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Structural properties of the price-to-earnings and price-to-book ratios

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Abstract We examine the structural properties of a firm's price-to-earnings (P/E) and price-to-book (P/B) ratios and the relation between these two ratios. A benchmark result is obtained under the hypothesis that firms use replacement cost accounting to value their operating assets, so that the P/B ratio coincides with Tobin's q. The firm's P/E ratio can then be expressed as a convex combination of the P/E ratios suggested respectively by the permanent earnings model and the Gordon growth model, with the relative weight to be placed on these two endpoints determined entirely by Tobin's q. Under current financial reporting rules, the accounting for operating assets is likely to be more conservative than replacement cost accounting. Our findings characterize how the magnitude and behavior of the P/E and P/B ratios are jointly shaped by several key variables, including both past and anticipated future growth, economic profitability, and accounting conservatism

Keywords Price-to-earnings · Price-to-book · Tobin's q · Growth · Profitability · Conservatism

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1 Introduction

The price-to-earnings (P/E) and price-to-book (P/B) ratios of publicly traded firms are quoted in a variety of contexts by analysts, managers, and academic researchers. The P/B ratio, calculated as the market value of a firm's equity divided by the book value of its equity, is frequently used as a proxy for Tobin's q, which, in turn, is regarded as a determinant of future investment expenditures.[1] In the industrial organization literature, Tobin's q is widely used as a measure of monopoly rents that firms are anticipated to earn in the future.[2] Similarly, the forward P/E ratio is calculated as the market value of a firm's equity at a particular date divided by the firm's expected earnings in the following year. The P/E ratio is used ubiquitously to calibrate the price of a particular stock or the overall valuation of the stock market.[3]

Our objective is to examine the structural properties of these two common market-based financial ratios. We explicitly model transactions of the firm as well as the accounting rules in use to represent these transactions. Our framework allows for both ratios to be determined by several explanatory variables, including pricing power in the firm's product markets, anticipated future market demand, and past capital investments. We explore the structural properties of each ratio and the relation between them, for instance, under what conditions is a "high" P/E ratio compatible with a "normal" P/B ratio?[4]

In terms of benchmarks, textbooks frequently view a P/B ratio equal to one as "normal," though it is commonly understood that both anticipated future profitability and conservative valuation of incumbent assets tend to push this ratio above one. According to the permanent earnings model, the normal forward P/E ratio is equal to the reciprocal of the firm's cost of capital.[5] The permanent earnings model applies under fair value accounting, which requires the book value of the firm's assets at each point in time to equal the present value of the firm's future cash flows. Deviations from the permanent earnings benchmark are usually explained by the fact that most firms use historical cost accounting to value their operating assets with the consequence of lower book values. An alternative benchmark value for the P/E ratio suggested in earlier literature is the Gordon growth model which implies an earnings multiple given by the reciprocal of the difference between the firm's cost of capital and the anticipated growth rate in earnings.[6]

[1] See, for example, Tobin (1969), Hayashi (1982), and Abel and Eberly (2011).

[2] For instance, Lindenberg and Ross (1981) submit that q *"exceeds one by the capitalized value of the Ricardian and monopoly rents which the firm enjoys."*

[3] See Basu (1977), Jaffe et al. (1989), and Lakonishok et al. (1994). At the aggregate level, the Fed model states that the stock market earnings yield (the inverse of the P/E ratio) should equal the 10-year nominal Treasury yield; see Asness (2003) and Bekaert and Engstrom (2010).

[4] The empirical relation between the P/E and P/B ratios is studied by Penman (1996).

[5] Alternative benchmarks for the P/E ratio are discussed by Feltham and Ohlson (1996), Ohlson and Juettner-Nauroth (2005), Penman (1996, 2013), and Zhang (2000).

[6] This benchmark can be justified in a setting where firm sales grow at a constant rate and the firm has only variable cash operating expenses. Accounting earnings are presumed equal to cash flow, and, as a consequence, firm value can be expressed as a multiple of forward earnings. See, for example, Beaver and Morse (1978), Zarowin (1990), and Damodaran (2006, p. 245).

Our first set of results focuses on the benchmark of replacement cost accounting, that is, when the book value of operating assets in use reflects the "market value" of these assets in a hypothetical competitive rental market for capacity services. The benchmark of replacement cost accounting is important for two reasons. First, Tobin's q is defined as the price-to-book ratio when assets are valued at their replacement values. In the investment literature, most commonly used methods of estimating Tobin's q, such as those proposed by Salinger and Summers (1983), Perfect and Wiles (1994), and Lewellen and Badrinath (1997), effectively "reconstruct" replacement cost book values based on the publicly available data. Second, the benchmark of replacement cost accounting will prove useful in calibrating the impact of the actual accounting rules on the P/E and P/B ratios.

We establish that the forward P/E ratio under replacement cost accounting can be expressed as a "convex combination" of the permanent-earnings and the Gordon growth formulas. The relative weight to be placed on these two "endpoints" turns out to be entirely a function of Tobin's q. For a firm operating in a competitive environment (Tobin's $q = 1$), the forward P/E ratio reduces to that under the permanent earnings model. If the firm enjoys a high degree of monopoly power, its forward P/E ratio will approach the value suggested by the Gordon growth model. Given replacement cost accounting, there will be a positive association between the P/E ratio and Tobin's q.

Our model envisions that firms make sequential investments in capital assets and use the production capacity of those assets to deliver output to the product market. Capital accumulation models in finance, economics, and industrial organization have frequently focused on a homogeneous capital stock, which obtains when the productive capacity of assets is assumed to decline in a geometric fashion. This assumption is usually imposed for analytical tractability, since it makes the vintage composition of the firm's assets irrelevant for future investment decisions. In contrast, our model allows for assets with unrestricted productivity patterns.[7] As a consequence, the firm's optimal capital stock becomes path dependent, and its market value depends not only on current capacity but also on the vintage composition of assets in place.

Growth in the firm's operations is a key determinant of financial ratios (Penman 2013). Our analysis points to the potentially opposite effects that *past* and anticipated *future* growth have on the P/E and P/B ratios. Higher future growth in the product market corresponds ceteris paribus to a higher P/B and a higher forward P/E ratio, simply because the value of future growth opportunities is reflected in the firm's stock price but neither in earnings nor the current book value of assets. In contrast, higher past growth in investments translates into a *lower* P/B ratio and a *lower* forward P/E ratio under replacement cost accounting. Intuitively, a firm with higher past investment growth has newer assets and a correspondingly greater replacement cost of assets in place (per unit of productive capacity in the current period). However, a higher replacement cost of assets in place also increases the

[7] Our model framework of capacity investments and replacement cost accounting builds on that of Rogerson (2008). This framework has been used in a number of recent studies spanning managerial performance evaluation (Rogerson 2008; Dutta and Reichelstein 2010), monopoly regulation (Rogerson 2011; Nezlobin et al. 2012), and financial statement analysis (Nezlobin 2012; McNichols et al. 2014).

firm's market value by a corresponding amount, as the discounted value of future economic profits is independent of the firm's investment history.[8] Therefore higher past investment growth translates into equal increases in the numerator and denominator of the price-to-book ratio and brings the overall ratio closer to one. Since our benchmark result establishes a positive association between the two ratios under replacement cost accounting, the P/E ratio must also be declining in past investment growth.

Generally accepted accounting principles for operating assets in the U.S. and other OECD countries are commonly viewed as a source of accounting conservatism. Investments in most intangible assets must be directly expensed. Furthermore, depreciation rules like the commonly used straight-line method will in many circumstances be accelerated, relative to the underlying pattern of decline in economic capacity.[9] The impact of more conservative accounting on the P/B ratio is unambiguous as only the denominator decreases with more conservative asset valuation rules. Regarding the P/E ratio, it is useful to recall the "Canceling Errors" theorem (Greenball 1969) asserting that for firms in a steady state with no growth, earnings must be invariant to the accounting methods used, and therefore the P/E ratio must be equal to the benchmark value identified under replacement cost accounting. We find that more conservative accounting tends to result ceteris paribus in a higher (lower) P/E ratio, provided the firm's investments have expanded (contracted) in the past.[10] Furthermore, with sufficiently conservative accounting rules, higher past growth in investments is shown to *increase* the forward P/E ratio.

A higher degree of economic profitability, as represented by the firm's pricing power in the product market, unequivocally increases both ratios under replacement cost accounting. However, we find that the effect of pricing power on q is attenuated for firms that have experienced high investment growth in the past. In general, the impact of higher economic profitability on the P/E ratio is shown to depend on the degree of accounting conservatism and the pattern of past investment growth. For firms that have no pricing power because they operate in a competitive environment, we obtain a positive (negative) association between the P/E and P/B ratios for growing (declining) firms.

Textbooks on financial statement analysis and equity valuation, such as those by Lundholm and Sloan (2013) and Penman (2013), emphasize the importance of

[8] Consistent with much of the investment literature in finance and economics, the firm's market price in our model is equal to the replacement cost of assets in place plus the discounted sum of future economic profits (e.g. Thomadakis 1976; Lindenberg and Ross 1981; Fisher and McGowan 1983; Salinger 1984; Abel and Eberly 2011). This result is obtained under the assumption that the firm's price is equal to the present value of future cash flows under the optimal investment policy. The price of a firm's stock can, of course, deviate from its fundamental value due to market inefficiencies or agency problems, two issues that are ignored in our analysis.

[9] Our analysis builds on the work of McNichols et al. (2014), who seek to obtain a measure of Tobin's q by applying a "conservatism correction" factor to the P/B ratio. The empirical part of their analysis shows that this measure of Tobin's q has better predictive power for future investments than the P/B ratio. Like McNichols et al. (2014), our focus is on unconditional conservatism, as contrasted with the conditional conservatism studies, e.g., Basu (1977) or Beaver and Ryan (2005).

[10] This finding is conceptually related to a "quadrant result" obtained in connection with the Accounting Rate-of-Return: see, for instance, Salamon (1985), Fisher and McGowan (1983), and Rajan et al. (2007).

growth in interpreting financial ratios. In particular, these authors argue that the price-to-book ratio is driven by the product of growth and excess profitability, while the price-to-earnings ratio is driven by the growth in future residual earnings (Lundholm and Sloan (2013), Chapter 11). In contrast to earlier studies, our model allows us to isolate the effect of *past* growth from the effect of future growth on the financial ratios. The finding that higher past growth has a negative effect on Tobin's *q* may partially explain the low predictive ability of empirical measures of Tobin's *q* for future growth in a firm's capital stock (see, e.g., Gomes 2001). Generally, our results demonstrate the importance of separately controlling for past and future growth in evaluating valuation multiples and shed light on the conflicting empirical evidence regarding the relation between growth rates and financial ratios. For example, Chan et al. (2003) find that (1) firms with high past growth in earnings trade at high P/E multiples, (2) "there is essentially no persistence or predictability in growth of earnings." Our findings show that high past growth can lead to a high P/E ratio even when it is not accompanied by high future growth and thus the observations of Chan et al. (2003) can be consistent with efficient equity pricing.

Earlier accounting literature on valuation multiples has linked the P/E and P/B ratios to the parameters of the residual earnings process, such as residual earnings persistence or growth in abnormal earnings; see, for instance, Feltham and Ohlson (1995), Zhang (2000), and Ohlson and Juettner-Nauroth (2005). This literature commonly assumes that earnings and residual earnings evolve according to some exogenous process. Yet, the corresponding time series will generally be determined jointly by the accounting rules in use and the economic fundamentals of the firm. By explicitly modeling the firm's transactions and the applicable reporting rules, our framework allows us to disentangle the effect of accounting and economic parameters, such as pricing power and demand growth for the firm's products, on the resulting financial ratios. To illustrate, given replacement cost accounting, we find that the growth rate in residual earnings will equal the growth rate in demand for the firm's product. That characterization, however, no longer applies for other accounting rules, e.g., straight-line depreciation for operating assets.

The remainder of the paper is organized as follows. Section 2 presents the model of capacity investments and defines the P/E and P/B ratios examined in the paper. Section 3 derives our benchmark characterization for these ratios under replacement cost accounting. The effects of conservative accounting on the P/E and P/B ratios are examined in Sect. 4, while Sect. 5 examines the impact of economic profitability. We conclude in Sect. 6.

2 Model description

Consider a single-product firm that makes periodic investments in productive capacity. Consistent with much of the finance and accounting literature on investment we assume that the market value of the firm's equity is equal to the present value of its future cash flows, and that the firm chooses its investments to maximize its value. Productive capacity is generated by operating assets that can be purchased in each period at a constant unit cost.[11] The useful life of operating assets is T periods. Specifically, a unit of asset purchased in period t provides capacity to produce x_τ units of the product in periods $t+\tau$ for $1 \le \tau \le T$. Without loss of generality, the acquisition cost of one asset unit is set equal to one. Denoting by I_t the investment in period t, the aggregate capacity available in period $t+1$, K_{t+1}, is determined by the investments over the past T periods:

$$K_{t+1} = x_1 \cdot I_t + x_2 \cdot I_{t-1} + \cdots + x_T \cdot I_{t-T+1},$$

where the vector $\boldsymbol{x} = (x_1, x_2, \ldots, x_T)$ will be referred to as the asset's productivity pattern and $\boldsymbol{I}_t = (I_t, I_{t-1}, \ldots, I_{t-T+1})$ as the relevant investment history at date t. The productivity of assets is assumed to decline weakly over their useful life, possibly reflecting physical wear and tear or increasing maintenance requirements:

$$1 = x_1 \ge x_2 \ge \cdots \ge x_T > 0.$$

In the special case of undiminished capacity, all x_τ are equal to one. This scenario is commonly referred to as the *one-hoss shay* pattern in the regulation literature, e.g., Laffont and Tirole (2000) and Rogerson (2011). An alternative specification in the literature assumes that assets remain productive indefinitely and that their capacity declines geometrically over time:

$$x_\tau = (1-\alpha) \cdot x_{\tau-1},$$

where $0 < \alpha \le 1$; see, for instance, Lindenberg and Ross (1981), Salinger (1984), Biglaiser and Riordan (2000), and Abel and Eberly (2011). We refer to such an asset decay pattern as the *geometric capacity decline* scenario.

Throughout this paper, we will impose the following regularity condition stipulating that the rate of asset declines increases over time.

$$\frac{x_\tau - x_{\tau+1}}{x_\tau} \ge \frac{x_{\tau-1} - x_\tau}{x_{\tau-1}} \tag{1}$$

for all $2 \le \tau \le T$, where $x_{T+1} \equiv 0$. We note that the inequalities in (1) are met in both the one-hoss shay and the geometric decline scenario.[12] This condition is also satisfied if capacity declines linearly over time.

In the basic version of our model, the firm and the capital market are assumed to have complete knowledge of the future demand for the firm's product.[13] To keep the model parsimonious, we assume that all production costs, incurred to deliver the product or service in question, are tied to the provision of productive capacity. In period t, the firm can produce and sell k_t units of product, subject to the capacity constraint, $k_t \le K_t$. The firm will then realize revenues of

[11] Rogerson (2008) has shown that the present model can be extended to settings where the cost of new assets changes over time.

[12] In the geometric scenario with $T = \infty$, all inequalities in (1) are satisfied as equalities. If the productive capacity declines geometrically over time but the useful life of assets is finite, then inequality (1) is strict for $\tau = T$.

[13] Extensions of the base model to stochastic environments are discussed in Sect. 3 below.

$$R_t(k_t) = H_t(k_t) \cdot k_t,$$

where $H_t(k_t)$ denotes the price per unit of output as a function of the quantity supplied. The revenue functions, $R_t(\cdot)$, are assumed to be increasing. The firm will therefore operate at capacity in every period, i.e., $k_t = K_t$. In addition, the inverse demand functions are assumed to have the following inter-temporal structure.

Assumption (A1): *Demand for the firm's product expands proportionately over time such that:*

$$H_{t+1}\big(k \cdot (1 + \mu_{t+1})\big) = H_t(k).$$

The significance of Assumption (A1) is that the firm can increase its sales volume by the factor $1 + \mu_{t+1}$ in period $t + 1$ while maintaining the same product price. It is readily verified that Assumption (A1) can be met by standard functional forms for demand curves, including constant elasticity demand curves.

For simplicity of exposition, our model focuses on an all-equity firm that disburses all free cash flows to its owners immediately. Cash flows either arise from investment expenditures or the revenues from sales. Accordingly, the market value of equity at date T is given by the present value of future cash flows under the optimal investment policy:

$$P_T \equiv \max_{\{I_{T+i}\}_{i=1}^{\infty}} \sum_{i=1}^{\infty} [R_{T+i}(K_{T+i}) - I_{T+i}] \cdot \gamma^i, \tag{2}$$

subject to $I_{T+i} \geq 0$. Here, $\gamma = \frac{1}{1+r}$ denotes the discount factor corresponding to the firm's cost of (equity) capital, r. Investments are made optimally in anticipation of future market demand for the firm's product, and the firm faces no frictions in issuing equity to finance these investments.

Earnings and book values reflect the underlying accounting rules through the depreciation rules in place. Depreciation expense for operating assets is recognized according to some schedule $\boldsymbol{d} = (d_1, \ldots, d_T)$, where d_τ is the share of investment that is expensed in period τ of its useful life, and the vector $\boldsymbol{d}$ satisfies:

$$\sum_{\tau=1}^{T} d_\tau = 1.$$

The aggregate depreciation expense in period t is then given by:

$$D_t = d_1 \cdot I_{t-1} + \cdots + d_T \cdot I_{t-T}.$$

Let $\boldsymbol{bv} = (bv_0, \ldots, bv_T)$ denote the corresponding asset valuation rule, so that the aggregate book value at date t is:

$$BV_t = bv_0 \cdot I_t + bv_1 \cdot I_{t-1} + \cdots + bv_T \cdot I_{t-T}.$$

We impose the condition that the book value of assets changes by the amount of depreciation expense recognized in a given period and that assets are fully expensed by the end of their useful life:

$$bv_0 = 1, \ \ bv_\tau = bv_{\tau-1} - d_\tau \text{ and } bv_T = 0.$$

For any given depreciation rule $\boldsymbol{d}$, the firm's accounting earnings in period t, E_t, are equal to the difference between revenues and the aggregate depreciation expense:

$$E_t = R_t(K_t) - D_t.$$

Given the preceding notation, we can now define the price-to-book and forward price-to-earnings ratios as:[14]

$$PB_T = \frac{P_T}{BV_T} \text{ and } PE_T = \frac{P_T}{E_{T+1}}. \tag{3}$$

Both of these ratios are determined by the operating assets in place, the applicable accounting (depreciation) rules, current profit margins, and investors' anticipation of future profitability. The latter depends on any future pricing power the firm is expected to have as well as growth expectations of the product market.

3 Replacement cost accounting

This section considers an accounting benchmark where assets in place are valued according to their replacement cost. The obvious interest in this benchmark derives from the fact that with replacement cost accounting the price-to-book ratio coincides with Tobin's q.[15] We explore how Tobin's q relates to the price-to-earnings ratio and how both ratios are affected by the age composition of productive assets. Specifically, we examine the impact of higher growth in past investments, and therefore a higher proportion of younger assets, on either of these ratios.

We start by characterizing the firm's optimal investment policy. Arrow (1964) and Rogerson (2008) have shown that the firm's infinite-horizon investment problem in (2) can effectively be decomposed inter-temporally. Following the approach of these two papers, we start by solving the maximization problem in (2) ignoring the non-negativity constraints on investments. We then provide conditions that are sufficient to ensure that the constraints in (2) are indeed not binding.

[14] Since we do not impose any assumptions on the composition of the firm's asset base in period 0, there is no loss of generality in evaluating the price-to-book and price-to-earnings ratios at date T. Our results hold for the financial ratios calculated at any date t, if the corresponding relevant investment history is understood to be $(I_t, \ldots, I_{t-T+1})$.

[15] Replacement cost accounting for operating assets, like plant, property and equipment, was permissible under U.S. Generally Accepted Accounting Principles in the 1970s. Lindenberg and Ross (1981) base their estimates of Tobin's q on companies that adopted this asset valuation rule. Subsequent literature suggested several methods for estimating the replacement cost of assets based on the information available in the published accounting reports; see, for instance, Salinger and Summers (1983), Perfect and Wiles (1994), and Lewellen and Badrinath (1997). Erickson and Whited (2006) evaluate the accuracy of different methods for computing Tobin's q.

The key concept in the intertemporal decomposition of the firm's investment problem is the user cost of capacity, which we denote by c.[16] To calculate the user cost, it is useful to imagine a hypothetical setting wherein the firm can rent capacity in a perfectly competitive rental market.[17] In such a market, suppliers would charge a price c for one unit of capacity *made available for one period of time*. Competition would ensure that the rental charge c is such that the discounted stream of future rental revenues would allow rental firms to break even. Thus,

$$-1 + \gamma \cdot c \cdot x_1 + \cdots + \gamma^T \cdot c \cdot x_T = 0, \tag{4}$$

or

$$c = \frac{1}{\gamma \cdot x_1 + \cdots + \gamma^T \cdot x_T}. \tag{5}$$

Now the optimal sequence of the firm's investments that solve (2) can be expressed in terms of the capacity levels that maximize:

$$\pi_t = R_t(K_t) - c \cdot K_t \tag{6}$$

in every period t, for $t > T + 1$.[18] We refer to $\pi_t = R_t(K_t) - c \cdot K_t$ as the *economic profit* of the firm in period t.

Denoting by K_t^o the optimal capacity levels that myopically maximize the economic profit in each period, it follows directly from Assumption (A1) that the optimal capacity levels grow at the rates μ_t:

$$K_{t+1}^o = \left(1 + \mu_{t+1}\right) K_t^o. \tag{7}$$

Given the anticipated future growth rates $\boldsymbol{\mu} \equiv (\mu_{T+2}, \mu_{T+3}, \ldots)$, the recursive Eq. in (7) define the target capacity levels $K_t^o(\boldsymbol{\mu})$ for $t \geq T + 2$. Clearly, the non-negativity constraint on new investments in (2) will not bind if the product market grows in each period, that is, $\mu_t \geq 0$. For our purposes, it suffices to assume that the firm's investment history and the trajectory of anticipated future growth satisfy a consistency condition. For an investment history $\boldsymbol{I}_T = (I_T, I_{T-1}, \ldots, I_1)$, we denote by $\boldsymbol{\lambda} \equiv (\lambda_2, \ldots, \lambda_T)$ the corresponding growth rates:

$$I_t = (1 + \lambda_t) \cdot I_{t-1}.$$

The *consistency condition* we impose between $\boldsymbol{\mu}$ and $\boldsymbol{I}_T = (I_T, I_{T-1}, \ldots, I_1)$ (or equivalently between $\boldsymbol{\mu}$, $\boldsymbol{\lambda}$, and I_1) requires that the induced investment levels, $I_t^o(\boldsymbol{\mu}, \boldsymbol{\lambda} | I_1)$, which yield the capacity levels $K_t^o(\boldsymbol{\mu})$, for $t \geq T + 1$ are indeed non-

[16] Arrow (1964) provided a general expression for the user cost of capital in terms of a certain series of recursively defined functions. The simple expression for c in Eq. (5) is due to Rogerson (2008).

[17] To be sure, our model does not assume the existence of such a rental market, yet the construct is useful in defining the user cost of capacity and the replacement cost of assets in place.

[18] We recall that at date T the investment decision I_T has been made, and therefore the capacity level for period $T + 1$ has already been decided. We further assume that at date T (when the P/E and P/B ratios are evaluated) the firm is already on the optimal investment path, i.e., the investment I_T was chosen so as to maximize π_{T+1}.

negative. Thus it would be possible to meet the preceding consistency condition despite a product market that is declining in some future periods.[19]

Under replacement cost accounting, assets are initially recorded at their acquisition cost. The depreciation expense is calculated such that, at each point in time, the remaining book value reflects the market value of the used asset in a (hypothetical) competitive rental market. Specifically, the value of an asset of age τ is set equal to the discounted value of its remaining revenue stream in the hypothetical rental market:

$$bv_{\tau}^{*} = \gamma \cdot c \cdot x_{\tau+1} + \cdots + \gamma^{T-\tau} \cdot c \cdot x_{T}. \tag{8}$$

Let $\boldsymbol{bv}^{*} = \left(bv_{0}^{*}, \ldots, bv_{T}^{*}\right)$ denote the corresponding sequence of replacement cost values. The depreciation charges under replacement cost accounting, $\boldsymbol{d}^{*} = \left(d_{0}^{*}, \ldots, d_{T}^{*}\right)$, are given by the identity $d_{t}^{*} = bv_{t-1}^{*} - bv_{t}^{*}$. Straightforward algebra then yields:

$$d_{\tau}^{*} = c \cdot x_{\tau} - r \cdot bv_{\tau-1}^{*}.$$

In particular, for assets exhibiting the one-hoss shay productivity pattern, replacement cost accounting amounts to the annuity depreciation rule with depreciation charges compounding at the cost of capital, that is, $d_{\tau}^{*} = (1+r) \cdot d_{\tau-1}^{*}$. In contrast, for a geometrically declining productivity pattern, replacement cost accounting calls for geometrically declining depreciation charges:

$$d_{\tau}^{*} = \alpha(1-\alpha)^{\tau-1}.$$

Finally, the straight-line depreciation rule, which is most commonly used for financial reporting purposes, corresponds to replacement cost accounting for assets that decline at a particular linear rate.

The firm's market value in (2) can be expressed as the replacement cost value of incumbent assets plus the discounted value of future maximized economic profits.[20] In our notation:

$$P_{T} = BV_{T}^{*} + \sum_{i=1}^{\infty} \gamma^{i} \cdot \pi_{T+i}^{o}, \tag{9}$$

where

$$\pi_{t}^{o} = H_{t}(K_{t}^{o}) \cdot K_{t}^{o} - c \cdot K_{t}^{o}. \tag{10}$$

It is an immediate consequence of the proportionate growth assumption in (A1) that the optimal product price is time invariant, that is, $H_{t}(K_{t}^{o}) = p^{o}$ for all $t \geq T+1$. Furthermore, economic profits grow at the same rate as the firm's product market, that is, $\pi_{t+1}^{o} = \left(1+\mu_{t+1}\right)\pi_{t}^{o}$. If $p^{o} = c$, the net present value of the firm's

[19] Specifically, the consistency condition will be met if $\mu_{t} \geq -\min_{1 \leq \tau \leq T} \frac{(x_{\tau}-x_{\tau+1})}{x_{\tau}}$ for all t.

[20] See the proof of Proposition 1 for details. This result generalizes similar findings of Lindenberg and Ross (1981) and Salinger (1984) to settings with a general vintage composition of assets.

investments is zero, and therefore we refer to such a firm as operating in a competitive environment. Pricing power and monopoly rents correspond to values of p^o exceeding the user cost of capacity, c.

A central finding of Rogerson (2008) is that the firm's economic profit in (6) can be expressed in accrual accounting terms. *Residual income* subtracts from earnings an imputed interest charge for the current book value of assets:

$$RI_t = E_t - r \cdot BV_{t-1} = R_t(K_t) - D_t - r \cdot BV_{t-1}.$$

In particular, Rogerson (2008) has shown that with replacement cost accounting residual income is equal to economic profit in each period, regardless of the firm's investment history.[21] Thus:

$$RI_t^* = E_t^* - r \cdot BV_{t-1}^* = (H(K_t) - c) \cdot K_t = \pi_t.$$

Thus $RI_t^* = \pi_t^o$ if the firm maximizes economic profit in period t. This connection establishes the desired link between firm value and economic profits on one side and book value and accounting earnings on the other. Provided economic profits grow at the rates μ_t, as determined by the growth in the firm's product market, equity value can be expressed succinctly as:

$$P_T = BV_T^* + \frac{RI_{T+1}^*}{r - s(\boldsymbol{\mu})} = BV_T^* + \frac{\pi_{T+1}^o}{r - s(\boldsymbol{\mu})}, \tag{11}$$

where

$$s(\boldsymbol{\mu}) = r - \frac{1}{\gamma + \gamma^2 \cdot (1 + \mu_{T+2}) + \gamma^3 \cdot (1 + \mu_{T+2}) \cdot (1 + \mu_{T+3}) + \cdots}. \tag{12}$$

We refer to $s(\boldsymbol{\mu})$ as the average future growth rate (beyond date $T + 1$), because $s(\boldsymbol{\mu}) = \mu$ in the special case where $\mu_{T+t} \equiv \mu$ for all $t \geq 2$.[22]

Since Tobin's q is defined as the ratio of the firm's market value to the replacement cost of its assets, the expression in (11) yields the following expression for q.

$$\frac{P_T}{BV_T^*} \equiv q_T = 1 + \frac{\pi_{T+1}^*}{(r - s(\boldsymbol{\mu}))BV_T^*}. \tag{13}$$

The expression in (13) is consistent with the verbal characterization of Lindenberg and Ross (1981), who state: "...for firms engaged in positive investment, in equilibrium, we expect q to exceed one by the capitalized value of

[21] This relation holds because for any history of investments,

$$\begin{aligned} D_t^* + r \cdot BV_{t-1}^* &= (d_1^* + r \cdot bv_0^*) \cdot I_{t-1} + \cdots + (d_T^* + r \cdot bv_{T-1}) \cdot I_{t-T} \\ &= c \cdot (x_1 \cdot I_{t-1} + \cdots + x_T \cdot I_{t-T}) = c \cdot K_t. \end{aligned}$$

[22] The first μ_t that matters in capitalizing future economic profits is μ_{T+2} because the baseline value for capitalizing future economic profits is π_{T+1}^o. We note that firm value, P_T, is well defined, provided the sequence $\boldsymbol{\mu}$ is such that the denominator on the right-hand side of (12) is positive.

the Ricardian and monopoly rents which the firm enjoys." We note that for a firm that operates in a competitive environment $\pi_T^o = 0$, and thus Tobin's q will be equal to one, regardless of the investment history.

At first glance, the expression in (13) is reminiscent of the representation of the market-to-book ratio in textbooks on equity valuation. Lundholm and Sloan (2013, Chapter 11), for instance, state that if future ROE, defined as $\frac{E_T}{BV_{T+1}}$, is constant and book values grow at a constant rate g, then as a consequence of the residual income valuation formula:[23]

$$\frac{P}{BV} = 1 + \frac{ROE - r}{r - g}. \tag{14}$$

While (14) can be aligned with (13) by substituting $\frac{\pi_{T+1}^*}{BV_T^*} = ROE^* - r$ and $s(\boldsymbol{\mu}) = \mu = g$, it should be kept in mind that the two characterizations of the market-to-book ratio follow from different assumptions. Our characterization in (13) relies primarily on the use of replacement cost accounting. Past growth in operating assets is effectively captured in BV_T^*, and the capitalization factor $r - s(\boldsymbol{\mu})$ reflects the anticipated future growth in market demand and sales revenue. In contrast, the expression in (14) relies on the assumption that ROE is constant and that operating assets (book values) grow at some constant rate. Yet that growth rate is determined jointly by the accounting rules in place, future growth in demand, and the history of investments. In particular, even if future capacity levels (and sales) were to grow at some constant rate μ from date T on, the resulting sequence of book values will generally not grow at μ.[24] An important separating feature of replacement cost accounting is that, in the calculation of the market-to-book ratio, the capitalization of future ROE's depends only on anticipated future demand growth, but not on the history of investments.[25]

The following result states a benchmark value for the P/E ratio under replacement cost accounting, $PE_T^* \equiv \frac{P_T}{E_{T+1}^*}$, in terms of Tobin's q.

Proposition 1 *Given replacement cost accounting, the forward price-to-earnings ratio is equal to*

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[23] In particular, $ROE = r$ under replacement cost accounting whenever the firm operates in a competitive environment, resulting again in a market-to-book ratio equal to one.

[24] The resulting sequence of book values will grow at the rate μ, irrespective of the accounting rules, in the special case of a constant growth rate for all investments, *both past and future*.

[25] To illustrate this point, assume that $T = 2$, $x_1 = x_2 = 1$ and $r = 10\%$. Assume further that $K_3^o = 100$ and the firm expects its sales to remain constant after period $T + 1$, i.e., $s(\boldsymbol{\mu}) = 0$, $K_4^o = 100$, $K_5^o = 100$, and so on. Consider the following investment history that leads to $K_3^o = 100$: $(I_2 = 0, I_1 = 100)$. To implement the optimal capacity levels going forward, the firm will need to make a replacement investment of 100 in years 3, 5, 7... Therefore, the firm's net cash flows will alternate between the values of $100 \cdot p^o - 100$ and $100 \cdot p^o$. It can be verified that, for this investment history, $P_T = 1{,}000 \cdot p^o - 1{,}000 \cdot 1.1/2.1$. Under the straight-line depreciation rule, $(d_1 = 0.5, d_2 = 0.5)$, $BV_T = 50$ and $E_{T+1} = 100 \cdot p^o - 50$, $ROE_{T+1} = 2p^o - 1$. It is straightforward to check that Eq. (14) does not hold under the straight-line rule if $g = 0$ (the demand growth rate). It will, however, hold under replacement cost accounting (annuity depreciation) where $d_1 = \frac{1}{2.1}$, $d_2 = \frac{1.1}{2.1}$.

$$PE_T^* = \frac{1}{r - s(\boldsymbol{\mu}) \cdot \frac{q_T - 1}{q_T}}. \tag{15}$$

Proposition 1 establishes that, given replacement cost accounting, a higher P/B ratio is associated with a higher P/E ratio provided the firm's product market is growing on average in the sense that $s(\boldsymbol{\mu}) \geq 0$. It is instructive to consider three special cases of Eq. (15). First, for a competitive firm ($q = 1$), the permanent earnings model applies in the sense that

$$PE_T^* = \frac{1}{r}.$$

This finding makes intuitive sense insofar as with a competitive product market there are no economic profits in the future, and therefore $RI_{T+1}^* = 0$, or $E_{T+1}^* = r \cdot BV_T^*$. At the same time, $BV_T^* = P_T$, and therefore the earnings multiple is $\frac{1}{r}$.

Second, and arguably less intuitive, the permanent earnings model continues to apply even if there are future economic profits, that is $q_T > 1$, yet the product market will be stationary in the future. As a consequence, $\mu_{T+i} = 0$ and $s(\boldsymbol{\mu}) = 0$. The firm's market value is then equal to the replacement cost of assets plus the capitalized value of the period $T + 1$ economic profit. Absent any future market growth, this capitalization factor is given by $\frac{1}{r}$, with the consequence that market value reduces to capitalized earnings.[26] We note that this finding applies regardless of the magnitude of the firm's profit margin in the future.

Third, for a firm that enjoys a high degree of pricing power, and therefore a high q, Eq. (15) approximates the Gordon growth formula:

$$\frac{P_T}{E_{T+1}^*} \approx \frac{1}{r - s(\boldsymbol{\mu})}.$$

As the firm's monopoly profits increase, capacity costs become relatively less important, and firm value is largely determined by the discounted value of future revenues. Growth in the product market directly translates into revenue growth and therefore increases the capitalization factor according to the Gordon growth formula.

"Growth in operations" is commonly seen as a key determinant of both the price-to-book and the price-to-earnings ratio. In the context of our model, it follows immediately that higher *future growth* in the product market will ceteris paribus

[26] Ohlson and Juettner-Nauroth (2005) derive the fundamental result that firm value can be expressed as capitalized forward earnings plus the capitalized value of future abnormal earnings growth. Their result is obtained irrespective of the accounting rules, provided the first difference of the residual income series grows or declines geometrically over time. This specification will be met in our model only in special cases. For instance, residual income grows at the same rate as market demand for the firm's product, given replacement cost accounting. For other accounting rules, though, the residual income series will no longer correspond to a geometric series, even if the future growth rates do. See Nezlobin (2012) for numerical examples illustrating this point.

translate into a higher q and, by Proposition 1, also into a higher P/E ratio. The present value of future growth opportunities is reflected in the firm's market value but not in the replacement cost of its assets in place at date T. Accordingly, Tobin's q is strictly increasing in μ_{T+i}, unless the firm operates in an environment of zero net present values. As the firm's earnings in period $T+1$ do not reflect future growth in the product market, the forward price-to-earnings ratio under replacement cost accounting is strictly increasing in μ_{T+i}, except in environments where $q_T = 1$.

To examine the impact of *past growth* on both ratios, it will be convenient to restate the expression for Tobin's q in (13), taking into account that residual income under replacement cost accounting is equal to economic profit. Thus:

$$q_T = 1 + \frac{(p^o - c)K^o_{T+1}}{(r - s(\boldsymbol{\mu}))BV^*_T}. \tag{16}$$

Recalling the definition of bv^*_τ in Eq. (8), we obtain the following equivalent representation of BV^*_t:

$$BV^*_T = \gamma \cdot c \cdot K^o_{T+1} + \gamma^2 \cdot c \cdot \bar{K}_{T+2} + \cdots + \gamma^T \cdot c \cdot \bar{K}_{2T}, \tag{17}$$

where $\bar{K}_{T+i}$ is the firm's capacity level in period $T+i$ assuming that no new assets are acquired after period T. Tobin's q then becomes:

$$q_T = 1 + \left(\frac{1}{r - s(\boldsymbol{\mu})}\right)\left(\frac{p^o - c}{c}\right)\left(\frac{K^o_{T+1}}{\gamma \cdot K^o_{T+1} + \gamma^2 \cdot \bar{K}_{T+2} + \cdots + \gamma^T \cdot \bar{K}_{2T}}\right). \tag{18}$$

Equation (18) indicates that q will exceed one by the product of three terms. The first term, $1/(r - s(\boldsymbol{\mu}))$, reflects the firm's cost of capital and future growth in demand for the firm's product. The term $(p^o - c)/c$ captures, on a percentage basis, the optimal markup that the firm charges in the product market above its long-run marginal cost. For a firm generating positive economic profits, this markup will reflect the degree of the firm's monopoly power, with zero as the benchmark for a firm operating under competitive conditions.[27] The third term determining q in (18) is the capacity that the firm's assets in place will generate in the next period divided by the discounted value of capacity that those assets will generate over their remaining lifetime. Ceteris paribus, the impact of past growth on Tobin's q is captured through this last term.

Proposition 2 *Given replacement cost accounting, Tobin's q and the forward price-to-earnings ratio* PE^*_T *are weakly decreasing in all past growth rates,* λ_t, *for* $1 < t \leq T$.

Empirical research in finance and economics commonly interprets both Tobin's q and the market-to-book ratio as indicators of "growth opportunities." This characterization also emerges in our model to the extent that higher profit margins

[27] The contribution margin ratio $(p^o - c)/c$ is a monotone transformation of the Lerner index of monopoly power, $L \equiv (p^o - c)/p^o$ (Martin 2002). In particular for a demand curve exhibiting constant price elasticity of demand, say ϵ, one obtains $\frac{p^o - c}{c} = \frac{1}{\epsilon - 1}$.

$(p^o - c)$ and higher growth rates in the product market $(s(\boldsymbol{\mu}))$ result in higher values of Tobin's q. At the same time, though, the firms that have expanded their investments in the past will ceteris paribus exhibit a lower q and a lower P/E ratio.

The intuition for Proposition 2 can be captured by considering two firms that operate at the same capacity in period $T+1$ and face identical future product market conditions. Suppose also that one of these firms has *newer assets* in the sense that the histories of investment growth rates for the two firms are the same except for one λ_t for some $1 < t \leq T$. The firm with the higher investment growth rate in period t can be viewed as having newer assets, since a larger share of its capacity is generated by assets purchased in period t or later.[28] The firm with newer assets will have a higher replacement cost of assets in place. Since future economic profits are equal for the two firms, the difference between their market values will be equal to the difference in the replacement cost of their assets in place. Tobin's q for the firm with older assets can be written as:

$$q_T^{(old)} \equiv \frac{P_T^{(old)}}{BV_T^{*(old)}} = \frac{BV_T^{*(old)} + \Delta}{BV_T^{*(old)}},$$

where Δ represents the present value of future economic profits. For the firm with newer assets, we then obtain:

$$q_T^{(new)} = \frac{BV_T^{*(old)} + M + \Delta}{BV_T^{*(old)} + M},$$

where M represents the increase in the replacement value of assets due to higher growth. The increment M in both the numerator and the denominator of $q_T^{(new)}$ pushes the q-ratio toward unity, and therefore $q_T^{(old)} > q_T^{(new)}$. Finally, the claimed monotonicity of the P_T/E^*_{T+1} ratio in past growth follows directly from Proposition 1.

Equation (18) demonstrates that the degree of the firm's monopoly power (as measured by the optimal markup, $\frac{p^o - c}{c}$) and past investment growth affect the firm's Tobin's q in a multiplicative fashion. Thus the cross-derivative of Tobin's q with respect to p^o and λ_t has the same sign as $\frac{\partial q_T}{\partial \lambda_t}$, which is negative. This means that the impact of monopoly power on Tobin's q, $\frac{\partial q_T}{\partial p^o}$, is greater for low growth firms (when λ_t is low) and smaller for high-growth firms (λ_t is high). Conversely, the impact of past growth on q is stronger (more negative) when the firm enjoys a strong degree of monopoly power. For a perfectly competitive firm, $p^o = c$, Tobin's q is identically equal to one for all values of λ_t.

Tobin's q and the forward price-to-earnings ratio under replacement cost accounting are *strictly* decreasing in each λ_t, except when either $q_T = 1$ or $s(\boldsymbol{\mu}) = 0$. Another degenerate case occurs in the geometric decline scenario that has been used widely in finance and economics studies alike, primarily because of its analytical

[28] To have equal capacity in period $T+1$, the firms then must have different investments in the first period, I_1. Yet I_1 cancels out from the calculation of Tobin's q in Eq. (18).

convenience.[29] It then turns out that q is constant in λ_t and, by implication, the PE^* ratio is unaffected by past growth. To see this, suppose assets have an indefinite useful life and their productive capacity declines geometrically with age: $\boldsymbol{x} = \left(1, (1-\alpha), (1-\alpha)^2, \ldots\right)$, where $0 \le \alpha \le 1$. The aggregate capacity in period $t+1$ will then be determined by the most recent investment and the aggregate capacity in the previous period:

$$K_{t+1} = (1-\alpha) \cdot K_t + I_t.$$

It is readily verified that the user cost of capacity reduces to $c = r + \alpha$ in the geometric setting.[30] Given the $\boldsymbol{d}^{\alpha}$ depreciation rule under replacement cost accounting, the book value of a unit investment of age τ is equal to the capacity this investment will generate in the next period: $bv^{\alpha}_{\tau} = (1-\alpha)^{\tau} = x_{\tau+1}$. Therefore the aggregate replacement cost of assets at each date is equal to the productive capacity in the following period:

$$BV^*_T = bv^{\alpha}_0 \cdot I_T + \cdots + bv^{\alpha}_{T-1} \cdot I_1 = x_1 \cdot I_T + \cdots + x_T \cdot I_1 = K^o_{T+1}. \tag{19}$$

Thus, when the capacity of assets declines geometrically, the firm's capital stock is homogeneous in the sense that capacity declines at the same rate for assets of all ages. As a consequence, current capacity is a sufficient statistic for the replacement cost of assets in place.

The firm's equity value also takes a particularly compact form in the geometric scenario. The firm's economic profit in period $T+1$ is equal to

$$p^o \cdot K^o_{T+1} - (\alpha + r) K^o_{T+1}.$$

From period $T+1$ onward, the firm's economic profits will increase at rate μ_t, since the optimal capacity levels will increase at this rate. Therefore the firm's value at date T is given by:

$$P_T = K^o_{T+1} + \frac{p^o - (\alpha + r)}{r - s(\boldsymbol{\mu})} K^o_{T+1} = \frac{p^o - \alpha - s(\boldsymbol{\mu})}{r - s(\boldsymbol{\mu})} K^o_{T+1} \tag{20}$$

and

$$q = \frac{p^o - \alpha - s(\boldsymbol{\mu})}{r - s(\boldsymbol{\mu})}.$$

Since Tobin's q does not depend on λ_t, Proposition 1 implies that the P/E ratio under replacement cost accounting also cannot depend on past investment growth. These findings are summarized as follows:

[29] See, for example, Dixit and Pindyck (1994, p. 374), Feltham and Ohlson (1996), and Biglaiser and Riordan (2000).

[30] Consistent with our characterization, Carlton and Perloff (2005) refer to $c = r + \alpha$ as the marginal cost of capital.

Observation 1 *In the geometric setting, both Tobin's q and the* PE_T^* *ratio are invariant to past investment growth.*

In light of Proposition 2, we find that the specification of geometrically declining capacity yields a degenerate case insofar as both financial ratios would be invariant to the vintage composition of incumbent assets.

To conclude this section, we note that our model can be extended to allow for certain forms of uncertainty about future market demand for the firm's product. The critical issue is whether the investment policy given by the myopically chosen capacity levels, K_t^o, is optimal such that the non-negativity constraint for the corresponding investment levels, I_t^o does not bind. It is well known that if the non-negativity constraint does bind for some of the I_t^o, then the characterization of the optimal policy for the constrained problem becomes computationally difficult in the case of general productivity patterns; see Arrow (1964) and Dixit and Pindyck (1994, p. 374). However, our results in this section continue to hold essentially unchanged if (1) the nature of uncertainty is such that the non-negativity constraint on I_t^o does not bind even in the most unfavorable market outcomes, or (2) if the firm can sell used assets in a competitive secondary market.

To elaborate on the first possibility, assume that $\tilde{\mu}_{t+1}$ is a random variable, which is observed by the firm just before investment I_t is made.[31] To ensure that the non-negativity of I_t^o is met, it is sufficient to assume that the support of the distribution of $\tilde{\mu}_{t+1}$ is bounded from below by zero. If the productivity of assets is strongly declining, $x_\tau > x_{\tau+1}$ for all τ, a weaker lower bound on $\tilde{\mu}_{t+1}$ can be imposed:

$$\tilde{\mu}_t \geq -\min_\tau \frac{(x_\tau - x_{\tau+1})}{x_\tau}.$$

For geometrically declining productivity, the bound above becomes $\tilde{\mu}_{t+1} \geq -\alpha$. Our result in Proposition 1 would then continue to hold if $s(\boldsymbol{\mu})$ is redefined as:

$$r - \frac{1}{\mathbb{E}_T\left[\gamma + \gamma^2\left(1 + \tilde{\mu}_{T+2}\right) + \gamma^3 \cdot \left(1 + \tilde{\mu}_{T+2}\right) \cdot \left(1 + \tilde{\mu}_{T+3}\right) + \cdots\right]},$$

where $\mathbb{E}_T[\cdot]$ denotes the expectation operator conditional on date T information.

Our results also continue to hold if the firm can divest and sell its used assets at competitive market prices.[32,33] The stochastic growth rates $\tilde{\mu}_t$ could then follow an essentially arbitrary process. Our result in Proposition 1 extends to this setting by substituting μ_{T+i} by their expected values at date T in the calculation of $s(\boldsymbol{\mu})$. The parameters λ_t should be interpreted as the ratio of the number of asset units of vintages t and $t-1$ that still belong to the firm at date T. Proposition 2, showing

[31] The firm then has enough information in period t to implement the optimal capacity level in period $t+1$, K_{t+1}^o.

[32] This assumption is frequently made in the investment literature, beginning with Arrow (1964). See also Abel and Eberly (2011).

[33] Assume that the secondary market satisfies the following "no-arbitrage" condition: for any two streams of asset purchases that result in the same capacity levels in all periods, the total discounted cost of the purchases must be the same. It can be verified that this condition implies that an asset of age τ will be priced at bv_τ^* in this market.

that, holding future expected growth constant, a firm with newer assets will have both a lower Tobin's q and a lower PE_T^* ratio compared to a firm with older assets, is unchanged.

4 Conservative accounting

The financial reporting rules employed in most OECD countries (U.S. GAAP and IFRS) differ from our baseline scenario of replacement cost accounting in at least two respects. First, some expenditures that arguably generate cash returns in future periods, such as those in research and development, are not recognized as assets and are expensed as incurred. Second, the depreciation schedules that are applied under current financial reporting rules to amortize capitalized assets usually ignore the time value of money. In our model, both of these factors will tend to make current accounting practice more conservative than replacement cost accounting.

A common criterion for ranking accounting rules by their degree of (unconditional) conservatism is that the more conservative rule yields consistently lower book values. In the context of our model, the depreciation schedule $\boldsymbol{d}$ will be called more *accelerated* than $\boldsymbol{d}'$ if for all $1 \leq t \leq T$: $bv_t \leq bv_t'$, or equivalently:

$$\sum_{t=1}^{\tau} d_t \geq \sum_{t=1}^{\tau} d_t',$$

for all $1 \leq \tau \leq T$. Clearly, this criterion provides only a partial ranking of alternative depreciation schedules. We will make use of the following conservatism criterion.

Definition 1 Depreciation rule $\boldsymbol{d}$ is more conservative than $\boldsymbol{d}'$ if for any $\tau \leq T - 1$

$$\frac{d_{\tau+1}}{d_\tau} \leq \frac{d'_{\tau+1}}{d'_\tau}. \tag{21}$$

Depreciation rule $\boldsymbol{d}$ is considered more conservative than $\boldsymbol{d}'$ if the depreciation charges according to $\boldsymbol{d}$ decline faster over time. Since these charges sum up to one for both rules, the more conservative rule must entail greater charges in earlier periods. Higher depreciation charges in earlier periods will in turn lead to consistently lower book values under the more conservative rule. The following result notes that our notion of conservatism is stronger than the criterion that ranks one depreciation schedule as more accelerated than another.

Observation 2 *Suppose* $\boldsymbol{d}$ *is more conservative than* $\boldsymbol{d}'$. *Then* $bv_\tau \leq bv_\tau'$ *for all* τ, *and* $BV_T(\boldsymbol{d}) \leq BV_T(\boldsymbol{d}')$.

In some industries, a major source of accounting conservatism is that firms directly expense expenditures related to intangible assets. To capture this effect in our framework, we say that $\boldsymbol{d}'$ is obtained from $\boldsymbol{d} = (d_1, \ldots, d_T)$ by increasing the share of investment directly expensed in the first year of operation, if for some $\eta > 0$,

$$d'_1 = (1 - \eta)d_1 + \eta,$$

and

$$d'_\tau = (1 - \eta)d_\tau$$

for $\tau > 1$. Depreciation rule $\boldsymbol{d}'$ directly expenses an η-share of investment initially and applies the same depreciation pattern as $\boldsymbol{d}$ to the remaining book value. The following result confirms the intuition that direct expensing does indeed correspond to a higher degree of conservatism according to the preceding definition.

Observation 3 *If* $\boldsymbol{d}'$ *is obtained from* $\boldsymbol{d}$ *by increasing the share of investment directly expensed, then* $\boldsymbol{d}'$ *is more conservative than* $\boldsymbol{d}$.

Another common source of accounting conservatism is that depreciation schedules used by firms usually ignore the time value of money. To model such rules in our framework, we consider the proportional depreciation rule given by:[34]

$$d^p_\tau = \frac{x_\tau}{x_1 + \cdots + x_T}.$$

Proportional depreciation coincides with the straight-line rule if the inter-temporal capacity pattern of assets corresponds to the one-hoss shay scenario (all $x_\tau = 1$). We also note that with proportional depreciation:

$$\frac{d^p_\tau}{d^p_{\tau+1}} = \frac{x_\tau}{x_{\tau+1}}.$$

Our next observation verifies that proportional depreciation is conservative relative to the replacement cost rule.

Observation 4 *The proportional depreciation rule is more conservative than replacement cost accounting.*

To illustrate the preceding result, for the one-hoss shay pattern the straight-line rule corresponds to proportional depreciation, while replacement cost accounting amounts to the annuity rule, where depreciation charges compounding at the rate r (Rajan and Reichelstein 2009). Clearly the straight-line depreciation rule is more conservative than annuity depreciation.

For the P/B ratio, the impact of conservatism is straightforward to the extent that assets in place tend to have a lower book value and therefore $P/B \geq 1$. For the P/E ratio, the effect of accounting conservatism depends on the firm's investment trajectory. First, it is well known that earnings are invariant to the accounting rules in use provided the firm operates in a steady state of no growth.[35] In addition, it has

[34] Proportional depreciation accords with the IAS 16 requirement that "... the depreciation method used shall reflect the pattern in which the asset's future economic benefits are expected to be consumed by the entity." In our model, the revenues generated by an asset are proportional to current productive capacity. The proportional depreciation rule allocates the cost of investment according to the (nominal) cash flows generated by the asset, ignoring the time value of those cash flows.

[35] See, for example, Penman (2013, p. 580).

been observed that conservative accounting results in lower earnings if investments have been increasing over the relevant history. Conversely, if investments have followed a declining trajectory, conservative accounting yields higher earnings and a correspondingly lower forward P/E ratio. These observations are explained by the fact that conservative accounting results in relatively high depreciation charges for newer assets. If the firm has been increasing its investments in operating assets, then higher depreciation charges will be applied to larger (more recent) investments, leading to a higher aggregate depreciation expense and lower earnings.

We denote the forward P/E ratio at date T by

$$PE_T(\boldsymbol{\lambda}, \boldsymbol{d}) = \frac{P_T(\boldsymbol{\lambda})}{E_{T+1}(\boldsymbol{\lambda}, \boldsymbol{d})}$$

to keep track of the applicable depreciation rule $\boldsymbol{d}$ and investment history $\boldsymbol{\lambda}$. Similarly, we will use the notation $q_T(\boldsymbol{\lambda})$. Finally, we refer to a *growing* firm as one where $\lambda_t \geq 0$ for all t, while a firm will be said to be *declining* if $\lambda_t \leq 0$. This leads to the following result relating investment growth to accounting conservatism.

Proposition 3 *The forward price-to-earnings ratio for a growing firm satisfies:*

$$PE_T(\boldsymbol{\lambda}, \boldsymbol{d}) \geq \frac{1}{r - s(\boldsymbol{\mu}) \cdot \frac{q_T(\boldsymbol{\lambda})-1}{q_T(\boldsymbol{\lambda})}}, \tag{22}$$

provided the depreciation rule $\boldsymbol{d}$ *is more conservative than replacement cost accounting. The inequality in* (22) *is reversed for declining firms.*

Proposition 3 combines our earlier finding in Proposition 1 with the observation that, ceteris paribus, aggregate earnings for a growing firm must decrease as the accounting becomes more conservative; that is:

$$E_{T+1}(\boldsymbol{\lambda}, \boldsymbol{d}') \geq E_{T+1}(\boldsymbol{\lambda}, \boldsymbol{d}),$$

provided $\boldsymbol{\lambda} \geq 0$ and $\boldsymbol{d}$ is more conservative than $\boldsymbol{d}'$. It follows from Proposition 3 that in the steady state of no growth ($\lambda_t = 0$), the P/E values generated by alternative accounting rules must all pass through the following *Pivot point*:

$$PE_T(\boldsymbol{0}, \boldsymbol{d}) = \frac{1}{r - s(\boldsymbol{\mu}) \cdot \frac{q_T(\boldsymbol{0})-1}{q_T(\boldsymbol{0})}}. \tag{23}$$

For the case of constant growth, that is $\lambda_t = \lambda$, Fig. 1 illustrates the effect of more conservative depreciation: the P/E ratios rotate counter-clockwise relative to the benchmark curve corresponding to replacement cost accounting.[36]

[36] The dashed line in Fig. 1 depicts the P/E ratio as a function of growth under the proportional depreciation rule, which is more conservative than replacement cost accounting. We will formally show that $PE_T(\boldsymbol{\lambda}, \boldsymbol{d}^p)$ is increasing in past growth in Proposition 4 below. Earlier accounting literature has considered "liberal" as opposed to conservative accounting; see, for example, Rajan et al. (2007) or Li (2013). We note that the inequality in Proposition 3 would be reversed for a depreciation schedule that is more liberal than replacement cost accounting.

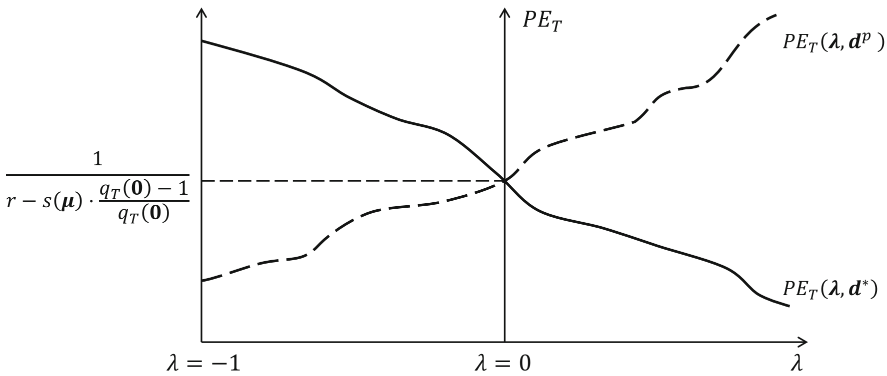

Fig. 1 Impact of growth on the P/E ratio under replacement cost accounting and proportional depreciation

The finding in Proposition 3 can be interpreted as a "quadrant result," akin to the one obtained in earlier accounting and economic studies for the return-on-investment ratio (ROI). These studies have shown that if a firm has expanded its past investments at a growth rate equal to the cost of capital, r, then $ROI = r$, regardless of the accounting rules.[37] With conservative accounting, ROI will exceed r if past growth rates have been consistently below r, and conversely ROI $\leq r$ if past growth rates have consistently exceeded r (Rajan et al. 2007).[38] For the ROI function, the quadrants are delineated by the horizontal and vertical lines passing through the Pivot point (r, r). For the P/E ratio, in contrast, the boundaries of the quadrants are delineated by the downward sloping curve $PE_T(\cdot, \boldsymbol{d}^*)$. Furthermore, more conservative accounting rotates the P/E ratio in a counter-clockwise fashion, when viewed as a function of λ. The opposite effect emerges for the ROI function, which rotates in a clockwise fashion as the accounting rules become more conservative.

If the applicable depreciation rules are sufficiently close to replacement cost accounting, the forces that drive $PE_T(\cdot, \boldsymbol{d}^*)$ to be decreasing in past growth will continue to prevail, and, as a consequence of Proposition 3, the benchmark value identified in (23) will then be an upper bound for the P/E ratio for a growing firm. On the other hand, if the accounting becomes sufficiently conservative, it is conceivable that the corresponding $PE_T(\cdot, \boldsymbol{d})$ curve will ultimately cease to be decreasing as suggested by the dashed line in Fig. 1. Our next result characterizes the behavior of the P/E and P/B ratios in past growth for accounting rules commonly used in practice, i.e., depreciation rules at least as conservative as the proportional rule.

In stating the following result, we impose an additional technical (and innocent) condition requiring earnings to be non-negative. This will be satisfied if the product

[37] See, for instance, Salamon (1985) or Fisher and McGowan (1983).

[38] These results have been obtained in a "representative project" model where the firm effectively invests in the same representative project, with exogenously determined growth rates. This framework is equivalent to our capacity model in the special case of zero economic profits, that is, $p^o = c$.

price is sufficiently high so as to cover the average depreciation charge per unit of capacity in each period:

$$p^o \geq \max_{\tau} \frac{d_\tau}{x_\tau}$$

for $1 \leq \tau \leq T$.[39]

Proposition 4

(1) *The price-to-book ratio,* $PB_T(\boldsymbol{\lambda}, \boldsymbol{d})$, *is decreasing in* λ_t *for* $1 < t \leq T$, *provided* $\boldsymbol{d}$ *is at least as conservative as the proportional depreciation rule.*

(2) *The price-to-earnings ratio,* $PE_T(\boldsymbol{\lambda}, \boldsymbol{d})$, *is increasing in* λ_t *for* $1 < t \leq T$, *provided* $\boldsymbol{d}$ *is at least as conservative as the proportional depreciation rule.*

Given sufficiently conservative accounting, that is, a depreciation schedule that is more conservative than the proportional depreciation rule, higher growth in past investments moves the P/E and P/B ratio in opposite directions. In particular, as the dashed line in Fig. 1 suggests, the corresponding P/E curve will be increasing everywhere, and, as a consequence, the earnings multiplier for a growing firm will always exceed the one corresponding to the permanent earnings model, that is, $\frac{1}{r}$.

The intuition for a monotonically increasing P/E ratio is most transparent in the one-hoss shay scenario. With straight-line depreciation, the aggregate depreciation expense will be proportional to the productive capacity regardless of whether the firm's assets are old or new. Therefore, if two firms operate at the same level of capacity, their forward earnings under the straight-line rule will be the same. However, the firm with newer assets will have a greater equity value, since it will have to replace its assets further into the future. The firm with newer assets will therefore have a higher P/E ratio.[40]

To provide further intuition for the second part of Proposition 4, we divide both the numerator and the denominator of the P/E ratio by K_{T+1}:

$$\frac{P_T}{E_{T+1}} = \frac{P_T / K_{T+1}}{E_{T+1} / K_{T+1}}. \tag{24}$$

As λ_t increases, so does the numerator in (24), since, by Eq. (11),

[39] If this assumption is not satisfied, the firm's accounting earnings can be negative for certain investment histories. Nonetheless, it can still be shown that the *earnings yield*, or the forward *E/P ratio*, is monotonic in each λ_t.

[40] The logic of this argument is related to the so-called *old plant trap* usually associated with biases in the Accounting Rate-of-Return (see, for instance Lundholm and Sloan 2013). The common feature is that differences in the age of incumbent assets may not be properly reflected in earnings, thus causing an accounting-induced bias in the respective financial ratios.

$$\frac{P_T}{K_{T+1}} = \frac{BV_T^* + \pi^o_{T+1}/(r - s(\boldsymbol{\mu}))}{K_{T+1}} = \frac{BV_T^*}{K_{T+1}} + \frac{p^o - c}{r - s(\boldsymbol{\mu})}, \tag{25}$$

and the average replacement cost per unit of capacity produced is greater for newer assets.

With proportional depreciation, the denominator in the right-hand side of (24) will not depend on λ_t. This is because the aggregate depreciation expense is given by:

$$D_{T+1} = \frac{x_1}{x_1 + \cdots + x_T} I_T + \cdots + \frac{x_T}{x_1 + \cdots + x_T} I_1 = \frac{K_{T+1}}{x_1 + \cdots + x_T}.$$

Accordingly, the forward P/E ratio will be increasing in past growth.

Taken together, Propositions 2 and 4 show that the directional impact of higher past growth on the P/B ratio does not depend on the degree of accounting conservatism. The proof of Proposition 4 makes use of the decomposition:

$$PB_T(\boldsymbol{\lambda}, \boldsymbol{d}) = q_T(\boldsymbol{\lambda}) \cdot \frac{BV_T^*(\boldsymbol{\lambda}, \boldsymbol{d})}{BV_T(\boldsymbol{\lambda}, \boldsymbol{d})}. \tag{26}$$

The second term on the right-hand side of (26) is less than one for conservative accounting rules, and, like the first term, it also decreases in each λ_t.[41]

In sum, this section has demonstrated that, in contrast to the P/B ratio, the impact of accounting conservatism on the P/E ratio depends on several factors. In particular, we conclude that the joint impact of past investment growth and accounting conservatism on the magnitude and directional change of the P/E ratio is richer than the numerical examples in financial statement analysis textbooks have suggested; see, for instance, Penman (2013, Chapter 17).

5 The impact of economic profitability

Our results in Sect. 3 have shown that both Tobin's q and the P/E ratio under replacement cost accounting are increasing in the firm's pricing power in the product market, captured by the economic profit margin $p^o - c$.[42] We now ask to what extent this finding continues to hold with conservative accounting. We also seek a tighter characterization of how the P/E and P/B ratios relate to each other in settings where the firm operates in a competitive environment.

While higher economic profitability clearly raises the P/B ratio, the impact on the P/E ratio under conservative accounting is ambiguous. Higher values of p^o increase the expression for firm value in the numerator, with p^o being capitalized. Yet this

[41] McNichols et al. (2014) refer to this ratio as the conservatism correction factor, since Tobin's q is obtained by dividing the price-to-book ratio by the conservatism correction factor. In their sample, the median value of the correction factor, $\frac{BV_T^*(\boldsymbol{\lambda},\boldsymbol{d})}{BV_T(\boldsymbol{\lambda},\boldsymbol{d})}$, was 1.37.

[42] It follows from Eq. (18) that Tobin's q is strictly increasing in p^o. Proposition 1 then implies that the P/E ratio under replacement cost accounting is also strictly increasing in p^o, unless there is no anticipation of growth in the product market (i.e., unless $s(\boldsymbol{\mu}) = 0$).

effect may be more than compensated by a higher denominator effect, in particular, if earnings are low due to conservative accounting and high-growth in past investments.

Observation 5 *The price-to-earnings ratio is increasing in* p^o *if*

$$PE_T(\boldsymbol{\lambda}, \boldsymbol{d}) < \frac{1}{r - s(\boldsymbol{\mu})}. \tag{27}$$

Conversely, the price-to-earnings ratio is decreasing in p^o *if the inequality in* (27) *is reversed.*

In conjunction with our findings in the previous section, Eq. (27) suggests that the P/E ratio will be increasing in the firm's pricing power under the following conditions: (1) the accounting is "close" to replacement cost accounting and $s(\boldsymbol{\mu}) > 0$, (2) the accounting is conservative, and (3) the firm has been declining ($\lambda_t < 0$), in which case:

$$PE_T(\boldsymbol{\lambda}, \boldsymbol{d}) \leq PE_T(\boldsymbol{\lambda}, \boldsymbol{d}^*) = \frac{1}{r - s(\boldsymbol{\mu}) \cdot \frac{q_T(\boldsymbol{\lambda}) - 1}{q_T(\boldsymbol{\lambda})}} \leq \frac{1}{r - s(\boldsymbol{\mu})}. \tag{28}$$

On the other hand, our finding in Proposition 4 suggests that the inequality in (27) will be reversed, and therefore the P/E ratio will be *decreasing* in the firm's pricing power provided the depreciation schedule is at least as conservative as the proportional depreciation rule and the firm has been growing at sufficiently high rates in the past.

We finally consider the case of a firm that operates under competitive conditions, that is, $p^o = c$. Under the additional assumption of constant past growth, that is, $\lambda_t = \lambda$ for all t, we then have the following result:

Proposition 5 *Suppose the firm operates in a competitive product market and past investments have grown at a constant rate,* λ. *The forward price-to-earnings ratio then relates to the price-to-book ratio as follows:*

$$PE_T(\boldsymbol{\lambda}, \boldsymbol{d}) = \frac{1}{r - \lambda \cdot \frac{PB_T(\boldsymbol{\lambda}, \boldsymbol{d}) - 1}{PB_T(\boldsymbol{\lambda}, \boldsymbol{d})}}. \tag{29}$$

The result is reminiscent of the finding in Proposition 1, with the P/B ratio replacing Tobin's q and the constant past growth rate, λ, replacing the average growth rate $s(\boldsymbol{\mu})$ in the product market. The latter is obviously of no importance once the firm's investments are all zero net-present value projects. Irrespective of the accounting rules, Proposition 5 establishes a positive association between the P/E and P/B ratios for growing firms and conversely a negative association for declining firms. Holding the accounting rules fixed, the sensitivity (slope) of this association is more pronounced for higher rates of past growth.

6 Conclusion

We have examined the structural properties of the price-to-book and price-to-earnings ratios, both of which play a central role in financial statement analysis. The explanatory variables included in our analysis comprise the accounting rules in use, the history of investments, future growth opportunities and the degree of competitiveness for the firm's products. In our framework of overlapping capacity investments, replacement cost accounting emerges as a natural benchmark insofar as residual earnings coincide with economic profit and the P/B ratio reduces to Tobin's q. The P/E ratio can then be expressed as a "convex combination" between the permanent earnings and the Gordon growth models. The relative weight on these two "endpoint" P/E ratios is determined entirely by Tobin's q. For firms in a competitive environment ($q \to 1$), the P/E ratio will approximate the permanent earnings model, while for a firm with strong pricing power ($q \to \infty$), the P/E ratio will tend towards that implied by the Gordon growth formula.

Higher growth in future periods in the product market will unambiguously increase the benchmark value for P/E ratio, unless the firm operates in a competitive industry where economic profits are zero regardless of growth opportunities. In contrast, we find that the impact of higher past growth in investments on the P/E ratio cannot be predicted without reference to the underlying accounting rules. Under replacement cost accounting, higher past growth leads to lower values of Tobin's q and a lower benchmark value for the P/E ratio. For conservative accounting rules, in contrast, the P/E ratio will exceed its benchmark value for a firm that has been growing in the past, with the opposite being true for a pattern of declining investments in the past. Once the accounting rules become sufficiently conservative, we find that, ceteris paribus, higher growth rates will actually increase the predicted P/E ratio. In particular, this will be true if assets have an undiminished productive capacity over a finite life span and depreciation is calculated according to the commonly used straight-line rule.

The findings in this paper suggest a number of promising directions for empirical testing. While for some of our explanatory variables the choice of empirical construct appears rather straightforward, e.g., past growth in investments, a number of empirical proxies come to mind for other variables such as a firm's current pricing power, anticipated future growth in the product market, or the degree of conservatism. Our model predicts a negative relation between past investment growth and Tobin's q and a positive relation between past investment growth and the P/E ratio under conservative accounting. Furthermore, we show that the sensitivity of Tobin's q to the firm's pricing power is inversely related to the past investment growth. Our model also demonstrates that for firms with high past investment growth, the PE ratio under conservative accounting rules can be inversely related to the pricing power in the product market. The relations between the P/E and P/B ratios and their determinants identified in this paper can be tested at both firm- and aggregate market levels.

Our model has relied on several simplifying assumptions. For example, we posited that the firm can always sell its product at a price that at least covers production costs. Yet an economically profitable firm may report negative accounting earnings if investments are expensed at a faster rate than their economic value declines. Since the P/E ratio is discontinuous at zero accounting earnings, we restricted attention to settings where the firm's accounting earnings are positive. Our monotonicity results continue to hold for economically profitable firms with negative accounting earnings if, instead of the P/E ratio, one considers the forward earnings yield or the E/P ratio.[43] Providing an interpretation for the P/E ratios of firms making economic losses would be an interesting direction for future research.

Finally, our analysis has ignored some of the determinants of the P/B and the P/E ratios suggested by earlier literature, including leverage, dividend policy, and financing constraints. We have also treated the firm's cost of capital as exogenous and independent of the projects undertaken by the firm. Incorporating these additional factors into our modeling framework would lead to a more complete understanding of the P/E ratio and its determinants.

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Appendix

Proof of Proposition 1 The residual earnings valuation model provides the identity:

$$P_T = BV_T + \sum_{t=1}^{\infty} RI_{T+t} \cdot \gamma^t,$$

for any accounting rules, provided the condition of comprehensive income measurement is met; see Preinreich (1935) and Feltham and Ohlson (1995, 1996). Since income is measured comprehensively in our model, we can apply the residual income formula to replacement cost accounting in particular. Thus:

$$P_T = BV_T^* + \sum_{t=1}^{\infty} RI_{T+t}^* \cdot \gamma^t = BV_T^* + \sum_{t=1}^{\infty} \pi_{T+t}^o \cdot \gamma^t.$$

As argued in the main text, the proportionate growth assumption in (A1) implies that $\pi_{t+1}^o = (1 + \mu_{t+1}) \cdot \pi_t^o$. We then obtain:

$$P_T = BV_T^* + \frac{RI_{T+1}^*}{r - s(\boldsymbol{\mu})} = BV_T^* + \frac{\pi_{T+1}^o}{r - s(\boldsymbol{\mu})}, \tag{30}$$

where

[43] In his empirical investigation, Penman (1996) also cites continuity considerations for studying the E/P rather than the P/E ratio.

$$\frac{1}{r-s(\boldsymbol{\mu})} \equiv \gamma + \gamma^2 \cdot \left(1+\mu_{T+2}\right) + \gamma^3 \cdot \left(1+\mu_{T+2}\right) \cdot \left(1+\mu_{T+3}\right) + \cdots$$

Dividing both sides in (30) by P_T yields

$$1 = \frac{1}{r-s(\boldsymbol{\mu})}\frac{E^*_{T+1}}{P_T} - \frac{s(\boldsymbol{\mu})/q_T}{r-s(\boldsymbol{\mu})},$$

which, in turn, implies

$$\frac{P_T}{E^*_{T+1}} = \frac{1}{r-s(\boldsymbol{\mu})\cdot\frac{q_T-1}{q_T}}. \qquad \square$$

Some of the proofs below will rely on the following auxiliary lemma.

Lemma *For any numbers* $a_1,\ldots,a_n$, *positive numbers* $b_1,\ldots,b_n$, *and growth rates* $\xi_2,\ldots,\xi_n \geq -1$, *the function*

$$f(\xi_1,\ldots,\xi_n) = \frac{a_n + (1+\xi_2)a_{n-1} + \cdots + (1+\xi_2)\ldots(1+\xi_n)a_1}{b_n + (1+\xi_2)b_{n-1} + \cdots + (1+\xi_2)\ldots(1+\xi_n)b_1}$$

is everywhere increasing (decreasing) in each ξ_i *for* $2 \leq i \leq n$, *if the sequence* $\frac{a_i}{b_i}$ *is decreasing (increasing) in* i.

Proof of Lemma The claim obviously holds for $n=2$. For $n>2$, the function $f(\xi_1,\ldots,\xi_n)$ can be written as:

$$f(\xi_1,\ldots,\xi_n) = \frac{A_2 + (1+\xi_i)A_1}{B_2 + (1+\xi_i)B_1}, \tag{31}$$

where

$$\begin{aligned}
A_2 &= a_n + (1+\xi_2)a_{n-1} + \cdots + (1+\xi_2)\ldots(1+\xi_{i-1})a_{n-i+2},\\
B_2 &= b_n + (1+\xi_2)b_{n-1} + \cdots + (1+\xi_2)\ldots(1+\xi_{i-1})b_{n-i+2},\\
A_1 &= \frac{1}{(1+\xi_i)}\{(1+\xi_2)\ldots(1+\xi_i)a_{n-i+1} + \cdots + (1+\xi_2)\ldots(1+\xi_n)a_1\},\\
B_1 &= \frac{1}{(1+\xi_i)}\{(1+\xi_2)\ldots(1+\xi_i)b_{n-i+1} + \cdots + (1+\xi_2)\ldots(1+\xi_n)b_1\}.
\end{aligned}$$

The fact that $\frac{a_i}{b_i}$ is decreasing in i implies that

$$\frac{A_2}{B_2} \leq \frac{a_{n-i+2}}{b_{n-i+2}} \leq \frac{a_{n-i+1}}{b_{n-i+1}} \leq \frac{A_1}{B_1}. \tag{32}$$

The representation in (31) and the inequality above reduce the problem for a general n to the special case of $n=2$. $\square$

Proof of Proposition 2 From Proposition 1, we know that:

$$\frac{P_T}{E^*_{T+1}} = \frac{1}{r - s(\boldsymbol{\mu}) \cdot \frac{q_{T-1}}{q_T}}.$$

It thus suffices to show that Tobin's q is decreasing in λ_t. Tobin's q can be rewritten as

$$\begin{aligned} q_T = & \frac{P_T}{BV^*_T} = \frac{E^*_{T+1} - s(\boldsymbol{\mu}) \cdot BV^*_T}{(r - s(\boldsymbol{\mu})) \cdot BV^*_T} \\ = & \frac{1}{(r - s(\boldsymbol{\mu}))} \cdot \frac{\left(x_1 p^o - d^*_1 - s(\boldsymbol{\mu}) \cdot bv^*_0\right) I_T + \cdots + \left(x_T p^o - d^*_T - s(\boldsymbol{\mu}) \cdot bv^*_{T-1}\right) I_1}{bv^*_0 I_T + \cdots + bv^*_{T-1} I_1}. \end{aligned}$$

If we show that

$$\frac{x_{i+1} p^o - d^*_{i+1} - s(\boldsymbol{\mu}) \cdot bv^*_i}{bv^*_i}$$

increases in i, then the monotonicity of Tobin's q will follow from Lemma A by setting $n = T$, $a_i = x_i p^o - d^*_i - s(\boldsymbol{\mu}) \cdot bv^*_{i-1}$, $b_i = bv^*_{i-1}$, and $\xi_i = \lambda_i$.

Observe that

$$\begin{aligned} \frac{x_{i+1} p^o - d^*_{i+1} - s(\boldsymbol{\mu}) \cdot bv^*_i}{bv^*_i} & = \frac{x_{i+1} p^o - d^*_{i+1} - r \cdot bv^*_i + (r - s(\boldsymbol{\mu})) bv^*_i}{bv^*_i} \\ & = \frac{p^o - c}{(bv^*_i / x_{i+1})} + r - s(\boldsymbol{\mu}). \end{aligned}$$

It remains to show that bv^*_i / x_{i+1} is decreasing in i. To that end, we note that

$$\frac{bv^*_{i-1}}{bv^*_i} = \frac{\gamma x_i + \cdots + \gamma^{T-i+1} x_T}{\gamma x_{i+1} + \cdots + \gamma^{T-i+1} x_{T+1}},$$

where $x_{T+1} = 0$.

We can apply Lemma A to the sequences defined by the following equations:

$$\begin{aligned} (a_1, \ldots, a_n) &= (x_T, \ldots, x_i), \\ (b_1, \ldots, b_n) &= (x_{T+1}, \ldots, x_{i+1}), \\ (1 + \xi_2, \ldots, 1 + \xi_n) &= (\gamma, \ldots, \gamma), \\ \left(1 + \xi'_2, \ldots, 1 + \xi'_n\right) &= (0, \ldots, 0). \end{aligned}$$

Since the productivity pattern satisfies the condition:

$$\frac{x_t - x_{t+1}}{x_t} \geq \frac{x_{t-1} - x_t}{x_{t-1}},$$

it follows that:

$$\frac{a_{t-1}}{b_{t-1}} \geq \frac{a_t}{b_t}.$$

Therefore the function f from Lemma A will be increasing in each ξ_i, and its value at $(\xi_2, \ldots, \xi_n)$ will be greater than its value at $\left(\xi_2', \ldots, \xi_n'\right)$. Hence,

$$\frac{bv_{i-1}^*}{bv_i^*} \geq \frac{\gamma \cdot x_i + 0 \cdot x_{i+1} + \cdots + 0 \cdot x_T}{\gamma \cdot x_{i+1} + 0 \cdot x_{i+2} + \cdots + 0 \cdot x_{T+1}} = \frac{x_i}{x_{i+1}},$$

and it follows that the sequence $\frac{bv_{i-1}^*}{x_i}$ is decreasing in i. □

Proof of Observation 2 Assume that $\boldsymbol{d}$ is more conservative than $\boldsymbol{d}'$. Since

$$\sum d_\tau = 1,$$

we can rewrite d_1 as:

$$d_1 = \frac{1}{1 + \frac{d_2}{d_1} + \cdots + \frac{d_2}{d_1}\frac{d_3}{d_2}\cdots\frac{d_T}{d_{T-1}}}. \tag{33}$$

Since $\boldsymbol{d}$ is more conservative than $\boldsymbol{d}'$,

$$\frac{d_{\tau+1}}{d_\tau} \leq \frac{d_{\tau+1}'}{d_\tau'}.$$

Equation (33) then implies that $d_1 \geq d_1'$. Note that if $d_\tau \leq d_\tau'$ for some τ, then $d_{\tau+1} \leq d_\tau \frac{d_{\tau+1}'}{d_\tau'} \leq d_{\tau+1}'$. Applying the same argument iteratively, one can verify that $d_\tau \leq d_\tau'$ implies $d_{\tau+i} \leq d_{\tau+i}'$ for any i.

Let $\delta(i) = bv_i - bv_i'$. We have the following observations:

1. $\delta(1) = bv_1 - bv_1' \leq 0$.
2. If $\delta(\tau) - \delta(\tau+1) \leq 0$ for some τ, then $\delta(\tau+i) - \delta(\tau+i+1) \leq 0$ for any $i \geq 0$.
3. $\delta(T) = 0$.

The function δ is negative at one, and once it becomes increasing, it continues to increase up to the end of the useful life. Therefore $\delta(i)$ can only cross zero once and this happens at $i = T$. The three observations hence imply that $\delta(i) \leq 0$ and $bv_i \leq bv_i'$ for all i. □

Proof of Observation 3 Assume that

$$d_1' = (1-\eta)d_1 + \eta,$$

and

$$d_\tau' = (1-\eta)d_\tau$$

for $\tau > 1$ and some $\eta > 0$.

Note that

$$\frac{d_\tau}{d_{\tau+1}} = \frac{d'_\tau}{d'_{\tau+1}}$$

for $\tau > 1$. For $\tau = 1$, we have

$$\frac{d'_1}{d'_2} = \frac{(1-\eta)d_1 + \eta}{(1-\eta)d_2} > \frac{d_1}{d_2}.$$

Therefore $\boldsymbol{d}'$ is more conservative than $\boldsymbol{d}$. □

Proof of Proposition 3 Let D_{T+1} and D'_{T+1} denote the aggregate depreciation expenses under rules $\boldsymbol{d}$ and $\boldsymbol{d}'$, respectively, where $\boldsymbol{d}$ is more conservative than $\boldsymbol{d}'$. We will show that $D_{T+1} \geq D'_{T+1}$ ($D_{T+1} \leq D'_{T+1}$) if investments $I_1, \ldots, I_T$ are monotonically increasing (decreasing). From this it will follow that if $\boldsymbol{d}$ is more conservative than replacement cost accounting and the firm is growing, then

$$PE_T(\boldsymbol{\lambda}, \boldsymbol{d}) = \frac{P_T(\boldsymbol{\lambda})}{E_{T+1}(\boldsymbol{\lambda}, \boldsymbol{d})} \geq \frac{P_T(\boldsymbol{\lambda})}{E^*_{T+1}} = \frac{1}{r - s(\boldsymbol{\mu}) \cdot \frac{q_T(\boldsymbol{\lambda})-1}{q_T(\boldsymbol{\lambda})}}.$$

Observe that

$$\begin{aligned} D_{T+1} =& I_1 \cdot d_T + I_2 \cdot d_{T-1} + \cdots + I_T \cdot d_1 \\ =& I_1 + (I_2 - I_1) \cdot (1 - bv_{T-1}) + \cdots + (I_T - I_{T-1}) \cdot (1 - bv_1). \end{aligned}$$

Therefore,

$$D_{T+1} - D'_{T+1} = (I_2 - I_1) \cdot \left(bv'_{T-1} - bv_{T-1}\right) + \cdots + (I_T - I_{T-1}) \cdot \left(bv'_1 - bv_1\right) \geq 0,$$

where the last inequality holds by Observation 2 if investments are increasing and $\boldsymbol{d}$ is more conservative than $\boldsymbol{d}'$. □

Proof of Observation 4 In the proof of Proposition 2, we have verified that $\frac{bv^*_{i-1}}{x_i}$ is decreasing in i. Recall that

$$\frac{d^*_i + r \cdot bv^*_{i-1}}{x_i} = c.$$

Since $\frac{bv^*_{i-1}}{x_i}$ is decreasing in i, the sequence $\frac{d^*_i}{x_i}$ must increase in i. Therefore,

$$\frac{d^*_i}{d^*_{i+1}} \leq \frac{x_i}{x_{i+1}} = \frac{d^p_i}{d^p_{i+1}}.$$ □

Proof of Proposition 4 We expand the P/E ratio as:

$$\frac{P_T}{E_{T+1}} = \frac{1}{(r-s(\boldsymbol{\mu}))} \cdot \frac{E^*_{T+1} - s(\boldsymbol{\mu}) \cdot BV^*_T}{E_{T+1}}$$
$$= \frac{1}{(r-s(\boldsymbol{\mu}))} \cdot \frac{I_T\left(x_1 p^o - d^*_1 - s(\boldsymbol{\mu}) \cdot bv^*_0\right) + \cdots + I_1\left(x_T p^o - d^*_T - s(\boldsymbol{\mu}) \cdot bv^*_{T-1}\right)}{I_T(x_1 p^o - d_1) + \cdots + I_1(x_T p^o - d_T)}.$$

To apply Lemma A, we need to show that

$$\frac{x_i p^o - d^*_i - s(\boldsymbol{\mu}) \cdot bv^*_{i-1}}{x_i p^o - d_i}$$

is decreasing in i. Note that

$$\frac{x_i p^o - d^*_i - s(\boldsymbol{\mu}) \cdot bv^*_{i-1}}{x_i p^o - d_i} = \frac{x_i p^o - d^*_i - r \cdot bv^*_{i-1} + (r - s(\boldsymbol{\mu}))bv^*_{i-1}}{x_i\left(p^o - \frac{d_i}{x_i}\right)}$$
$$= \frac{p^o - c + (r - s(\boldsymbol{\mu}))\frac{bv^*_{i-1}}{x_i}}{p^o - \frac{d_i}{x_i}}. \tag{34}$$

Assumption (1) implies that $\frac{bv^*_{i-1}}{x_i}$ is decreasing in i (see the proof of Proposition 2), and since $r > s(\boldsymbol{\mu})$, the numerator is decreasing in i. If $\boldsymbol{d}$ corresponds to proportional depreciation, then the denominator does not depend on i. If $\boldsymbol{d}$ is more conservative than the proportional depreciation rule, then $\frac{d_i}{x_i}$ is decreasing in i, and the denominator is increasing in i. Therefore the ratio (34) is decreasing in i.

To show that the price-to-book ratio is decreasing in each λ_t, we recall that the firm's market value is given by:

$$P_T = BV^*_T + \frac{(p^o - c)K^o_{T+1}}{r - s(\boldsymbol{\mu})}.$$

PB_T is therefore equal to:

$$PB_T = \frac{BV^*_T}{BV_T} + \frac{(p^o - c)}{r - s(\boldsymbol{\mu})} \cdot \frac{K^o_{T+1}}{BV_T}. \tag{35}$$

It remains to show that both $\frac{BV^*_T}{BV_T}$ and $\frac{K^o_{T+1}}{BV_T}$ decrease in each λ_t.[44] Note that

$$\frac{BV^*_T}{BV_T} = \frac{bv^*_0 I_T + \cdots + bv^*_{T-1} I_1}{bv_0 I_T + \cdots + bv_{T-1} I_1}.$$

By Lemma A, to show that BV^*_T/BV_T is declining in λ_t, it suffices to check that $\frac{bv^*_i}{bv_i}$ is increasing in i. Recall that

$$bv^*_i = \gamma c x_{i+1} + \cdots + \gamma^{T-i} c x_T$$

[44] A related argument, which relies on a weaker notion of accounting conservatism, is provided in the proof of Proposition 2 in McNichols et al. (2014).

and

$$bv_i = d_{i+1} + \cdots + d_T.$$

Using the assumption that $\frac{x_\tau}{x_{\tau+1}}$ increases in τ and Lemma A, we obtain:[45]

$$\frac{bv_i^*}{bv_{i+1}^*} = \frac{\gamma c x_{i+1} + \cdots + \gamma^{T-i} c x_T}{\gamma c x_{i+2} + \cdots + \gamma^{T-i-1} c x_T} \le \frac{x_{i+1} + \cdots + x_T}{x_{i+2} + \cdots + x_T}.$$

If we now show that

$$\frac{bv_i}{bv_{i+1}} \ge \frac{x_{i+1} + \cdots + x_T}{x_{i+2} + \cdots + x_T}, \tag{36}$$

it will follow that $\frac{bv_i^*}{bv_i}$ increases in i.

Inequality (36) is equivalent to:

$$\frac{d_{i+1} + \cdots + d_T}{x_{i+1} + \cdots + x_T} \ge \frac{d_{i+2} + \cdots + d_T}{x_{i+2} + \cdots + x_T}.$$

The inequality above holds by Lemma A, since $\frac{d_\tau}{x_\tau}$ decreases in τ. This concludes the proof that $\frac{BV_T^*}{BV_T}$ declines in each λ_t.

To verify that $\frac{K_{T+1}^o}{BV_T}$ also decreases in λ_t, we will check that $\frac{K_{T+1}^o}{BV_T^*}$ declines in λ_t. Note that

$$\frac{K_{T+1}^o}{BV_T^*} = \frac{x_1 I_T + \cdots + x_T I_1}{bv_0^* I_T + \cdots + bv_{T-1}^* I_1}. \tag{37}$$

We have shown earlier that $\frac{bv_{\tau-1}^*}{x_\tau}$ is decreasing in τ, and therefore $\frac{x_\tau}{bv_{\tau-1}^*}$ is increasing in τ. Then, by Lemma A, the right-hand side of (37) decreases in each λ_t. □

Proof of Observation 5 $\frac{\partial PE_T}{\partial p^o}$ has the same sign as $\frac{\partial \ln(PE_T)}{\partial p^o}$.

$$\frac{\partial \ln(PE_T)}{\partial p^o} = \frac{\partial \ln P_T}{\partial p^o} - \frac{\partial \ln(E_{T+1})}{\partial p^o} = \frac{1}{P_T}\left(\frac{\partial P_T}{\partial p^o}\right) - \frac{1}{E_{T+1}}\left(\frac{\partial E_{T+1}}{\partial p^o}\right),$$

$$\frac{\partial P_T}{\partial p^o} = \frac{K_{T+1}^o}{r - s(\boldsymbol{\mu})},$$

$$\frac{\partial E_{T+1}}{\partial p^o} = K_{T+1}^o.$$

Therefore $\frac{\partial PE_T}{\partial p^o}$ has the same sign as

$$\frac{K_{T+1}^o}{(r - s(\boldsymbol{\mu}))P_T} - \frac{K_{T+1}^o}{E_{T+1}} = \left(\frac{1}{r - s(\boldsymbol{\mu})} - PE_T\right)\frac{K_{T+1}^o}{P_T}. \quad □$$

[45] Recall that $x_{T+1} = 0$.

Proof of Proposition 5 We show that if investments have grown at the constant rate λ, the difference

$$E_{T+1} - \lambda \cdot BV_T \tag{38}$$

is invariant to the accounting rules. To that end, it suffices to demonstrate that

$$D_{T+1} + \lambda \cdot BV_T = (1+\lambda)^T I_1,$$

for any depreciation schedule, $\boldsymbol{d}$. By definition:

$$\begin{aligned} D_{T+1} &= d_1 \cdot I_T + d_2 \cdot I_{T-1} + \cdots + d_t \cdot I_1 \\ &= [d_1 \cdot (1+\lambda)^{T-1} + d_2 \cdot (1+\lambda)^{T-2} + \cdots + d_T] \cdot I_1. \end{aligned}$$

Similarly,

$$\begin{aligned} BV_T &= bv_0 \cdot I_T + bv_1 \cdot I_{t-1} + \cdots + bv_{T-1} \cdot I_1 \\ &= [bv_0 \cdot (1+\lambda)^{T-1} + bv_1 \cdot (1+\lambda)^{T-2} + \cdots + bv_{T-1}] \cdot I_1 \end{aligned}$$

Thus

$$D_{T+1} + \lambda \cdot BV_T = (1+\lambda)^T \cdot \left[\sum_{t=1}^{T} (d_t + \lambda \cdot bv_{t-1}) \cdot (1+\lambda)^{-t} \right] I_1 \tag{39}$$

For any depreciation schedule and any $\lambda > -1$, the expression in brackets in the right-hand side of (39) is equal to one.[46] We may evaluate (38) by supposing replacement cost accounting. Proposition 1 then yields $E^*_{T+1} = r \cdot P_T$ and $BV^*_T = P_T$. Thus,

$$E_{T+1} - \lambda \cdot BV_T = (r - \lambda) \cdot P_T, \tag{40}$$

and the claim follows immediately. □

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[46] If one interprets λ as the interest rate, this equivalence relation relies on the identity between the initial investment and the present value of future depreciation- and imputed interest charges (Preinreich 1935).

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