# **SEPARATION OF ENERGY FRACTIONS OF AN ELECTRON BEAM BY A LOCALIZED NONUNIFORMITY OF MAGNETIC FIELD IN THE COLLECTOR REGION OF GYRODEVICES**

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To separate the energy fractions of an electron beam, we propose that a ferromagnetic ring coaxial to the solenoid is inserted in the collector region, which can increase the efficiency of energy recovery in the case of electron deposition onto the multistage collector.

### **1. INTRODUCTION**

One of the methods for improving the gyrotron efficiency is deposition of the used electron beam onto the collector whose potential is lower than the cavity potential [1–3]. In this case, a significant deceleration of electrons without their reflection is possible if the minimum energy of electrons in the used beam is sufficiently high. For this purpose, the high-frequency field in the interaction space should have favorable "adiabatic" longitudinal structure [4].

Additional possibilities for energy recovery appear if the energy fractions of an electron beam are spatially separated and each fraction is deposited onto the collector part with the corresponding potential [3, 5–9]. To separate the fractions, it was proposed to pass an electron beam in the collector region of the gyrotron through the magnetic-field nonuniformity created by an additional solenoid connected oppositely with respect to the basic magnet [7]. However, calculations showed that an excessively large nonuniformity of the field leads to the appearance of reflected electrons. A smaller and hopefully "reflectionless" nonuniformity can be realized by using a ferromagnetic ring (Fig. 1), which is replaced by a cylindrical rod with large magnetic permeability ( $\mu \gg 1$ ) in the two-dimensional theoretical model considered herein (Fig. 2).



Fig.1. Diagram of a gyrotron with two-stage energy recovery:  $1$  is the tube body,  $2$  is the insulator, 3 is the ferromagnetic ring, 4 is the first stage of the collector, 5 is the second stage of the collector,  $6$  is the solenoid,  $7$  is the trajectory of "slow" electrons, and  $8$  is the trajectory of "fast" electrons.

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Fig. 2. Magnetic-field lines and electron trajectories in the separator region.

#### **2. EQUATIONS OF MOTION**

In the model shown in Fig. 2, the magnetic field outside the ferromagnetic rod is the superposition of a uniform field and a field of the two-dimensional dipole:

$$
\mathbf{H} = H\mathbf{z}_0 + R^2 H \operatorname{rot}\left(\mathbf{y}_0 \frac{x}{x^2 + z^2}\right),\,
$$

where  $y_0$  and  $z_0$  are the unit vectors of the y and z axes, respectively. The equations of motion of an electron with allowance for its energy constancy have the form

$$
dv/dt = \mathbf{v} \times \boldsymbol{\omega}_H, \qquad dr/dt = \mathbf{v}, \qquad \boldsymbol{\omega}_H = e\mathbf{H}/(mc\gamma), \qquad \gamma = 1/\sqrt{1 - v^2/c^2},
$$

where **r** and **v** are, respectively, the radius vector and velocity of an electron,  $e$  and  $m$  are its charge and mass, respectively, and  $c$  is the speed of light in free space. In the dimensionless variables

$$
\tau = \frac{\omega_{\rm c}}{\gamma} t, \qquad {\bf r}' = \frac{\bf r}{R} \,, \qquad {\bf u} = \frac{\gamma}{\omega_{\rm c} R} {\bf v},
$$

where  $\omega_c = eH/(mc)$  is the cyclotron frequency in the collector region and R is the ferromagnetic-cylinder radius, the above equations take the form

$$
\frac{d\mathbf{u}}{d\tau} = \mathbf{u} \times \mathbf{h}, \qquad \frac{d\mathbf{r}'}{d\tau} = \mathbf{u}.
$$
 (1)

The magnetic-field structural factor

$$
\mathbf{h}(\mathbf{r}') = \frac{\mathbf{H}}{H} = \mathbf{z}_0 + \text{rot}\left(\mathbf{y}_0 \frac{x'}{x'^2 + z'^2}\right)
$$
 (2)

far from the ferromagnetic cylinder (for  $|\mathbf{r}'| \gg 1$ ) tends to  $\mathbf{z}_0$ .

In what follows, we omit the primes and assume that  $x, y$ , and  $z$  are new dimensionless coordinates.

### **3. INITIAL CONDITIONS**

As a rule, the spread of the guiding centers of the electron orbits in gyrotrons is small compared with the Larmor radius. Correspondingly, we assume that the axes of all electron trajectories at the separator input (the region of an unperturbed magnetic field, in which the collector generatrix is parallel to the magnetic-field lines and  $|\mathbf{r}| \gg 1$  are at the same distance  $d = D/R$  from the collector. In actual gyrotrons, the minimum distance between the internal surface of the collector and the ferromagnet is limited by the thickness of both the tube wall and the gap with the cooling liquid, which can totally amount to several centimeters. Allowing for a variety of existing gyrotrons, a special way for realizing the dipole magnetic field should be used in each particular case, for example, ferromagnetic rings of arbitrary cross section, a system of opposite turns with current, etc. Such systems can be calculated by modern packages such as HFSS and KARAT, but this is beyond the scope of this work.

The magnetic field in the collector region of a gyrotron is much smaller than that in the cavity:

$$
H \ll H_{\text{cav}}.\tag{3}
$$

Hence, by virtue of the fact that the adiabatic invariant is constant [10], almost all the electron energy near the collector is due to the longitudinal velocity component, i.e.,

$$
u \sim u_{\parallel},\tag{4}
$$

and the pitch factor of electrons at the separator input is much smaller than unity

$$
g = \frac{u_{\perp}}{u_{\parallel}} = \frac{u_{\perp \text{cav}} \sqrt{H/H_{\text{cav}}}}{u_{\parallel}} < g_{\text{cav}} \sqrt{\frac{H}{H_{\text{cav}}}} = g_{\text{cav}} \frac{R_{\text{res}}}{R} \sim \frac{R_{\text{cav}}}{R} \ll 1,\tag{5}
$$

since  $u_{\parallel} > u_{\parallel \text{cav}}$  and  $g_{\text{cav}} \sim 1$ .

## **4. CONCEPT OF SEPARATION OF THE VELOCITY FRACTIONS OF AN ELECTRON BEAM**

Electrons with low velocities ( $u \ll 1$ ) adiabatically travel in a nonuniform magnetic field and are deposited onto the collector plate along the magnetic-field lines (see Fig. 2). Electrons with sufficiently high velocities  $(u \gg 1)$  having passed through the nonuniformity region undergo only a small deviation, but a part of the energy of their longitudinal motion is converted to the transverse (rotational) velocity component.

Perturbation of the fast-electrons trajectory during the separator passage can be estimated by approximating the transverse magnetic field by the derivative of a delta function  $\delta(z)$  as follows:

$$
h_x = -\pi \frac{\mathrm{d}\delta(z)}{\mathrm{d}z} \,. \tag{6}
$$

Assuming that  $h_z = 1$  and  $u_{\parallel} = \text{const}$ , we obtain an equation for the oscillatory velocity of an electron in the form

$$
\ddot{u}_x + u_x = -\pi \frac{\mathrm{d}\delta(u_{\parallel}\tau)}{\mathrm{d}\tau} \,. \tag{7}
$$

Therefore, when passing the nonuniform part of the field, the electron acquires the correction

$$
u_x = -\pi/u_{\parallel} \tag{8}
$$

to the transverse velocity. This estimate can be used as an approximation for electrons with velocities  $u \sim 1$ . For such electrons, Eqs. (1) do not allow simplification. Therefore, a more accurate study of this velocity fraction is possible only by numerical simulation.

### **5. RESULTS OF NUMERICAL SIMULATION**

#### **5.1. Separation of fractions of a direct electron beam**

If electrons at the separator input had a zero transverse velocity, then any impact parameter d would correspond to such an electron velocity  $u^*$  that all particles with velocities  $u < u^*$  are deposited onto the collector surface, while all electrons with higher velocities pass through the separator. An electron is assumed deposited onto the collector if its coordinate x becomes equal to 1 at some time. The dependence of the cutoff velocity  $u^*$  on the impact parameter d, which was obtained by integrating Eq. (1), is shown in Fig. 3. The plot shows that the cutoff velocity  $u^*$  increases with decreasing impact parameter d (increasing magnetic-field nonuniformity). Of course, this diagram can only be used for approximate estimates since electrons of the used beam in an actual gyrotron have small (see Eq. (5)), but nonzero transverse velocity.

### **5.2. Separation of fractions with nonzero transverse velocities of electrons**



Fig.3. Electron passage through the separator as a function of the impact parameter  $d$  and velocity  $u$  for the direct electron beam.

For nonzero transverse velocity at the separator input, electrons of any energy fraction have different transverse coordinates depending on the cyclotron rotation phase. Correspondingly, the ratio  $T = N_{\text{pass}}/N$  of the number of electrons having passed through the separator to the number of electrons at the separator input can vary from 0 to 1. The velocity range  $u_{\parallel(0)} < u < u_{\parallel(1)}$  (limited by the dashed vertical lines in Fig. 4) corresponding to the range  $0 < T < 1$  increases and the minimum energy of electrons having passed through the separator decreases with increasing pitch factor.

In principle, the proposed system involves the possibility of reflecting a part of the electron beam from the magnetic-field nonuniformity. However, no reflection was observed in the calculations for a large number of electrons with different energies and phases. This allows us to assume that the number of electrons reflected from the magnetic separator in the electron-beam parameter range

typical of the existing gyrotrons is small and such electrons do not significantly influence the device operation.

Using Eq. (5) for the given relationship between the magnetic fields in the cavity and the collector region, we can plot the curves corresponding to the maximum possible pitch factor of electrons at the separator input. Typical maximum values of the pitch factor  $g$  are about 0.1 for a process gyrotron with direct output of microwave radiation and of the order of 0.05 for a high-power gyrotron for controlled thermonuclear fusion with a built-in quasioptical converter [11]. Since part of the energy of longitudinal motion of electrons is converted to the rotational component of velocity (the value of this effect can be estimated from Eq. (8)) when passing through the separator, additional "demagnetization" of an electron beam in the gradually decreasing magnetic field is required prior to the subsequent deceleration at the last stage of the collector (Fig. 1).

All electrons in the collector region are concentrated below the line of the maximum possible pitch factor in the plane  $(u_{\perp}, u_{\parallel})$ . Projection of the point of intersection of this line and isoline  $T = 0$  onto the axis of longitudinal velocities yields the value of the decelerating potential  $U_{\text{dec}} = m (\omega_c R u_{\parallel(0)})^2/(2e\gamma^2)$  of the subsequent collector stage since all electrons with  $u < u_{\parallel(0)}$  (shaded region in Fig. 4) are deposited onto the collector in the separator region regardless of their velocity ratio. All the electrons which passed through the separator have energies exceeding that corresponding to the decelerating potential and, therefore, can be intercepted without risk of reflection by the electrostatic field of the collector.

### **6. CONCLUSIONS**

The described magnetic separator can be used for

(i) additional increase in the efficiency of energy recovery in gyrotrons with favorable structure of the high-frequency field [12];



Fig. 4. Fraction of passed electrons as a function of the longitudinal and transverse velocities  $(d = 0.8)$ .

(ii) energy recovery in gyrotrons at gyrofrequency harmonics and in gyroamplifiers, in which minimum energy of electrons at the output of the interaction space is so small that the simplest one-stage energy recovery [1–4] becomes inefficient.

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