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THE THEORY OF LASER BATHYMETRY USING THE EFFECT OF MULTIPLE REFLECTION OF A LIGHT PULSE FROM THE SEAFLOOR

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This work deals with the problem of using lidar for measuring the extremely small depths at which the difference between the times of arrival of the echo signals from the sea surface and the seafloor becomes smaller than the sounding-pulse duration. To solve this problem, we propose to use the effect of multiple reflection of light from the seafloor, which leads to the appearance of additional echo signals with delay times exceeding the time of the double light-pulse transmission through the water layer. It is shown that the doubly reflected signal can actually be used for decreasing the minimum measured sea depth by a factor of about 2.5 with unchanged sounding-pulse duration.

1. INTRODUCTION

An airborne bathymetric lidar is known to be an efficient tool for mapping the shallow-water regions of the seafloor and ecological monitoring of the ocean coastal area [1–3]. The joint use of the laser bathymeter with the measurer of the spectral radiance of the solar radiation emerging from water, which is realized in the system SHOALS+CASI (Scanning Hydrographic Operational Airborne Lidar Survey+Compact Airborne Spectrographic Imager), allowed one not only to record the topography of the seafloor, but also synthesize its color images and obtain the spectral data on the optical characteristics of the sea water [4–6]. This information is successfully used to observe the shore-erosion process, the seafloor-topography variations, and the shallow-water suspension transfer, determine the precipitation type and the seafloor-vegetation presence, monitor the water turbidity, and record contamination. The developers of the above-mentioned system note that one of the system disadvantages is related to the difficulties in measuring the sea depth in the immediate vicinity of the shore edge, where the difference between the arrival times of the echo signals from the sea surface and the seafloor becomes smaller than the signal duration.

This work is devoted to analyzing the possibility of improving the accuracy of measuring small depths by the lidar method using the effect of multiple reflection of the light pulse from the seafloor. Formulas for calculating the characteristics of the echo signals, which result from single, double, and triple reflections of the pulse from the seafloor, are given. Comparative analysis of the characteristics of the singly and doubly reflected signals is performed and the possibilities of determining the sea depth from the singly and doubly reflected signals are estimated under conditions where the sounding-pulse duration exceeds the time of double transmission of the signal through the water layer.

2. EXPRESSIONS FOR ECHO SIGNALS

The echo signals from the seafloor are calculated under the following assumptions.

1. Lidar is located at the height H above the plane sea surface (see Fig. 1).

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2. The seafloor (the plane $z = \text{const}$) diffusely scatters the incident light and is characterized by the diffuse-reflection coefficient (albedo) R_b.

3. The source and receiver directional patterns are coaxial, oriented along the normal to the sea surface, and have the Gaussian shape and widths (at the level $1/e$) $2\vartheta_\text{s}$ and $2\vartheta_\text{r}$, respectively.

4. To simplify the problem, the light scattering in water was disregarded and the radiance attenuation of the radiation transmitted through the water layer was determined using Bouguer's law.

Under the above-formulated assumptions, the power P_1 of the singly reflected signal is determined from the equation (see the Appendix)

$$P_1(t) = A_1 \frac{\Omega_\text{r}}{\Sigma} f(1 - 2z/v_\text{w}), \tag{1}$$

where $A_1 = P_\text{s} R_\text{b} \Sigma_\text{r} \exp(-2cz)/(\pi m^2)$, $\Sigma = \Sigma_\text{s} + \Sigma_\text{r} + (\Omega_\text{s} + \Omega_\text{r}) L^2$, $L = H + z/m$, $\Omega_\text{s} = \pi\vartheta_\text{s}^2$, $\Omega_\text{r} = \pi\vartheta_\text{r}^2$, P_s is the peak power of the source, m is the refractive index of water, c is the water attenuation coefficient, and Σ_s and Σ_r are the areas of the source and receiver apertures, respectively, Ω_s and Ω_r are the radiation and reception solid angles, v_w is the speed of light in water, v is the speed of light in free space, and the time t is reckoned from the instant of the laser-pulse incidence on the sea surface. The function $f(t - r/v)$ describes the shape of the echo signal from the point target located in free space at the distance r from the lidar. This function satisfies the condition $f(0) = 1$ (this function is a convolution of the radiated signal and the receiver response to the delta-pulse signal).

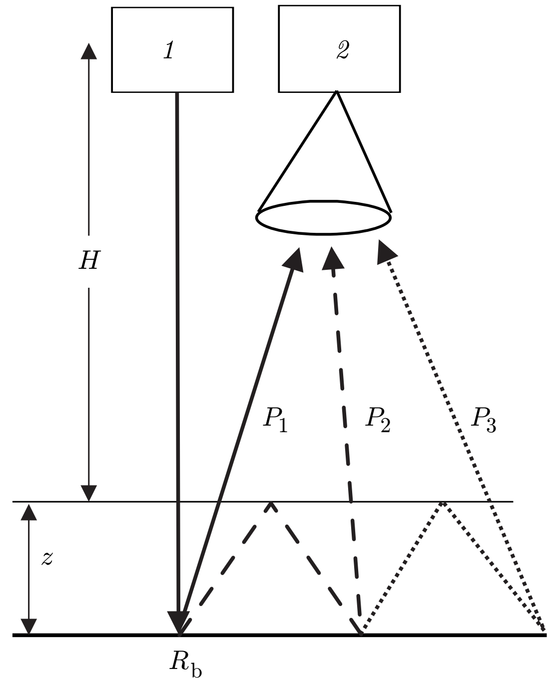

Fig. 1. Location diagram: radiation source (*1*), optical receiver (*2*), H is the carrier height, z is the sea depth, and P_1, P_2, and P_3 are the singly, doubly, and triply reflected signals.

To calculate the power of the doubly reflected signal, we should calculate the integral (see the Appendix)

$$P_2(t) = A_2 \frac{\Omega_\text{r}}{\Sigma} 2\pi \int_0^\infty \exp\left(-\frac{\pi\rho^2}{\Sigma}\right) \Phi(\mu) f \left[t - \frac{2z}{v_\text{w}} \left(1 + \frac{1}{\mu}\right)\right] \rho \, \mathrm{d}\rho, \tag{2}$$

where

$$A_2 = A_1 R_\text{b}/(4\pi z^2), \qquad \Phi(\mu) = \mu^4 R_f(\mu) \exp(-2cz/\mu), \qquad \mu(\rho) = 2z/\sqrt{\rho^2 + 4z^2}. \tag{3}$$

Here, R_f is the Fresnel reflection coefficient for the nonpolarized light incident to the water–air interface from below at the angle $\vartheta = \arccos \mu$:

$$R_f(\mu) = (R_1^2 + R_2^2)/2, \tag{4}$$

where

$$R_1 = \frac{\mu - m\sqrt{1 - m^2 + m^2\mu^2}}{\mu + m\sqrt{1 - m^2 + m^2\mu^2}} \textit{and} \qquad R_2 = \frac{m\mu - \sqrt{1 - m^2 + m^2\mu^2}}{m\mu + \sqrt{1 - m^2 + m^2\mu^2}} \tag{5}$$

for $\mu \geq \sqrt{m^2 - 1}/m$,

$$R_1 = R_2 = 1 \tag{6}$$

for $\mu < \sqrt{m^2 - 1}/m$.

A similar expression for the power of the triply reflected signal has the form (see the Appendix)

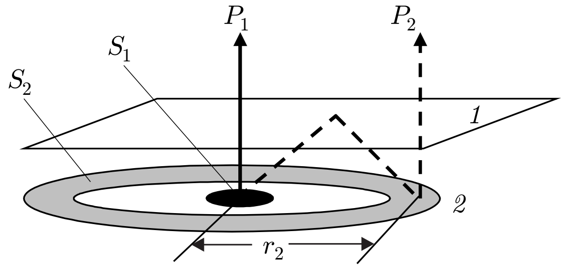

Fig. 2. The ray pattern of the single and double signal reflection from the seafloor (*2*): S_1 is the located seafloor element, P_1 is the singly reflected signal, S_2 is the seafloor area to which the light singly reflected by the seafloor returns as a result of total internal reflection from the water–air interface (*1*), P_2 is the doubly reflected signal, and r_2 is the radius of the ring-shaped region of the seafloor from which the maximum of the signal P_2 arrives.

$$P_3(t) = A_3 \frac{\Omega_{\mathrm{r}}}{\Sigma}(2\pi)^2 \int_0^\infty \int_0^\infty \exp\left[-\frac{\pi(\rho_1^2+\rho_2^2)}{\Sigma}\right] I_0\left(\frac{2\pi}{\Sigma}\rho_1\rho_2\right) \Phi(\mu_1)\Phi(\mu_2) \times f\left[t - \frac{2z}{v_{\mathrm{w}}}\left(1+\frac{1}{\mu_1}+\frac{1}{\mu_2}\right)\right] \rho_1\rho_2 \,\mathrm{d}\rho_1\,\mathrm{d}\rho_2, \quad (7)$$

where $A_3 = A_2 R_{\mathrm{b}}/(4\pi z^2)$, $\mu_i = \mu(\rho_i)$, $i = 1, 2$ and $I_0(\xi)$ is a modified Bessel function of the first kind of order zero.

3. COMPARATIVE ANALYSIS OF THE CHARACTERISTICS OF THE SINGLY AND DOUBLY REFLECTED SIGNALS

When performing the calculations, $f(t)$ was specified in the form of the Gaussian function

$$f(t) = \exp[-(2t/\Delta t)^2]. \quad (8)$$

The parameter Δt characterizes the effective duration of the sounding pulse (with allowance for its distortion by an optical receiver) and determines the longitudinal size of the pulsed volume:

$$\Delta z = v_{\mathrm{w}}\Delta t/2. \quad (9)$$

It is also assumed that the condition $\vartheta_{\mathrm{r}} \gg \vartheta_{\mathrm{s}}$, which allows one to generalize Eq. (2) to the case of “stepwise” directional patterns of the source and the receiver, is fulfilled.

To analyze the differences of the characteristics of the singly and doubly reflected signals (by their power and arrival times), it is convenient to use the dimensionless variables

$$t_\Delta = t/\Delta t, \qquad z_\Delta = z/\Delta z, \qquad \Delta\tau = 2c\Delta z = cv_w\Delta t \quad (10)$$

instead of the variables t, z, and c, respectively. In this case, the singly reflected signal power normalized to the maximum is given by the formula

$$P_1(t_\Delta)/P_1(t_\Delta = z_\Delta) = \exp[-4\,(t_\Delta - z_\Delta)^2], \quad (11)$$

and the ratio of the power of the doubly reflected signal to the peak power of the singly reflected signal is calculated using the formula

$$P_2(t_\Delta)/P_1(t_\Delta = z_\Delta) = 2R_{\mathrm{b}} \int_0^X \mu^4 R_{\mathrm{f}}(\mu) \exp\{-\Delta\tau\, z_\Delta \mu^{-1} - 4\,[t_\Delta - z_\Delta(1+\mu^{-1})]^2\}\, x\,\mathrm{d}x, \quad (12)$$

$$\mu = (1+x^2)^{1/2}, \qquad X = \vartheta_{\mathrm{r}} L/(2z). \quad (13)$$

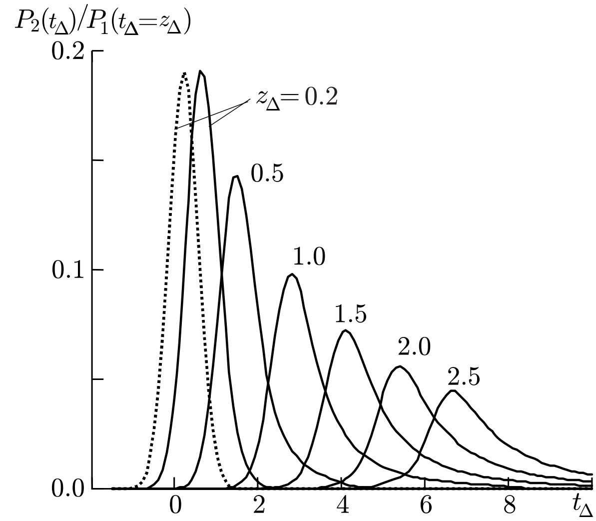

Fig. 3. Relative power of the signal that is doubly reflected from the seafloor (solid curves) as a function of the dimensionless time t_Δ for $\Delta\tau = 0.1$, $R_\mathrm{b} = 0.5$, and various values of z_Δ (indicated in the figure). The dotted curve corresponds to the singly reflected signal for $z_\Delta = 0.2$, Δt is the effective duration of the sounding pulse, Δz is the resolution-element size with respect to the depth, and $\Delta\tau$ is the optical length of the light pulse.

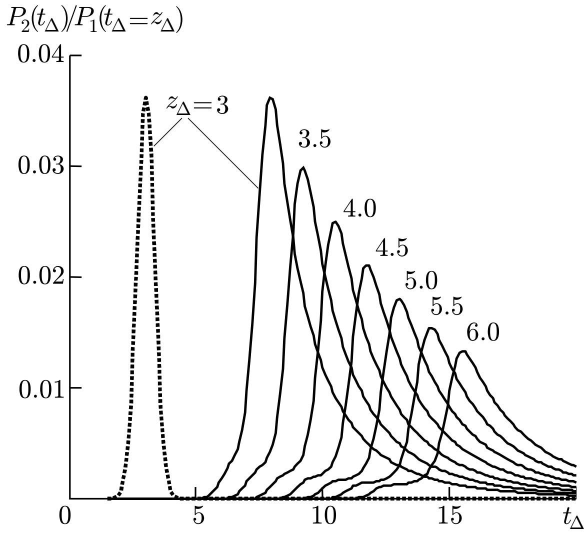

Fig. 4. The same as in Fig. 3, but for other values of z_Δ.

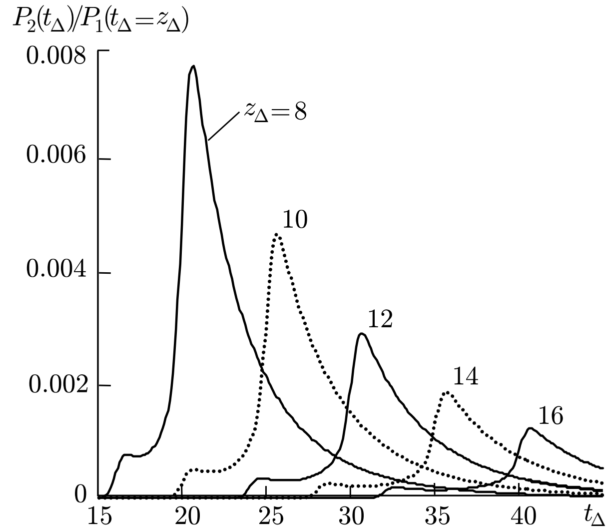

Fig. 5. The same as in Fig. 3, but for the doubly reflected signals for $\Delta\tau = 0.1$ and $R_\mathrm{b} = 0.5$. The values of z_Δ are shown in the figure.

For small depths, the maximum of the signal P_2 corresponds to the ring-shaped region S_2 of the seafloor on which the light reflected by the surface to the seafloor at angles close to the total internal reflection angle is incident (see Fig. 2). The radius of this ring-shaped region is estimated from the formula $r_2 \approx 2z$. Therefore, the upper integration limit X on the right-hand side of Eq. (12) can be considered as the ratio of the radius of the seafloor region, which falls within the field of view of the receiver, to the quantity r_2, i.e., $X \approx \vartheta_\mathrm{r} L / r_2$.

Figures 3–7 show the results of calculating the signal P_2 using Eq. (12) for $X = 5$. Along with the signals P_2, the dotted lines in Figs. 3 and 4 show the signals P_1 arriving from the same depth as the signals P_2s, which immediately follow them. For convenience of comparing the signal shapes, the maximum value of the power of the signal P_1 is reduced to be equal to that of the signal P_2.

Figure (3) and Eq. (12) show that the ratio of the maximum powers of the signals P_2 and P_1 is approximately equal to $0.4R_\mathrm{b}$ for very small depths ($z \ll \Delta z$). The difference of the arrival times of the corresponding signals becomes pronounced even for the depths $z \approx 0.2\,\Delta z$. With increasing depth, the ratio of the maximum powers of the singly and doubly reflected signals is reduced to about $0.01R_\mathrm{b}$ for $z \approx 10\,\Delta z$ (see Figs. 4 and 5). The signal P_2 has the form of a pulse with a "precursor," which is formed by the light

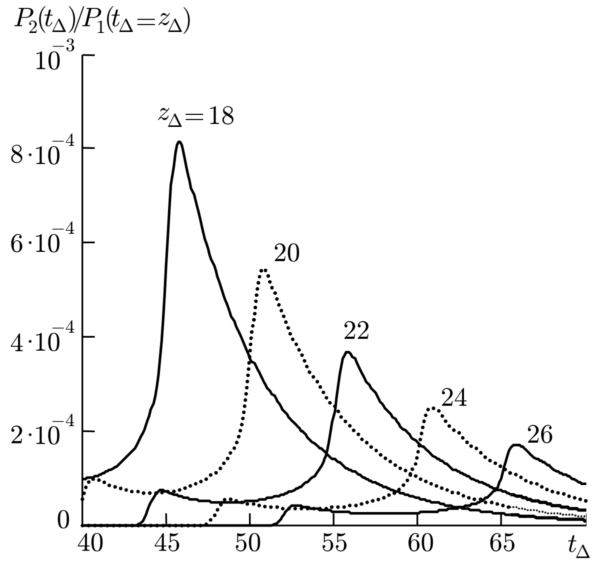

Fig. 6. The same as in Fig. 5, but for other values of z_Δ.

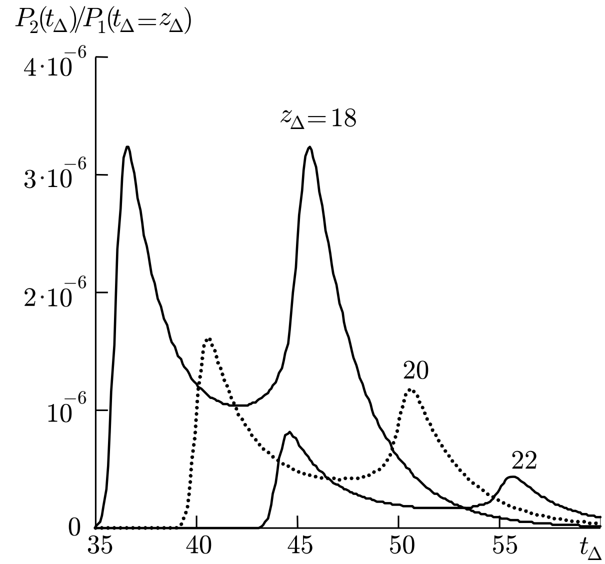

Fig. 7. The same as in Fig. 6, but for $\Delta\tau = 0.3$ and $R_\mathrm{b} = 0.5$. The values of z_Δ are given in the figure.

reflected from the surface at the angles that are smaller than the total internal reflection angle. The relative contribution from this light to the signal P_2 increases with increasing depth and water-attenuation coefficient (see Figs. 5–7). As is evident from Fig. 7, for large depths, the doubly reflected signal can be divided into two pulses with almost identical amplitudes. The first and the second pulses come from the seafloor element S_1 (see Fig. 2) and the surface S_2, respectively.

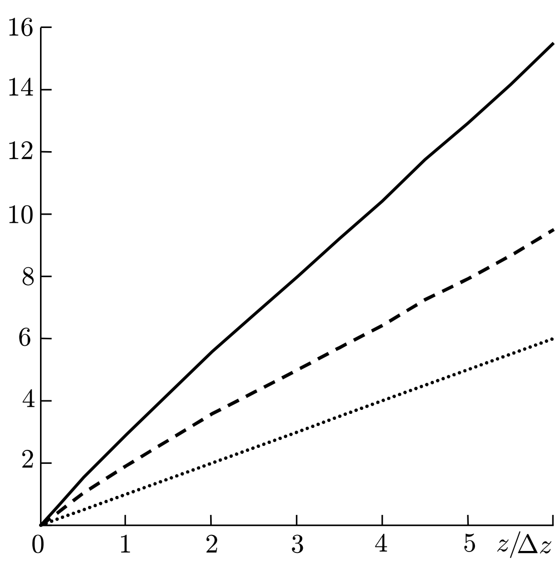

Fig. 8. Comparative data on the delay times of the singly and doubly reflected signals for $2c\,\Delta z = 0.1$. The solid, dashed, and dotted lines correspond to the dependences of $t_2/\Delta t$, $(t_2{-}t_1)/\Delta t$, and $t_1/\Delta t$ on $z/\Delta z$, respectively.

The solid curve in Fig. 8 shows the arrival time t_2 of the maximum of the signal P_2 as a function of the depth z. For comparison, this figure also shows the delay time $t_1 = 2z/v_\mathrm{w}$ of the singly reflected signal and the delay time of the signal P_2 with respect to the signal P_1, i.e., the difference $t_2 - t_1$ of the arrival times of the maxima of these signals, as functions of z. It is obvious from the figure that the delay time of the signal P_2 is more sensitive to the depth variation than that of the signal P_1.

The problem of the wind-wave influence on the characteristics of the pulsed signals of various multiplicity of scattering from the water-reservoir seafloor is a separate issue beyond the scope of this study. It should be noted that this problem is rather mathematically complicated. However, simple estimates for the water-surface waves with wave lengths of about the doubled depth of the reservoir and greater show that the presence of the waves does not almost influence the arrival time of the maximum of the signal P_2. In this case, all the main conclusions of this work also hold true under the conditions of the water-surface waves.

4. CONCLUSIONS

Comparative analysis of the characteristics of the echo signals resulting from the single and double reflections of the light pulse from the seafloor allows one to draw the following conclusions.

1. For a small (about several lengths of the pulsed volume) sea depth, the signal P_2, which is doubly reflected from the seafloor, can be recorded under the condition of separate reception of the singly (P_1) and doubly reflected signals.

2. The singly and doubly reflected signals arrive from different seafloor regions, which makes their separate reception in principle possible. To receive the signal P_2, the special optical receiver with the field of view in the form of a ring adjacent to the observed seafloor element can be used (see Fig. 2).

3. Under the condition of separate reception of the partial signals P_1 and P_2 and the reference signal P_0, the water-reservoir depth z can be estimated by three methods:

(a) conventionally, i.e., by the difference $t_1 = 2z/c$ of the arrival times of the singly reflected signal (P_1) and the reference signal (P_0);

(b) by the difference t_2 of the arrival times of the doubly reflected signal (P_2) and the reference signal (P_0);

(c) by the difference $t_2 - t_1$ of the arrival times of the doubly and singly reflected signals P_2 and P_1, respectively.

If the arrival times of the three above-mentioned signals identically fluctuate and are measured with the same error, then, using the second method instead of the conventional one, we can by a factor of 2.5 to 3 reduce the minimum water-reservoir depth, which can be measured by a lidar with a given longitudinal resolution. Under the same conditions, the third method allows one to reduce the minimum measured depth by a factor of 1.5 to 2. However, in the case of strong fluctuations of the signal P_0, due to, e.g., the waves, the third method can turn out to be the most useful, because it allows one to measure the reservoir depth without the signal reflected by the surface.

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APPENDIX. DERIVATION OF THE FORMULAS FOR CALCULATING ECHO SIGNALS

Let a pulsed light source with the radiance distribution $B_{\mathrm{s}} D_{\mathrm{s}}^{(0)}(\mathbf{r}, \mathbf{n})$, where B_{s} is the central-ray radiance and the aperture functions of the source satisfy the conditions $D_{\mathrm{s}}^{(0)}(0,0) = 1$ and $D_{\mathrm{s}}(0) = 1$, be located at the height H above the water surface. Hereafter, $\mathbf{r}$ is the two-dimensional radius vector of the points in the horizontal plane and $\mathbf{n}$ denotes the projections of the unit vectors characterizing the light-beam direction onto the horizontal plane. The light source irradiates the water surface along the normal to the latter. The light radiation transmitted through the surface reaches the seafloor and is isotropically scattered from it (the seafloor reflection coefficient is equal to R_{b}). A portion of the scattered radiation reaches the surface, passes through it, and arrives at the optical receiver of a lidar with the aperture function $D_{\mathrm{r}}^{(0)}(\mathbf{r}, \mathbf{n}) D_{\mathrm{r}}(t)$ (the same normalizations are used).

The power P_1 of the signal of single scattering from the seafloor is described by the well-known relationship of the small-angle radiative transfer theory:

$$P_1(t) = B_{\mathrm{s}} \frac{R_{\mathrm{b}}}{\pi m^2} \iint\limits_{-\infty}^{+\infty} E_1(\mathbf{r}) E_{\mathrm{r}}(\mathbf{r}) D_{sr}(t - l_1/v_{\mathrm{w}})\, \mathrm{d}^2 r, \tag{A1}$$

where $E_1(\mathbf{r})$ is the irradiance distribution at the point $\mathbf{r}$ on the seafloor, which is created by the light source, $E_\mathrm{r}(\mathbf{r})$ is the irradiance distribution which would be created by the light source with the aperture function of the lidar optical receiver, D_sr is the convolution of the time functions of the light source and the optical receiver, $l_1 = 2z$ is the distance covered by the light pulse under water, and v_w is the speed of light in water.

A part of the light radiation, which is scattered by the seafloor and reaches the surface, is reflected back to water by the surface, reaches the seafloor, and is again scattered upwards. Having reached the surface, this radiation is transmitted through the latter and arrives at the lidar optical receiver, forming the signal of the double scattering from the seafloor. The power of this signal (the signal of double reflection from the seafloor) can be described by a relationship that is similar to Eq. (A1):

$$P_2(t) = B_\mathrm{s}\frac{R_\mathrm{b}}{\pi m^2}\iint\limits_{-\infty}^{+\infty} E_2(\mathbf{r})E_\mathrm{r}(\mathbf{r})D_\mathrm{sr}(t - l_1/v_\mathrm{w} - l_2/v_\mathrm{w})\,\mathrm{d}^2r, \tag{A2}$$

where $E_2(\mathbf{r})$ is the irradiance distribution on the seafloor, which is created by the Lambert source located at the point $\mathbf{r}_1$ on the seafloor (this is an effective source corresponding to the light that is singly reflected from the seafloor), with allowance for the reflecting surface of the water–air interface, $l_2 = 2z/\mu_1$, $\mu_1 = \cos\theta_1$, and θ_1 is the angle of the first reflection on the water–air interface.

When deriving the formula for E_2, we ignore the light-scattering effects in water. In this case, the calculation performed in accordance with the classical photometry laws yields the following expression:

$$E_2(\mathbf{r}) = \frac{R_\mathrm{b}}{4\pi z^2}\iint\limits_{-\infty}^{+\infty} E_1(\mathbf{r}_1)\Phi(\mu_1)\,\mathrm{d}^2r_1, \tag{A3}$$

where $\Phi(\mu_1) = \mu_1^4 R_\uparrow(\mu_1)\exp(-2cz/\mu_1)$, $R_\uparrow$ is the Fresnel coefficient of the light reflection from the water–air interface, $mu_1 = 2z/\sqrt{(\mathbf{r}_1 - \mathbf{r})^2 - (2z)^2}$, and c is the light-attenuation index in water.

A portion of the light radiation, which is doubly reflected from the seafloor, is reflected back to water, reaches the seafloor, and is scattered upwards for the third time. After reaching the surface, this radiation is transmitted through the surface and arrives at the lidar optical receiver, thereby forming the signal of triple reflection from the seafloor. The power of this signal is described by the relationship that is similar to Eq. (A2):

$$P_3(t) = B_\mathrm{s}\frac{R_\mathrm{b}}{\pi m^2}\iint\limits_{-\infty}^{+\infty} E_3(\mathbf{r})E_\mathrm{r}(\mathbf{r})D_\mathrm{sr}(t - l_1/v_\mathrm{w} - l_2/v_\mathrm{w} - l_3/v_\mathrm{w})\,\mathrm{d}^2r, \tag{A4}$$

where $E_3(\mathbf{r})$ is the irradiance distribution on the seafloor, which is created by the Lambert source located at the point $\mathbf{r}_2$ on the seafloor (this is an effective source corresponding to the light that is doubly reflected from the seafloor), with allowance for the reflecting surface of the air–water interface, $l_3 = 2z/\mu_2$, $\mu_2 = \cos\theta_2$, and θ_2 is the angle of the second reflection on the water–air interface.

Since

$$E_3(\mathbf{r}) = \frac{R_\mathrm{b}}{4\pi z^2}\iint\limits_{-\infty}^{+\infty} E_2(\mathbf{r}_1)\Phi(\mu_1)\,\mathrm{d}^2r_1,$$

with allowance for Eq. (A3), we obtain

$$E_3(\mathbf{r}) = \left(\frac{R_\mathrm{b}}{4\pi z^2}\right)^2\iint\limits_{-\infty}^{+\infty}\mathrm{d}^2r_1\iint\limits_{-\infty}^{+\infty} E_1(\mathbf{r}_2)\Phi(\mu_1)\Phi(\mu_2)\,\mathrm{d}^2r_2, \tag{A5}$$

where $\mu_2 = 2z/\sqrt{(\mathbf{r}_2^2 - \mathbf{r}_1) - (2z)^2}$.

In what follows, it is assumed that the lidar is designed according to the monostatic scheme (the source–receiver base is absent) with coaxial geometry (the axes of the source and receiver directional patterns coincide).

Let us specify the functions in the expressions for the power of the echo signal of the single-, double-, and triple radiation reflections from the water-reservoir seafloor. The aperture functions of the light source and the lidar optical receiver are approximated by the Gaussian functions

$$D_\alpha^{(0)}(\mathbf{r},\mathbf{n}) = \exp\left[-\left(\frac{\pi r^2}{\Sigma_\alpha} + \frac{\pi n^2}{\Omega_\alpha}\right)\right], \tag{A6}$$

where the index α is s (source) or r (receiver), Σ_α is the aperture area, and Ω_α is the solid angle of radiation or reception. Under the conditions of ignoring the scattering effects in water, the expressions for E_1 and E_r have the following form:

$$E_1(\mathbf{r}) = B_\mathrm{s}\exp(-cz)\iint\limits_{-\infty}^{+\infty} D_s^{(0)}(\mathbf{r}-L\mathbf{n},\mathbf{n})\,\mathrm{d}^2 n, \tag{A7}$$

$$E_\mathrm{r}(\mathbf{r}) = \exp(-cz)\iint\limits_{-\infty}^{+\infty} D_r^{(0)}(\mathbf{r}-L\mathbf{n},\mathbf{n})\,\mathrm{d}^2 n, \tag{A8}$$

where c is the water attenuation index.

Substituting Eqs. (A6)–(A8) into Eq. (A1), after some algebra we obtain the expression for the power of the echo signal of single reflection from the seafloor:

$$P_1(t) = A_1\frac{\Omega_\mathrm{r}}{\Sigma}D_\mathrm{sr}(1-2z/v_\mathrm{w}), \tag{A9}$$

where $A_1 = P_\mathrm{s}R_\mathrm{b}\Sigma_\mathrm{r}\exp(-2cz)/(\pi m^2)$, $P_\mathrm{s} = B_\mathrm{s}\Sigma_\mathrm{s}\Omega_\mathrm{s}$, $\Sigma = \Sigma_\mathrm{s} + \Sigma_\mathrm{r} + (\Omega_\mathrm{s}+\Omega_\mathrm{r})\,L^2$, and $L = H + z/m$.

Substituting Eqs. (A3) and (A6)–(A8) into Eq. (A2), after some algebra we obtain the expression for the power of the echo signal of double reflection from the seafloor:

$$P_2(t) = A_2\frac{\Omega_\mathrm{r}}{\Sigma}\iint\limits_{-\infty}^{+\infty}\exp\left(-\frac{\pi\rho^2}{\Sigma}\right)\Phi(\mu_1)D_\mathrm{sr}\left[t-\frac{2z}{v_\mathrm{w}}\left(1+\frac{1}{\mu_1}\right)\right]\mathrm{d}^2\rho, \tag{A10}$$

where $A_1 = A_1R_\mathrm{b}/(4\pi z^2)$ and $\mu_1 \equiv \mu_1(\boldsymbol{\rho})/\sqrt{\rho^2+4z^2}$.

Substituting Eqs. (A5)–(A8) into Eq. (A4), some algebra we obtain the expression for the power of the echo signal of triple reflection from the seafloor:

$$P_3(t) = A_3\frac{\Omega_\mathrm{r}}{\Sigma}\iint\limits_{-\infty}^{+\infty}\mathrm{d}^2\rho_1\iint\limits_{-\infty}^{+\infty}\exp\left[-\frac{\pi\left(\rho_1^2+\rho_2^2\right)}{\Sigma}\right]\Phi(\mu_1)\Phi(\mu_2)D_{sr}\left[t-\frac{2z}{v_\mathrm{w}}\left(1+\frac{1}{\mu_1}+\frac{1}{\mu_2}\right)\right]\mathrm{d}^2\rho_2, \tag{A11}$$

where $A_3 = A_2R_\mathrm{b}/(4\pi z^2)$, $\mu_i \equiv \mu(\boldsymbol{\rho}_i)$, and $i = 1, 2$.

The multiplicity of the integral in Eqs. (A10) and (A11) can be reduced if we note that their integrands depend on the absolute value of the vector $\boldsymbol{\rho}$. Simple transformations yield the above-presented Eqs. (2) and (7).

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