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SYNTHESIS OF MODE CONVERTERS ON THE BASIS OF THE FDTD METHOD

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We propose a universal technique for synthesizing mode converters, which is based on numerical integration of the Maxwell equations on a space-time mesh by the FDTD method. The new technique is an iterative algorithm, in which the correction to the converter profile at each iteration is calculated via the fields on the converter surface that are obtained from two conjugate problems, specifically, by direct and inverse (with an inversion of the time-integration direction) solution of the equations for electromagnetic fields. The efficiency of the synthesis algorithm is illustrated by examples that are of practical importance. The technique is compared with that proposed earlier, which used the solution of a system of equations for coupled waves.

1. INTRODUCTION

In the context of the scalar problem of two phase correctors in free space, which form the specified field, the concept of iterative synthesis was formulated for the first time in [1]. The proposed concept allows one to convert a monochromatic paraxial wave beam with a given transverse distribution of the field amplitude to a beam with a desired distribution by using the correction of only phase distributions of the above-mentioned fields. Later, this concept was developed further to solve the problems of synthesizing quasioptical mirrors of complex shapes [2] and weakly irregular waveguides [3, 4]. It is intensely used to develop millimeter-wave gyrotrons for controlled fusion devices [5]. All these variants of the synthesis are based on the common iterative scheme formulated for mirrors with weak deviations of the profile from the plane one (phase-corrector approximation) and for weakly irregular waveguides (approximation of the perturbation-theory method on the basis of a system of coupled waves). In accordance with this scheme, the profile of a mirror or a waveguide converter is corrected at each iteration on the basis of the difference of two types of fields on the synthesized surface, namely, the field obtained by solving the direct diffraction problem with the source in the form of specified input radiation and the field obtained by solving the conjugate diffraction problem with the source in the form of the desired output radiation. The conjugate problem can be interpreted as a solution of the time-inverted Maxwell equations.

All the above-mentioned works discuss synthesis problems, which are mathematically incorrect. First, one cannot state beforehand whether a solution of the problem exists. Paper [1] contains only a proof of the fact that the degree of coincidence of the synthesized and desired solutions do not decrease from iteration to iteration. Second, it remains unproved that the solution is unique. Moreover, convincing examples were found that demonstrated the existence of several solutions in some cases. In addition, one cannot prove that the synthesized solution depends continuously on the starting conditions of the problem. Despite this fact, synthesis methods continue developing rapidly, which can be attributed to both the complexity and the great practical significance of solving the problems of multimode converter design, when each found solution is worth literally its weight in gold.

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In this paper, synthesis methodology is developed further in order to widen the class of synthesized systems and generalize the approach by abandoning the approximations of the phase corrector for mirrors and weakly irregular waveguides.

In the absence of small parameters in the geometry of an electrodynamic system, the use of calculation methods of the Finite-Difference Time Domain (FDTD) type becomes adequate. The FDTD method is currently a powerful tool for calculating a wide class of waveguide systems [6, 7]. It is based on solving the Maxwell equations directly on a space–time mesh. In the case of the Cartesian coordinate system, various components of the electric and magnetic fields are calculated in an elementary cell of the space mesh at different corners of the parallelepiped formed by the mesh nodes. Here, the entire volume of the analyzed object is filled with this mesh. The field at each mesh node is calculated via the field at this node at the previous iterative step and the fields at the adjacent nodes at the current time step. For example, at the nodes which fall within a perfect conductor, the field is identically zero, which ensures automatically the required boundary conditions at the surface of this conductor.

High accuracy of field calculations by the FDTD method requires a mesh whose spacing is one or two orders of magnitude smaller than the wavelength. Therefore, the computation time needed to design multimode systems becomes a major obstacle to using this method. The latter fact is especially important when developing iterative synthesis procedures, in which one should solve the problem of field analysis many times.

When solving the problem at a given frequency, the computation of complex values of the fields at each node of the mesh requires a computation time comparable with the time costs for implementation of the FDTD method itself. Therefore, in order to avoid excessive time costs, the procedures of calculating complex fields can be used only for a limited number of cross sections and surfaces. Analysis of the modal content is an even more costly procedure which is usually expedient and possible in only the initial and final cross sections of a waveguide converter.

In what follows we consider a synthesis algorithm allowing for these features of the FDTD method.

2. STATEMENT OF SYNTHESIS PROBLEMS FOR WAVEGUIDES

In the most general statement of the problem, we will consider solutions of the Maxwell equations at a given frequency ω. Consider the problem (in the general case, a three-dimensional one) of a waveguide with perfectly conducting walls and the length L, whose surface should be found (i.e., synthesized) from the condition of complete transformation of the specified input fields to the required output fields (Fig. 1a). We assume that semiinfinite regular waveguides, which ensure perfect matching, i.e., the absence of reflected waves coming to the synthesized wavegide from the regions $z > L$ and $z < 0$, are joined to the considered waveguide on the left and on the right. In the cross section $z = 0$, only the input complex fields $\mathbf{E}_{\rm in}(0, \mathbf{r}_\perp)$ and $\mathbf{H}_{\rm in}(0, \mathbf{r}_\perp)$ are specified, which correspond to the radiation propagating to the region $z \in [0, L]$. Here, $\mathbf{r}_\perp$ is the radius vector in the plane $z = \text{const}$. In the cross section $z = 0$, these fields can be represented as a superposition of the modes of the above-mentioned regular waveguide, which propagate in the positive direction of the z axis. The required, or desired fields $\mathbf{E}_{\rm out}(L, \mathbf{r}_\perp)$ and $\mathbf{H}_{\rm out}(L, \mathbf{r}_\perp)$ are specified in the cross section $z = L$.[1] They form the radiation propagating from the region $z \in [0, L]$. In the cross section $z = L$, these fields are represented by another superposition of the modes, which also propagate in the positive direction of the z axis. The input and required fields should be specified self-consistently, so as to ensure the fulfillment of the conservation law in the form of equality of power fluxes for all ingoing and outgoing waves in the region $z \in [0, L]$.

Application of the FDTD method for solving the specified problem requires taking the features of the method into account. Specifically, within the framework of the FDTD method, the matching semiinfinite waveguide sections are replaced by sections of finite lengths, which are filled with an absorber (Fig. 1b). Such

[1] In the general case, the desired wave beams propagating from the region $z \in [0, L]$ can be specified in the cross sections $z = 0$ and $z = L$. For $z = 0$, the desired fields form the reflected radiation, which can be the objective in, e.g., the problems of reflector synthesis.

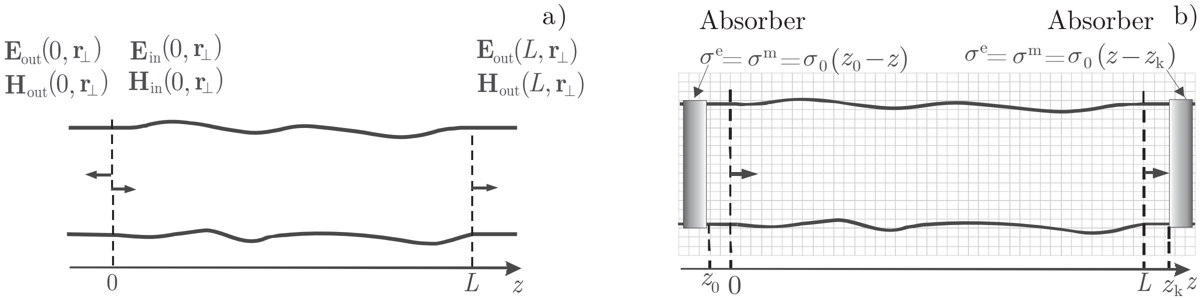

Fig. 1. Statement of the problem of synthesis in a waveguide (*a*); simulation of the matching by means of absorbers and the mesh on whose basis the ladder structure of boundaries is formed (*b*).

absorbers ensure a wide angular matching spectrum and are formed by electrical and magnetic conductivities, which are identical in value and increase linearly in the direction of propagation of the absorbed radiation. Usually, one can achieve a reflection level of about 10^{-3} and smaller by using such absorbers whose thickness is 10–20 times greater than the sizes of elementary cells of the mesh (the case where one of the waves incident on the absorber has a frequency close to the cutoff is one the only exception).

To simulate processes at a given frequency, field sources should be taken in the form of monochromatic emitters (fields or equivalent currents) which are switched on at some time. Before this time, all the fields in the considered volume are zero. The switch-on process is excitation of a field with some frequency spectrum whose width is inversely proportional to the switch-on time. One can wait till the processes of wave propagation and diffraction reach a stationary level, which is provided by the presence of the above-mentioned absorbers, and then state (with some degree of accuracy) that only one frequency has “survived.” Usually, a different approach is used in the FDTD algorithms, namely, in order to calculate the complex amplitude $a = |a| \exp(i\phi)$, only the spectral component at the frequency ω is singled out of the time dependence $a(t)$ of the field by means of Fourier analysis:

$$a' = \frac{\omega}{\pi n} \int\limits_0^{2\pi n/\omega} a(t)\cos(\omega t)\,\mathrm{d}t, \tag{1}$$

$$a'' = \frac{\omega}{\pi n} \int\limits_0^{2\pi n/\omega} a(t)\sin(\omega t)\,\mathrm{d}t, \tag{2}$$

where $n \geq 1$ is an integer. Then, $|a| = \sqrt{a'^2 + a''^2}$ and $\phi = \arctan(a''/a')$. This approach means frequency filtration. The filtration is more efficient, the longer is the time interval on which the Fourier analysis is performed, i.e., the greater the value of n.

Among the excited frequencies, there arise those corresponding to evanescent waves. Quasistatic fields of such waves are concentrated only near the source. The effect of these fields on the synthesis of the converter surface is weakened further by the use of spatial filters, which are discussed later.

Figure 1*b* shows schematically the space mesh used to analyze the fields within the framework of the FDTD method. When performing synthesis, this mesh is chosen such that it covers the volume of the initial waveguide with sufficient margin, since in the process of iterations during the synthesis, the shape of the surface changes and the waveguide cross section becomes at some points wider or narrower than the initial crosssection (Fig. 1*b*).

To develop the iterative algorithm, we will need two types of solutions of the Maxwell equations. Each of the solutions satisfies the boundary conditions on metal. The fields $\mathbf{E}^+(t, z, \mathbf{r}_\perp)$ and $\mathbf{H}^+(t, z, \mathbf{r}_\perp)$ of the first above-mentioned solution are found by solving the Maxwell equations directly, as they are integrated in the positive direction of the time axis. In this case, the source is the fields whose transverse distributions correspond to $\mathbf{E}_{\rm in}(0,\mathbf{r}_\perp)$ and $\mathbf{H}_{\rm in}(0,\mathbf{r}_\perp)$ and time dependence is described by the oscillations at the frequency ω (with accuracy up to the switch-on process). Such a source forms a power flow from the left to the right (Fig. 1*b*). The fields $\mathbf{E}^-(t,z,\mathbf{r}_\perp)$ and $\mathbf{H}^-(t,z,\mathbf{r}_\perp)$ of the second solution are obtained by integration of the Maxwell equations with the reverse direction of the time axis, in which case the source is the fields with the distributions $\mathbf{E}_{\rm out}(L,\mathbf{r}_\perp)$ and $\mathbf{H}_{\rm out}(L,\mathbf{r}_\perp)$, which are also simulated by the switched-on oscillations at the frequency ω. The waves radiated from such a source propagate from the right to the left.

The correction to the surface profile is calculated at each iteration from the complex-valued tangential components of the magnetic field at the waveguide walls and the normal components of the electric field:

$$\Delta l = \alpha\,\mathrm{Im}(\mathbf{H}_\tau^{+*}\mathbf{H}_\tau^- + \mathbf{E}_n^{+*}\mathbf{E}_n^-), \tag{3}$$

where α is a certain constant and the asterisk denotes complex conjugation. The practical choice of this constant and the choice of the form of the correction are justified below.

Calculation of the complex amplitudes in Eq. (3) means finding the amplitude and phase of the corresponding Fourier harmonic at the frequency ω, but the use of this procedure in accordance with the above discussion is necessary only at the mesh nodes falling on the waveguide surface, rather than in the entire volume.

3. ITERATIVE SYNTHESIS ALGORITHM

To use the iterative synthesis procedure, one has to specify some initial approximation for the surface profile. For example, it can be a cylindrical surface for the problem of conversion of the modal content in a circular waveguide. Then we solve the direct problem in which the fields $\mathbf{E}^+(t,z,\mathbf{r}_\perp)$ and $\mathbf{H}^+(t,z,\mathbf{r}_\perp)$ are calculated. Subsequently (or simultaneously), we solve the conjugate problem in which the fields $\mathbf{E}^-(t,z,\mathbf{r}_\perp)$ and $\mathbf{H}^-(t,z,\mathbf{r}_\perp)$ are found. Fourier analysis of these fields on the surface allows one to use Eq. (3) to find a small correction to the surface profile. Further, at the next iterative step, new fields are calculated in the already changed geometry and the found fields are used to determine the new surface profile, etc. This procedure can be repeated until the obtained fields coincide with the desired ones with sufficient accuracy. A block diagram of the iterative synthesis procedure is shown in Fig. 2.

To estimate the degree of coincidence at each iterative step in the input and output cross sections, i.e., at $z = 0$ and $z = L$, one can calculate mutual power coefficients which characterize the degree of coincidence of the obtained solutions with the desired ones:

$$\eta_i^- = \left| \int\limits_S ([\mathbf{E}_i^- \times \mathbf{H}_{\rm in}] + [\mathbf{E}_{\rm in} \times \mathbf{H}_i^-])\,\mathrm{d}\mathbf{S} \right|^2, \tag{4}$$

$$\eta_i^+ = \left| \int\limits_S ([\mathbf{E}_i^+ \times \mathbf{H}_{\rm out}] + [\mathbf{E}_{\rm out} \times \mathbf{H}_i^+])\,\mathrm{d}\mathbf{S} \right|^2, \tag{5}$$

where i is the iteration number and S is the side surface of the converter. We assume in Eqs. (4) and (5) that the complex fields ($\mathbf{E}_{\rm in}$, $\mathbf{H}_{\rm in}$) and ($\mathbf{E}_{\rm out}$, $\mathbf{H}_{\rm out}$) in them are normalized to a unit power flux.

Let us dwell on choosing the form of the profile correction in more detail. It is evident that this correction should satisfy at least two criteria: (i) the calculated value of the correction should tend to zero as the obtained solution approaches the desired one, and (ii) the correction should generate such spatial harmonics of the profile that increase the conversion efficiency if the solutions do not coincide.

Surely, the set of correction forms includes an infinite number of solutions which satisfy the above-mentioned criteria. Among the corrections with the simplest form, two corrections were studied. One of them is expressed by Eq. (3). It can easily be verified that if $\mathbf{H}^+(z) = \mathbf{H}^-(z)$ and $\mathbf{E}^+(z) = \mathbf{E}^-(z)$, then $\Delta l(z) = 0$. If the fields $\mathbf{H}^+$ and $\mathbf{E}^+$ correspond to one mode with the propagation constant h_1, i.e.,

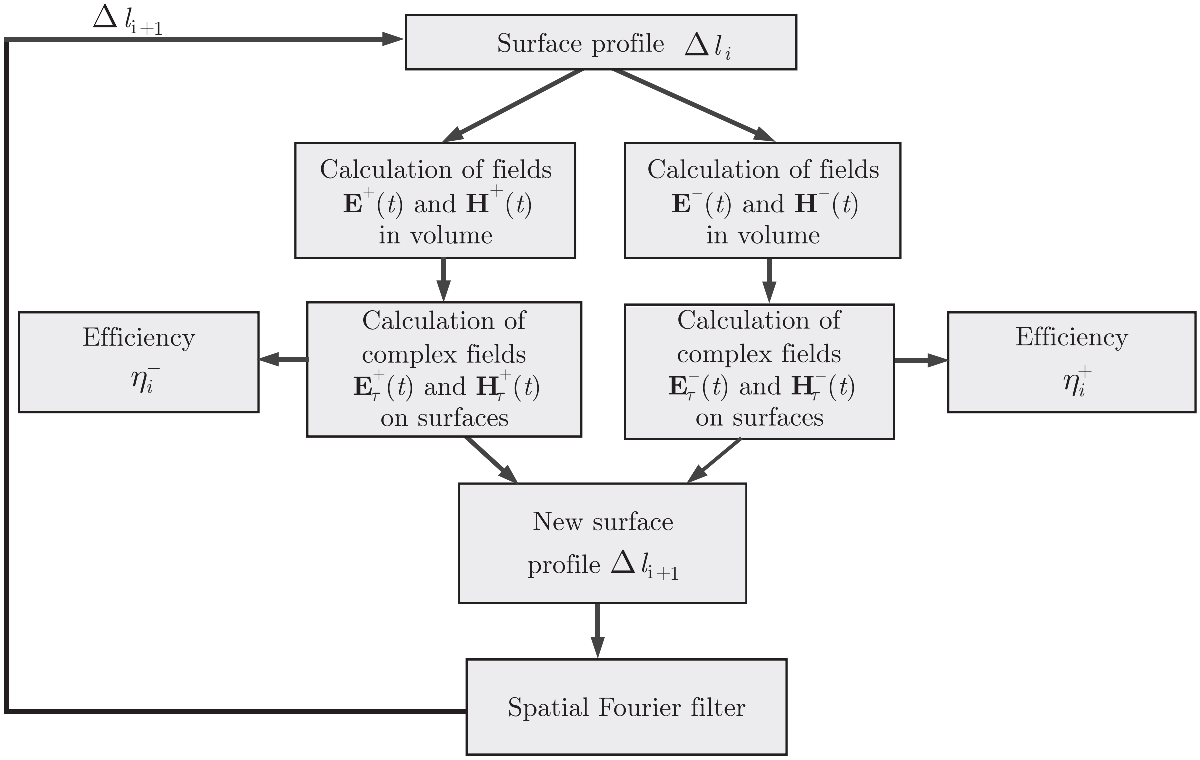

Fig. 2. Block diagram of the iterative synthesis algorithm.

$\{\mathbf{H}^+, \mathbf{E}^+\} \propto \exp(ih_1 z)$, and $\mathbf{H}^-$ and $\mathbf{E}^-$ correspond to another mode with the propagation constant h_2, i.e., $\{\mathbf{H}^-, \mathbf{E}^-\} \propto \exp(ih_2 z)$, then $\Delta l \propto \sin[(h_2 - h_1)\, z]$. Such a corrugation profile of the waveguide surface within the framework of perturbation theory ($\Delta l \ll \lambda$) is the required solution which ensures the maximum energy exchange between the two modes along the z coordinate [8].

Another form of the correction, which was considered, is given by

$$\Delta l(z, \mathbf{r}_\perp) = \alpha\, |U^+ - U^-|^2 \tag{6}$$

and it also ensures that Δl tends to zero when the fields U^+ and U^-, which can be both tangential magnetic-field components and normal electric-field components, coincide. However, in the case of two modes with different propagation constants, which has already been considered, Eq. (6) yields a formula for the correction in the form $\Delta l \propto \sin[(h_2 - h_1)\, z] + \text{const}$. Thus, an "extra" constant term is present in Δl. This example shows that the use of the correction in the form of Eq. (3) is more accurate since the formed spatial spectrum of the profile is not widened by additional harmonics. Therefore, in what follows, we use this form of the correction in all numerical examples.

It is of methodical interest to compare the proposed correction (3) to the profile with the correction proposed in [3], in which an iterative synthesis algorithm was developed on the basis of solving a system of equations for coupled waves. In the waveguide converters considered in [3], the mode energy exchange is described in the one-dimensional case by a system of equations for complex amplitudes of a_j modes:

$$\frac{\mathrm{d}a_j}{\mathrm{d}z} - ih_j a_j = i \sum_s \chi_{js} l(z) a_s, \tag{7}$$

where χ_{js} is the coupling coefficient of the modes with the numbers j and s, and $l(z)$ is a function describing the surface profile. The coupling coefficient in a waveguide close to a cylindrical one is expressed via the fields of modes with the numbers j and s, which are taken on the walls of a reference (unperturbed cylindrical) waveguide and are normalized to the unit power flux [9]:

$$\chi_{js} = \frac{1}{2k} \int\limits_0^{2\pi r_0} (H_{jz} H_{sz} + H_{j\varphi} H_{s\varphi} + E_{jr} E_{sr})\, \mathrm{d}p. \tag{8}$$

Here, the integral is taken over the perimeter of the cross section of an unperturbed waveguide with the radius r_0, $k = \omega/c$ is the wave number in free space, and φ is the azimuthal angle in the cylindrical coordinate system. The correction to the profile in [3] (written in our notation) is proposed in the form

$$\Delta l = \frac{\pi}{4L} \sum_{j,s} \mathrm{Im} \left(\frac{a_j^{+*} a_s^- - a_j^{-*} a_s^+}{\chi_{js}} + \frac{a_s^{+*} a_j^- - a_s^{-*} a_j^+}{\chi_{sj}} \right), \tag{9}$$

where the summation is performed over all propagated modes and L is the length of the synthesized section.

To compare both methods, we now discuss a special case of an axisymmetric waveguide, for which we will be interested in only axisymmetric modes of magnetic type. We will consider only the solutions for small surface corrections ($\Delta l \ll \lambda$), such that the application of perturbation theory is valid. In this case, Eq. (3) is simplified as much as possible, since only the longitudinal (with respect to the z axis) component of the magnetic field is nonzero. Let us also simplify Eq. (9). In the absence of reflected waves, all coupling coefficients, as follows from the power-flux conservation law, satisfy the equality $\chi_{sj} = \chi_{js}^*$ and are expressed from Eq. (8) via the magnetic fields of modes with the numbers j and s, taken on the walls:

$$\chi_{js} = \frac{\pi r_0}{k} H_{jz}(r_0) H_{sz}(r_0) = -\frac{\mu_j \mu_s}{r_0^4 \,(h_j - h_s) \sqrt{h_j h_s}}, \tag{10}$$

Here, as in Eq. (8), r_0 is the radius of an unperturbed waveguide, $h_j = \sqrt{k^2 - (\mu_j/r_0)^2}$ and $h_s = \sqrt{k^2 - (\mu_s/r_0)^2}$ are the propagation constants of modes with the numbers j and s, respectively, μ_j and μ_s are the roots of the equation $J'(\mu) = 0$, where $J(\mu)$ is a Bessel function and the prime denotes the derivative with respect to the argument). Since, according to Eq. (10), all coupling coefficients are real, the formula for the correction to the profile can be rewritten in the form

$$\Delta l(z) = \frac{\pi}{2L} \sum_{j,s} \mathrm{Im} \Big(a_j^{+*} a_s^- - a_j^{-*} a_s^+ \Big) \frac{1}{\chi_{js}} \tag{11}$$

or, if the terms under the sign of the imaginary part are rearranged, in a simpler form:

$$\Delta l(z) = \frac{\pi}{L} \sum_{j,s} \mathrm{Im} \Big[a_j^{+*}(z) a_s^-(z) \Big] \frac{1}{\chi_{js}}. \tag{12}$$

In particular, in the system of two coupled modes in the problem of complete conversion of one mode to another, Eq. (12) yields the required profile in the form

$$\Delta l(z) = \frac{\pi}{\chi_{12} L} \sin[(h_1 - h_2)\, z]. \tag{13}$$

As was shown in [8], this formula is an exact solution of the problem within the framework of perturbation theory, and no further iterations are needed.

On the other hand, Eq. (3) for the correction to the surface profile can be written, in accordance with our proposal, by using the same complex amplitudes of waves if the fields are represented as an expansion in terms of waveguide modes:

$$H_z(z, \mathbf{r}_\perp) = \sum_j a_j(z) H_{jz}(\mathbf{r}_\perp). \tag{14}$$

This representation is inapplicable for practical calculations due to the reason discussed above, namely, enormous time costs of calculations for averaging of the fields and their expansion in modes. However, it allows one to compare immediately the synthesis formulas. In particular, substituting Eq. (14) into Eq. (3), we obtain

$$\Delta l(z) = \alpha \sum_{j,s} \mathrm{Im}\left[a_j^{+*}(z)a_s^-(z)\right] H_{jz}(r_0)H_{sz}(r_0) = \tilde{\alpha} \sum_{j,s} \mathrm{Im}\left[a_j^{+*}(z)a_s^-(z)\right] \chi_{js}, \tag{15}$$

where $\tilde{\alpha}$ is a certain constant independent of the coordinates. Thus, Eq. (15) proposed by us for the correction is close in its structure to Eq. (12), to which the equation proposed in [3] is reduced. The only difference is that in Eq. (12), the imaginary part of each product of complex amplitudes under the summation sign is divided by the corresponding coupling coefficient, whereas in Eq. (15), we have multiplication by it. The presence of the coupling coefficient in the denominator leads to difficulties in the case where uncoupled or weakly coupled modes happen to be among the modes included in the synthesis procedure. In this case, the correction to the surface profile can increase indefinitely, and one has to limit it artificially by excluding pairs of modes of the above-mentioned type. Equation (15) has no such drawback. On the other hand, Eq. (15) proposed by us comprises the constant $\tilde{\alpha}$, which should be determined somehow for practical calculations. We propose to choose it from the estimations yielded by perturbation theory. In particular, for the example considered above, it follows from the analytical solution within the framework of perturbation theory [8] that in the case where one mode should be completely converted to another copropagating mode, the constant $\tilde{\alpha}$ is expressed in the form

$$\tilde{\alpha} = \pi/(\chi_{1\,2}^2 L). \tag{16}$$

Now, for $\tilde{\alpha}$ chosen in the form of Eq. (16), correction (15) in the considered special case completely coincides with correction (12) obtained in accordance with [3]. In the same case, in Eq. (3), the factor $\alpha = \pi^2 r_0/(\chi_{1\,2}^2 Lk)$, in accordance with Eq. (16). Evidently, the choice of smaller values of α is admissible, but leads to a greater total number of iterative steps required to achieve the acceptable conversion level due to the smallness of corrections. As the calculations show, the choice of overrated corrections also leads to an increase in the number of iterations and unreasonably great deviations of the synthesized profile from the unperturbed one.

Note that if undesired reflections occur, coefficients (4) and (5) also turn out to be less than unity. It follows from the above discussion that in the presence of reflections, the procedure itself starts exciting the profile harmonics which favor a decrease in this reflection.

4. APPLICATION OF FILTERS TO THE SYNTHESIZED SURFACE

As in most other methods, the field calculation in the FDTD method is associated with some errors. Specific errors related only to the FDTD method can conventionally be divided into three groups. The first group includes Schottky-type wideband noise which is related to the errors caused by the resolution of the time–space mesh. The second group is related to finite times of the source switch-on and finite onset times of oscillations in the system. The third group is related to parasitic reflections, in particular those caused by imperfect matching of the end absorbers (Fig. 1*b*). All these sources of imperfections lead to errors in field calculations, thereby affecting the shape of the synthesized profile.

We now consider profile aberrations using an example of a Schottky noise source. Let the only nonzero fields on the surface be the magnetic fields having a single component and depending only on the longitudinal coordinate z (such a case was already considered when we compared the profile corrections). Let the synthesis problem also consist in the conversion of a mode with the propagation constant h_1 to the mode with the propagation constant h_2:

$$\mathbf{H}^+(z) = [1 + \delta(z)] \exp(ih_1 z), \qquad \mathbf{H}^-(z) = [1 + \delta(z)] \exp(ih_2 z), \tag{17}$$

where the formula for the fields is supplemented by a source $\delta(z)$ of the Schottky noise whose amplitude is small, such that $|\delta(z)| \ll 1$. Then the correction to the profile will have the form

$$\Delta l(z) = \alpha\, \mathrm{Im}[(\mathbf{H}_\tau^+)^* \mathbf{H}_\tau^-] \approx \alpha\, [1 + 2\delta(z)]\, \sin[(h_2 - h_1)\, z]. \tag{18}$$

As follows from Eq. (18), the relative value of the noise in the profile is twice as great as the noise in the formulas for the fields. If the Schottky noise source is assumed random from iteration to iteration, then the noise in the profile will increase even more at the next iterative step. Thus, to suppress the noise amplification from iteration to iteration, one should filter the harmonics of the synthesized profile.

It is reasonable to single out the useful signal (i.e., the surface profile) against the background of noise and interference by using Fourier filters since the considered sources of noise and errors have some pronounced spectral features. For example, the Schottky noise is wideband. Therefore, one should reject the high- and low-frequency spatial harmonics. The errors related to finite times of oscillation switch-on usually contribute to the low-frequency components. Parasitic reflections lead to the appearance of the profile harmonics with the typical transverse scale π/h, where h is the propagation constant of the incident wave.

Consider the development of the simplest two-dimensional Fourier filter using an example of a waveguide close to the cylindrical one. To do this, we represent the correction to the cylinder surface as a Fourier series expansion over the azimuthal coordinate φ:

$$\Delta l(\varphi, z) = \sum_{n=-\infty}^{+\infty} a_n(z) \exp(in\varphi), \tag{19}$$

where $a_n(z)$ are complex amplitudes of the Fourier harmonics. Taking into account that the profile is a real value and, hence, $a_{-n} = a_n^*$, we rewrite Eq. (19) as

$$\Delta l(\varphi, z) = a_0(z) + 2 \sum_{n=N_1}^{N_2} \mathrm{Re}[a_n(z) \exp(in\varphi)], \tag{20}$$

where the summation limits should be chosen in accordance with the above-mentioned physical considerations, to ensure the best filtration. As will be shown by examples, in some cases it is sufficient to allow for only one or two azimuthal harmonics. We also note that in the numerical realizations of synthesis algorithms, the surface profile was stored in computer memory in the form of amplitudes of the Fourier harmonics in each cross section determined by the mesh spacing along the longitudinal coordinate.

To filter the profile along the longitudinal coordinate, we use the Fourier integral transform of each azimuthal Fourier harmonics:

$$a_n(z) = \int_{-K_{\max}}^{K_{\max}} S_n(k) \exp(ikz)\, \mathrm{d}k, \tag{21}$$

in which the integration limits should also be restricted, and the spectrum $S_n(k)$ is calculated by the formula

$$S_n(k) = \frac{1}{2\pi} \int_0^L a_n(z) \exp(-ikz)\, \mathrm{d}z. \tag{22}$$

When developing a numerical filter on the basis of Eqs. (20)–(22), it is expedient to put the zeroth harmonic equal to zero since the input and output of the converter are usually specified in the form of a waveguide having a circular cross section and should not be changed in the synthesis process.

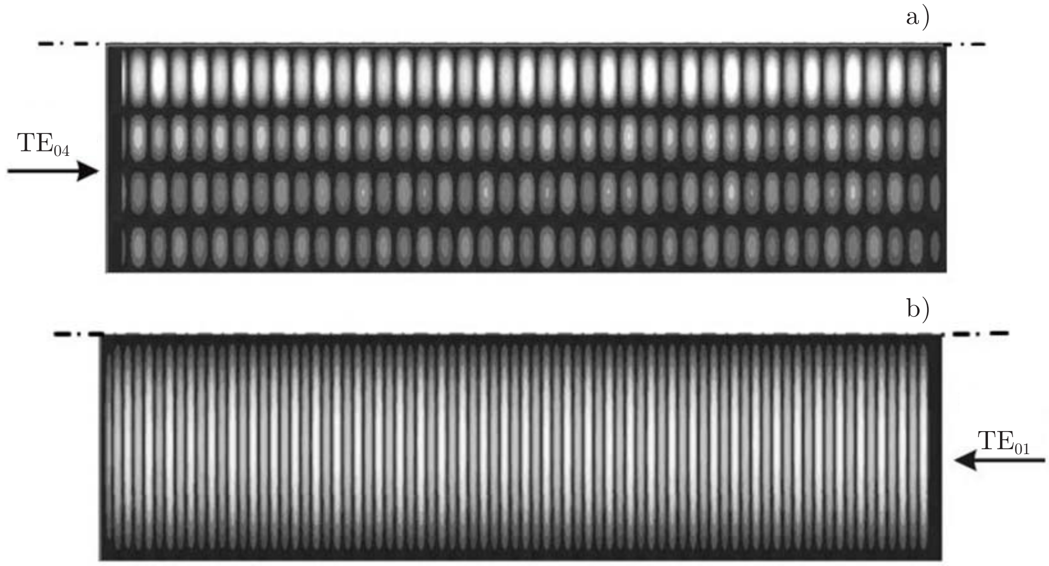

Fig. 3. Initial instantaneous distributions of the azimuthal component of the electric field (at the zeroth iterative step) in the longitudinal section of the waveguide for the incidence of the TE_{04} mode from the left (*a*) and of the TE_{01} mode from the right (*b*).

5. AXISYMMETRIC TE_{01}–TE_{04} MODE CONVERTER

The problem of the efficient TE_{01}–TE_{04} mode conversion is complicated by the existence of at least two "parasitic" modes, namely, TE_{02} and TE_{03}. Despite the fact that axisymmetric modes of the electric type do not appear in an axisymmetric waveguide if it is excited by any magnetic-type wave, the presence of even the two above-mentioned parasitic modes makes solving the conversion problem rather difficult. This is due to the fact that the coupling of the TE_{01} and TE_{04} modes is not the strongest for any small perturbation of the profile of a regular waveguide, which leads to its tapering. For example, the coupling of the TE_{04} and TE_{03} modes is greater due to a higher cutoff frequency of the TE_{03} mode compared with the TE_{01} mode. Therefore, efficient conversion of the modes that are of interest to us is possible only in a relatively long waveguide. Its length cannot be shorter than the longest length of the beats in the modes of the system (in our case, they are beats of the lower-order TE_{01} and TE_{02} modes).

We will use the two-dimensional realization of the proposed synthesis algorithm and choose the length of the synthesized converter section to be equal to three lengths of the lower-mode beats. For a frequency of 30 GHz and the waveguide diameter equal to 49 mm, this length amounts to 400 mm. The initial field distributions corresponding to the incidence of the TE_{01} mode from the right and of the TE_{04} mode from the left are shown in Fig. 3 (halves of the cross section of a circular waveguide are presented and the waveguide axis is shown by a dash-dot line).

As a result of the synthesis procedure, we obtain the solution (see Fig. 4) which ensures efficient conversion of one mode to another at a level of 99% (1% is spent for the loss due to scattering to other modes). The calculation accuracy estimated by the fulfillment of the conservation law amounts to 0.5–1%.

As the initial surface profile (zeroth iterative step), we took a smooth cylindrical waveguide. As is seen in Fig. 5*a*, at the next iterative step, the profile becomes periodic (with the period corresponding to the maximum energy exchange between the TE_{01} and TE_{04} modes) and has an almost constant amplitude along the length. At the subsequent iterative steps, the profile is enriched with harmonics, which are, in particular, responsible for the transfer of energy among all other propagated axisymmetric modes. In this case, the efficiency of conversion of the desired modes increases on the average from iteration to iteration (Fig. 5*b*). The highest conversion efficiency was achieved at the 18th iteration.

When finding the solution, we used a one-dimensional Fourier filter which rejects the profile harmonics

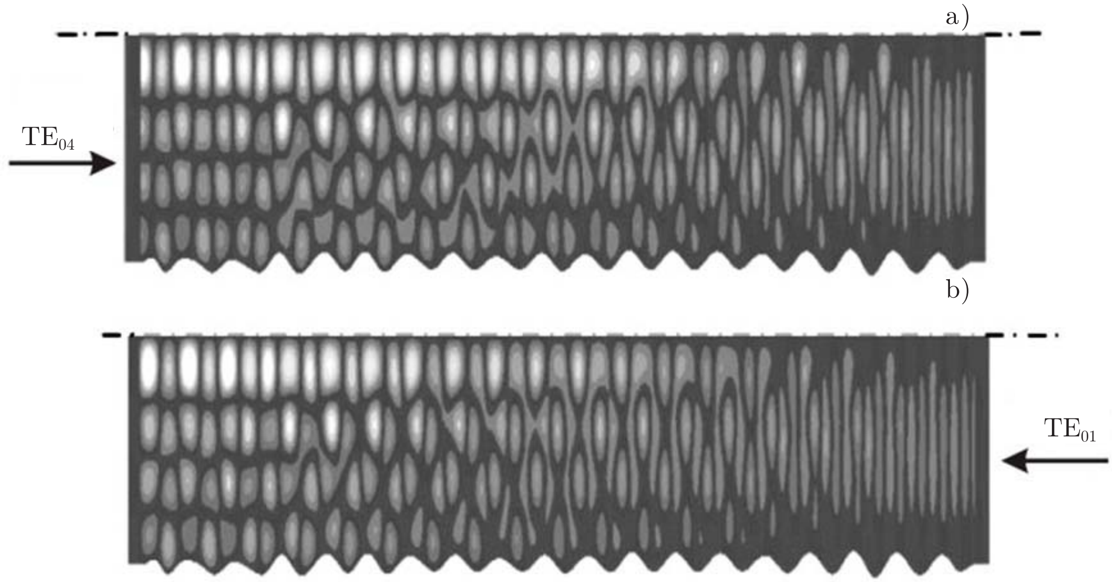

Fig. 4. Instantaneous distributions of the azimuthal component of the electric field at the final (18th) iterative step in the longitudinal section of the waveguide for the incidence of the TE_{04} mode from the left (*a*) and of the TE_{01} mode from the right (*b*).

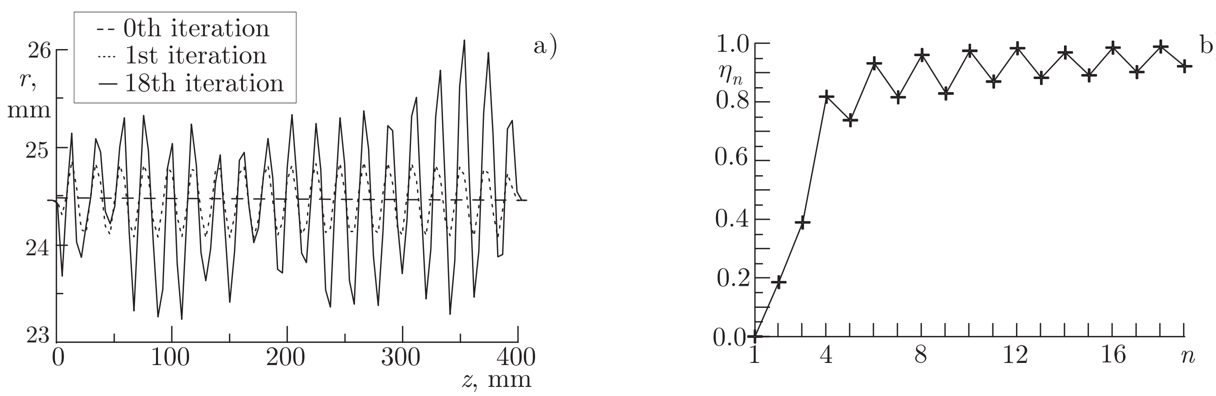

Fig. 5. Profile of the synthesized surface over the length (r is the waveguide radius) at several iterative steps (*a*) and the efficiency of conversion as a function of the iteration number (*b*).

capable of causing Bragg reflection. We also ignored the constant correction to the profile at each iterative step and used a smoothing procedure similar to that proposed in [4] at the ends of the converter. The maximum change in the radius of the synthesized profile amounted to 0.3λ, which corresponded to a sufficiently deep profile, which, strictly speaking, could not be described within the framework of perturbation theory.

To check the accuracy of the obtained solution, we used the cross-section method based on solving a system of equations for coupled modes with allowance for both propagated modes and some evanescent modes, including reflected modes [9]. The solution obtained by this method (Fig. 6) is in good qualitative and quantitative agreement with the solution found by the FDTD method. In this case, the efficiency of conversion also amounted to 99%, but minor differences were revealed. Specifically, in the solution based on the cross-section method, the maximum-efficiency conversion is achieved at 75 MHz, i.e., at a frequency that is lower (by 2.5%) than the calculated value (Fig. 7*a*). One can see in Fig. 7 that in the case of incidence of the TE_{01} mode, the fact that the conversion efficiency differs from 100% is caused by both reflection into the counterpropagating TE_{04} and TE_{02} modes (about 0.6%) and by the presence of an incompletely converted copropagating TE_{02} mode at the output (approximately 0.6% in addition).

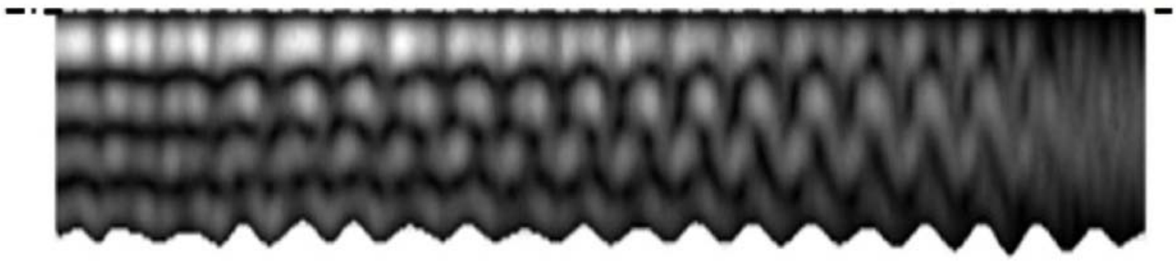

Fig. 6. RMS distribution of the electric-field amplitude in the longitudinal section of the waveguide.

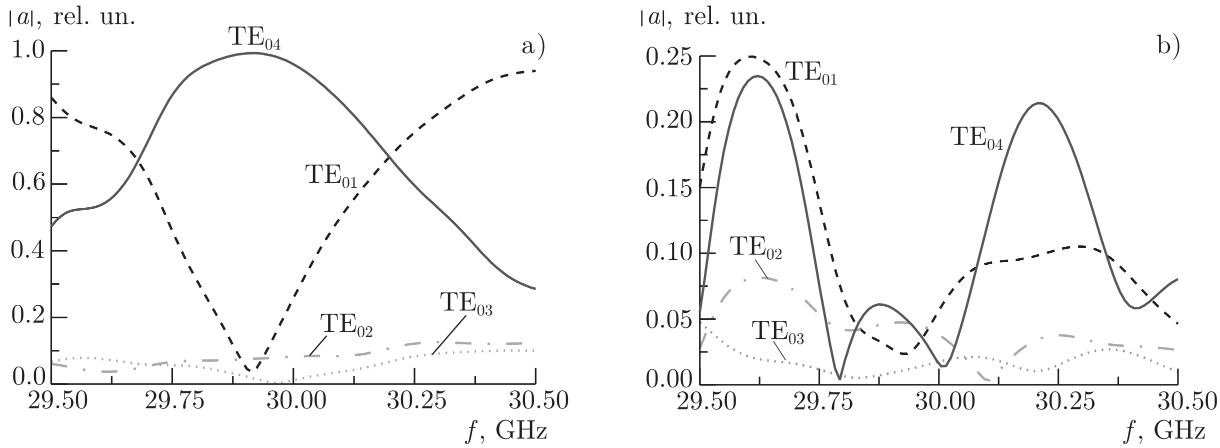

Fig. 7. Conversion to copropagating and counterpropagating modes (*a* and *b*, respectively) at the final iterative step as a function of the frequency for the incidence of the TE_{01} mode from the right.

6. SMALL-SIZE TE_{11}—TE_{01} MODE CONVERTER

The converter of the fundamental mode of a waveguide with a rectangular cross section to the lower-order axisymmetric TE_{01} mode is of interest for many applications, both in the case of high-power microwave radiation (in view of the electric-strength reasons) and for low-power devices (due to a low ohmic loss). Among many variants of such a conversion, we will consider a converter in the first section of which the TE_{10} mode of a rectangular waveguide is converted adiabatically to the TE_{11} mode of a circular waveguide and only then the TE_{11} mode is converted to the TE_{01} mode in the next section representing a periodically bent waveguide. A successful realization of such a converter at a frequency of 30 GHz is described in [10]. The structure of the fields in this converter is shown in Fig. 8.

Before the development of powerful computation tools such as HFSS, "Microwave Studio," etc., the calculation of a periodically bent waveguide was possible only within the framework of the methods of cross sections and perturbation theory. In the latter case, it is convenient for calculations to describe the converter surface in the cylindrical coordinates (r, z, φ) by a simple analytical formula

$$r(z, \varphi) = r_0 + l \sin(2\pi z/D) \cos\varphi, \tag{23}$$

where r_0 is the average radius of the converter, l is the deformation amplitude, and D is the beating period of the TE_{11} and TE_{01} modes in an unperturbed waveguide with the radius r_0. The length L of the converter is related to the deformation amplitude by the complete-conversion condition $L \times l = \text{const}$. However, the problem is that one cannot decrease the length, while increasing the deformation amplitude, due to the unavoidable increase in the scattering to parasitic modes. Among the propagated parasitic modes, the TE_{21} mode, whose coupling to the TE_{11} mode is sufficiently strong, is the most dangerous. The propagation constant of the TE_{21} mode lies strictly between the propagation constants of the TE_{11} and TE_{01} modes ($h_{TE_{01}} < h_{TE_{21}} < h_{TE_{11}}$). It is this parasitic mode that prevents decreasing the length

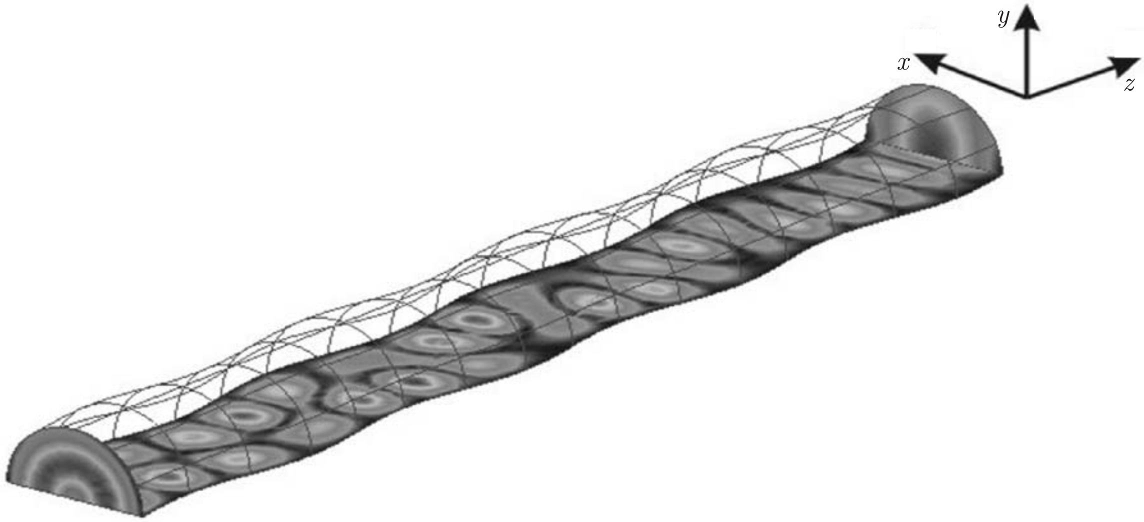

Fig. 8. TE_{11}–TE_{01} converter and the instantaneous distribution of the absolute value of the electric field in its longitudinal section.

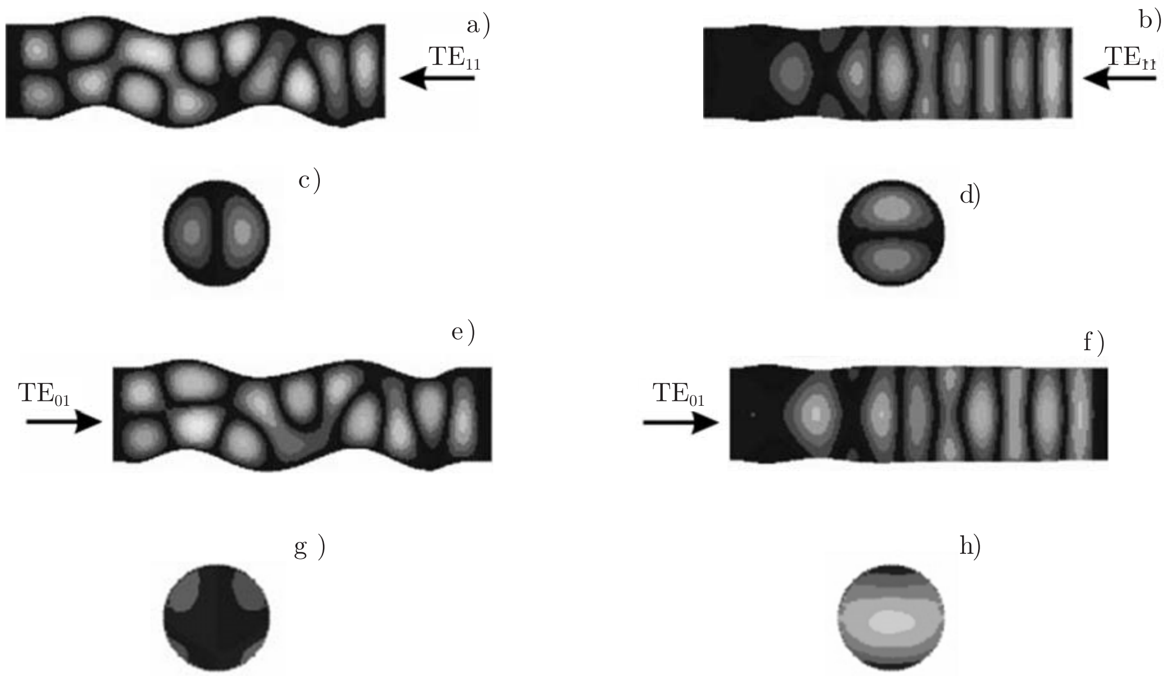

Fig. 9. Field distribution in the longitudinal and cross sections of the converter for the incidence of the TE_{11} mode from the right (*a–d*) and of the TE_{01} mode from the left (*e–h*): E_y component in the xz plane (*a*), E_y component in the yz plane (*b*), E_y component in the plane parallel to the xy plane at the converter output (*c*), E_x component in the same plane (*d*), E_y component in the xz plane (*e*), E_y component in the yz plane (*f*), E_y component in the xy plane (*g*), and E_x component in the xy plane (*h*).

of the converter. In particular, in [10], the converter length was limited by four beating periods of the operating modes (about 12λ). Along with the above-mentioned obstacle related to parasitic scattering, the applicability of perturbation theory is violated with increasing amplitude of bending. In particular, the above-mentioned converter was calculated, for the sake of comparison, by several methods including the perturbation method (MCW), FDTD, and HFSS [10]. The calculations by the two latter methods yield close results, and perturbation theory yields a systematic shift of the center of frequency conversion by 1% upwards, which is due evidently to an excessively high amplitude of the surface perturbation.

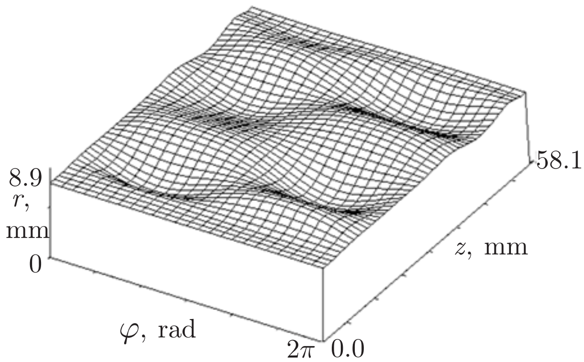

Fig. 10. Development of the converter surface.

Thus, it was confirmed that the calculation by the FDTD method ensures satisfactory accuracy. Then a problem was specified to synthesize a converter twice as short as compared with that considered in [10].

The results of using the three-dimensional variant of the synthesis procedure are presented in Fig. 9 showing instantaneous distributions of the electric fields obtained in two ways: for the incidence on the converter of the TE_{11} mode from the right and of the TE_{01} mode from the left. In the notations of the field components and cross sections, we used the system of coordinates shown in Fig. 8. The obtained conversion efficiency amounted to 99% and was achieved in 16 iterative steps.

The shape of the three-dimensional synthesized surface in the form of a waveguide development is shown in Fig. 10. It is seen that the profile describes a surface similar to that yielded by Eq. (23). However, one can observe an elliptical addition for large values of z.

The complete mesh used for the converter synthesis consisted of $120 \times 120 \times 400$ nodes. In each cross section, the number of azimuthal harmonics of the profile was limited to three harmonics: $\cos\varphi$, $\cos(2\varphi)$, and $\cos(3\varphi)$. The number of field recalculation steps per synthesis iteration (number of time steps) amounted to $3 \cdot 10^3$. The synthesized converter (for $f = 30$ GHz) had the following parameters: a length of 58.1 mm (about 6λ), an average radius of 7.43 mm (about 0.8λ), and a maximum radius variation of 2.94 mm (about 0.3λ).

7. CONCLUSIONS

The proposed iterative algorithm for synthesizing the waveguide converter allows one to achieve high efficiency of conversion of the spatial structure of radiation and its transportation in waveguide systems. The method is based on direct solution of the Maxwell equations on a space–time mesh and does not require the presence of small parameters in the geometry of the waveguide system. It is applicable for calculation of a wide class of devices including waveguide converters and mirror converters. The generality of the method makes it applicable when using other programs for field calculations (HFSS, "Microwave Studio," MAFIA, etc.). Examples of the converter synthesis demonstrate high efficiency and were checked by the methods of cross sections and HFSS, as well as in experiments [11, 12].

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