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Earnings vs. stock-price based incentives in managerial compensation contracts

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Abstract We develop a theory of stock-price-based incentives even when the stock price does not contain information unknown to the firm. In our model, a manager must search for and decide on new investment projects when the market may have a difference of opinion about the quality of the firm's investment opportunities. The firm optimally provides incentives based solely on realized earnings, leading to an efficient investment policy, when the market has congruent or pessimistic beliefs; however, the firm optimally introduces stock-price-based incentives, leading to an inefficient investment policy, when the market has optimistic beliefs. If the firm can raise equity capital on favorable terms, negative NPV projects from the perspective of the firm may be positive NPV projects from the perspective of current shareholders. The firm motivates the manager to take such projects by basing some compensation on the current stock price.

Keywords Compensation · Investment policy · Mispricing

JEL Classification D86 · G31 · G32

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1 Introduction

It is common for senior managers to be compensated based on both accounting and stock-price measures of performance.[1] But, if markets are efficient and firms have access to all information known to the market, stock prices should be redundant in the determination of optimal contracts (Dybvig and Zender 1991). Consequently, theoretical explanations for the inclusion of both accounting and stock-price-based incentives have focused primarily on the contract-relevant information conveyed by each (e.g., Kim and Suh 1993; Feltham and Xie 1994; Dutta and Reichelstein 2005).[2] In this paper, we argue that, if markets are inefficient, optimal contracts may include stock-price incentives even when the stock price does not contain information unknown to the firm because such incentives motivate managers to take actions that exploit mispricing.

We develop our theory in a model of a firm whose objective is to maximize long-run value for its current risk-neutral shareholders. The firm's risk-neutral manager is required to run its assets-in-place and must be motivated to expend privately costly but unobservable effort to search for new investment projects. If the manager conducts the search she identifies a specific project, learns private (and unverifiable) information about its quality, and chooses to invest in the project if its quality is sufficiently high. We assume the firm must raise equity financing from outside investors to fund the project. Importantly, outside investors do not observe the precise quality of the project but only observe whether the firm chooses to invest; therefore the financing terms will depend on their beliefs about the quality of the firm's investment opportunities. The key novel feature of our model is that we allow outside investors (the market) to have differences of opinion with the manager and current shareholders about the quality of these opportunities.

We show that the optimal managerial compensation contract consists of a fixed salary and incentives based only on realized earnings, leading to an efficient investment policy, when the market has either congruent or pessimistic beliefs. Surprisingly, earnings-based incentives are preferred to stock-price-based incentives even when the market has congruent beliefs because the former provide stronger incentives for the manager to search for new projects. Our main result shows that the optimal contract introduces stock-price-based compensation, leading to an inefficient investment policy, when the market is optimistic about the firm's investment opportunities. The firm's investment policy is inefficient because projects may be negative NPV from the perspective of the entire firm but positive NPV from the perspective of current shareholders when the market is optimistic and equity can be raised on favorable terms (i.e., less dilutive of current shareholders). As in Fischer and Merton (1984), overvaluation reduces the cost of equity capital for current shareholders and makes otherwise marginally negative NPV projects

[1] See, e.g., Lambert and Larcker (1987) for an early cross-sectional analysis of the observed weights on accounting and stock-price measures of performance in executive compensation contracts.

[2] Other theories of stock-price-based incentives in rational markets consider differences in patience between the manager and shareholders (e.g., DeMarzo and Fishman 2007) and information manipulation (e.g., Goldman and Slezak 2006). Bolton et al. 2006 develop a theory, discussed below, of stock-price incentives in a market with overconfident investors.

attractive.[3] The firm motivates the manager to invest in such projects by providing stock-price incentives so that she benefits from the increase in the stock price accompanying the announcement of a new investment.

We show that stock-price incentives are stronger and the firm's overinvestment problem is more severe when the market is more optimistic. These effects are attenuated, however, when a larger proportion of the firm's value comes from assets-in-place because the benefit of reduced dilution will be small relative to the amount of money raised in an equity offering. Hence we predict that large, mature firms will tend to provide weaker stock-price incentives and will have less severe overinvestment problems relative to small, growth firms. We also predict that equity-dependent firms—those with less internal cash and costlier access to debt financing—will put more weight on stock-price incentives in managerial compensation contracts, consistent with the evidence in Ittner et al. (1997). And, investment expenditures in equity-dependent firms will be more sensitive to stock mispricing, consistent with the evidence in Baker et al. (2003).

Importantly, the firm exploits optimistic market sentiment by issuing equity *and* overinvesting, not by simply issuing new equity and hoarding the cash (or paying a dividend), because there is no disagreement in the market about the value of the firm's assets-in-place. Several empirical studies support the prediction that firms simultaneously issue more equity and invest more when their stock is overvalued (e.g., Chirinko and Schaller 2001; Baker et al. 2003; Gilchrist et al. 2005; and Campello and Graham 2013.)

1.1 Literature review

This paper's main contribution is to the theoretical literature examining the relative weights on accounting and stock-price-based measures of performance in managerial compensation contracts. Building on the insights of Holmstrom and Tirole (1993), Kim and Suh (1993) show that in a noisy rational expectations equilibrium both stock prices and accounting information (e.g., earnings) provide valuable signals about managerial effort, so the optimal incentive contract will depend on both performance measures largely according to their informativeness.

When the manager's actions are multi-dimensional, Paul (1992) shows that the firm's stock price is an efficient aggregator of information about its value, but it is not generally an efficient aggregator of information about managerial performance. In an efficient market, the stock price weighs an information signal (e.g., the performance of a particular project) to the extent that it resolves uncertainty about the future *value* of the firm; however, optimal incentives require that the performance measure weighs an information signal to the extent that it measures

[3] An illustrative example is the company JetFax, Inc., which changed its name to EFax.com on February 8, 1999, to coincide with the introduction of a free fax-to-email Web service. The company's stock price more than tripled in the following two months, and the company announced it would raise $15 million in new equity financing in May 1999 to "secure additional funds to fuel [its] next phase of growth."s Recently, it has been argued that access to cheap equity capital led to over-investment in natural gas drilling even when managers had private doubts about the quality of the projects (*The New York Times*, October 20, 2012).

the manager's unobservable effort or *value added* to the firm. In a multi-task setting, stock-price-based compensation may then distort the manager's effort provision across tasks. Nonetheless, the stock price should be included as a performance measure when the individual pieces of information contained in it cannot be contracted upon directly.

Bushman and Indjejikian (1993) develop a model in which basing compensation on earnings promotes the efficient allocation of effort across tasks, but the optimal contract also includes the stock price as a performance measure because it contains other information that is useful for evaluating managerial effort—for example, about the expected long-run performance of the firm's investments. Feltham and Xie (1994) generalize these results and show that performance measures should be included in compensation contracts if they reduce noise, resulting in increased equilibrium effort intensity, or improve the congruence between the impact of the manager's actions and the principal's expected gross payoff, resulting in more efficient allocation of effort across tasks. Datar et al. (2001) extend this analysis to consider the optimal weights on different performance measures.

Dutta and Reichelstein (2005) consider a multi-period, multi-task model in which the manager provides privately costly effort and must be given incentives to undertake investments. They show that accounting earnings and the firm's stock price are included among the performance measures in an optimal contract even when it can be based on realized cash flows.[4] An accrual accounting system capitalizes investment expenditures and therefore provides investment incentives by reducing the effect of current investment on income but introduces an agency cost due to measurement error when, for example, investment expenditures are difficult to distinguish from operating expenses. The stock price provides investment incentives because it is forward looking but introduces an agency cost because it contains all value-relevant information and therefore exposes the manager to risks outside her control. The weights on current earnings and the stock price in the manager's compensation contract are shown to depend on the accounting measurement errors and the variability of the investment payoffs.

Finally, compensation contracts may be based on various measures of performance depending on the manager's ability to manipulate these measures (e.g., Goldman and Slezak 2006).

Our paper is also related to the literature examining managerial myopia. Narayanan (1985) argued that, if the market cannot observe project choice, managers may overinvest in short-term projects to improve the market's perception about their ability. Stein (1989) showed that, if the stock market uses current earnings to forecast firm value, managers may attempt to manipulate current earnings even if the market is rational. Von Thadden (1995) argued that overinvestment in short-term projects may be caused by the fear of early termination of long-run projects when there is asymmetric information. Shleifer and Vishny (1990) argued that managers may forego long-term projects if managers are averse to current stock price mispricing. Our paper

[4] Paul (1992), Kim and Suh (1993), and Bushman and Indjejikian (1993) find that the optimal contract would ideally depend on the firm's terminal cash flow but they assume it is not available for contracting purposes.

differs from this work in two important respects. First, these authors assumed the firm provides short-run incentives, whereas we show that the firm will optimally choose to provide short-run incentives even when maximizing long-run value for current shareholders. Second, we argue that asymmetric information, when combined with market mispricing, leads to overinvestment in long-run projects, not underinvestment.

Similar to our paper, Bebchuk and Stole (1993) showed that short-term managerial incentives may lead to overinvestment in long-run projects if the manager has private information and the investment choice is observable. By overinvesting, the firm can increase its current stock price by signaling to the market that its long-run prospects are strong—an action that firms with low-quality projects will choose not to mimic. Stein (1996) argued that managers with short horizons will prefer to overinvest when their stock is overvalued because the cost of capital is lower. Building on the work of Stein (1996), Polk and Sapienza (2009) argued that, if the market misprices firms according to their level of investment, managers will "cater" to the market by increasing investment. The manager destroys long-run value by overinvesting, but by catering to the wishes of the market, the current share price goes up, which is desirable to shareholders who are assumed to have short horizons. In contrast, we provide an explanation for why firms would choose to offer stock-price incentives even when it leads to overinvestment and a mechanism by which this creates value for existing shareholders.

Our paper is most closely related to the work of Bolton et al. (2006) who showed that optimal compensation contracts may include short-run stock-price incentives to motivate managers to engage in activities that lead to greater divergence of investor beliefs and higher stock prices in the short run, at the expense of long-run value, when some investors are overconfident. Our paper is distinct in several important respects. First, in our model, the optimal contract does not include stock-price incentives when the market has congruent beliefs. Second, we explicitly model the activity (investment) that exploits market sentiment in the short run, yielding a rich set of empirical implications relating firm characteristics, such as indicators of mispricing, the book-to-market ratio, and uncertainty about the quality of investment opportunities to its investment, equity issuance, and compensation policies. Third, we predict that firms exploit market optimism about the quality of its investment opportunities by issuing equity *and* overinvesting. In contrast, Bolton, Scheinkman, and Xiong predict that firms exploit market sentiment by simply issuing equity and predict that firms underinvest in long-run projects because managers divert their energy into short-run activities that boost the current stock price. Finally, we assume the firm's objective is to maximize the long-run value to current shareholders, not the current stock price. Nonetheless, the firm still motivates managers to take projects that destroy long-run firm value.

2 The model

We consider an all-equity firm with risk-neutral shareholders and a risk-neutral manager. The manager is needed to run the firm's assets-in-place and, if properly motivated, may also expend privately costly but unobservable effort to search for new investment opportunities. If the manager does not provide search effort, the firm generates earnings, denoted $X > 0$, only from its assets-in-place where X is common knowledge. However, if the manager does provide search effort at a private cost of $g > 0$, the firm will also have access to an investment project that costs I dollars and generates earnings (before managerial compensation) equal to $y + \epsilon$. We assume the value of y is drawn from a uniform distribution on the interval $[-\gamma, \theta]$, where $\gamma > 0$ and $\theta > 0$ are exogenous parameters, and ϵ is a mean zero noise term.[5] The distribution over the random variable y represents the prior beliefs of the firm's current shareholders and the manager about the quality of the firm's investment opportunities. In the search process, we assume the manager observes the value of y precisely but does not observe any information about the noise term. Therefore from the manager's perspective, the NPV of the project is given by y, which can be either positive or negative. Based on her observation of y, the manager then decides whether to invest in the project. Other market participants (current shareholders and potential new investors) do not observe y precisely but update their beliefs about its distribution depending on the manager's decision to invest. Therefore our model includes both moral hazard (the manager can provide unobservable effort to find a new investment project) and asymmetric information (the manager has superior information about project quality).

We assume the firm is "equity dependent"—that is, it must finance new projects with external equity.[6] The firm may need to rely on external equity because it has little incremental debt capacity—for example, because of high cash flow volatility or small tax benefits—and little internal cash.[7] We further assume that the firm must seek outside investors to finance the new project rather than tapping current shareholders via a rights offering. This assumption is reasonable if, for example, current shareholders already have large, poorly diversified ownership in the firm.

The key novel feature of our model is that we allow outside investors to have differences of opinion with the manager and current shareholders about the quality of the firm's investment opportunities. In this framework, investors may "agree to disagree" even in the presence of commonly observed information.[8] Specifically, we assume that outside investors believe that the random variable y, the NPV of potential investments, is drawn from a uniform distribution on the support $[-\gamma, \rho\theta]$,

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[5] The noise term is required only to rule out forcing contracts that would be optimal if y could be observed precisely. As we show later, the specific distribution of the noise term does not impact the optimal contract.

[6] We do not consider internal cash or external debt as a source of financing. On one hand, our main results continue to hold with debt financing because market sentiment would also impact the terms at which the firm can raise debt; on the other hand, if the firm has access to multiple sources of finance, then the choice of financing may signal information to the market, which makes it more difficult for the firm to exploit market sentiment. Signaling associated with multiple sources of financing greatly complicates the analysis so we restrict our attention to equity-dependent firms.

[7] In their empirical study of the relation between stock prices and investment in "equity-dependent" firms, Baker et al. (2003) use the Kaplan and Zingales (1997) index of financial constraints—including variables such as scaled cash flow, scaled dividends, cash balances, leverage, and Tobin's Q—to proxy for equity-dependence.

[8] The "differences of opinion" framework has been used extensively to study asset prices and trading volume (e.g., Harrison and Kreps 1978; Harris and Raviv 1993; Kandel and Pearson 1995).

where ρ parameterizes the differences of beliefs. We will consider three cases: (1) $\rho = 1$ corresponds to the case where outside investors' beliefs are congruent with those of the manager and current shareholders, i.e., they have the same prior distribution about the firm's investment opportunities; (2) $\rho > 1$ corresponds to the case where outside investors are optimistic, i.e., they overestimate the upside potential of the firm's investment opportunities; and (3) $\rho < 1$ corresponds to the case where outside investors are pessimistic, i.e., they underestimate the upside potential of the firm's investment opportunities. For example, the case $\rho > 1$ might represent the latter stages of the Internet boom when the market continued to hold optimistic views about growth opportunities in the industry even though many insiders were concerned that growth opportunities were limited.[9] For simplicity, we assume the discount rate is zero.

The firm's earnings depend on whether the manager invests in the new project which we indicate with $j \in \{0, 1\}$. If the manager does not invest in the new project, then $j = 0$, the firm does not raise capital I, and its earnings (before compensating the manager) are:

$$e_0 = X.$$

In this case, there are symmetric beliefs about the value of the firm because earnings from the assets-in-place are common knowledge. We denote the market's assessment of firm value upon learning that the manager has chosen not to invest in a new project by P_0, which is simply given by:

$$P_0 = e_0 - w_0,$$

where w_0 denotes the manager's total compensation if she chooses not to invest in a new project. (We determine this endogenously below.)

If the manager does invest in the new project, then $j = 1$, the firm raises capital I, the project's incremental earnings are $v_1 = y + \epsilon$,[10] and the firm's total earnings (before compensating the manager) are:

$$e_1 = X + v_1.$$

In this case, the market's assessment of firm value upon learning that the manager has chosen to invest in a new project, but prior to the firm receiving the funding I, is denoted by P_1 and is given by:

$$P_1 = \hat{E}_{y,\epsilon}[e_1 - w_1 | j = 1],$$

where w_1 represents the manager's total compensation if she chooses to invest in the new project (also determined endogenously below) and the expectation, $\hat{E}_{y,\epsilon}[\cdot]$, is

[9] Gervais et al. (2011) characterize optimal compensation contracts when the *manager* is overconfident and the firm is rational.

[10] Since we have a one-period model we are effectively assuming the investment is fully depreciated and expensed at the end of the period.

taken over y and ϵ according to new investors' beliefs about the distribution of y conditional on the manager choosing to invest in the new project.[11]

The new investors' assessment of firm value when the manager chooses to invest in a new project, P_1, determines the proportional ownership the new investors receive in return for their investment, I. In particular, if the new investors provide equity capital I to invest in the project, they receive the fraction $I/(P_1 + I)$ of the firm's total earnings net of the manager's wage (i.e., $e_1 - w_1$), and the current shareholders receive the remaining fraction $P_1/(P_1 + I)$. This illustrates why market beliefs impact the firm's optimal investment policy: raising new equity is less dilutive (is cheaper) when the market is more optimistic about the firm's prospects.

We assume that investment is observable and contractible, therefore, the compensation contract may depend on whether the manager chooses to invest. The compensation contract may also depend on the the equilibrium share price at date 1, P_1, and the firm's earnings, e_1. Therefore, if the manager invests in a new project, she is paid $w_1(P_1, e_1)$. If the manager does not invest in a new project, then the firm's future earnings are non-stochastic, and, without lost of generality, the firm pays the manager only a fixed salary, w_0 (determined optimally below).[12] The manager has a reservation utility, $\bar{U}$, reflecting her outside employment opportunities.

We assume the firm's current shareholders hold their shares until total earnings are realized and the firm wishes to maximize the long-run value to current shareholders. Therefore, assuming the firm wishes to motivate the manager to search, we can express its optimization problem as:

$$\max_{w_0, w_1(P_1, e_1)} \Pi \equiv E\left[(e_0 - w_0) \cdot (1 - j) + \frac{P_1}{P_1 + I}(e_1 + I - w_1(P_1, e_1)) \cdot j\right] \qquad \text{Program (P)}$$

such that

(IC1) $\quad j(y) \in \arg\max_{j \in \{0,1\}} U(j, y) = w_0(1 - j) + w_1(P_1, e_1) \cdot j - g;$

(IC2) $\quad EU \equiv E_y[U(j(y), y)] \geq w_0 \geq \bar{U}.$

(MC) $\quad E[w_1(P_1, e_1)] \geq w_0; \quad \partial w_1(P_1, e_1)/\partial e_1 \geq 0.$

The first incentive constraint (IC1) requires that the manager makes the investment decision that is in her best interest given the signal, y.

The second incentive constraint (IC2) requires that the manager finds it optimal to provide search effort. The first inequality also ensures that the ex ante participation constraint is satisfied, i.e., the manager expects to receive at least as much as she would from her outside opportunities. The second inequality ensures that the interim participation constraint is satisfied, i.e., the manager will choose to stay with the firm to manage the assets-in-place even when the firm does not invest in a new project.

[11] We assume that potential new investors are the marginal investors. Hence, the firm's market value is determined by the beliefs of the new investors about its future earnings.

[12] Consistent with the differences of opinion framework, we assume that new investors do not update their beliefs about the quality of the firm's investment opportunities by observing the managerial compensation contract. (See Levine and Hughes (2005) for a model in which the contract conveys such information.) That is, agents agree to disagree, as opposed to the rational expectations framework in which beliefs converge based on market signals.

The monotonicity constraint (MC) requires that the manager expects to receive at least the same pay when she invests in a project as when she does not. We also require that the manager's compensation is non-decreasing in the final earnings so that she does not have an incentive to sabotage the new project.

In summary, the sequence of events in our model is as follows:

Date 0: The firm offers the manager a compensation scheme $\{w_0, w_1(P_1, e_1)\}$. If the manager accepts, she chooses whether to expend effort to search for new investment opportunities. If the manager expends effort, she finds a new project and observes its NPV, y.

Date 1: If the manager chooses to invest in the project, then new investors assign the valuation P_1 to the firm, and the firm raises the necessary capital, I, by issuing equity. If the manager chooses not to invest in the project, then the firm valuation is P_0.

Date 2: Earnings are realized and distributed to shareholders net of the compensation to the manager.

For what follows, we make the following regularity assumptions to obtain interior solutions:

(A1) $X - \bar{U}$ is sufficiently large;
(A2) γ is sufficiently large; and
(A3) g is sufficiently small.

The precise constraints on these parameters are determined in the proofs in the Appendix 1. Assumption (A1) ensures that the firm's optimization problem is concave; (A2) implies that investment in the new project will occur with positive probability but not with certainty; and (A3) requires that the cost of effort, g, is sufficiently small that the firm motivates the manager to search for new projects. Together, these assumptions ensure that the firm's optimization problem is well behaved and allow us to focus on the parameter region with interior solutions for the effort and investment decisions.

3 The optimal contract

3.1 A relaxed problem

We first consider a relaxed version of the current problem described in Program (P). As in our current problem, the firm hires the manager to expend search effort to find an investment project, outside equity is needed if the investment is to be made, and the outsider investors' beliefs are as described before. However, in our relaxed problem, we now assume the firm (not just the manager) knows the expected future earnings of the new project, y, i.e., there is no asymmetric information between the manager and the current shareholders. Operationally, the firm's problem is similar to Program (P) except for the incentive constraint (IC1); that is, the firm can now commit to an investment policy based on the signal y. The investment policy is represented by a threshold NPV (or hurdle) H where the firm invests ($j = 1$) if $y > H$ and the firm does not invest ($j = 0$) if $y \leq H$. The firm pays the manager a total wage of w_0 if no investment is made and $w_1(P_1, e_1)$ if the investment is made. Importantly, compensation $w_1(P_1, e_1)$ can be a function of the known signal y. Hence, the program relaxes Program (P) even when menu contracts can be used to elicit information from the manager in our original problem. Clearly, the firm's payoff in this relaxed problem represents an upper bound for the firm's payoff in the original optimization problem, Program (P).

Consider the case in which the market has either optimistic or congruent beliefs.[13] Since the firm doesn't need to motivate the manager to make appropriate investment decisions, it is straightforward to show that the firm will optimally set $w_1(P_1, e_1)$ equal to a fixed wage, denoted w_1^R. This wage must be set high enough relative to w_0 to encourage search effort (IC2) but will satisfy the (MC) constraint ($\partial w_1(P_1, e_1)/\partial e_1 \geq 0$) with equality, because optimistic investors will otherwise overestimate the expected value of $w_1(P_1, e_1)$, leading to an underpricing of the firm's equity when the firm seeks outside financing.

The firm's optimization problem in this case, which is indicated as Program (R), can therefore be expressed as follows:

$$\max_{w_0, w_1^R} \Pi \equiv E\left[(e_0 - w_0) \cdot (1 - j) + \frac{P_1}{P_1 + I}(e_1 + I - w_1^R) \cdot j\right] \qquad \text{Program (R)}$$

such that

(IC2) $\quad EU \equiv E_y[U(j(y), y)] \geq w_0 \geq \bar{U}.$

(MC) $\quad w_1^R \geq w_0.$

The following result characterizes the solution to the relaxed Program (R).

Proposition 1 *In Program (R), the compensation, w_0 and w_1^R, the stock price, P_1^R, and the firm's payoff, Π^R, can be expressed as functions of the investment threshold, H.*

$$w_0 = \bar{U},$$

$$w_1^R = \bar{U} + g\Big/\left(\frac{\theta - H}{\theta + \gamma}\right),$$

$$P_1^R = X + \frac{\rho\theta + H}{2} - \bar{U} - g\Big/\left(\frac{\theta - H}{\theta + \gamma}\right),$$

$$\Pi^R = (X - \bar{U}) \cdot \left(\frac{H + \gamma}{\theta + \gamma}\right) + \frac{P_1^R}{P_1^R + I}\left[\left(X + I - \bar{U} + \frac{\theta + H}{2}\right) \cdot \left(\frac{\theta - H}{\theta + \gamma}\right) - g\right].$$

The firm chooses H to maximize Π^R.

[13] The case in which the market has pessimistic beliefs proceeds along somewhat similar lines. The proofs for this case are derived in Appendix 2.

Proof See the Appendix 1. □

3.2 Implementation with a linear contract

We now show that a linear compensation contract can implement the optimal investment policy, defined by the investment threshold H, in Proposition 1 when only the manager has the information signal y about project quality. We also show that the firm's payoff under this contract approximates the payoff Π^R, which is an upper bound for the firm's payoff in Program (P), and therefore the linear compensation contract approximates the optimal contract in our original problem.

If the manager does not invest in a new project, then the firm's future earnings are non-stochastic, and, without lost of generality, the firm pays the manager only a fixed salary, w_0. If the firm invests in the new project, the linear compensation contract pays the manager a fixed salary and provides stock-price and earnings-based incentives according to:

$$w_1(P_1, e_1) = a + b \cdot P_1 + c \cdot e_1,$$

where $b \in [0, 1]$ and $c \in [0, 1]$ are no short-position constraints on the manager's contract.

It will be helpful to re-write the compensation contract as follows:

$$w_1(P_1, e_1) = a + b \cdot P_1 + c \cdot e_1 \equiv A + B \cdot \hat{y} + C \cdot v_1$$

where $\hat{y} \equiv \hat{E}_y(y|j = 1)$ is the market's expectation of the project NPV conditional on the firm choosing to invest. The following lemma shows there is a one-to-one mapping from the parameters $\{a, b, c\}$ to the parameters $\{A, B, C\}$.

Lemma 1 *There is a one-to-one mapping from the parameters* $\{a, b, c\}$ *to the parameters*
$A = \frac{a+(b+c)\cdot X}{1+b}$, $B = b\left(\frac{1-c}{1+b}\right)$, *and* $C = c$, *where* $B \in [0, (1 - C)/2]$ *and* $C \in [0, 1]$.

Proof See the Appendix 1. □

This alternative expression for w_1 shows that the manager's compensation depends on the market's perception of the project NPV, $\hat{y}$, which is an important determinant of the stock price, as well as the project's incremental earnings, v_1.

Proposition 2 *The following linear compensation contract implements the optimal investment threshold, H, as specified in Proposition 1*:

$$
\begin{aligned}
w_0 &= \bar{U}, \\
w_1 &= A + B \cdot \hat{y} + C \cdot v_1 \quad \textit{where} \\
A &= \bar{U}, \\
B &= \frac{-2CH}{\rho\theta + H}, \\
C &= 2g(\theta + \gamma)/(\theta - H)^2.
\end{aligned}
$$

If the firm invests the stock price is given by $P_1 = X + 0.5 \cdot (1 - B - C) \cdot (\rho\theta + H) - A$. *The firm's payoff,* Π, *relative to its upper bound,* Π/Π^R, *approaches one when (1) managerial compensation is small relative to firm value* ($X - \bar{U}$ *is large) or (2) the cost of effort is small (g is small).*

Proof See the Appendix 1. □

The linear compensation contract implements the optimal investment policy in the relaxed Program (R) exactly and yields a payoff that approximates the upper bound payoff to the firm. The approximation is exact when the value of the firm P_1 in Proposition 2 is equal to the value P_1^R in Proposition 1. The proof of Proposition 2 shows that the discrepancy between P_1 and P_1^R is due to the difference in beliefs between outside investors and insiders about the expected value of the wage contract. That is, the value of the firm is determined net of compensation costs, and, since managerial compensation $w_1(P_1, e_1)$ has a stochastic component, there may be differences of opinion about the expected value of managerial compensation. As we show in Proposition 2, these differences are relatively small when the value of assets-in-place is large relative to compensation ($X - \bar{U}$ is large) and when the stochastic component of the linear compensation contract is small (the cost of effort g is small). In reality, managerial compensation constitutes a small proportion of total firm value, and discrepancies in the value of these contracts between investors with different opinions are likely to be much smaller still. Thus we believe the linear compensation contract provides an economically meaningful approximation to the optimal contract.

Moreover, note that the relaxed Program (R) allows compensation $w_1(P_1, e_1)$ to be a general function of P_1 and e_1. In particular, the form of the contract may depend on the quality of the investment project, y. Consequently, the linear contract implements the optimal investment policy and closely approximates the optimal firm payoff even when menu contracts designed to elicit the manager's private information are allowed.

The following corollary characterizes the key results in our paper.

Corollary 1

(i) *The manager invests efficiently* ($H = 0$) *if the market has congruent or pessimistic beliefs* ($\rho \leq 1$), *but the manager over-invests* ($H < 0$) *if the market has optimistic beliefs* ($\rho > 1$).

(ii) *The manager receives no stock-price-based incentives* ($B = 0$) *if the market has congruent or pessimistic beliefs* ($\rho \leq 1$), *but receives stock-price-based incentives* ($B > 0$) *if the market has optimistic beliefs* ($\rho > 1$).

Proof See the Appendix 1. □

The intuition for the firm's investment policy is as follows. New investments generate two potential benefits for current shareholders: the NPV of the project and the NPV from equity issuance. If new investors have congruent or pessimistic beliefs ($\rho \leq 1$), equity issuance is not a positive NPV transaction for current shareholders and the manager invests efficiently (i.e., the threshold NPV is $H = 0$).[14] However, if new investors have optimistic beliefs ($\rho > 1$), the firm's shares are overvalued, equity issuance is a positive NPV transaction for current shareholders, and the manager invests in projects that may otherwise be negative NPV (i.e., the threshold NPV is $H < 0$).

The firm implements the optimal investment policy when new investors have congruent or pessimistic beliefs by basing the manager's compensation only on realized earnings ($B = 0$ and $C > 0$). It may be surprising that stock-price incentives are not a perfect substitute for earnings-based incentives when the market has congruent beliefs. The reason for this is that the two types of incentives have different effects on the manager's decision to provide search effort. To illustrate, suppose $B > 0$ and $C = 0$ so that the manager only receives stock-price incentives. In this case, the manager has no incentive to provide search effort because her decision to invest is independent of y—new investors only see whether an investment is made when setting the stock price, and the manager's compensation is unrelated to the project's earnings (i.e., independent of y). Conversely, when $B = 0$ and $C > 0$, the manager's investment decision does depend on y, and, therefore, she has more of an incentive to provide search effort. Consequently, earnings-based incentives are preferred to stock-price incentives even when the market has congruent beliefs.[15]

Our key finding is that the firm introduces stock-price incentives ($B > 0$) when new investors are optimistic. In this case, the firm's equity is overvalued, and the firm wishes to motivate the manager to take otherwise negative NPV projects ($H < 0$). The firm's cost of equity capital is low when its shares are overvalued (i.e., issuing equity leads to little dilution of current shareholders), and projects that are negative NPV from the perspective of the *entire firm* may be positive NPV from the perspective of *current shareholders* (Fischer and Merton 1984). By benefiting from

[14] The linear compensation contract implements the first-best investment policy *conditional on the manager providing search effort*. However, it does not necessarily implement the first-best search effort by the manager.

[15] Dybvig and Zender (1991) find that stock-price-based and earnings-based incentives are perfect substitutes when the market is rational. Our model differs from theirs because we assume the manager must provide privately costly effort to learn about the investment, whereas they assume the manager is endowed with this information. In our model, earnings-based incentives and stock-price incentives are not perfect substitutes when the market has congruent beliefs because of their differential ability to motivate search effort.

the stock price increase accompanying the announcement of a new investment, the manager is motivated to invest in such projects. The manager cannot be motivated to take such projects if incentives are based only on earnings.

Importantly, the firm cannot exploit mispricing by issuing equity and foregoing the investment because the market is optimistic about the firm's investment opportunities, not the value of its assets-in-place. Thus issuing equity to raise cash to pay a dividend is not a value-enhancing strategy for current shareholders as in Bolton et al. (2006). This allows us to develop several implications linking market mispricing to the firm's financing, compensation, and investment policies, as we describe in the next section.

4 Implications

We now derive comparative statics results and discuss the empirical implications of our model. We first characterize the important variables in our model and suggest reasonable empirical proxies for them.

Our model applies best to an equity-dependent firm. If, for example, the firm has access to internal cash and risk-free debt, the market might interpret equity issuance as a negative signal about the quality of the firm's investment opportunities, diminishing its willingness to exploit the market's initial optimism by issuing shares (e.g., Myers and Majluf 1984). Baker et al. (2003) use the Kaplan and Zingales (1997) index of financial constraints to proxy for equity-dependence. The index attaches negative weight (less equity-dependent) to variables such as scaled cash flow, scaled dividends, and cash balances, and positive weight (more equity-dependent) to leverage and Tobin's Q.

The parameter X represents earnings from assets-in-place. This should be measured relative to the value of the firm's investment opportunities. Hence a good empirical proxy is the firm's book-to-market ratio. The parameter γ describes the downside risk, and the parameter θ represents the upside potential of the firm's investment opportunities. Thus higher values of these parameters are associated with greater uncertainty about the quality of the firm's investment opportunities. Finally, the parameter ρ represents market sentiment and is a measure of mispricing in our model. The empirical literature suggests several possible proxies: for example, Gilchrist et al. (2005) use analyst earnings forecast dispersion to measure mispricing, Polk and Sapienza (2009) use discretionary accruals and ex post stock performance to measure mispricing, and Baker and Wurgler (2006) summarize variables associated with investor sentiment from the academic literature to develop a cross-sectional measure of mispricing.

4.1 Investment policy

The following results show how the manager's investment policy depends on the firm's characteristics and the market environment.

Corollary 2 *If the market is optimistic* ($\rho > 1$), *the investment threshold* (H) *decreases in the market's optimism* (ρ), *the cost of effort* (g), *and investment downside potential* (γ); *and increases in the value of assets-in-place* (X). *The sensitivity of the threshold to market sentiment* ($|dH/d\rho|$) *decreases in the value of assets-in-place* (X).

Our model predicts that equity-dependent firms will issue more equity and invest more when their stock is more overvalued (ρ increases). This result is consistent with numerous empirical findings. For example, Loughran and Ritter (1995) find evidence that companies respond to changes in investor sentiment by issuing equity during periods of overvaluation.[16] There is also considerable evidence that mispricing leads to greater firm investment. Although early time-series (Blanchard et al. 1993) and cross-sectional (Morck et al. 1990) studies support the view that mispricing leads to modest increases in firm investment, recent studies such as Polk and Sapienza (2009) find stronger evidence relating firm investment to measures of mispricing. Furthermore, several studies document an explicit link between measures of mispricing and equity issuance and firm investment. For example, Chirinko and Schaller (2001) find that equity issuance and investment surged at the peak of the Japanese stock market boom in the late 1980s. Baker et al. (2003) find that investment in equity-dependent firms is more sensitive to measures of equity mispricing than firms with greater access to internal cash and external debt financing. Gilchrist et al. (2005) find evidence that new equity issuance and investment increase when analyst forecast dispersion, a measure associated with mispricing, increases. And Campello and Graham (2013) find evidence that technology firms had a greater propensity to invest the proceeds from equity issuance during the technology "bubble" in the 1990s than did non-tech manufacturing firms. This result also suggests a novel explanation for the excessive volatility of investment over the business cycle. If market sentiment is overly optimistic in good times and overly pessimistic in bad times, then investment will be more volatile than predicted by changes in fundamentals.

In the presence of market optimism, the firm's investment policy is more efficient (the hurdle NPV, H, moves toward zero) when the firm has greater assets-in-place, X. The reason for this was noted above: market sentiment has less impact on the NPV of new equity financing when the firm has greater assets-in-place. Thus the firm's investment policy is driven more by the project fundamentals (investment is more efficient). This result also implies that the firm's investment policy is more efficient when the manager has lower effort costs, g, and the potential project downside, γ, is smaller. The latter result implies that firms with more dispersion in the quality of their investment opportunities have less efficient investment policies when the market is optimistic.

Our model also predicts that the sensitivity of investment to market sentiment is attenuated when the firm has more assets-in-place (X increases). The reason is that, when a firm raises I dollars of new equity, only a small portion of the value is

[16] Kim and Weisbach (2008) find supporting evidence in a study of IPOs and SEOs in 38 countries, although De Angelo et al. (2010) argue that the economic significance of the mispricing effect on equity issuance is small because most firms choose not to issue equity when it is overvalued.

impacted by the market's optimism when the firm's existing assets have greater value. In other words, the benefits of issuing overvalued equity are small when the earnings from new investments are small relative to the earnings from the firm's existing businesses. Consequently, market optimism has a smaller effect on equity issuance and investment in large, mature firms (i.e., high book-to-market) than it does on small, growth firms.

4.2 Compensation

The optimal compensation contract (A, B, C) must (1) motivate the manager to provide search effort and (2) implement the optimal investment policy. To understand the roles played by the various components of the compensation contract, consider the case in which new investors have congruent beliefs ($\rho = 1$). In this setting, the firm chooses earnings-based incentives, C, sufficiently large to motivate the manager to provide search effort. Since the manager benefits from generating higher earnings ($C > 0$), the manager also has an incentive to invest in the new project if and only if it is positive NPV ($H = 0$). Hence long-term incentives are chosen to be just large enough to motivate search effort, and these incentives alone are sufficient to implement the optimal investment policy (i.e., $A = \bar{U}$ and $B = 0$). However, if new investors are optimistic, earnings-based incentives motivate search effort but are insufficient to implement the optimal investment policy: with only earnings-based incentives, the manager's optimal investment policy is to set $H = 0$, which is sub-optimal for current shareholders. In this case, stock-price incentives, B, provide an added incentive to invest because the manager benefits from the stock price increase associated with an investment. The following results show how the manager's earnings-based incentives depend on the firm's characteristics and the market environment.

Corollary 3 *If the market is optimistic* ($\rho > 1$), *the manager's earnings-based incentives* (C) *decrease in the market's optimism* (ρ); *and increase in the cost of effort* (g), *the investment downside potential* (γ), *and the value of assets-in-place* (X).

The optimal earnings-based incentives, C, must be sufficiently large to motivate the manager to provide search effort, but the manager's willingness to search also depends on the optimal investment policy because the manager will have less incentive to search for a new project if she is unlikely to learn information that will motivate her to invest. An increase in market sentiment (ρ) increases the likelihood of investment (H decreases), and, hence, weaker earnings-based incentives are required to motivate search effort. If investors are optimistic, an increase in the value of assets-in-place (X) decreases the likelihood of investment and, hence, stronger earnings-based incentives are required to motivate search effort. If the project downside (γ) increases, the likelihood of investment decreases, and stronger earnings-based incentives are required to motivate search effort. Finally, if the cost of effort (g) increases, stronger earnings-based incentives are required to motivate search effort.[17]

[17] These comparative statics results hold in the region where it is optimal for the firm to motivate search effort. If, for example, the cost of effort becomes too high, then it may be optimal for the firm not to motivate search effort, in which case it is straightforward to show that the firm chooses $B = C = 0$.

By Lemma 1, the stock-price incentives are given by $b = B/(1 - B - C)$. From Proposition 2, we have $b = 0$ when $\rho = 1$ and $b = \frac{-2CH}{(1+C)H+(1-C)\rho\theta}$ when $\rho > 1$.

Corollary 4 If the market is optimistic ($\rho > 1$), the manager's stock-price incentives (b) increase in the market's optimism (ρ), the cost of effort (g), and the investment downside potential (γ); and decrease in the value of assets-in-place (X).

The firm wishes to motivate the manager to take some negative NPV projects when its stock is overvalued ($H < 0$ when $\rho > 1$). Earnings-based incentives alone cannot implement this investment policy, so the firm optimally introduces stock-price incentives because this motivates the manager to invest in order to benefit from the increase in the stock price accompanying the announcement of a new investment (even when the project is marginally negative NPV). The firm optimally puts more weight on stock-price incentives when the market is more optimistic because this motivates the manager to invest in even lower quality projects (H decreases) financed with overvalued equity. The firm optimally puts less weight on stock-price incentives when the firm has more assets-in-place (X increases) because the benefit of issuing equity is small when the earnings from new investments are small relative to the earnings from the firm's existing businesses. Thus firms likely to be associated with mispricing (e.g., greater dispersion in analyst earnings forecasts, high discretionary accruals) and firms with relatively high growth opportunities (e.g., high market-to-book ratios) will compensate managers with stronger stock-price incentives. Furthermore, since it is relatively cheap for the firm (acting on behalf of current shareholders) to motivate effort via stock-price incentives when new investors are optimistic, the firm optimally provides stronger stock-price incentives when the cost of effort (g) is high and when the investment downside (γ) is high.[18]

4.3 Announcement effects

The firm's stock price will react to the announcement of a new investment because it reveals information about the firm's future earnings. If the firm does not invest, the stock price is P_0 but if the firm does invest, the stock price is P_1. Hence, the initial stock price (based on new investors' valuation) can be expressed as

$$P_{initial} = \frac{H+\gamma}{\rho\theta+\gamma}P_0 + \frac{\rho\theta - H}{\rho\theta+\gamma}P_1.$$

Therefore the price reaction to the announcement that the firm will invest is:

$$\triangle P \equiv P_1 - P_{initial} = \frac{H+\gamma}{\rho\theta+\gamma}(P_1 - P_0).$$

[18] The choice of stock-price (short-run) incentives and earnings-based (long-run) incentives can be related to compensation vesting periods. For example, stock-based compensation often vests over a period of time, typically 5 years. In our model, a longer vesting period is consistent with relatively more earnings-based incentives compared to stock-price incentives.

Corollary 5 If the market is optimistic ($\rho > 1$), the stock price impact when the manager chooses to invest, measured by $\triangle P \equiv P_1 - P_{initial}$, is positive and increases in ρ, X and γ; and decreases in g.

The stock price impact is greater when the market's expectation of project quality, conditional on the manager choosing to invest, is greater. Thus the stock price impact is greater when the value of the firm's assets-in-place (X) is greater and the cost of effort (g) is smaller because, from Corollary 2, each has the effect of increasing the investment threshold (H). The investment downside (γ) has two effects on the stock-price reaction to new investment: greater downside reduces the threshold for investment (reducing the announcement effect) but also decreases the ex ante likelihood of investment (increasing the announcement effect). In our model, the latter effect dominates, so the stock price reaction to new investment is greater when the investment downside is greater. The degree of market optimism (ρ) also has two effects on the stock price reaction to new investment: greater market optimism reduces the threshold for investment but also increases the market's perception of the upper bound on the firm's investment opportunities. In our model, the latter effect dominates, so the stock price reaction to new investment is greater when the market is more optimistic.

5 Conclusions

We develop a theory to explain why managers are compensated with stock-price incentives even when stock prices convey no contract-relevant information unknown to the firm. If the market is optimistic about a firm's prospects, the firm can raise equity capital on favorable terms, and otherwise negative NPV projects may enhance value to *current* shareholders even though they destroy long-run firm value. By basing some of the manager's compensation on the current stock price, the firm motivates the manager to invest in such projects because the stock price increases upon the announcement of a new investment. We derive numerous novel empirical implications relating measures of market sentiment and firm characteristics such as the book-to-market ratio and the dispersion in the quality of investment opportunities to the firm's investment, security issuance, and compensation policies.

We restricted our attention to equity-dependent firms, i.e., firms that must raise new equity financing to fund new projects because they have neither sufficient cash nor sufficient access to debt financing. It would be interesting to extend the model to allow for cash, debt financing, or both. For example, it is plausible that if new equity investors are optimistic about a firm's investment opportunities, then new debt investors are also likely to be optimistic, in which case access to "cheap" debt financing encourages firms to invest in negative NPV projects, particularly if the debt is risky and the firm has few assets-in-place. However, the choice of financing may send signals to the market about the manager's private information (as noted by Myers and Majluf 1984) which hinders the firm's ability to take advantage of security mispricing. We leave these issues to future research.

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Appendix 1: Proofs

Proof of Proposition 1 Note that $w_1^R \geq w_0 \geq \bar{U}$. It must be true that $w_0 = \bar{U}$, otherwise, the firm can lower both w_1^R and w_0 and increase its payoff.

The firm chooses w_1^R to satisfy the (IC2) constraint. Thus,

$$EU = w_0 \cdot prob(y \leq H) + w_1^R \cdot prob(y > H) - g = \bar{U},$$

which implies

$$w_1^R = [\bar{U} + g - w_0 \cdot prob(y \leq H)]/prob(y > H) = \bar{U} + g \Big/ \left(\frac{\theta - H}{\theta + \gamma}\right). \quad (1)$$

The stock price can be expressed as

$$P_1^R = \hat{E}_{y,\epsilon}[X + y - w_1^R | y > H] = X + \frac{\rho\theta + H}{2} - \bar{U} - g \Big/ \left(\frac{\theta - H}{\theta + \gamma}\right),$$

and the firm's payoff can be expressed as

$$\begin{aligned} \Pi^R &= E[e_0 - w_0 | y \leq H] \cdot prob(y \leq H) \\ &\quad + \frac{P_1^R}{P_1^R + I} E[e_1 + I - w_1^R | y > H] \cdot prob(y > H) \\ &= (X - \bar{U}) \cdot prob(y \leq H) \\ &\quad + \frac{P_1^R}{P_1^R + I} \left[E[e_1 + I | y > H] \cdot prob(y > H) - w_1^R \cdot prob(y > H)\right] \\ &= (X - \bar{U}) \cdot \frac{H + \gamma}{\theta + \gamma} + \frac{P_1^R}{P_1^R + I} \left[(X + \frac{\theta + H}{2} + I - \bar{U}) \cdot \frac{\theta - H}{\theta + \gamma} - g\right]. \end{aligned} \quad (2)$$

The firm chooses H to maximize Π^R. □

Proof of Lemma 1 Re-write the linear compensation contract as:

$$\begin{aligned} w_1(P_1, e_1) &= a + b \cdot (X + p_1) + c \cdot (X + v_1) \\ &= a^* + b \cdot p_1 + c \cdot v_1. \end{aligned}$$

where

$$a^* = a + (b + c) \cdot X,$$

and

$$p_1 = \hat{E}_{y,\epsilon}[v_1 - w_1(P_1, e_1) | j = 1] = \hat{E}_{y,\epsilon}[v_1 - (a^* + b \cdot p_1 + c \cdot v_1) | j = 1].$$

In this form, the fixed salary (a^*) is non-stochastic, stock-price incentives ($b \cdot p_1$)

depend on the market's expectation of the project NPV net of compensation costs, and the earnings-based incentives ($c \cdot v_1$) depend on the realized earnings from the new project. Thus the earnings-based compensation is measured according to the new project's economic value added (EVA) since v_1 measures its earnings net of capital costs.

If we denote $A = \frac{a^*}{1+b}, B = b\left(\frac{1-c}{1+b}\right)$, and $C = c$, then it is straightforward to show that

$$w_1(P_1, e_1) = A + B \cdot \hat{y} + C \cdot v_1,$$
$$p_1 = (1 - B - C)\hat{y} - A,$$

where $\hat{y} \equiv \hat{E}_y(y|j = 1)$ is the market's expectation of the project NPV conditional on the firm choosing to invest. It is easy to verify that there is a one-to-one mapping from the parameters $\{a, b, c\}$ to the parameters $\{A, B, C\}$, and it is also easy to verify that the short-sale constraints $b \in [0, 1]$ and $c \in [0, 1]$ imply the constraints $B \in [0, (1 - C)/2]$ and $C \in [0, 1]$. □

Proof of Proposition 2 We construct a linear contract to implement the optimal payoff, Π^R, as specified in Proposition 1, in five steps.

(i) Consider the manager's investment decision as described in the incentive condition (IC1). If she does not invest in the new project, her payoff is simply $U(j = 0, y) = w_0 - g$. If she invests in the new project, her expected payoff is $U(j = 1, y) = A + B\hat{y} + Cy - g$. Since $U(j = 1, y)$ is increasing in y, the manager's optimal investment decision can be represented by a threshold $H \in [-\gamma, \theta] : j(y) = 1$ if and only if $y > H$. Clearly, she chooses $j = 1$ if $U(j = 1, y) > U(j = 0, y)$, or

$$y > H = -\hat{y}B/C + (w_0 - A)/C.$$

Since

$$\hat{y} = \hat{E}(y|y \geq H) = \frac{\rho\theta + H}{2} = \frac{\rho\theta - \hat{y}B/C + (w_0 - A)/C}{2},$$

it follows that

$$\hat{y} = \frac{\rho\theta + (w_0 - A)/C}{2 + B/C},$$

and

$$H = \frac{-B}{2C + B}\rho\theta + \frac{2(w_0 - A)}{2C + B}.$$

Imposing H at the level specified in Proposition 1 implies

$$B = 2\frac{-CH + w_0 - A}{\rho\theta + H}. \quad (3)$$

(ii) Consider the manager's decision whether to expend search effort. If she does not, the manager will get a payoff of w_0. (She can claim she put in effort but y was too small.)
If she expends effort her expected payoff is given by:

$$\begin{aligned} EU &= E[(1-j)w_0 + j(A + B\hat{y} + Cy)] - g \\ &= E[j(A - w_0 + B\hat{y} + Cy)] + w_0 - g \\ &= CE[j(y-H)] + w_0 - g \\ &= C\frac{(\theta - H)^2}{2(\theta+\gamma)} + w_0 - g \end{aligned}$$

where the third equality follows from the fact that $A - w_0 + B\hat{y} + CH = 0$. Therefore, to motivate the manager to expend search effort, the following condition must be satisfied:

$$C\frac{(\theta - H)^2}{2(\theta+\gamma)} \geq g.$$

If the inequality is binding, it follows that

$$C = 2g(\theta+\gamma)/(\theta-H)^2. \tag{4}$$

(iii) According to the ex ante participation constraint, let

$$\begin{aligned} \bar{U} &= E[(1-j)w_0 + j(A + B\hat{y} + Cy)] - g \\ &= w_0 \cdot prob(y \leq H) + E[A + B\hat{y} + Cy | y > H] \cdot prob(y > H) - g, \end{aligned}$$

which has two implications. First,

$$\begin{aligned} \bar{U} &= w_0 \cdot prob(y \leq H) + (A + B\frac{\rho\theta + H}{2} + C\frac{\theta + H}{2}) \cdot prob(y > H) - g \\ &= w_0 \cdot prob(y \leq H) + (-CH + w_0 + C\frac{\theta + H}{2}) \cdot prob(y > H) - g \\ &= w_0 + C\frac{\theta - H}{2} \cdot \frac{\theta - H}{\theta + \gamma} - g \\ &= w_0, \end{aligned}$$

where the second equality follows from the expression of B in Eq. (3), and the last equality follows from the expression of C in Eq. (4). This fixes $w_0 = \bar{U}$.
Second,

$$\begin{aligned} E[A + B\hat{y} + Cy | y > H] &= [\bar{U} + g - w_0 \cdot prob(y \leq H)]/prob(y > H) \\ &= \bar{U} + g \Big/ \left(\frac{\theta - H}{\theta + \gamma}\right). \end{aligned} \tag{5}$$

It follows immediately that

$$A = \bar{U} + g\Big/\left(\frac{\theta - H}{\theta + \gamma}\right) - B\frac{\rho\theta + H}{2} - C\frac{\theta + H}{2}.$$

This equation holds if $A = \bar{U}$ and B is expressed by Eq. (3). Therefore fix $A = \bar{U}$.

(iv) It follows from $A = \bar{U}$, the expression of B in Eq. (3), and the expression of C in Eq. (4) that

$$\begin{aligned} P_1 = X + p_1 &= X + (1 - B - C)\frac{\rho\theta + H}{2} - A \\ &= X + \frac{\rho\theta + H}{2} - \bar{U} - g\Big/\left(\frac{\theta - H}{\theta + \gamma}\right) + C\frac{\theta(1-\rho)}{2} \\ &= P_1^R + C\frac{\theta(1-\rho)}{2}, \end{aligned} \tag{6}$$

where P_1^R is from Proposition 1. Clearly, if $\rho > 1$, then $P_1 < P_1^R$. The firm's payoff is given by:

$$\begin{aligned} \Pi &= E[e_0 - w_0 | y \le H] \cdot prob(y \le H) \\ &\quad + \frac{P_1}{P_1 + I} E[e_1 + I - w_1(P_1, e_1) | y > H] \cdot prob(y > H) \\ &= (X - \bar{U}) \cdot prob(y \le H) \\ &\quad + \frac{P_1}{P_1 + I}[E[e_1 + I | y > H] \cdot prob(y > H) - E[w_1(P_1, e_1) | y > H] \\ &\quad \cdot prob(y > H)]. \end{aligned} \tag{7}$$

Compare the expressions of Π in Eq. (7) and Π^R in Eq. (2). Note that $E[w_1(P_1, e_1)|y > H] = w_1^R$ from Eqs. (1) and (5). When managerial compensation is small relative to firm value ($X - \bar{U}$ is large) or the cost of effort is small (g is small so C as expressed in Eq. 4 is small), we have $\frac{P_1}{P_1+I} \approx \frac{P_1^R}{P_1^R+I}$ from Eq. (6). Therefore the firm's payoff, Π, relative to its upper bound, Π/Π^R, approaches one.

(v) We will show in the proof of Corollary 1 that if $\rho \ge 1$ then $H \le 0$, which implies $B \ge 0$. It is easy to verify that with reasonable assumptions about parameter values that the no short-sale conditions are met.

□

Proof of Corollary 1 Consider the case where $\rho \ge 1$. Following Proposition 1, it is straightforward to show after some algebra that

$$\Pi^R = (X-\bar{U})\cdot\frac{H+\gamma}{\theta+\gamma} + (X+I+\frac{\theta+H}{2}-\bar{U})\cdot\frac{\theta-H}{\theta+\gamma} - g$$
$$-\frac{I}{P_1^R+I}\left[X+I+\frac{\rho\theta+H}{2}+\frac{(1-\rho)\theta}{2}-\bar{U}-g\Big/\frac{\theta-H}{\theta+\gamma}\right]\frac{\theta-H}{\theta+\gamma}$$
$$= (X-\bar{U})\cdot\frac{H+\gamma}{\theta+\gamma} + \left(X+I+\frac{\theta+H}{2}-\bar{U}\right)\cdot\frac{\theta-H}{\theta+\gamma} - g$$
$$-I\cdot\frac{\theta-H}{\theta+\gamma} - \frac{I}{P_1^R+I}\frac{(1-\rho)\theta}{2}\frac{\theta-H}{\theta+\gamma}$$
$$= X-\bar{U}-g+\frac{\theta^2-H^2}{2(\theta+\gamma)} - \frac{I}{P_1^R+I}\frac{(1-\rho)\theta}{2}\frac{\theta-H}{\theta+\gamma}.$$

Let $\bar{\Pi} = (\theta+\gamma)\Pi^R$. The optimal interior H is given by the following first-order condition (foc).

$$0 = \frac{d\bar{\Pi}}{dH}$$
$$= -H + \frac{I}{P_1^R+I}\frac{(1-\rho)\theta}{2} + \frac{I}{(P_1^R+I)^2}\frac{(1-\rho)\theta}{2}(\theta-H)\left(\frac{1}{2}-\frac{g(\theta+\gamma)}{(\theta-H)^2}\right). \tag{8}$$

Therefore,

$$H = \frac{I}{P_1^R+I}\frac{(1-\rho)\theta}{2}\left[1+\frac{1}{P_1^R+I}(\theta-H)\left(\frac{1}{2}-\frac{g(\theta+\gamma)}{(\theta-H)^2}\right)\right]. \tag{9}$$

If $\rho = 1$, then it is obvious that $H = 0$. If $\rho > 1$, then the right hand side of the above equation is clearly less than zero. Therefore, $H < 0$.

It is easy to show that

$$\frac{d^2\bar{\Pi}}{dH^2} = -1 + o(1/(P_1^R+I)^2), \tag{10}$$

where $o(1/(P_1^R+I)^2)$ indicates higher or similar order of $1/(P_1^R+I)^2$. Under the mild condition that $X-\bar{U}$ is relatively large so P_1^R+I is relatively large, the $d^2\bar{\Pi}/dH^2 < 0$, so the firm's objective function is globally concave. The case $\rho < 1$ is considered separately in Appendix 2.

(ii) The statements for $B = -2CH/(\rho\theta+H)$ follows immediately from (i). □

Proof of Corollary 2 Note that the optimal H is determined by Eq. (8). Under the condition that $X-\bar{U}$ is relatively large, we have

$$\frac{\partial^2\bar{\Pi}}{\partial H\partial\rho} = -\frac{I}{P_1^R+I}\frac{\theta}{2} + o(1/(P_1^R+I)^2) < 0.$$

Combining this with Eq. (10), and we have

$$\frac{dH}{d\rho} = -\frac{\frac{\partial^2 \bar{\Pi}}{\partial H \partial \rho}}{\frac{d^2 \bar{\Pi}}{dH^2}} = -\frac{-\frac{I}{P_1^R + I}\frac{\theta}{2} + o\left(1/(P_1^R + I)^2\right)}{-1 + o\left(1/(P_1^R + I)^2\right)} < 0. \tag{11}$$

It follows that

$$\frac{\partial}{\partial X}\left(\left|\frac{dH}{d\rho}\right|\right) = \frac{-\frac{I}{(P_1^R + I)^2}\frac{\theta}{2} + o(1/(P_1^R + I)^3)}{\left[1 + o(1/(P_1^R + I)^2)\right]^2} < 0.$$

Also under the condition that $X - \bar{U}$ is relatively large, we have from Eq. (8):

$$\frac{\partial^2 \bar{\Pi}}{\partial H \partial g} = -\frac{2I}{(P_1^R + I)^3}\frac{(1-\rho)\theta}{2}(\theta - H)\left(\frac{1}{2} - \frac{g(\theta+\gamma)}{(\theta - H)^2}\right)\left(-\frac{\theta+\gamma}{\theta - H}\right),$$

$$\frac{\partial^2 \bar{\Pi}}{\partial H \partial \gamma} = -\frac{2I}{(P_1^R + I)^3}\frac{(1-\rho)\theta}{2}(\theta - H)\left(\frac{1}{2} - \frac{g(\theta+\gamma)}{(\theta - H)^2}\right)\left(-\frac{g}{\theta - H}\right),$$

both of which are negative if $\rho > 1$. We have $d^2\bar{\Pi}/dH^2 < 0$ from Eq. (10). Thus H decreases in g and γ if $\rho > 1$.

Similarly, we have

$$\frac{\partial^2 \bar{\Pi}}{\partial H \partial X} = -\frac{I}{(P_1^R + I)^2}\frac{(1-\rho)\theta}{2} + o(1/(P_1^R + I)^3),$$

which is positive if $\rho > 1$. Thus H increases in X if $\rho > 1$. □

Proof of Corollary 3 Note that if $\rho > 1$, H decreases in ρ and increases in X by Corollary 2. Since $C = 2g(\theta+\gamma)/(\theta - H)^2$ and $dC/dH = 2C/(\theta - H) > 0$, we have:

$$\frac{dC}{d\rho} = \frac{dC}{dH}\frac{dH}{d\rho} < 0,$$
$$\frac{dC}{dX} = \frac{dC}{dH}\frac{dH}{dX} > 0.$$

Then, like H, C decreases in ρ and increases in X if $\rho > 1$.

Under the assumption that $X - \bar{U}$ is relatively large,

$$\frac{dC}{dg} = \frac{2(\theta+\gamma)}{(\theta - H)^2} + \frac{dC}{dH}\frac{dH}{dg} = \frac{2(\theta+\gamma)}{(\theta - H)^2} + o(1/(P_1^R + I)^3) > 0,$$

where the last equality follows from the proof of Corollary 2. Thus C is increasing in g if $\rho > 1$. Similarly C is increasing in γ if $\rho > 1$. □

Proof of Corollary 4 For $\rho > 1$, write $b = \frac{B}{1-B-C} = \frac{-2CH}{(1+C)H + (1-C)\rho\theta}$. Thus,

$$\frac{db}{dH} = -\frac{2C}{[(1+C)H + (1-C)\rho\theta]^2}\frac{2H^2 + \rho\theta[(1-C)\theta + (1+C)H]}{\theta - H}.$$

Under the assumption that $X - \bar{U}$ is relatively large, $P_1^R + I$ is relatively large. According to Eq. (9), the absolute value of H is relatively small compared with θ, so that $(1-C)\theta + (1+C)H > 0$. Thus $db/dH < 0$.

We also have:

$$\frac{db}{d\rho} = -\frac{2C}{[(1+C)H + (1-C)\rho\theta]^2}\left[\frac{2H^2 + \rho\theta[(1-C)\theta + (1+C)H]}{\theta - H}\frac{dH}{d\rho} - H(1-C)\theta\right].$$

For sufficiently large $X - \bar{U}$, we have $H \approx \frac{I}{P_1^R + I}\frac{(1-\rho)\theta}{2}$ from Eq. (9) and $\frac{dH}{d\rho} \approx -\frac{I}{P_1^R + I}\frac{\theta}{2}$ from Eq. (11). Hence,

$$\frac{db}{d\rho} \approx \frac{C}{[(1+C)H + (1-C)\rho\theta]^2}\frac{I\theta}{(P_1^R + I)(\theta - H)}\left[2H^2 + \theta[(1-C)\theta + (2\rho + C - 1)H]\right].$$

Since the absolute value of H is relatively small compared to θ, the above expression is positive, and b is increasing in ρ.

Note that

$$\frac{db}{dX} = \frac{db}{dH}\frac{dH}{dX}.$$

It follows from $db/dH < 0$ and by Corollary 2, $dH/dX > 0$ that $db/dX < 0$. Thus b is decreasing in X.

Note that

$$\frac{db}{dg} = \frac{\partial b}{\partial g} + \frac{db}{dH}\frac{dH}{dg}.$$

$db/dH < 0$ and by Corollary 2, $dH/dg < 0$. It is clear that $\partial b/\partial g$ is greater than zero because b is increasing in C. Thus b is increasing in g. Similarly, b is increasing in γ. □

Proof of Corollary 5 Write the price reaction to investment as

$$\triangle P \equiv P_1 - P_{initial} = \frac{H+\gamma}{\rho\theta+\gamma}(P_1 - P_0) = \frac{1}{2}\frac{H+\gamma}{\rho\theta+\gamma}[(1-C)\rho\theta + (1+C)H].$$

Under the assumption that $X - \bar{U}$ is relatively large, the absolute value of H is relatively small compared to $\rho\theta$, so that $(1-C)\rho\theta + (1+C)H > 0$. Thus $\triangle P > 0$.

$$\begin{aligned}\frac{d(\triangle P)}{dH} = &\frac{1}{2}\frac{1}{\rho\theta+\gamma}[(1-C)\rho\theta + (1+C)H] \\ &+ \frac{1}{2}\frac{H+\gamma}{\rho\theta+\gamma}\frac{\theta(1+C-2\rho C) - (1-C)H}{\theta - H}.\end{aligned}$$

Under the assumption that g is relatively small, C is relatively small so that $1 + C - 2\rho C > 0$. Thus $d(\triangle P)/dH > 0$.

When $X - \bar{U}$ is relatively large, using the approximation $\frac{dH}{d\rho} \approx -\frac{I}{P_1^R + I}\frac{\theta}{2}$ from Eq. (11), we have

$$\begin{aligned}\frac{d(\triangle P)}{d\rho} &= -\frac{1}{2}\frac{(H+\gamma)\theta}{(\rho\theta+\gamma)^2}[(1-C)\rho\theta + (1+C)H] + \frac{1}{2}\frac{H+\gamma}{\rho\theta+\gamma}(1-C)\theta + \frac{\partial(\triangle P)}{\partial H}\frac{dH}{d\rho} \\ &= \frac{1}{2}\frac{(H+\gamma)\theta}{(\rho\theta+\gamma)^2}[(1-C)\gamma - (1+C)H] + o(1/(P_1^R + I)) \\ &> 0.\end{aligned}$$

If $\rho > 1$, by Corollary 2, H is increasing in X and decreasing in g and γ.

$$\begin{aligned}\frac{d(\triangle P)}{dX} &= \frac{d(\triangle P)}{dH}\frac{dH}{dX} > 0, \\ \frac{d(\triangle P)}{dg} &= \frac{1}{2}\frac{H+\gamma}{\rho\theta+\gamma}(H-\rho\theta)\frac{2(\theta+\gamma)}{(\theta-H)^2} + \frac{d(\triangle P)}{dH}\frac{dH}{dg} < 0.\end{aligned}$$

Thus $\triangle P$ increases in X and decreases in g.

Finally,

$$\begin{aligned}\frac{d(\triangle P)}{d\gamma} &= \frac{1}{2}\frac{\rho\theta - H}{(\rho\theta+\gamma)^2}[(1-C)\rho\theta + (1+C)H] \\ &\quad - \frac{1}{2}\frac{H+\gamma}{\rho\theta+\gamma}(\rho\theta - H)\frac{2g}{(\theta-H)^2} + \frac{d(\triangle P)}{dH}\frac{dH}{d\gamma} \\ &= \frac{1}{2}\frac{(\rho\theta)^2 - H^2}{(\rho\theta+\gamma)^2} + o(g) + o(1/(P_1^R + I)^3),\end{aligned}$$

where the last equality follows from the expression of $C = 2g(\theta+\gamma)/(\theta-H)^2$ and from the proof of Corollary 2. Under the assumptions that $X - \bar{U}$ is relatively large and that g is relatively small, $d(\triangle P)/d\gamma > 0$ and thus $\triangle P$ increases in γ. □

Appendix 2: The case with $\rho < 1$

We proceed as in Sect. 3.1, but in the case $\rho < 1$, the firm will optimally set $w_1(P_1, e_1) = \bar{w}_1 + (e_1 - X)$ where $\bar{w}_1$ is a constant. The advantage of this contract is that pessimistic outsiders will underestimate the value of this compensation, leading to a higher valuation of the firm.

The manager's interim participation constraint is $\min E[\bar{w}_1 + (e_1 - X) | y > H] = \bar{w}_1 + H \geq w_0 \geq \bar{U}$. We also relax this constraint using $E[\bar{w}_1 + (e_1 - X) | y > H] = \bar{w}_1 + (\theta + H)/2 \geq w_0 \geq \bar{U}$.

The firm's optimization problem in this case, which is indicated as Program (R), can therefore be expressed as follows:

$$\max_{w_0,\bar{w}_1} \quad \Pi \equiv E\left[(e_0 - w_0)\cdot(1-j) + \frac{P_1}{P_1+I}(e_1 + I - \bar{w}_1 - (e_1 - X))\cdot j\right]$$

Program (R)

such that

(IC2) $EU \equiv E_y[U(j(y),y)] \geq w_0 \geq \bar{U}.$
(MC) $\bar{w}_1 + (\theta + H)/2 \geq w_0.$

The following result characterizes the solution to the relaxed Program (R).

Proposition 3 *In Program (R), the compensation,* w_0 *and* $w_1 = \bar{w}_1 + (e_1 - X)$, *the stock price,* P_1^R, *and the firm's payoff,* Π^R, *can be expressed as functions of the investment threshold,* H.

$$\begin{aligned} w_0 &= \bar{U}, \\ \bar{w}_1 &= \bar{U} + g\Big/\left(\frac{\theta - H}{\theta + \gamma}\right) - \frac{\theta + H}{2}, \\ P_1^R &= X + \frac{\theta + H}{2} - \bar{U} - g\Big/\left(\frac{\theta - H}{\theta + \gamma}\right), \\ \Pi^R &= (X - \bar{U})\cdot\frac{H + \gamma}{\theta + \gamma} + P_1^R\cdot\frac{\theta - H}{\theta + \gamma}. \end{aligned}$$

The firm chooses H *to maximize* Π^R.

Proof of Proposition 3 Note that $\bar{w}_1 + (\theta + H)/2 \geq w_0 \geq \bar{U}$. It must be true that $w_0 = \bar{U}$, otherwise, the firm can lower both $\bar{w}_1$ and w_0 and increase its payoff.

The firm chooses $\bar{w}_1$ to satisfy the (IC2) constraint. Thus,

$$EU = w_0 \cdot prob(y \leq H) + \left(\bar{w}_1 + \frac{\theta + H}{2}\right)\cdot prob(y > H) - g = \bar{U},$$

which implies

$$\bar{w}_1 + \frac{\theta + H}{2} = [\bar{U} + g - w_0 \cdot prob(y \leq H)]/prob(y > H) = \bar{U} + g/\left(\frac{\theta - H}{\theta + \gamma}\right). \tag{12}$$

The stock price can be expressed as

$$\begin{aligned} P_1^R &= \hat{E}_{y,\epsilon}[e_1 - \bar{w}_1 - (e_1 - X)|y > H] = X - \bar{w}_1 \\ &= X + \frac{\theta + H}{2} - \bar{U} - g\Big/\left(\frac{\theta - H}{\theta + \gamma}\right), \end{aligned}$$

and the firm's payoff can be expressed as

$$
\begin{aligned}
\Pi^R &= E[e_0 - w_0 | y \le H] \cdot prob(y \le H) \\
&\quad + \frac{P_1^R}{P_1^R + I} E[e_1 + I - \bar{w}_1 - (e_1 - X) | y > H] \cdot prob(y > H) \\
&= E[X - \bar{U} | y \le H] \cdot prob(y \le H) \\
&\quad + \frac{P_1^R}{P_1^R + I} \left[E[e_1 + I | y > H] \cdot prob(y > H) - (\bar{w}_1 + \frac{\theta + H}{2}) \cdot prob(y > H) \right] \\
&= (X - \bar{U}) \cdot prob(y \le H) + \frac{P_1^R}{P_1^R + I} (X + I - \bar{w}_1) \cdot prob(y > H) \\
&= (X - \bar{U}) \cdot \frac{H + \gamma}{\theta + \gamma} + P_1^R \cdot \frac{\theta - H}{\theta + \gamma}.
\end{aligned}
\tag{13}
$$

The firm chooses H to maximize Π^R. □

Note that the expression for Π^R in the proof of Proposition 3 can be rewritten as:

$$
\begin{aligned}
\Pi^R &= (X - \bar{U}) \cdot \frac{H + \gamma}{\theta + \gamma} + \left[X + \frac{\theta + H}{2} - \bar{U} - g \Big/ \left(\frac{\theta - H}{\theta + \gamma} \right) \right] \cdot \frac{\theta - H}{\theta + \gamma} \\
&= X - \bar{U} - g + \frac{\theta^2 - H^2}{2(\theta + \gamma)}.
\end{aligned}
\tag{14}
$$

Therefore the optimal hurdle rate is $H = 0$.

We now show that a linear compensation contract can implement the optimal investment policy, H, in Proposition 3 when only the manager has the information signal y about project quality. We also show that the firm's payoff under this contract approximates the payoff Π^R, which is an upper bound for the firm's payoff in Program (P), and therefore the linear compensation contract approximates the optimal contract in our original problem.

If the manager does not invest in a new project, the firm's future earnings are non-stochastic, and, without lost of generality, the firm pays the manager only a fixed salary, w_0. If the firm invests in the new project, the linear compensation contract pays the manager a fixed salary and provides stock-price and earnings-based incentives according to:

$$w_1(P_1, e_1) = a + b \cdot P_1 + c \cdot e_1,$$

where $b \in [0, 1]$ and $c \in [0, 1]$ are no short-position constraints on the manager's contract.

It will be helpful to re-write the compensation contract as follows:

$$w_1(P_1, e_1) = a + b \cdot P_1 + c \cdot e_1 \equiv A + B \cdot \hat{y} + C \cdot v_1$$

where $\hat{y} \equiv \hat{E}_y(y | j = 1)$ is the market's expectation of the project NPV conditional on the firm choosing to invest. As in Lemma 1, there is a one-to-one mapping from the parameters $\{a, b, c\}$ to the parameters $\{A, B, C\}$.

Proposition 4 *The following linear compensation contract implements the optimal investment threshold*, $H = 0$, *as specified in Proposition 3*:

$$\begin{aligned} w_0 &= \bar{U}, \\ w_1 &= A + B \cdot \hat{y} + C \cdot v_1 \ \ \textit{where} \\ A &= \bar{U}, \\ B &= \frac{-2CH}{\rho\theta + H} = 0, \\ C &= 2g(\theta+\gamma)/(\theta - H)^2 = 2g(\theta+\gamma)/\theta^2. \end{aligned}$$

If the firm invests the stock price is given by $P_1 = X + 0.5 \cdot (1 - B - C) \cdot (\rho\theta + H) - A$. The firm's payoff, Π, relative to its upper bound, Π/Π^R, approaches one when managerial compensation is small relative to firm value ($X - \bar{U}$ is large).

Proof of Proposition 4 We construct a linear contract to implement the optimal payoff, Π^R, as specified in Proposition 3, in five steps.

(i) Consider the manager's investment decision as described in the incentive condition (IC1). If she does not invest in the new project, her payoff is simply $U(j = 0, y) = w_0 - g$. If she invests in the new project, her expected payoff is $U(j = 1, y) = A + B\hat{y} + Cy - g$. Since $U(j = 1, y)$ is increasing in y, the manager's optimal investment decision can be represented by a threshold $H \in [-\gamma, \theta] : j(y) = 1$ if and only if $y > H$. Clearly, she chooses $j = 1$ if $U(j = 1, y) > U(j = 0, y)$, or

$$y > H = -\hat{y}B/C + (w_0 - A)/C.$$

Since

$$\hat{y} = \hat{E}(y|y \geq H) = \frac{\rho\theta + H}{2} = \frac{\rho\theta - \hat{y}B/C + (w_0 - A)/C}{2},$$

it follows that

$$\hat{y} = \frac{\rho\theta + (w_0 - A)/C}{2 + B/C},$$

and

$$H = \frac{-B}{2C + B}\rho\theta + \frac{2(w_0 - A)}{2C + B}.$$

Imposing H at the level specified in Proposition 3 implies

$$B = 2\frac{-CH + w_0 - A}{\rho\theta + H}. \tag{15}$$

(ii) Consider the manager's decision whether to expend search effort. If she does not, the manager will get a payoff of w_0. (She can claim she put in effort but y was too small.) If she expends effort, her expected payoff is given by:

$$\begin{aligned} EU &= E[(1-j)w_0 + j(A + B\hat{y} + Cy)] - g \\ &= E[j(A - w_0 + B\hat{y} + Cy)] + w_0 - g \\ &= CE[j(y - H)] + w_0 - g \\ &= C\frac{(\theta - H)^2}{2(\theta + \gamma)} + w_0 - g \end{aligned}$$

where the third equality follows from the fact that $A - w_0 + B\hat{y} + CH = 0$. Therefore, to motivate the manager to expend search effort, the following condition must be satisfied:

$$C\frac{(\theta - H)^2}{2(\theta + \gamma)} \geq g.$$

If the inequality is binding, it follows that

$$C = 2g(\theta + \gamma)/(\theta - H)^2. \tag{16}$$

(iii) According to the ex ante participation constraint, let

$$\begin{aligned} \bar{U} &= E[(1-j)w_0 + j(A + B\hat{y} + Cy)] - g \\ &= w_0 \cdot prob(y \leq H) + E[A + B\hat{y} + Cy | y > H] \cdot prob(y > H) - g, \end{aligned}$$

which has two implications. First,

$$\begin{aligned} \bar{U} &= w_0 \cdot prob(y \leq H) + (A + B\frac{\rho\theta + H}{2} + C\frac{\theta + H}{2}) \cdot prob(y > H) - g \\ &= w_0 \cdot prob(y \leq H) + (-CH + w_0 + C\frac{\theta + H}{2}) \cdot prob(y > H) - g \\ &= w_0 + C\frac{\theta - H}{2} \cdot \frac{\theta - H}{\theta + \gamma} - g \\ &= w_0, \end{aligned}$$

where the second equality follows from the expression of B in Eq. (15), and the last equality follows from the expression of C in Eq. (16). This fixes $w_0 = \bar{U}$.
Second,

$$\begin{aligned} E[A + B\hat{y} + Cy | y > H] &= [\bar{U} + g - w_0 \cdot prob(y \leq H)]/prob(y > H) \\ &= \bar{U} + g/\left(\frac{\theta - H}{\theta + \gamma}\right). \end{aligned} \tag{17}$$

It follows immediately that

$$A = \bar{U} + g\Big/\left(\frac{\theta - H}{\theta + \gamma}\right) - B\frac{\rho\theta + H}{2} - C\frac{\theta + H}{2}.$$

This equation holds if $A = \bar{U}$ and B is expressed by Eq. (15). Therefore fix $A = \bar{U}$.

(iv) It follows from $A = \bar{U}$, the expression of B in Eq. (15), and the expression of C in Eq. (16) that

$$\begin{aligned} P_1 &= X + p_1 = X + (1 - B - C)\frac{\rho\theta + H}{2} - A \\ &= X + \frac{\rho\theta + H}{2} - \bar{U} - g\Big/\left(\frac{\theta - H}{\theta + \gamma}\right) + C\frac{\theta(1-\rho)}{2} \\ &= P_1^R + (1 - C)\frac{\theta(\rho - 1)}{2}, \end{aligned} \tag{18}$$

where P_1^R is from Proposition 3. Clearly, if $\rho < 1$ then $P_1 < P_1^R$. The firm's payoff is given by:

$$\begin{aligned} \Pi &= E[e_0 - w_0 | y \le H] \cdot prob(y \le H) \\ &\quad + \frac{P_1}{P_1 + I} E[e_1 + I - w_1(P_1, e_1) | y > H] \cdot prob(y > H) \\ &= (X - \bar{U}) \cdot prob(y \le H) \\ &\quad + \frac{P_1}{P_1 + I}[E[e_1 + I | y > H] \\ &\quad \cdot prob(y > H) - E[w_1(P_1, e_1) | y > H] \cdot prob(y > H)]. \end{aligned} \tag{19}$$

Compare the expressions of Π in Eq. (19) and Π^R in Eq. (13). Note that $E[w_1(P_1, e_1)|y > H] = \bar{w}_1 + (\theta + H)/2$ from Eqs. (12) and (17). When managerial compensation is small relative to firm value ($X - \bar{U}$ is large), we have $\frac{P_1}{P_1+I} \approx \frac{P_1^R}{P_1^R+I}$ from Eq. (18). Therefore the firm's payoff, Π, relative to its upper bound, Π/Π^R, approaches one.

(v) As $H = 0$ from Eq. (14), this implies $B = 0$. It is easy to verify that with reasonable assumptions about parameter values that the no short-sale conditions are met.

□

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