{C}_2\text{H}_5$  reactions ( $n=1-8$ ) are shown in Fig. 3. Fewer TS structures than the  $\text{Pd}_n$  structures are presented because the TS search starting from two different  $\text{Pd}_n$  structures may result in the same TS structure. For example, when a planar  $\text{Pd}_4$  atomic cluster inserts into a C–H bond of ethane, the four Pd atoms always become non-planar (nearly tetrahedral) in the TS structure. For each TS structure, four spin states ( $S=0-3$ ) were studied. The lowest-energy TS structures for nearly all sizes adopt the singlet electronic state ( $S=0$ ) except for size 7 ( $S=1$ ). This is likely because the introduction of ethane into the system breaks the high symmetry of the molecular orbitals (MOs) of the Pd atomic clusters and eliminates the (near) degeneracy of the MOs of the Pd clusters; the energy spacings between adjacent MOs become so large that the electrons occupy the lowest possible MOs, two by two, dictated by the Aufbau principle. As expected, the  $S=3$  high spin state of every TS structure has much higher energy than all other spin states in all cases. This is because too many electrons are forced to occupy high-energy MOs in the  $S=3$  electronic state. This energy increase cannot be sufficiently compensated by the energy decrease due to the extra negative exchange energy and the smaller e–e repulsion in the high-spin state. For this reaction, no higher spin state than  $S=3$  is of interest or further examined in this study.

Fig. 4 compares the CCSD(T)/TZ and B3LYP-D3BJ/DZ electronic energies for the Pd clusters (4a) and the TS structures (4b). The correlation is nearly perfect with  $R^2=1$  in both cases. The CCSD(T) energy is approximately 99.53% of the B3LYP-D3BJ energy for Pd atomic clusters and 99.55% for the TS structures. The very small 0.02% difference in percentage is mostly due to the introduction



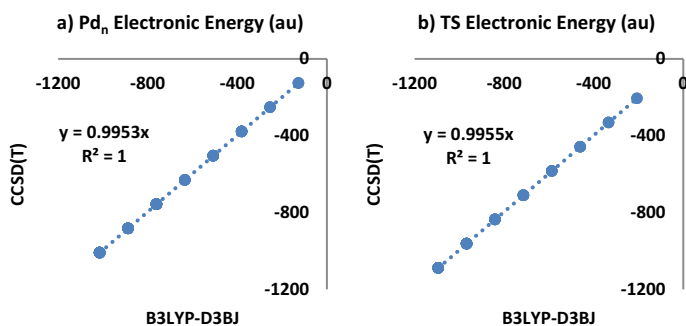
**Fig. 3** The optimized TS structures of the  $\text{Pd}_n + \text{C}_2\text{H}_6 \rightarrow \text{H-Pd}_n - \text{C}_2\text{H}_5$  reactions ( $n=1-8$ ). The CCSD(T) global minimum structures are underscored. The spin quantum number  $S$  is 1 for size 7 and 0 for all other sizes

of ethane in the TS structure. The CCSD(T)/TZ singlet point energy of a free ethane molecule is 99.73% of its B3LYP-D3BJ/DZ energy.

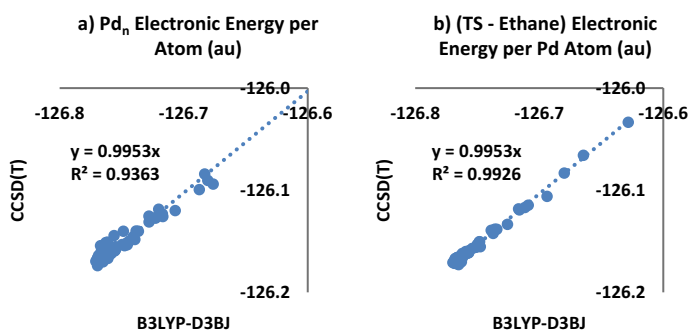
The correlations between the CCSD(T) and B3LYP-D3BJ energies are virtually perfect partly because of the vast difference in energy between the Pd clusters of different sizes. Therefore, we also compared the energy per Pd atom for all cluster sizes in Fig. 5. For the TS structures, we first subtract the energy of free ethane from the total energy, and then divide the remaining energy by the number of Pd atoms.

Fig. 5 demonstrates satisfactory linear correlations between the CCSD(T) and B3LYP-D3BJ energies per Pd atom. The CCSD(T) energy per Pd atom is 99.53% of the B3LYP/D3BJ energy in both Pd atomic clusters and in TS structures (the energy of free ethane is subtracted from the TS energy). The correlation between the CCSD(T) and B3LYP-D3BJ results for the TS structures ( $R^2=0.9926$ ) is notably





**Fig. 4** CCSD(T)/TZ vs B3LYP/DZ electronic energies for **a** the Pd<sub>n</sub> atomic clusters and **b** the TS structures of the Pd<sub>n</sub>+C<sub>2</sub>H<sub>6</sub>→H–Pd<sub>n</sub>–C<sub>2</sub>H<sub>5</sub> reactions



**Fig. 5** CCSD(T)/TZ vs B3LYP/DZ electronic energies per Pd atom for **a** the Pd<sub>n</sub> atomic clusters and **b** the TS structures (the ethane energy is subtracted)

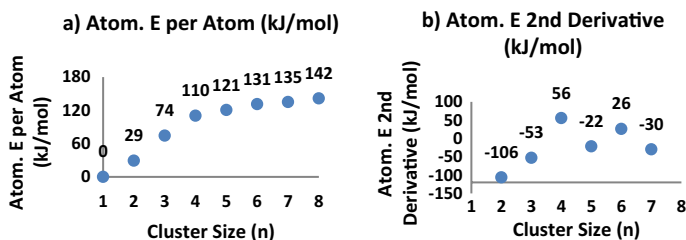
stronger than that for the Pd clusters ( $R^2 = 0.9363$ ). This is because the near degeneracy among multiple electronic states and the resulting spin contamination prevalent in the Pd atomic clusters are suppressed in the TS structures due to the introduction of the ethane. As expected, less spin contamination in the TS structures results in a better correlation between the density functional theory and the coupled cluster theory. The strong correlation between the CCSD(T)/TZ and B3LYP-D3BJ/DZ energies, in total or per Pd atom, serves as indirect evidence of the satisfactory accuracy of the B3LYP-D3BJ optimized geometries. We further compared the CCSD(T)/TZ and B3LYP-D3BJ/DZ optimized structures for Pd<sub>2</sub> in the singlet and triplet electronic states. For the singlet electronic state, the CCSD(T)/TZ and B3LYP-D3BJ/DZ bond lengths are 2.820 and 2.764 Å, respectively. Compared to the benchmark CCSD(T)/TZ result, the B3LYP-D3BJ/DZ method underestimates the bond length by 0.056 Å. Note that the CCSD(T)/TZ bond distance of Pd<sub>2</sub> in the singlet electronic state (2.820 Å) is longer than the Pd–Pd distance in the face-centered cubic (FCC) crystalline structure of bulk Pd (2.748 Å) [68], which suggests an unusually weak bond in the Pd dimer in the singlet electronic state. For the triplet electronic state,

the CCSD(T)/TZ and B3LYP-D3BJ/DZ bond lengths are 2.506 and 2.554 Å, respectively. Compared with the experimental bond length (2.48 Å) [69], the CCSD(T)/TZ and B3LYP-D3BJ/DZ methods overestimate the bond length by 0.026 and 0.074 Å, respectively. Others' PW91, B3LYP, and BLYP calculations also overestimated the Pd–Pd bond distance by 0.01 [64], 0.06 [48], 0.05 [70], and 0.08 Å [71]. The PW91 [64] and CCSD(T) bond distances are more accurate than some other DFT calculations [48, 70, 71] and our own B3LYP-D3BJ calculations for the Pd dimer in the triplet electronic state. The discrepancies between the CCSD(T) and B3LYP-D3BJ bond distances (2.506 vs. 2.554 Å) of the Pd dimer do not significantly impact the accuracy of the CCSD(T) electronic energy: the CCSD(T)/TZ optimized energy is merely 0.4 kJ/mol or 0.8 kJ/mol lower than the CCSD(T)/TZ//B3LYP-D3BJ/DZ energy for the singlet or triplet electronic state, respectively. Both CCSD(T)/TZ and B3LYP-D3BJ/DZ calculations suggest that the Pd–Pd bond distance in the triplet state is significantly shorter than that of the singlet electronic state. This is because the theoretical Pd–Pd bond order is one in the triplet electronic state but zero in the singlet electronic state (see reasoning in “The Pd<sub>n</sub> atomic clusters”).

We also compared the CCSD(T)/TZ and B3LYP-D3BJ/DZ optimized structures for Pd<sub>3</sub> in the singlet and triplet electronic states. For the singlet electronic state, the CCSD(T)/TZ and B3LYP-D3BJ/DZ optimized structures are both equilateral triangles (D<sub>3h</sub> point group) with a bond length of 2.531 and 2.509 Å, respectively. The B3LYP-D3BJ/DZ method slightly underestimates the bond length by 0.022 Å. For the triplet electronic state, the CCSD(T)/TZ and B3LYP-D3BJ/DZ structures are not equilateral triangles but isosceles triangles (C<sub>2v</sub> point group) due to the Jahn–Teller effect. The CCSD(T)/TZ bond lengths are 2.596, 2.596, and 2.496 Å, whereas the B3LYP/DZ bond lengths are 2.573, 2.573, and 2.566 Å. The discrepancies between the CCSD(T) and the B3LYP/DZ bond lengths of Pd<sub>3</sub> (− 0.023, − 0.023, and 0.070 Å) do not significantly affect the accuracy of the CCSD(T) single-point energy either: the CCSD(T)/TZ optimized energy is only 0.5 kJ/mol or 1.9 kJ/mol lower than the CCSD(T)/TZ//B3LYP-D3BJ/DZ energy for the singlet or triplet electronic state, respectively.

### The Pd<sub>n</sub> atomic clusters

To understand the size effect on the energetics of these Pd clusters, we computed the atomization energy of these clusters with  $S=0, 1, 2, 3$  at the CCSD(T)/TZ level of theory. The Pd atom in the ground electronic state ([Kr]4d<sup>10</sup>5s<sup>0</sup>) is used as the zero reference in the calculation of atomization energies. Although multiple atomic structures, each with multiple electronic spin states, were calculated, only the maximum atomization energies per atom of the Pd clusters for each cluster size, and the corresponding second derivatives with respect to the cluster size, are shown in Fig. 6 for visual clarity. The singlet electronic state ( $S=0$ ) has the maximum atomization energy for sizes 1, 3, 7, and 8. Sizes 2, 4, 5, and 6 prefer the triplet state ( $S=1$ ). The lowest-energy structures of Pd<sub>3–8</sub> adopt triangular, tetrahedral, square pyramidal, octahedral, pentagonal bipyramidal, and twisted cubic shapes, respectively. The expression of the energy second derivative for size  $n$  is  $[E(n+1) + E(n-1) - 2E(n)]$ .



**Fig. 6** a Atomization energies per atom, abbreviated as Atom E., of the Pd atomic clusters, b the atomization energy second derivatives with respect to the atomic cluster size

A positive energy second derivative indicates energetic stability relative to the nearest neighboring sizes because the disproportionation reaction  $2\text{Pd}_n \rightarrow \text{Pd}_{n-1} + \text{Pd}_{n+1}$  is endothermic and thus less likely to occur at a finite temperature, whereas a negative second derivative indicates energetic instability relative to the nearest neighboring sizes.

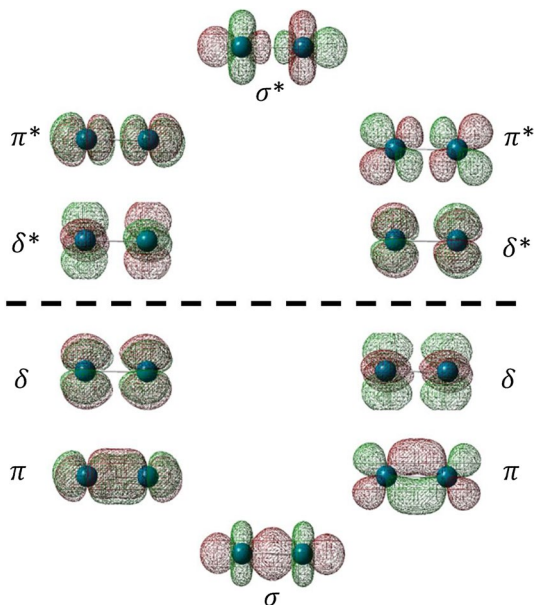
Fig. 6a shows that the atomization energy per atom increases monotonically with the cluster size as expected because the average number of bonds increases as the average number of surrounding Pd atoms increases as the cluster size increases. We were not able to calculate the energy second derivative for size 1 or 8 because they each have only one nearest neighbor in cluster size. In agreement with a global optimization study at the BLYP level [49], sizes 2, 3, 5, and 7 have negative second derivatives and thus are energetically unstable relative to their nearest neighbors.  $\text{Pd}_2$  is particularly unstable with the smallest atomization energy (29 kJ/mol) and the most negative energy second derivative ( $-106$  kJ/mol) at the CCSD(T) level.

$\text{Pd}_4$  and  $\text{Pd}_6$  have a positive second derivative (56 and 26 kJ/mol, respectively) and are relatively stable. Specifically, size 4 might be considered the magic number of the small Pd clusters as  $\text{Pd}_4$  has the largest energy second derivative. Although we cannot calculate the energy second derivative for  $\text{Pd}_8$ , the aforementioned global search at the BLYP level suggests that this size is also relatively stable [49].

Our CCSD(T) calculations suggest that  $^1\text{Pd}_3$  has much larger atomization energy per atom than  $^3\text{Pd}_2$  (74 vs 29 kJ/mol), which agrees with the multi-reference singles plus doubles configuration interaction (MRSDCI) calculations for the Pd dimer and trimer [72]. The Pd dimer in the singlet electronic state ( $^1\text{Pd}_2$ ) has even smaller atomization energy per atom (17 kJ/mol) than the triplet electronic state (29 kJ/mol). The very weak bond strength of  $^1\text{Pd}_2$  can be well understood by visualizing its ten highest occupied MOs in Fig. 7.

Among the 10 highest occupied MOs of  $^1\text{Pd}_2$  in Fig. 7, from the bottom up, there is one  $\sigma_{d-d}$  orbital, two degenerate  $\pi_{d-d}$  orbitals, two degenerate  $\delta_{d-d}$  orbitals, two degenerate  $\delta_{d-d}^*$  orbitals, two degenerate  $\pi_{d-d}^*$  orbitals, and one  $\sigma_{d-d}^*$  orbital. The bond order (B. O.) is zero in  $^1\text{Pd}_2$  because the ten d orbitals from the two Pd atoms form five bonding orbitals and five corresponding antibonding orbitals, all of which are doubly occupied. Note that  $\text{Pd}_2$  in the triplet electronic state ( $^3\text{Pd}_2$ ) has a bond order of 1. This is because when an excited  $^3\text{Pd}$  atom with two unpaired electrons ( $[\text{Kr}]4d^95s^1$ ) interacts with the  $^1\text{Pd}$  atom ( $[\text{Kr}]4d^{10}$ ) in the ground electronic state

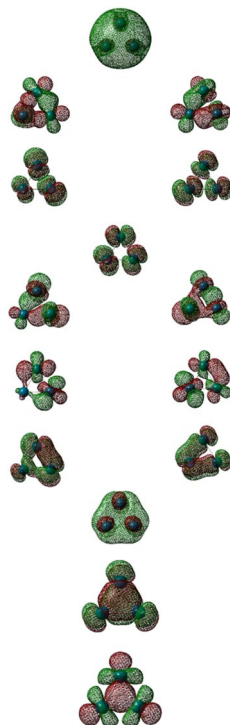
**Fig. 7** The ten highest occupied MOs of  ${}^1\text{Pd}_2$ . The dashed line separates the five d-d antibonding MOs from the five d-d bonding MOs



to form  ${}^3\text{Pd}_2$ , the  $\sigma_{4d-4d}$  and  $\sigma_{5s-5s}$  bond orders are both one half and therefore the overall bond order of  ${}^3\text{Pd}_2$  is one. The higher bond order of  ${}^3\text{Pd}_2$  is also evidenced by its higher CCSD(T) vibrational frequency ( $203\text{ cm}^{-1}$ ) than that of  ${}^1\text{Pd}_2$  ( $107\text{ cm}^{-1}$ ). Note that a higher vibrational frequency indicates a larger force constant ( $k$ ) and a deeper potential energy well ( $D_e$ ). Note that the Taylor expansion of the Morse expression,  $D_e(1 - e^{-\alpha(r-r_e)})^2$ , suggests that the depth of the potential energy well of a diatomic molecule  $D_e$  is approximately proportional to its force constant  $k$ . While the Pd–Pd bond strength is anticipated to be much greater in the triplet electronic state (B. O. = 1) than in the singlet electronic state (B. O. = 0) in the Pd dimer,  ${}^3\text{Pd}_2$  sits only 25 kJ/mol below  ${}^1\text{Pd}_2$  in the potential energy surface (PES). This is because the bond energy of the  ${}^3\text{Pd}_2$  is relative to a singlet Pd atom and a much higher-energy triplet Pd atom ( ${}^3\text{Pd}$  lies 68 kJ/mol higher than  ${}^1\text{Pd}$ ). Given the above data, we estimate the average strength of the  $\sigma_{4d-4d}$  and  $\sigma_{5s-5s}$  bonds between two Pd atoms is approximately  $25 + 68 = 93\text{ kJ/mol}$ .

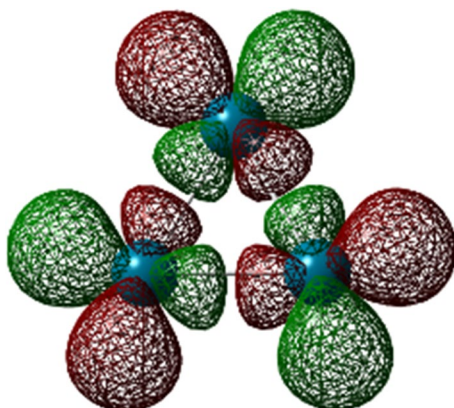
But if the Pd atoms in the ground singlet electronic state have only completely filled subshells, why can  ${}^1\text{Pd}_3$  have a much larger atomization energy per atom (74 kJ/mol) than  ${}^1\text{Pd}_2$  (17 kJ/mol) and even  ${}^3\text{Pd}_2$  (29 kJ/mol)? The 15 highest occupied MOs of  ${}^1\text{Pd}_3$  in Fig. 8 shed light: while 14 out of the 15 highest occupied MOs of  ${}^1\text{Pd}_3$  are linear combinations of the d orbitals as expected, the HOMO, however, is a linear combination of three 5 s atomic orbitals, one from each Pd atom. The total bond order of  ${}^1\text{Pd}_3$  is thus two. Both of these two bonds are three-center two-electron  $\sigma$  bonds, one being the overlap of three 5 s atomic orbitals and the other being the overlap of three 4d atomic orbitals. The latter results from the removal of the two electrons from the antibonding  $\sigma_{4d}^*$  orbital illustrated in Fig. 9. The empty  $\sigma_{4d}^*$  orbital (which is the LUMO of  ${}^1\text{Pd}_3$ ) allows for the net  $\sigma$  bonding among the 4d orbitals (B. O. = 1) in  ${}^1\text{Pd}_3$ .

**Fig. 8** The top 15 highest occupied molecular orbitals of  $^1\text{Pd}_3$ , among which 14 are interactions among the 4d orbitals, whereas the HOMO is the interaction among the 5s orbitals



In addition, there is also net  $\sigma$  bonding among the 5s orbitals (B. O. = 1) in  $^1\text{Pd}_3$ . Given the above data (the atomization energy per atom is 74 kJ/mol and the total bond order is 2), we estimate the average strength of the  $\sigma_{4d}$  and  $\sigma_{5s}$  bonds among the three Pd atoms in  $^1\text{Pd}_3$  is approximately  $74 \cdot 3/2 = 111$  kJ/mol, which is notably larger than that of  $^3\text{Pd}_2$  (93 kJ/mol). This is because in a Pd dimer, each  $\sigma_{4d}$  or  $\sigma_{5s}$  bond results from only one region of constructive interaction between the two Pd atoms, while in a Pd trimer each  $\sigma_{4d}$  or  $\sigma_{5s}$  bond results from three regions of constructive interaction among the

**Fig. 9** The LUMO of  $^1\text{Pd}_3$ , an empty  $\sigma_{4d}^*$  orbital that allows for the net  $\sigma$  bonding among the 4d orbitals (B. O. = 1) in  $^1\text{Pd}_3$



three Pd atoms in a triangle. In general, the atomization energy per atom in a Pd cluster increases as the cluster size increases mainly because the number of neighboring Pd atoms increases on average. For comparison, each Pd atom is surrounded by 12 nearest Pd atoms in the face-centered cubic structure (FCC) structure of bulk Pd, which has significantly larger atomization per atom (377 kJ/mol) than the small clusters studied in this work [68].

The CCSD(T) global minimum of Pd<sub>4</sub> adopts a nearly tetrahedral structure (with slight distortion due to the Jahn–Teller effect) in the triplet electronic state with an atomization energy per atom of 110 kJ/mol. Its singlet counterpart (<sup>1</sup>Pd<sub>4</sub>) has a higher energy mainly because all d-d antibonding orbitals are fully occupied in <sup>1</sup>Pd<sub>4</sub>. The atomization energy per atom slowly increases as the cluster size increases from sizes 5 to 8: 121 kJ/mol, 131 kJ/mol, 135 kJ/mol, and 142 kJ/mol, respectively. Since these four values are still significantly smaller than the atomization energy per atom of bulk Pd (377 kJ/mol) [68], we suspect that the atomization energy per atom will continue to slowly increase well beyond size 8 and even well beyond size 55 as suggested by a PW91 study [64].

### The TS of Pd<sub>n</sub> + C<sub>2</sub>H<sub>6</sub> → H–Pd<sub>n</sub>–C<sub>2</sub>H<sub>5</sub>

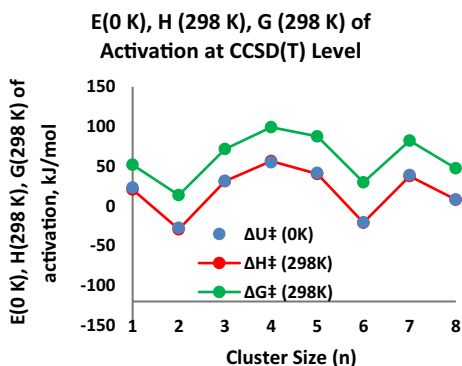
The internal energy (0 K), enthalpy (298 K), and Gibbs energy (298 K) of activation of the Pd<sub>n</sub> + C<sub>2</sub>H<sub>6</sub> → H–Pd<sub>n</sub>–C<sub>2</sub>H<sub>5</sub> reactions are presented in Fig. 10. Their values along with the electronic energy of activation are also provided in Table 2.

The 0 K internal energy of activation (including the zero-point vibrational energy) is calculated using the following equation:

$$\Delta U^\ddagger = U(\text{TS}) - U(\text{Pd}_n) - U(\text{ethane}) \quad (1)$$

In this equation,  $U(\text{TS})$  is the lowest possible internal energy of all TS structures optimized for the Pd<sub>n</sub> + C<sub>2</sub>H<sub>6</sub> → H–Pd<sub>n</sub>–C<sub>2</sub>H<sub>5</sub> reaction for each cluster size  $n$ ;  $U(\text{Pd}_n)$  is the lowest possible energy of all Pd<sub>n</sub> local minimum (LM) structures. Although other higher-energy TS structures and other higher-energy Pd<sub>n</sub> structures also make contributions to the C–H activation by Pd clusters, Eq. 1 provides

**Fig. 10** The internal energy of activation at 0 K, enthalpy of activation at 298 K, and Gibbs energy of activation at 298 K at the CCSD(T)/TZ level



**Table 2** The CCSD(T) electronic energy of activation, the internal energy of activation at 0 K, the enthalpy of activation at room temperature, and the Gibbs free energy of activation at room temperature. All values are in kJ/mol

<i>n</i>	$\Delta E_{elec}^{\ddagger}$	$\Delta U^{\ddagger}$ (0 K)	$\Delta H^{\ddagger}$ (298 K)	$\Delta G^{\ddagger}$ (298 K)
1	37	23	21	52
2	−12	−27	−29	14
3	47	32	31	72
4	69	55	56	99
5	56	42	40	88
6	−5	−21	−21	30
7	52	39	38	82
8	21	8	8	47

a simple yet representative means of quantifying the abilities of the Pd<sub>*n*</sub> clusters to activate a C-H bond in ethane. The electronic energy of activation ( $\Delta E_{elec}^{\ddagger}$ ) is equivalent to  $\Delta U^{\ddagger}$  excluding the zero-point vibrational energy (ZPVE).  $\Delta U^{\ddagger}$  is consistently smaller than  $\Delta E_{elec}^{\ddagger}$  by ~14 kJ/mol mainly because of one less C-H stretching vibrational mode in the TS.  $\Delta H^{\ddagger}$  and  $\Delta G^{\ddagger}$  at 298 K were calculated using the following equations:

$$\Delta H^{\ddagger} = H(TS) - H(Pd_n) - H(\text{ethane}) \quad (2)$$

$$\Delta G^{\ddagger} = G(TS) - G(Pd_n) - G(\text{ethane}) \quad (3)$$

In the above two equations,  $H(TS)$  or  $G(TS)$  is the lowest possible enthalpy or Gibbs energy of all TS structures for each cluster size *n*;  $H(Pd_n)$  or  $G(Pd_n)$  is the lowest possible enthalpy or Gibbs energy of all Pd<sub>*n*</sub> local minimum (LM) structures. Although the internal energy or Helmholtz energy of activation at room temperature is not presented in Fig. 10 or Table 2, they can be easily derived using the following equations:

$$\Delta U^{\ddagger} = \Delta H^{\ddagger} - \Delta(PV)^{\ddagger} = \Delta H^{\ddagger} - RT\Delta n^{\ddagger} = \Delta H^{\ddagger} + RT \quad (4)$$

$$\Delta A^{\ddagger} = \Delta G^{\ddagger} - \Delta(PV)^{\ddagger} = \Delta G^{\ddagger} - RT\Delta n^{\ddagger} = \Delta G^{\ddagger} + RT \quad (5)$$

where  $\Delta n^{\ddagger}$  is the change in the number of the moles of gas species ( $\Delta n^{\ddagger} = -1$  in a bimolecular reaction) and  $R$  is the gas constant. At 298 K,  $\Delta U^{\ddagger}$  or  $\Delta A^{\ddagger}$  is ~2.5 kJ/mol larger than  $\Delta H^{\ddagger}$  or  $\Delta G^{\ddagger}$ , respectively, assuming the ideal behavior of all gas particles.

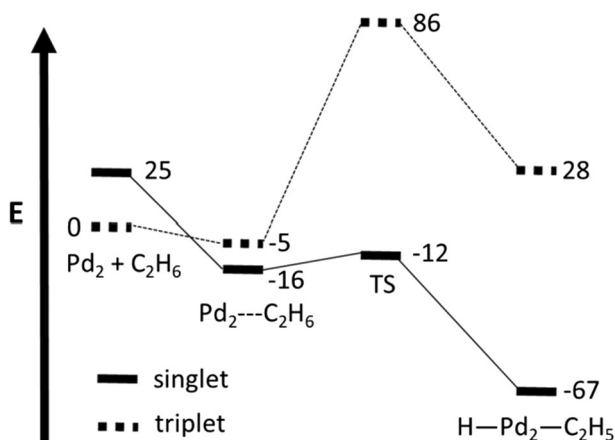
Pd<sub>4</sub> has the largest 0 K energy of activation (55 kJ/mol) and 298 K enthalpy of activation (57 kJ/mol) among all sizes likely due to its ultrastability. Pd<sub>2/6/8</sub> have the lowest energy and enthalpy of activation likely due to the large energy decrease when the Pd<sub>*n*</sub>-C<sub>2</sub>H<sub>6</sub> reactant complex is formed before overcoming the energy barrier. In particular, sizes 2 and 6 both have negative energy and enthalpy of activation. A negative energy or enthalpy of activation is possible mainly because the energy released in the formation of the Pd<sub>*n*</sub>-C<sub>2</sub>H<sub>6</sub> reactant complex

can be greater than the energy barrier this complex overcomes to form the TS structure. Fig. 11 shows that the singlet TS of  $\text{Pd}_2 + \text{C}_2\text{H}_6 \rightarrow \text{H-Pd}_2\text{-C}_2\text{H}_5$  lies 37 kJ/mol below the separate reactants in the singlet state and 12 kJ/mol below the separate reactants in the triplet state. Either way, the energy barrier is negative. The minimum energy path of this reaction starts from the triplet  $\text{Pd}_2$  and ethane. In the formation of the  $\text{Pd}_2\text{-C}_2\text{H}_6$  reactant complex, a spin flip can take place (due to the spin-orbit coupling or the spin exchange with nearby species) and all electrons become paired. The singlet  $\text{Pd}_2\text{-C}_2\text{H}_6$  then overcomes a small 4 kJ/mol energy barrier for the hydrogen shift from ethane to  $\text{Pd}_2$  and forms the singlet  $\text{H-Pd}_2\text{-C}_2\text{H}_5$ .

Negative energy barriers are not a unique phenomenon for  $\text{Pd}_n + \text{C}_2\text{H}_6 \rightarrow \text{H-Pd}_n\text{-C}_2\text{H}_5$ . They are also observed in the interaction between ethane and the Pt, Rh, or Ir atomic clusters when the  $\text{TM}_n\text{-C}_2\text{H}_6$  complex ( $\text{TM} = \text{Pt/Rh/Ir}$ ) has a lower energy than the TS of the  $\text{TM}_n + \text{C}_2\text{H}_6 \rightarrow \text{H-TM}_n\text{-C}_2\text{H}_5$  reaction for selected TM cluster sizes [15, 22, 23].

While  $\text{Pd}_2$  has the lowest energy, enthalpy, and Gibbs energy of activation and thus is considered the most active toward ethane, its small atomization energy per atom (29 kJ/mol) and negative second derivative of the atomization energy ( $-106$  kJ/mol) indicate its very poor thermal stability and a strong tendency to agglomerate or to undergo a disproportionation reaction. Among all studied cluster sizes,  $\text{Pd}_6$  and  $\text{Pd}_8$  well balance thermodynamic stability and catalytic activity. Their room-temperature Gibbs energies of activation were determined to be 30 and 47 kJ/mol ( $12RT$  and  $19RT$ ), respectively, when we considered only the global minima of the Pd cluster and TS for each of these two sizes.

The Gibbs energy of activation may be defined more thoroughly and thereby more accurately via a statistical means assuming the  $NPT$  condition in which the number of particles ( $N$ ), the pressure ( $P$ ), and temperature ( $T$ ) remain constant. For



**Fig. 11** The CCSD(T) potential energy surface of  $\text{Pd}_2 + \text{C}_2\text{H}_6 \rightarrow \text{H-Pd}_2\text{-C}_2\text{H}_5$  in the singlet (solid line) and triplet states (dotted line). ZPVE is excluded from the potential energy. All numbers are in kJ/mol



example, the Gibbs energy of activation of  $\text{Pd}_n$  under the isobaric condition may be calculated using the following equation:

$$\Delta G^\ddagger = G^{\text{apparent}}(\text{TS}) - G^{\text{apparent}}(\text{Pd}_n) - G(\text{ethane}) \quad (6)$$

where  $G^{\text{apparent}}$  is defined statistically as follows:

$$e^{-\frac{G^{\text{apparent}}}{RT}} = \sum_i^{\text{allstructures}} e^{-\frac{G_i}{RT}}$$

Note that  $\sum_i^{\text{allstructures}} e^{-\frac{G_i}{RT}}$  provides a reasonable estimate of the thermodynamic contribution of all structures in an ensemble under the constant  $PT$  condition. Using Eq. 6,  $\Delta G^\ddagger$  for  $\text{Pd}_{1-8}$  were determined statistically over the very large number of  $\text{Pd}_n$  and TS structures studied in this work. The resulting statistical  $\Delta G^\ddagger$  values are virtually the same as those obtained with Eq. 3 within a negligible discrepancy (0.2 kJ/mol for size 3 and 0.0 kJ/mol for all other sizes). Equations 3 and 5 may differ significantly when there exist multiple  $\text{Pd}_n$  structures with similar Gibbs energies or when there exist multiple TS structures with similar Gibbs energies. Equation 3 underestimates  $\Delta G^\ddagger$  in the former case and overestimates  $\Delta G^\ddagger$  in the latter case.

### Two electron-flow mechanisms of $\text{Pd}_n + \text{C}_2\text{H}_6 \rightarrow \text{H-Pd}_n\text{-C}_2\text{H}_5$

How is the C–H bond in ethane weakened by the Pd clusters? Does the Pd cluster take away C–H bonding electrons or donate electrons to the C–H antibonding orbitals? The calculations of the natural bonding orbital (NBO) charges of the lowest-energy TS structure of the  $\text{Pd}_n + \text{C}_2\text{H}_6 \rightarrow \text{H-Pd}_n\text{-C}_2\text{H}_5$  ( $n = 1-8$ ) reaction were carried out at the B3LYP-D3BJ level of theory to reveal the direction of the electron flow between the ethane molecule and the Pd cluster. Table 3 presents the total NBO charge on the Pd cluster in the TS structure. The total charges on the Pd clusters are less than 0.1 in magnitude for all eight sizes. This indicates that the *net* electron flow between the Pd cluster and the ethane molecule is minimal in their interactions.

We further examined the NBO charges on the individual Pd atoms in the  $\text{Pd}_6$  and  $\text{Pd}_8$  TS structures because they have a large atomization energy per atom (131 and

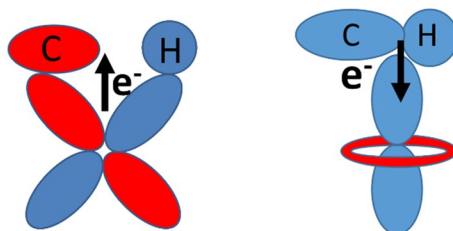
**Table 3** The total charge on  $\text{Pd}_n$  in the lowest-energy TS structures of the  $\text{Pd}_n + \text{C}_2\text{H}_6 \rightarrow \text{H-Pd}_n\text{-C}_2\text{H}_5$  ( $n = 1-8$ ) reaction

$n$	Charge on $\text{Pd}_n$
1	0.095
2	0.086
3	0.012
4	−0.025
5	0.063
6	0.061
7	0.001
8	−0.010

142 kJ/mol, respectively) and a small enthalpy of activation toward ethane ( $-21$  and  $8$  kJ/mol, respectively). The Pd atom partially bonded to the C and H atoms in the TS has a very small NBO charge ( $-0.097$  and  $-0.050$  for sizes 6 and 8 respectively). This minimal electron flow in the  $\text{Pd}_n + \text{C}_2\text{H}_6 \rightarrow \text{H-Pd}_n\text{-C}_2\text{H}_5$  reaction contrasts the much larger electron flow between ethane and  $\text{Ir}_8$  ( $a - 0.314$  NBO charge on the Ir atom partially bonded to C and H) [15]. For a similar  $\text{Rh}_8 + \text{C}_2\text{H}_6$  reaction, the Rh atom partially bonded to C and H also has a large  $-0.287$  NBO charge [23]. It is clear that the net electron flow is from  $\text{C}_2\text{H}_6$  to the empty d orbitals of the Ir and Rh clusters but this cannot be the entire story for the Pd clusters. Since the *net* electron flow between  $\text{Pd}_n$  and  $\text{C}_2\text{H}_6$  is very small compared to Ir and Rh systems, we conjecture that the Pd clusters weaken the C–H bond of ethane via two mechanisms concurrently. The first involves electron flow from the 4d orbitals of the Pd cluster to the  $\sigma^*$  antibonding orbitals in ethane, and the second involves electron flow from the occupied  $\sigma$  bonding orbital of ethane to the empty 4d or 5 s orbitals of the Pd atom(s). The charge transfer between  $\text{Pd}_n$  and  $\text{C}_2\text{H}_6$  is small because the  $\text{Pd}_n$  cluster accepts and donates electrons simultaneously. Despite the minimal net charge transfer between the Pd cluster and ethane, a very important change transpired: some C–H  $\sigma$  bonding electrons become  $\sigma^*$  antibonding electrons in ethane and thereby a C–H bond is activated. Fig. 12 provides a rudimentary illustration of these two concurrent electron-flow mechanisms: A  $\text{Pd}_n$  cluster can donate its 4d electrons to the antibonding orbitals of ethane and meanwhile the same  $\text{Pd}_n$  cluster can also accept the C–H bonding electrons concurrently. Although the 4d orbitals of the free Pd atom in the ground electronic state are completely occupied, there are partially empty 4d orbitals in a  $\text{Pd}_n$  cluster that can accommodate electrons from ethane. The NBO charge analysis for other cluster sizes suggests that the same two mechanisms account for the activation of the C–H bond in ethane.

Further NBO charge analysis of the separated ethane and  $\text{Pd}_6$  indicates that the shifted H atom, among all atoms, undergoes the most dramatic change in the atomic charge. The NBO charge of H in free ethane is calculated to be 0.22. In the reaction between ethane and  $\text{Pd}_6$ , the shifted H accepts  $0.06 e^-$  from the Pd cluster and  $0.14 e^-$  from the  $\text{C}_2\text{H}_5$  group during the formation of the TS structure. This shifted H atom has a very small positive NBO charge (0.02) in the TS structure partly because its Pauling electronegativity is the same as that of Pd (2.2). For  $\text{Pd}_8$ , the shifted H atom donates  $0.01 e^-$  to the Pd cluster and accepts  $0.15 e^-$  from the  $\text{C}_2\text{H}_5$  group in the formation of the TS structure. The shifted H atom also has a very small positive NBO charge (0.08) in the TS structure.

**Fig. 12** Two mechanisms account for the  $\text{Pd}_n$ -activated C–H bond in ethane



## Conclusions

We used the CCSD(T)/TZ//B3LYP-D3BJ/DZ dual-level hybrid method to study the various local minima of the Pd atomic clusters ( $\text{Pd}_n$ ) and their abilities to activate a C–H bond in ethane. The global minimum structures of  $\text{Pd}_n$  ( $n=3-8$ ) are triangular, tetrahedral, triangular bipyramidal, octahedral, pentagonal bipyramidal, and twisted cubic, respectively. The global minima of  $\text{Pd}_n$  ( $n=1, 3, 7, 8$ ) are in the singlet electronic state, whereas  $\text{Pd}_n$  ( $n=2, 4, 5, 6$ ) are in the triplet electronic state. The shapes of the  $\text{Pd}_n$  in the lowest-energy TS of  $\text{Pd}_n + \text{C}_2\text{H}_6 \rightarrow \text{H}-\text{Pd}_n-\text{C}_2\text{H}_5$  largely resemble the corresponding free  $\text{Pd}_n$  clusters (with noticeable distortion in the TS structure for sizes 7 and 8). Due to the interaction between ethane and  $\text{Pd}_n$  that eliminates the (near) degeneracy of molecular orbitals, all lowest-energy TS structures are in the singlet electronic state except for size 7 (which is a triplet). Among the eight cluster sizes studied in this work, size 2 has the lowest energy, enthalpy, and Gibbs energy of activation but it also has poor thermal stability indicated by its small atomization energy and negative second derivative of atomization energy. Size 4 exhibits satisfactory thermal stability, but its ability to activate a C–H bond is significantly poorer than other sizes.  $\text{Pd}_6$  and  $\text{Pd}_8$  well balance the thermal stability and catalytic ability and thus are likely the ideal sizes for the effective activation of the C–H bonds in ethane.  $\text{Pd}_6$  and  $\text{Pd}_8$  are considered stable as they both have a large atomization energy per atom: 131 and 142 kJ/mol, respectively. They are considered active toward ethane as their room-temperature Gibbs energies of activation of  $\text{Pd}_n + \text{C}_2\text{H}_6 \rightarrow \text{H}-\text{Pd}_n-\text{C}_2\text{H}_5$  are only 30 and 47 kJ/mol (12RT and 19RT), respectively. In this work, we also quantified the C–H activation abilities of each Pd cluster size in two ways under constant *PT* conditions. One way is to take the difference between the lowest Gibbs energy of the TS structures and the lowest Gibbs energy of the reactants. The other is to take the contribution of all reactant structures and all TS structures into account in a statistical manner under the constant *TP* condition. These two different ways yield virtually the same result because of the lack of (near) degeneracy among the lowest-energy reactants and the lack of (near) degeneracy among the lowest-energy TS structures.

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**Data availability** The authors declare that the data supporting the findings of this study are available within the paper and its Supplementary Information file. Should any raw data files be needed in another format they are available from the corresponding author upon reasonable request.

## Declarations

**Conflict of interest** The authors declare no competing financial or non-financial interest.

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