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POSSIBILITIES FOR CONTINUOUS FREQUENCY TUNING IN TERAHERTZ GYROTRONS WITH NONTUNABLE ELECTRODYNAMIC SYSTEMS

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Large ohmic losses in the cavities of terahertz gyrotrons may lead to the overlapping of the axial mode spectra. In a number of gyrotron experiments, this effect has been used to provide a fairly broadband frequency tuning by changing appropriately the operating magnetic field and/or accelerating voltage of the gyrotron. Similar to the systems with nonfixed axial structure of the RF electromagnetic field and low diffraction quality, which are due to weak reflections of the operating wave from the collector end of the electrodynamic system, this changing leads to a monotonic change in the axial index of the operating wave and transition from the gyrotron regime to the gyro-BWO regime. According to a theoretical comparison of these two methods performed on the basis of generalization of self-consistent gyrotron equations with allowance for variations in the axial electron momenta, low-reflection systems can provide a higher efficiency and monotonicity of the frequency tuning.

1. INTRODUCTION

The development of coherent terahertz radiation sources for high-field EPR spectroscopy and dynamic polarization of nuclei in NMR spectroscopy stimulates the increasing interest in high-frequency gyrotrons (see, e. g., [1-5]). These sources are based on the use of stimulated cyclotron radiation of a beam of electrons moving along helical trajectories in a very strong uniform magnetic field. In principle, terahertz gyrotrons are able to ensure oscillation powers much greater than the needs of the mentioned applications. However, despite the encouraging advances in the development of much more compact terahertz devices, such as clinotrons [6], orotrons [7], and klystrons with distributed interaction [8], which require much weaker magnetic fields for their operation, these devices have not been effectively used in high-field spectroscopy for now, giving place to gyrotrons in the majority of experiments in this area.

From the point of view of spectroscopic applications, along with the high stability of the frequency and power of the output radiation, the possibility of tuning the terahertz-oscillator frequency, or at least its continuous adjustment to the resonance values by about 1%, is much more attractive than the magnetic field variation in the spectrometer. It should be mentioned that such a tuning band is impossible in most of the millimeter-wave gyrotrons since they use frequency-nontunable cavities with a relatively high Q-factor. Therefore, the continuous frequency tuning band $\Delta\omega \sim \omega/Q$, which can be provided by varying the operating magnetic field or voltage at a fixed axial operating mode, usually does not exceed 0.1% (ω is the operating frequency). However, a very high level of ohmic losses in the walls of a terahertz cavity may lead to the overlapping of the bands corresponding to the neighboring axial modes. This effect has made

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it possible to demonstrate a relatively broadband continuous frequency tuning of the radiation by varying successively the axial index of the operating mode [9–13].

It should be mentioned that this "ohmic" method of frequency tuning has an obvious disadvantage related to high losses in the cavity walls and, therefore, low oscillation efficiency. In this regard, in accordance with [14–18] (see also [19–20]), it would be natural that, along with more or less unavoidable "ohmic" losses, useful "diffraction" losses are also used for the frequency tuning. In other words, the part of losses related to the radiation output should be increased as much as possible. This is achieved by transition from high-Q cavities to low-Q electrodynamic systems with small reflection of the operating wave from the collector end. Such an approach increases the source efficiency and ensures a smoother tuning of the radiation frequency.

In this paper, we study the role of diffraction and ohmic losses in the frequency tuning of terahertz gyrotrons, which is due to the change in the axial mode index. In Sec. 2, we consider the mechanisms of quasi-continuous tuning of the gyrotron frequency. This tuning can be achieved if the wave excitation regime is of "mode" nature, such that the axial structure of the operating wave at the output of the interaction space is not fixed by the electrodynamic system and is determined by the conditions of the electron–wave interaction. To study such systems, in Sec. 3 we generalize the system of stationary gyrotron equations with nonfixed axial structure of the operating mode to the case of notable variations in the axial momenta of particles for a simultaneous description of excitation of both the quasi-critical axial modes (gyrotron regime) and the modes which are far enough from the cutoff (gyro-BWO regime). In Sec. 4, based on the equations obtained and by modeling the oscillator whose parameters are close to those of a tunable gyrotron with operating frequencies near 330 GHz, which was studied in the experiment [13], the capabilities and limitations of the quasi-continuous frequency tuning are illustrated.

2. CONTINUOUS GYROTRON FREQUENCY TUNING

The theory of frequency tuning in gyrotrons with nontunable electrodynamic systems was developed in [14–18] mainly for cavities with low ohmic losses and low (including the minimum one) diffraction quality. In particular, these papers studied the influence of the electron beam on the axial structure of a microwave field and operating gyrotron frequency. It was shown there that by varying the operating magnetic field and/or electron energy for sufficiently high currents, it is possible to provide a continuous and monotonic variation of the axial field structure and oscillation frequency even in high-Q cavities. However, such a variation is much smoother in electrodynamic systems with low Q-factor.

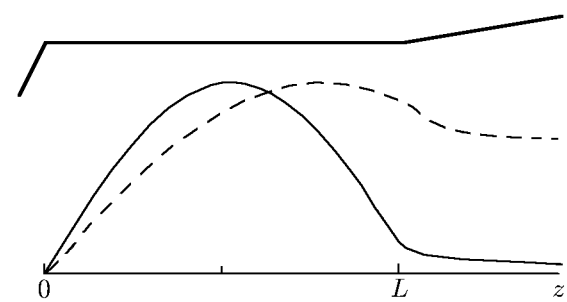


Fig. 1. Scheme of the gyrotron cavity (bold line) and the field structure of the lower-order axial mode for strong (solid line) and weak (dashed line) reflections from the collector end of the cavity. Hereafter, z is the axial coordinate and L is the waveguide length.

To discuss the frequency tuning problem, we recall the main properties of the conventional gyrotron cavity formed by a segment of a quasi-regular axially symmetric waveguide (Fig. 1). The transverse electric mode TE_{mpq} with frequency ω close to the cutoff frequency ω_{cr} of the regular part of the waveguide is generally used as the operating mode in such a cavity. For such modes, the mutual multiple reflection of the co- and counter-propagating components of the standing operating wave at the waveguide ends is fairly effective even in the case of small cross-sectional irregularities at the waveguide ends. As a rule, the cathode end of the waveguide is limited by a subcritical narrowing, which converts the counter-propagating component of the standing wave into the co-propagating one with reflection coefficient close to unity. At the collector end, the regular part of the waveguide transforms into an expanding output waveguide, which ensures the diffraction output of the co-propagating wave and, simultaneously, its multiple reflection into the counter-propagating wave.

The diffraction quality Q_{d} of a gyrotron cavity is determined by the length L of its regular part, group velocity $V_{\rm gr}$ of the operating wave, and also the coefficients K_1 and K_2 of reflection of the wave from the cathode and collector ends, respectively. In the absence of ohmic losses and under the assumption that reflection from both ends is negligibly small and the transverse operating mode is excited within a waveguide of length L, the effective Q-factor (the so-called minimum diffraction quality) is determined by the ratio of the characteristic time of the wave emission $L/V_{\rm gr}$ and the wave period $2\pi/\omega$ [14–18, 21]:

$$Q_{\rm min} = \omega L/V_{\rm gr}. \tag{1}$$

With ideal reflection of the wave from the supercritical cathode narrowing ($|K_1| = 1$) and reflection from the collector narrowing with the coefficient $K = |K_2|$, the characteristic time of output of radiation from the cavity and the diffraction quality of the latter increase by a factor of α, where $\alpha = (1-K)^{-1}$, compared with the value in Eq. (1):

$$Q_{\rm d} = \alpha Q_{\rm min}. \tag{2}$$

Under conditions where the transformation of transverse modes inside the cavity can be neglected and the approximation of a fixed transverse structure can be used, in the stationary oscillation regime of a gyrotron the frequency ω and the function $f(z)$, which describes the axial structure of the cavity eigenmode, can be found for any reflection coefficient K from a self-consistent system of equations [14] that includes the inhomogeneous string equation [22] for the axial structure of the cavity mode:

$$\frac{{\rm d}^2 f}{{\rm d}z^2} + h^2 f = \rho. \tag{3}$$

In Eq. (3),

$$h(z) = \sqrt{(\omega/c)^2 - k_\perp^2(z)}, \qquad k_\perp = \mu_{m,p}/R_{\rm w}(z) \tag{4}$$

are the axial and transverse wave numbers, respectively, for the operating mode, $\mu_{m,p}$ is the pth positive root of the equation $J'_m(\mu) = 0$, where $J_m(\mu)$ is a first-kind Bessel function of order m, $R_{\rm w}(z)$ is the radius of the electrodynamic system in the transverse cross section with the coordinate z, c is the speed of light in free space, and m and p are the azimuthal and radial mode indices, respectively. The factor ρ of the wave excitation by an electron beam can be found from the equations of motion of particles in a static magnetic field and in the field of the resonant component of the resonant mode [14] (see also Sec. 3). As the boundary conditions for Eq. (3), we will use the simplest zero condition for the wave field at the input ($z = 0$) of the operating cavity:

$$f(0) = 0, \tag{5}$$

as well as the condition of reflectionless output of radiation (radiation condition), which we impose in a smoothly broadening output waveguide at a sufficient distance from the collector end of the cavity in the cross section with the coordinate $z_{\rm out}$, where the wave frequency is well away from the cutoff frequency:

$$\frac{{\rm d}f}{{\rm d}z} = ihf|_{z=z_{\rm out}}. \tag{6}$$

In the case of strong reflection from the collector end of the cavity ($K \approx 1$), when the diffraction quality is high compared with its minimum value and the operating currents are not too much higher than the starting value, the axial structures of the eigenmodes can be considered fixed and close to the "cold" ones (i. e., those found without allowance for the electron beam) mode structures of the closed cavity, in which the "zero" boundary condition (5) is set both at the input and output ends:

$$f_q \approx \sin h_q z, \tag{7}$$

where $h_q \approx \pi q/L$, $q = 1, 2, \ldots$ is the axial index. Rigorously speaking, only the lower-order axial modes with the indices $q = 1, 2$, whose frequencies are closest to the cutoff frequency and axial wave numbers are minimal (Fig. 2*a*), can be called the gyrotron oscillation regimes. For these modes, the electron–wave

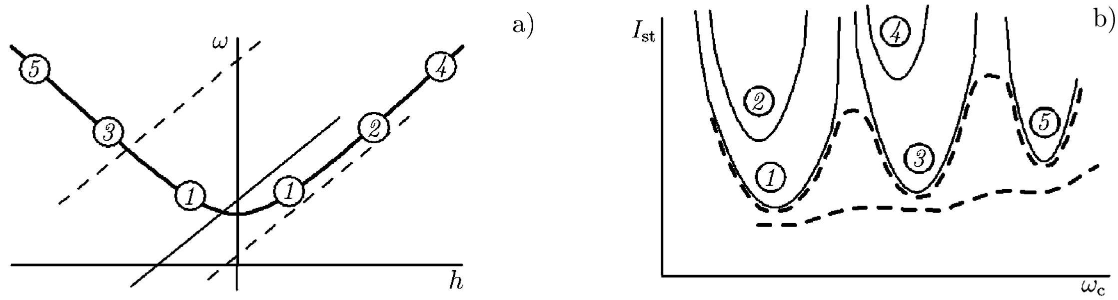


Fig. 2. Dispersion diagrams (a) and the corresponding dependences of the starting current strength $I_{\rm st}$ on the cyclotron frequency (b) for the modes in both the gyro-TWT and gyro-BWO regimes. Points *1* correspond to the mode $q = 1$, points *2* and *3*, to the mode $q = 2$, which propagates concurrently and oppositely to the beam particles, respectively, and points *4* and *5*, to the mode $q = 3$, which propagates concurrently and oppositely to the beam particles, respectively. Dashed curves in Fig. (b) illustrate the evolution of the starting diagrams (from discrete to monotonic) when passing from the regime of the cavity-fixed structure of the microwave field to the regime with nonfixed structure of the wave field.

resonance condition at the sth cyclotron harmonic

$$\omega \approx s\omega_{\rm c}, \tag{8}$$

where $s = 1, 2, \ldots$, $\omega_{\rm c}$ is the cyclotron frequency, is fulfilled simultaneously for the co- and counter-propagating waves. For the higher-order axial modes (Fig. 2a) there is selective resonant interaction between the particles and the co-propagating component of the standing wave (gyro-TWT regime):

$$\omega - hv_\parallel \approx s\omega_{\rm c}, \tag{9}$$

or with a counter-propagating component (gyro-BWO regime):

$$\omega + hv_\parallel \approx s\omega_{\rm c}. \tag{10}$$

Here, $v_\parallel$ is the axial velocity of the particles. The Doppler shift effect described by the terms $hv_\parallel$ on the right-hand sides of Eqs. (9) and (10) shifts (with respect to the case of gyrotron interaction) the resonant values of magnetic fields for such regimes (Fig. 2a), and also makes them more sensitive to the axial-velocity spread of the particles.

We study the possibility of changing the frequency due to transitions from one axial mode to another. For the quasi-continuity of such a tuning, the frequency bands of the neighboring modes should overlap. In the presence of considerable ohmic losses in the walls, this is possible, in principle, even for a high diffraction quality. If the spectrum of axial wave numbers is close to the "cold" spectrum of eigenmodes with the wave numbers h_q, then, in accordance with Eq. (2), their diffraction quality can be described by the relation

$$Q_{\rm d} = \alpha Q_0/q, \tag{11}$$

where $Q_0 = 4\pi(L/\lambda)^2$ is the minimum factor for the lower-order axial mode $q = 1$, and λ is the wavelength. In the case of a cavity-fixed frequency spectrum of axial modes

$$\frac{\omega_{\rm q}}{c} \approx \sqrt{k_\perp^2 + \left(\frac{q\pi}{L}\right)^2} \approx k_\perp \left[1 + \frac{q^2}{8}\left(\frac{\lambda}{L}\right)^2\right],$$

the distance between the neighboring modes is determined by the formula

$$\frac{\omega_q - \omega_{q-1}}{ck_\perp} = \frac{\pi(2q-1)}{2Q_0}. \tag{12}$$

We neglect the difference between the ohmic Q-factor Q_{ohm} and the axial-mode number and assume that it is a factor of β less than the minimum diffraction quality: $Q_{\text{ohm}} = Q_0/\beta$. The width of the resonant band of the mode $\Delta\omega_q$ is determined by the losses which are inversely proportional to the total Q-factor:

$$\Delta\omega_q \approx \frac{2}{Q_0}\left(\frac{q}{\alpha} + \beta\right). \tag{13}$$

The "cold" resonant curves of two neighboring axial modes overlap when the absolute value of their frequency difference is less than the half-sum of the mode bandwidths. Correspondingly, overlapping of the bands of a fixed spectrum of axial modes and quasi-continuous frequency tuning are possible only with very high losses:

$$\beta \geq \left(q - \frac{1}{2}\right)\left(\frac{1}{\alpha} - \frac{\pi}{2}\right). \tag{14}$$

As an example, we consider tuning when the axial-mode index is changed from $q = 1$ to $q = 4$. Even with the minimum reflections, when α is of the order of unity, the ohmic-loss factor β needed for the continuous tuning is about 2. In this case, the ratio of the ohmic-loss power P_{ohm} to the microwave radiation power P_Σ of the electron beam

$$\frac{P_{\text{ohm}}}{P_\Sigma} = \frac{\beta}{\beta + q/\alpha} \tag{15}$$

is equal to 2/3 for the first and is 1/3 for the fourth axial mode. If the coefficient of reflection from the output end of the cavity is $K = 0.7$ ($\alpha = 2$), then the ohmic-loss factor that is necessary for the continuous tuning increases to $\beta \approx 4$, while the ohmic-loss fraction determined in Eq. (15) is close to 0.9 for the first and is 2/3 for the fourth mode.

These estimates are valid if the electrodynamic system of a gyrotron determines a nearly discrete spectrum of axial modes. Such a situation occurs when the reflection from the output end of the cavity is strong enough and the operating current of the gyrotron is not too much higher than the starting value. However, if the indices q are high, the gyrotron regime transforms into the gyro-BWO or gyro-TWT regime (Fig. 2*a*) [15, 16, 18]. In this case, the mode frequency is far from the cutoff and the corresponding waves are weakly reflected from the irregularities of the electrodynamic system, which in this case is similar in its properties to the waveguide with a continuous spectrum of axial modes.

A fixed quasi-discrete spectrum of axial modes corresponds to isolated areas of the gyrotron self-excitation (Fig. 2*b*), at whose boundaries the starting current tends to infinity. These areas are separated by "forbidden" bands, inside which the gyrotron operation is impossible. If, unlike the idealized boundary condition considered above, the zero of the wave field is not fixed at the collector end of the cavity, then a change in the operating magnetic field or accelerating voltage leads to the transformation of the axial structure of the microwave field and, correspondingly, the transformation of the oscillation frequency under the action of the electron beam even for negligibly small ohmic losses [16, 18]. For a high diffraction quality (i.ė., large values of α), transformation of the mode structure and frequency tuning occur mainly in the forbidden bands. A decrease in the wave reflections from the collector end makes the axial structure less fixed and facilitates its variation under the action of the electron beam. If the magnetic field decreases and the gyrotron regime converts into the gyro-TWT regime, then the maximum of the electric field of the lower-order axial mode shifts from the middle of the cavity to its collector end (Fig. 1). In the opposite case of increasing magnetic field, the gyrotron regime converts into the gyro-BWO regime. In this case, the maximum of the microwave field shifts towards the cathode end of the cavity, and other maxima appear gradually, so that the lower-order axial mode with the structure similar to one arc (half-period) of a sinusoid, converts into the second mode, then to the third mode, etc. Such a continuous change of operating modes permits the gyrotron excitation for any magnetic field corresponding to the gyro-BWO region. The dependence of the starting current on the magnetic field remains nonmonotonic, such that the starting current in the forbidden bands is about a factor of α higher than its minimum values. It should be mentioned that the corresponding quasi-continuous frequency tuning is observed in the experiment only with a sufficiently high operating current. For an extremely low diffraction quality, where α is of the order of unity, transformation of the mode structure and the frequency tuning are monotonic over the entire range of magnetic fields corresponding to the gyro-BWO regime, and the dependence of the starting current on the magnetic field is also smooth and monotonic. With small reflections from the collector end, the frequency tuning is slightly nonmonotonic.

Thus, to achieve a monotonic quasi-continuous frequency tuning in the gyrotron, it suffices to minimize reflections from the collector end of the electrodynamic system and thus provide conditions under which the axial structure of the microwave field is not fixed.

3. THE EQUATIONS DESCRIBING THE OSCILLATOR OPERATION IN GYROTRON AND GYRO-BWO REGIMES

If the operating-mode frequency is close to the cutoff frequency, then the electron–wave interaction in a gyro-device does not lead to variations in the electron's axial momentum. Exactly this approximation was used in the equations of a gyrotron with nonfixed structure of the wave electric field in [14]. However, with broadband frequency tuning, changing the magnetic field may lead to a change in the axial mode index from $q = 1$ to such a high value that the mode becomes far from the cutoff. The transverse component of the high-frequency magnetic field of the wave increases and the axial momentum of the electron is no longer constant, which makes it necessary to generalize the gyrotron equations to such a case.

If the axial structure of the microwave is not fixed and the radius $R_{\rm w}(z)$ of the axially symmetric electrodynamic system (waveguide) is changed gradually with the axial coordinate, then in the polar coordinate system $(r,\ \phi)$, where r is the polar radius and ϕ is the azimuthal angle, the field of the transverse electric mode of the waveguide can be represented in the form

$$E_+ = -ik\exp(i\phi)\left\{J'_m(k_\perp r){\rm Re}[iA(z)\exp(i\varphi)] - \frac{imJ_m(k_\perp r)}{r}{\rm Re}[A(z)\exp(i\varphi)]\right\},$$
$$B_+ = \exp(i\phi)\left\{J'_m(k_\perp r){\rm Re}\left[\frac{{\rm d}A}{{\rm d}z}\exp(i\varphi)\right] + \frac{imJ_m(k_\perp r)}{r}{\rm Re}\left[i\frac{{\rm d}A}{{\rm d}z}\exp(i\varphi)\right]\right\},$$
$$B_z = k_\perp^2 J_m(k_\perp r){\rm Re}[A\exp(i\varphi)].$$

Here, $E_+ = E_x + iE_y = \exp(i\phi)(E_r + iE_\phi)$ and $B_+ = B_x + iB_y$ are complex combinations of the transverse components of the electric and magnetic fields, respectively (the x and y axes of the Cartesian coordinate system lie in the transverse plane), k is the wave number, $\varphi = m\phi - \omega t$ is the phase of the wave, and t is the time. The complex wave amplitude $A(z)$ is so normalized that the power $P_{\rm wave}$ transferred by the wave is described by the following expression:

$$P_{\rm wave} = \frac{ck}{4}N{\rm Re}\left(iA\frac{{\rm d}A^*}{{\rm d}z}\right), \tag{16}$$

where $N = J_m^2(\mu_{m,p})(\mu_{m,p}^2 - m^2)$ is the coordinate-independent wave norm and the asterisk denotes complex conjugation.

In the particular case of a paraxial electron beam, which is used in the so-called large-orbit gyrotrons, all the electrons move along helical trajectories around the waveguide axis, and their oscillatory transverse motion is described by the formulas

$$r_+ = r_{\rm c}\exp(i\Psi), \qquad V_+ = iV_\perp\exp(i\Psi),$$

where Ψ, $r_{\rm c}$, and $V_\perp$ are the electron's gyro-phase, gyro-radius, and transverse velocity, respectively, $r_+ = x + iy$, $V_+ = V_x + iV_y$, V_x, and V_y are the projections of the transverse electron velocity on the x and y axes. with this beam configuration, the electron–microwave interaction at the sth cyclotron harmonic occurs only if the number of this harmonic coincides with the azimuthal index m of the operating transverse mode. The corresponding relativistic equations of motion for the Lorentz factor (dimensionless energy) $\gamma = (1 - V^2/c^2)^{-1/2}$, where V is the electron velocity, of the normalized components $p_\perp = \gamma V_\perp/c$ and $p_z = \gamma V_z/c$ of the electron momentum and the electron phase $\theta = s\Psi - \omega t$ with respect to the wave can be written in the following form:

$$\frac{\mathrm{d}\gamma}{\mathrm{d}z} = -\chi \mathrm{Re}[f \exp(i\theta)], \qquad \frac{\mathrm{d}p_z}{\mathrm{d}z} = -\frac{\chi}{k}\mathrm{Re}\left[\frac{\mathrm{d}f}{\mathrm{d}z}\exp(i\theta)\right],$$
$$\frac{\mathrm{d}p_\perp}{\mathrm{d}z} = -\chi_\perp \mathrm{Im}\left[\left(\frac{\gamma f}{p_z} + \frac{i}{k}\frac{\mathrm{d}f}{\mathrm{d}z}\right)\exp(i\theta)\right], \qquad \frac{\mathrm{d}\theta}{\mathrm{d}z} = \frac{sk_\mathrm{c} - \gamma k}{p_z} + F. \tag{17}$$

Here, $\chi_\perp = k_\perp J_s'(k_\perp r_\mathrm{c})$, $\chi = \chi_\perp p_\perp/p_z$ is the electron–wave interaction factor, $k_\mathrm{c} = eB_0/(m_\mathrm{e}c^2)$ is the normalized magnetic field, B_0 is the external magnetic field, m_e is the electron mass, and e is the elementary charge. On the right-hand side of the equation for the electron phase, along with the "inertial" first term, the so-called "force" term F was also taken into account.

In the more general case of a tubular electron beam, the transverse complex electron component has a more complex form:

$$r_+ = R_\mathrm{e}\exp(i\varphi) + r_\mathrm{c}\exp(i\Psi),$$

where R_e is the radius of the guiding centers of the electron beam. In this case, averaging of the equations of motion over the azimuthal coordinate ϕ again leads to equations of the form (17), in which the electron–wave coupling factor is determined by the expression $\chi_\perp = k_\perp J_{s-m}(k_\perp R_\mathrm{e}) J_s'(k_\perp r_\mathrm{c})$.

In the case of a fairly smooth profile of the electrodynamic system, the axial structure of a dimensionless complex amplitude of the operating wave $f(z) = ekA(z)/(m_\mathrm{e}c^2)$ is determined by the nonuniform-string equation (3) with boundary conditions (5) and (6). The excitation factor in Eq. (3) has the form

$$\rho = -G\,\langle\chi\exp(-i\theta)\rangle,$$

where $G = 4eIk/(m_\mathrm{e}c^3N)$, the angle brackets denote averaging over initial electron phases, and I is the beam current.

If the ohmic losses are neglected, then the square of the axial wave number $h^2(z)$ is a real-valued function. In this case, the electronic efficiency of the system

$$\eta_\mathrm{el} = \frac{\langle\gamma_0 - \gamma\rangle}{\gamma_0 - 1},$$

coincides with the output-wave power normalized to the electron-beam power

$$\eta_\mathrm{wave} = \frac{1}{(\gamma_0 - 1)G}\mathrm{Re}\left(if\frac{\mathrm{d}f^*}{\mathrm{d}z}\right),$$

so that the energy conservation law

$$P_\mathrm{el} = m_\mathrm{e}c^2 I\,\langle\gamma_0 - \gamma\rangle/e = P_\mathrm{wave}$$

follows from Eqs. (16) and (17). Here, γ_0 is the Lorentz factor of the particles at the input of the interaction space, P_el and P_wave are the loss of the kinetic power of the electron beam and the power transferred by the wave, respectively. To allow for ohmic losses in the cavity, it is needed to add the corresponding imaginary term to the wave number squared: $h^2 \to h^2 + ik^2/Q_\mathrm{ohm}$. In this case, the inequality $\eta_\mathrm{el} > \eta_\mathrm{wave}$ is fulfilled, and the ohmic-loss power (also normalized to the electron-beam power) is given by the formula $\eta_\mathrm{ohm} = \eta_\mathrm{el} - \eta_\mathrm{wave}$.

For the quasi-critical wave, the estimate $\mathrm{d}f/\mathrm{d}z \sim hf \ll kf$ is valid and the axial momentum variations

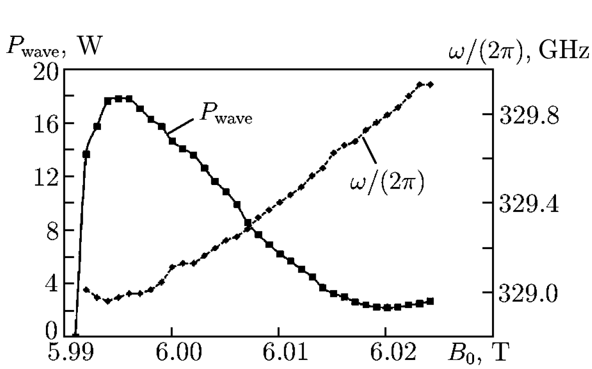


Fig. 3. Results of the experiment [13] with a frequency-tunable gyrotron: power and frequency of the output radiation as functions of the operating magnetic field.

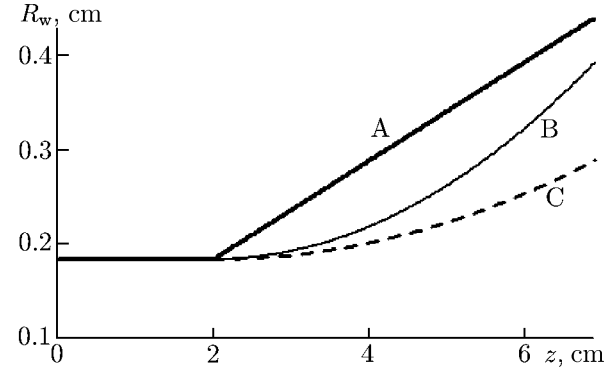


Fig. 4. Three profiles of the operating gyrotron cavity studied in the numerical calculations.

of a particle are negligibly small. In this case, Eqs. (3) and (17) reduce to the equations of a gyrotron with nonfixed structure of the microwave field, which were derived in [14]. In the opposite limiting case of a far-from-cutoff co- or counter-propagating resonant component of the wave when $f(z) = \hat{f}(z)\exp(\pm ihz)$, where $\hat{f}(z)$ is a certain function, Eqs. (3) and (17) transform to the gyro-TWT or gyro-BWO equations [23].

4. MODELING OF FREQUENCY TUNING IN A TERAHERTZ GYROTRON

Let us apply Eqs. (3) and (17) with boundary conditions (5) and (6) for calculation of a gyrotron with parameters similar to those of a gyrotron with an operating frequency of about 330 GHz, which was realized in the experiment [11]. In that gyrotron with an accelerating voltage of 10.1 kV, the tubular electron beam with a current of 200 mA and a pitch factor of 1.8 excited the $\text{TE}_{-4,3}$ mode at the second cyclotron harmonic (the minus sign in the azimuthal index means that the direction of azimuthal rotation of the electric field of the mode is opposite to the direction of cyclotron rotation of the electrons). The ohmic Q-factor for this mode is estimated as $Q_{\rm ohm} = 10^4$, while the minimum diffraction quality for a 3-cm long cavity that was used in the experiment was $Q_{\rm ohm} = 1.4 \cdot 10^4$. Such Q-factors correspond to the ohmic loss fraction 60% for the lower-order axial mode, in agreement with the estimates (14) and (15) obtained for weak reflections from the output end of the cavity (α is of the order of unity).

In this experiment, we observed a continuous frequency tuning during the successive change of the axial mode from $q = 1$ to $q = 7$ (Fig. 3). The tuning occurred under conditions where the operating current was about 200 mA and significantly exceeded the starting threshold equal to about 30 mA. It should be mentioned that the study of frequency tuning for an operating current much greater than the starting value on the basis of stationary single-frequency equations (3) and (17) is not quite correct since the competition of "hot" axial modes excited at different frequencies is neglected (see the text below). For this reason, we will examine the excitation of a shorter cavity with a length of 2 cm, for which the starting current is close to 60 mA and the exceeding of this value over the threshold is not too large. Consider three forms of broadening of the output waveguide (Fig. 4): in the form of a cone with a large opening angle 6° (profile A) and two smoother parabolic profiles, B and C.

In the starting characteristics for profile A, four axial modes $q = 1$–4 are clearly separated from each other (Fig. 5), while for the smoother profiles B and C they convert gradually one into another. However, the minimum values of the starting currents for each axial mode do not change with the variation in the output profile.

According to calculations, in the nonlinear regime with an operating current of 200 mA for profile A, an increase in the magnetic field also leads to the successive excitation of three axial modes $q = 1$–3 with separated oscillation bands (Fig. 6). The dependences of the output and electronic efficiencies on the

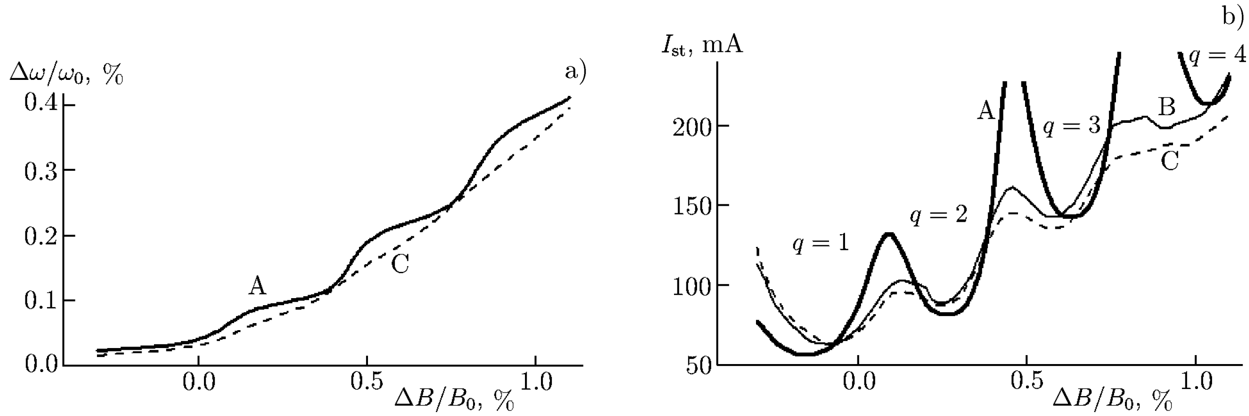
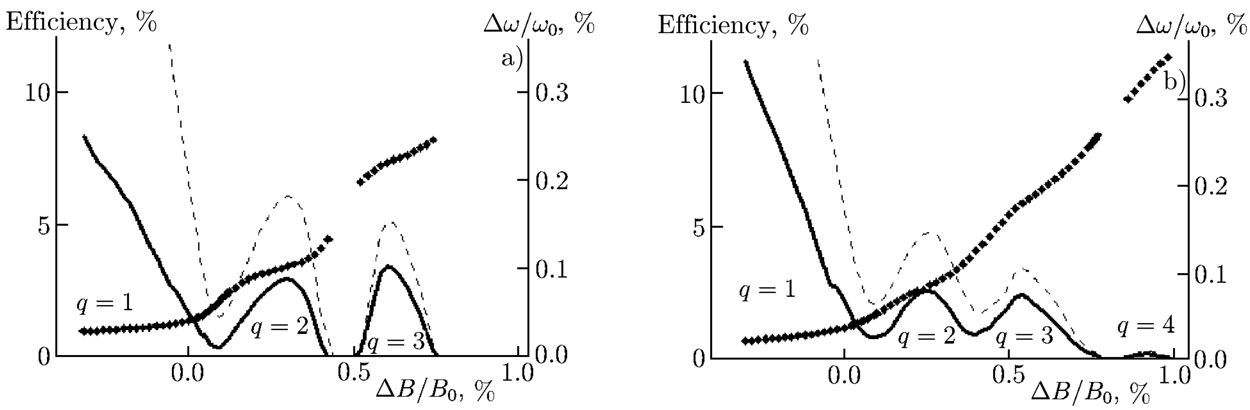


Fig. 5. The gyrotron eigenfrequencies (a) and starting current (b) as functions of the operating magnetic field for the cavities with profiles A (bold line), B (thin line), and C (dashed line).



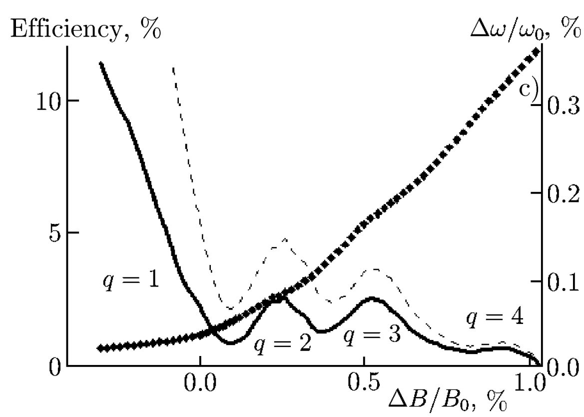


Fig. 6. Output (solid curves) and electronic (dashed lines) efficiencies of the gyrotron, as well as the normalized shift of oscillation frequency (bold dotted curves) for the cavities with profiles A (a), B (b), and C (c) at an operating current of 200 mA.

magnetic field have maxima corresponding to the eigenfrequency of each of the three modes. In the gaps between these maxima, the output signal power steps down almost to zero or is absent. The fourth axial mode is not excited since its starting current exceeds the chosen operating current. The dependence of the oscillation frequency on the magnetic field, $\omega(B)$, is slow in the regions corresponding to the eigenmode generation with a high power of the output signal and is fast in the regions with small oscillation power corresponding to the transition from one axial mode to another. All this is evidence for the “mode” nature of the gyrotron operation when strong reflection from a relatively sharp irregularity at the output opening fixes the discrete spectrum and structure of the cavity modes to a considerable degree.

When a smoother profile B is used, the axial modes $q = 1$–3 are still separated from each other (Fig. 6), but the transition from one mode to another appears to be smoother than in the case of profile

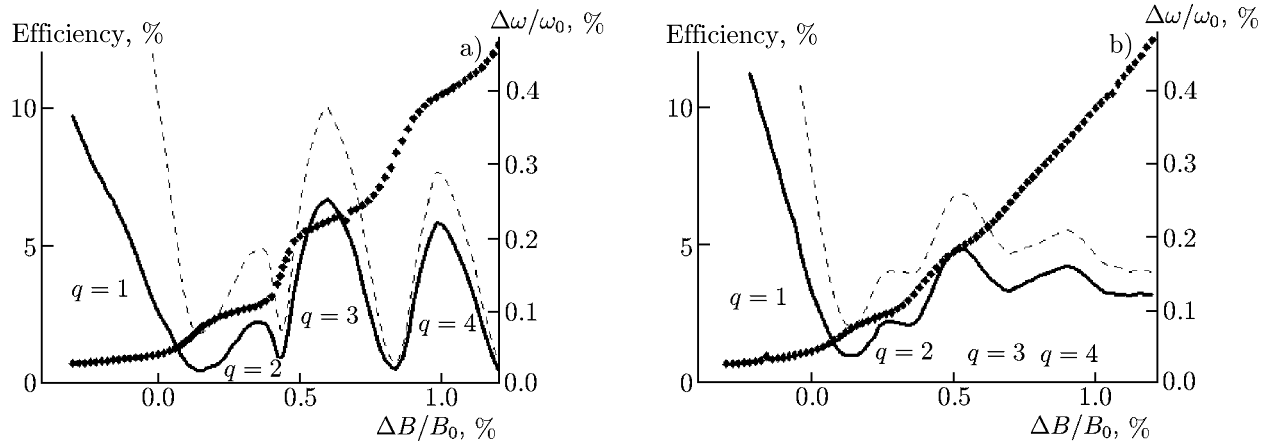


Fig. 7. Output (solid curves) and electronic (dashed curves) gyrotron efficiencies, as well as the normalized frequency shift (bold dotted curves) for the cavities with profiles A (a) and C (b) for an operating current of 400 mA.

A. Moreover, with the further increase in the magnetic field after the oscillation cutoff, a transition to the oscillation of the mode $q = 4$ occurs. In the case of a still smoother profile C, the oscillation bands of the modes $q = 2$–4 are virtually not separated, so that the frequency dependence of the output radiation power is smooth and the frequency tuning is monotonic. This means that the electrodynamic system with output profile C does not fix the axial structure of modes and the discrete spectrum of their frequencies.

It is important to mention that the relation between the electronic and output efficiencies and, therefore, the fraction of ohmic losses in the cases of profiles B and C are about the same. This means that the transition from the opening B to the smoother opening C does no lead to a notable decrease in the wave reflection from the output end and in the diffraction quality. In other words, the transition from profile B to profile C does not lead to a considerable variation in the "cold" properties of the electrodynamic system which is close to the waveguide section. Therefore, a smoother frequency tuning in the case of profile B is caused by the smooth output of electrons from their interaction with the electromagnetic wave.

For the current 200 mA, oscillation at the fourth axial mode is characterized by a low power of the output signal since this current is close to the starting current corresponding to this mode. To achieve a broader-band tuning, it is needed to increase the operating current. For example, for the current 400 mA in the profile C cavity, the discrete structure of axial modes almost disappears, except for the first mode, and the oscillation frequency changes continuously, monotonically, and smoothly in the successive transitions from the mode $q = 1$ to the mode $q = 4$ (Fig. 7).

Thus, a smooth continuous frequency tuning is provided by the following conditions: 1) the operating current increases significantly the starting current of excitation of a few axial modes, 2) a smooth transition from the interaction space to the broadening output waveguide provides a smooth output of electrons from their resonant interaction with the microwave field, and 3) the axial structure of the wave is of "hot" nonfixed nature, which is very different from the cavity-fixed distributions for a discrete set of "cold" modes (Fig. 8).

The importance of smooth termination of the electron–wave-field interaction can additionally be illustrated by changing the magnetic field strength in the output waveguide for a fixed profile C (Fig. 9). Output efficiency in the case of a uniform field and smooth termination of interaction in the output broadening due to an increase in the axial wave number and violation of the electron cyclotron resonance with the wave is higher than in the case of abrupt termination of interaction due to a jump-like change in the magnetic field. If the magnetic field is uniform, then the region of resonant interaction of particles with the lower-order, closest-to-cutoff mode $q = 1$ is limited by the regular part of the cavity, while for the higher-order axial modes the interaction region also comprises part of the waveguide broadening and is longer. Thus, a smooth termination of the electron interaction with the wave should be provided not only by a smooth broadening of the output waveguide, but also by a slow magnetic field variation at the start of the output section.

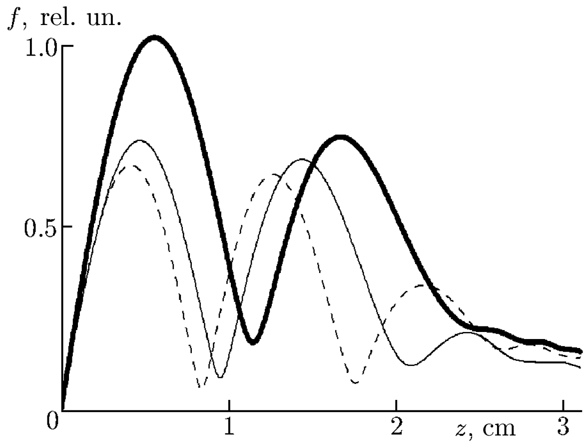


Fig. 8. Evolution of the axial structure of the excited microwave field during variation of the operating magnetic field in the range corresponding to the transition from the mode $q = 2$ to the mode $q = 3$ in the profile C cavity for the currents 200 and 400 mA. The bold line corresponds to the case $\Delta B/B_0 = 0.26\%$, the thin line, to the case $\Delta B/B_0 = 0.38\%$, and the dashed line, to the case $\Delta B/B_0 = 0.46\%$.

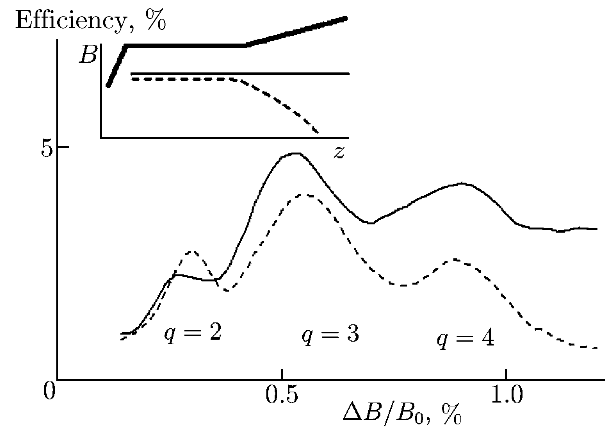


Fig. 9. Transition from the mode $q = 2$ to the mode $q = 4$ in the profile C cavity (bold line in the inset) for the current 400 mA. Dashed lines illustrate the case where the electron–wave interaction is "switched off" abruptly at the cavity output, which corresponds to a jump-like change in the operating magnetic field in this region.

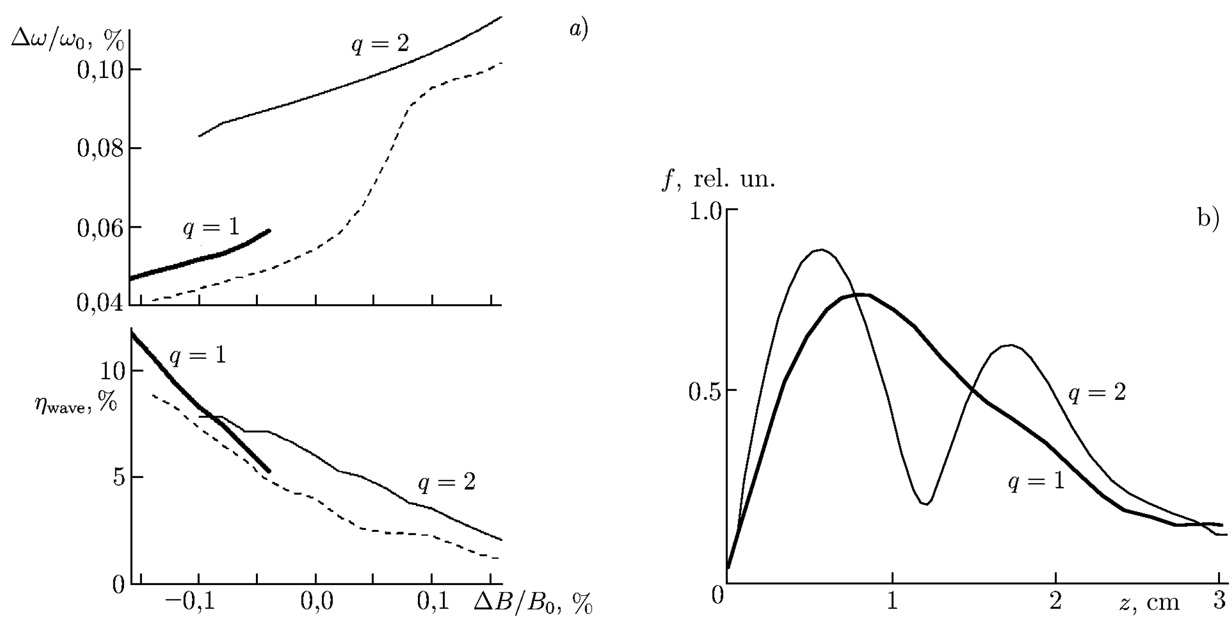


Fig. 10. Transition from the mode $q = 1$ to the mode $q = 2$ in the profile C cavity for the currents 600 (dashed lines) and 800 mA (solid lines) (a). Axial structures of the electric fields for two "hot" eigenmodes, which can be excited for the current 800 mA in the same magnetic field corresponding to $\Delta B/B_0 = -0.04\%$ (b).

It has already been mentioned that the stationary single-frequency model described by Eqs. (3) and (17) becomes incorrect in the case where the current exceeds considerably the starting value. For example, as the current increases from 600 to 800 mA, this model demonstrates splitting of the continuous solution of the equations into two branches (Fig. 10). In this case, in a certain range of the magnetic field values there are two solutions of the single-frequency equations corresponding to different frequencies and powers of the output radiation, as well as different axial structures of the microwave field (Fig. 10*b*). One of these "hot" modes is close in structure to the first, "one-hump" mode of the "cold" cavity and the other is a counterpart of the second, "two-hump" mode. Such a result agrees well with the linear theory [18], which predicts the existence of different modes for the same operating magnetic field (Fig. 2*b*). In this situation, the answer to the question "Which of the two modes is actually established with these oscillator parameters?" can be given on the basis of a nonstationary multifrequency analysis (see, e. g., [24]). Two competing eigenmodes are characterized by close oscillation powers, but slightly different frequencies (Fig. 10*a*). For a high current, the transition from the first to the second axial mode is accompanied by a slight frequency jump. Possibly, such an effect is observed with a small field variation around 6 T in Fig. 3 taken from the experimental paper [13].

5. CONCLUSIONS

The open collector end of a conventional gyrotron cavity, through which the diffraction output of radiation is performed, admits, for any reflection coefficient, the restructuring of the axial wave field with the variation in the operating magnetic field and/or accelerating voltage, accompanied by a continuous tuning of the oscillation frequency [16–18]. An increase in the magnetic field (or accelerating voltage) leads to the transition from the gyrotron oscillation regime of the lower-order mode with one axial variation to the gyro-BWO regime, in which higher axial modes are excited. For high coefficients of reflection from the output end, the transitions of oscillation from one axial mode to another occur through the "forbidden" bands, whose existence is due to the fact that the cavity largely determines the axial structure of the modes and their quasi-discrete spectrum. Under such conditions, the continuous frequency tuning requires that the current greatly exceeds its starting value. This is an important factor that affects the smoothness of the frequency tuning of the gyrotron. The stationary approach, which is valid with a relatively small exceeding of the starting value of the current, shows that as the operating current increases, the "hot" structure of the field excited in the cavity becomes more and more different from the structures of the axial cavity modes. Correspondingly, the axial-mode spectrum changes to a quasi-continuous spectrum of "hot" modes. The further increase in the operating current leads to the occurrence of a competition between two or several "hot" axial modes with different eigenfrequencies that exist in the same operating magnetic field. In this situation, the modeling of the system requires the numerical codes developed on the basis of a nonstationary multifrequency analysis (see, e. g., [24, 25]).

For weak reflections of the wave from the output end, the change in the axial structure of the excited wave and its frequency tuning are reached at a smaller exceeding of the operating current over the oscillation threshold and are smoother. Along with this mechanism of tuning, high ohmic losses, which provide the overlapping of the axial modes of the cavity and also help reach a relatively monotonic frequency tuning, are an important factor at high frequencies. These conclusions were confirmed experimentally [9–13]. Since the ohmic losses reduce the output-signal power generated by the electrons, the simultaneous achievement of a smooth frequency tuning and a relatively high efficiency in a terahertz gyrotron requires that the diffraction quality is of the order of or lower than the ohmic Q-factor.

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