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A NEW METHOD FOR DETERMINATION OF THE ELECTRON NUMBER DENSITY IN THE E REGION OF THE IONOSPHERE FROM RELAXATION TIMES OF ARTIFICIAL PERIODIC INHOMOGENEITIES

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We propose a new method for determination of the electron number density in the E region of the ionosphere on the basis of scattering of radio waves from artificial periodic inhomogeneities formed by the high-power radio emission at two frequencies and having different spatial periods. The ratio of relaxation times of the artificial periodic inhomogeneities at a given altitude is determined only by the ratio of their spatial periods, which makes it possible to determine electron number density. The paper presents the corresponding calculations and the estimates of possible measurement errors.

1. INTRODUCTION

Artificial periodic inhomogeneities (APIs) are formed in the field of a high-power standing radio wave produced by interference of two waves, of which one is emitted vertically upwards by a high-power transmitter, while the other is reflected from the ionosphere. A periodic structure is sounded by probing radio pulses, and the signals backscattered by APIs are received. The scattering from APIs is resonant, i.e., the intensity of the API-scattered signals is maximum when the waves scattered from individual inhomogeneities interfere constructively. In this case, the intense-scattering condition (Bragg condition) results in equality of the wavelengths of the waves emitted by heating and probing transmitters. The set of methods employing APIs to diagnose the ionosphere and the atmosphere is described in [1, 2] along with the main results obtained using those methods.

Experimental and theoretical studies of the processes of API formation and relaxation showed that the largest body of information on the parameters of the medium can be obtained in the altitude range from 90 to 130 km. Artificial periodic inhomogeneities can be used to study simultaneously the temperature and density of the neutral atmosphere, the velocity of the plasma vertical motions in the D and E regions, the vertical component of the turbulent velocity near the turbopause, and ionization irregularities in the E region of the ionosphere, including sporadic layers and their ion content [1–8]. The power of the signals scattered from the APIs located in this region exceeds the noise level by 10–100 times.

The above-mentioned measurements employ the method within the framework of which the frequencies and polarizations of the high-power and probing waves coincide. The characteristics determined initially are the backscattered-signal amplitudes as functions of time and of the apparent height from which they arrive. The API relaxation time is determined as the characteristic time of an e-fold decrease in the amplitude of the signal backscattered by API. The durations of the formation and relaxation of APIs depend on the physical processes which are dominant in a given altitude range in the environment.

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A method for determination of the electron number density in the ionosphere by using APIs was already proposed by the authors earlier [9, 10]. The idea of the method is as follows. Artificial periodic inhomogeneities are formed by an extraordinary-mode wave with the frequency f_1 and are probed by ordinary-mode pulses at the frequency f_2. To measure the profile $N(h)$ of electron number density in the range from 10^4 to 10^6 cm^{-3}, one of the frequencies (usually that of the probing wave) is scanned in the range from 0.7 to 1 MHz, which is adjacent to the heating-wave frequency f_1. The electron number density is determined from the condition $\lambda_1 = \lambda_2$ of equality of the wavelengths of the heating and probing waves:

$$f_1 n_1 = f_2 n_2, \tag{1}$$

where n_1 and n_2 are the refractive indices of radio waves. The amplitudes of the signals scattered from APIs and coming from the altitudes, at which this relationship is satisfied interfere constructively. The corresponding apparent height is determined from the delay of the probing pulsed signals. In particular, this method was used to study the main features of the behavior of the interlayer E–F valley [3, 4].

It should be noted that this measurement method is impeded by a number of technical difficulties. First, the probing radar should be tuned in a certain frequency range which is often filled with noise. Second, the fact that the heating and probing transmitters are located close to each other and the operating frequencies differ only slightly can result in cross-talk interference in the receiving channel of the radar. These factors impede detection of the scattered signals and practical application of the method.

Technically, it is easier to perform measurements when the inhomogeneities are formed and probed by the waves with the same frequency and polarization. In this case, Eq. (1) is satisfied at all the altitudes at which a periodic structure was formed, and we can study the time dependence of the characteristics of the API-scattered signals in any altitude interval of interest. To do this, the high-power radio emission forms APIs during several seconds. After the "heating" transmitter is switched off, the periodic structure relaxes. At this time, it is probed by short radio pulses. This can be done by using the heating transmitter emitting pulses with the same frequency and polarization as those used to produce the APIs. The apparent heights from which the backscattered signals are reflected are also determined from their delay times.

For example, the temperature and density of the neutral atmosphere and the velocity of the vertical motions are determined in such a way [2, 5–7]. However, if the apparent and true heights of reflection in the lower ionosphere (D region) are close to each other, then the difference between them increases with increasing electron number density. Therefore, we meet the problem of finding true heights from the known apparent heights. Solving this problem requires knowledge of the profile $N(h)$ of electron number density. This measurement technique does not allow determining the electron number density simultaneously with the atmospheric parameters. Therefore, when determining the temperature and density at the altitudes of the ionospheric E region, model profiles of $N(h)$ are usually used, which can reduce the measurement accuracy.

This paper proposes a new method of diagnosing the ionosphere at the altitudes of the E region by measuring the parameters of APIs formed (and probed) in turn by two waves whose frequencies are sufficiently separated. Use of the proposed method makes it possible to bypass the above-mentioned technical difficulties. Moreover, the profile $N(h)$ of electron number density at the altitudes of the E layer is useful information in itself. And, finally, the experimentally obtained profile $N(h)$ improves the accuracy of determining the temperature and density of the atmosphere and extends the altitude range of their determination.

2. DETERMINATION OF THE ELECTRON NUMBER DENSITY AT THE ALTITUDES OF THE E REGION BY THE TWO-FREQUENCY METHOD

At altitudes ranging from 90 to 190 km, a periodic structure is formed as a result of diffusion redistribution of the ionospheric plasma, and relaxation of the arising inhomogeneities after the switch-off of high-power heating takes place as a result of ambipolar diffusion. The API relaxation time is equal to

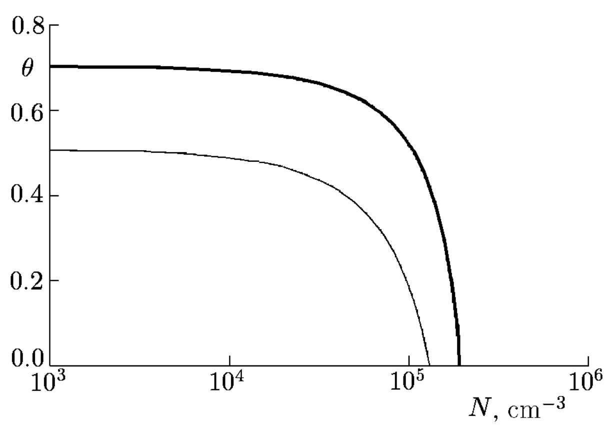

Fig. 1. Ratio of the API relaxation times as a function of the electron number density for frequencies $f_1 = 5.6$ MHz and $f_2 = 4.7$ MHz (upper curve) and $f_1 = 5.6$ MHz and $f_2 = 4.0$ MHz (lower curve).

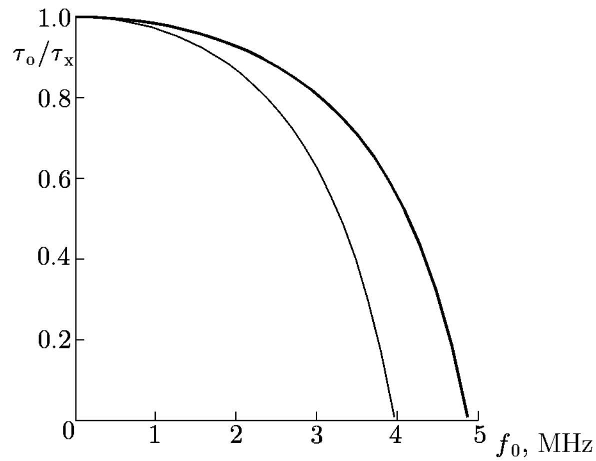

Fig. 2. Ratio of the API relaxation times as a function of the plasma frequency if the ionosphere is modified by waves of ordinary and extraordinary polarizations at a frequency of 5.6 MHz (upper curve) or 4.7 MHz (lower curve).

$\tau = (4k^2 D_\alpha)^{-1}$, where D_α is the ambipolar-diffusion coefficient, $k = 2\pi/\lambda = 2\pi f n/c$ is the wave number of the high-power wave, λ is the wavelength of the heating wave, and c is the speed of light. Thus, the API relaxation time depends not only on the ambipolar-diffusion coefficient, but also on the wavelength λ of the heating wave or the API spatial period $\Lambda = \lambda/(2n)$, where n is the refractive index. As is known, the refractive index of radio waves in a plasma depends on the electron number density.

We will create a periodic structure in the ionosphere and probe it in turn at the frequencies f_1 and f_2 (e.g., odd heating and probing cycles at the frequency f_1 and even cycles are at the frequency f_2). If the API relaxation times are determined at the same altitude with a certain ambipolar-diffusion coefficient, then the ratio of API relaxation times at the frequencies f_1 and f_2 is equal to

$$\theta = \frac{\tau_1}{\tau_2} = \frac{k_2^2}{k_1^2} = \frac{f_2^2}{f_1^2}\,\frac{n_2^2}{n_1^2}\,. \tag{2}$$

As a rule, APIs are created by means of a high-power extraordinary-mode radio wave to prevent excitation of artificial ionospheric turbulence. Within the quasi-longitudinal approximation, the refractive index for the extraordinary wave is equal to

$$n_{1,2}^2 = 1 - \frac{f_0^2}{f_{1,2}\,(f_{1,2} - f_\mathrm{L})}\,, \tag{3}$$

where $f_0 = 4\pi e^2 N/m$ is the plasma frequency at a given altitude, e and m are the electron charge and mass, respectively, $f_\mathrm{L} = f_H \cos\alpha$, α is the angle between the magnetic field and the vertical, and f_H is the electron gyrofrequency. Substituting Eq. (3) into Eq. (2) we obtain the following expression for the ratio of relaxation times:

$$\theta = \frac{f_2}{f_1}\,\frac{(f_1 - f_\mathrm{L})}{(f_2 - f_\mathrm{L})}\,\frac{[f_2\,(f_2 - f_\mathrm{L}) - f_0^2]}{[f_1\,(f_1 - f_\mathrm{L}) - f_0^2]}\,. \tag{4}$$

The calculated dependences $\theta(N)$ for two pairs of frequencies are shown in Fig. 1. The bold line shows the quantity $\theta(N)$ for $f_1 = 5.6$ MHz and $f_2 = 4.7$ MHz, and the thin line, for $f_1 = 5.6$ MHz and $f_2 = 4.0$ MHz. The "longitudinal" gyrofrequency at an altitude of 100 km was assumed equal to $f_\mathrm{L} = 1.35$ MHz. The ratio $\theta = \tau_1/\tau_2$ of the API relaxation times is greater, the greater is the difference between the operating frequencies f_1 and f_2. For a given ratio of the frequencies, a variation in θ is determined by a variation in the electron number density. For example, for the frequencies $f_1 = 5.6$ MHz and $f_2 = 4.7$ MHz (the upper curve in Fig. 1), the value θ decreases from 0.65 to 0.1 as the electron number density varies from $N = 4 \cdot 10^4$ cm^{-3} to $N = 1.8 \cdot 10^5$ cm^{-3}. For the second pair of frequencies ($f_1 = 5.6$ MHz and $f_2 = 4.0$ MHz; the lower curve in Fig. 1), the ratio of the relaxation times changes from 0.45 to 0.1 as the electron number density varies from $N = 3 \cdot 10^4$ cm^{-3} to $N = 1.3 \cdot 10^5$ cm^{-3}. Measuring the dependence of θ on the altitude h, one can find $N(h)$ using the performed calculations.

There can be another, simpler measurement option, when waves with the same frequency and the polarization varied from cycle to cycle are used to form and probe APIs. Let $f_1 = f_2 = f$. Then, for the given (for definiteness) extraordinary polarization of the first wave and the ordinary polarization of the second wave, we find by analogy with Eq. (4) that

$$\theta = \frac{\tau_{\rm o}}{\tau_{\rm x}} = \frac{n_{\rm x}^2}{n_{\rm o}^2} = \frac{(f + f_{\rm L})\,[f(f - f_{\rm L}) - f_0^2]}{(f - f_{\rm L})\,[f(f + f_{\rm L}) - f_0^2]}\,, \tag{5}$$

where the subscripts o and x refer to the ordinary and extraordinary waves, respectively.

Figure 2 shows the ratio of the API relaxation times as a function of the plasma frequency for this method of measurement. The upper and lower curves correspond to modification of the ionosphere by waves with the frequencies $f = 5.6$ MHz and $f = 4.7$ MHz, respectively.[1]

To estimate in which altitude interval the considered method can be used, we calculated the dependence $\theta(h)$ for a known profile $N(h)$. Figure 3 shows the profile of electron number density calculated within the framework of the model of [12] for the midday of April 7, 2004. The electron number density is maximum at an altitude of 109 km. The dependence $\theta(h)$ was calculated for the same two pairs of frequencies as previously, namely, for $f_1 = 5.6$ MHz and $f_2 = 4.7$ MHz and 4.0 MHz. Equation (4) was used for calculations. Figure 4 shows the dependence $\theta(h)$ corresponding to the given profile $N(h)$. The upper curve was calculated for $f_1 = 5.6$ MHz and $f_2 = 4.7$ MHz, and the lower curve, for $f_1 = 5.6$ MHz and $f_2 = 4.0$ MHz. It is seen in Fig. 4 that the quantity θ starts to decrease at an altitude of about 89 km and reaches its minimum value at an altitude of 109 km, at which the electron number density is maximum. Determination of $N(h)$ by the above-described method can be realized within this altitude interval if the impeding factors are absent. Among such factors, we can mention sporadic layers and turbulence (in the lower part of the altitude interval).[2] The upper limit of the measurement interval is stipulated by a decrease in the relaxation times and an increase in the relative error of their measurement.

3. ESTIMATE OF THE MEASUREMENT ERROR FOR ELECTRON NUMBER DENSITY

The measurement error is determined by two factors. First, it depends on the method of registration of the initial data, i.e., the amplitudes of the received signals, and on their preliminary processing aiming at determination of the relaxation times. Second, it is influenced by measurement conditions, i.e., the noise level, the presence of turbulent phenomena, and/or natural variations in the ionospheric and atmospheric parameters. Since $\theta = \tau_1/\tau_2$, the relative error of determining this quantity is equal to $\delta\theta = \delta\tau_1 + \delta\tau_2$. Let us estimate the relative error $\delta\tau$ related to the registration rate and an exponential approximation of the time dependence of the amplitude A of the received signal.

It has already been mentioned that the amplitude $A(t)$ is approximated by the exponential function to determine τ. According to [13], under the exponential approximation of the experimental data, the error is determined by the variance σ of the measured values of the scattered-signal amplitude and the number

[1] Equation (5) was also obtained in the quasi-longitudinal approximation. However, the calculations performed using a more rigorous Appleton-Hartree formula for the refractive index demonstrate good agreement with the results of the approximate calculations.

[2] The influence of these factors on measurement of the API relaxation times was earlier analyzed in [5, 6], in which the measurements of the temperature and density of the neutral atmosphere by using APIs were described. These parameters are also determined from the altitude dependence of the API relaxation times in the given altitude interval.

of readouts during the scattered-signal relaxation. In this case, the relative error is equal to

$$\delta\tau = \frac{2\sqrt{3}\,\delta A}{\sqrt{F\tau}}\,, \tag{6}$$

where F is the data reading rate and $F\tau$ is the number of readouts.

To find the error of determining the electron number density, we find the dependence $N(\theta)$ from Eq. (4):

$$N = \frac{m}{4\pi e^2}\,\frac{\theta b_1 - a b_2}{\theta - a}\,, \tag{7}$$

where

$$a = \frac{1 - f_{\rm L}/f_1}{1 - f_{\rm L}/f_2}\,, \qquad b_1 = f_1\,(f_1 - f_{\rm L}), \qquad b_2 = f_2\,(f_2 - f_{\rm L}). \tag{8}$$

From Eq. (7), it is easy to find the relative error of determining the electron number density:

$$\delta N = K\,\delta\theta, \tag{9}$$

where $K = K(f_1, f_2, \theta) = N^{-1}/|\partial N/\partial\theta|$.

Table 1 presents the estimates of the relative error of determining the electron density for three values of θ. The estimates were made for frequencies $f_1 = 5.6$ MHz and $f_2 = 4.7$ MHz. For definiteness, the profile $N(h)$ shown in Fig. 3 and the values of $\theta(h)$ calculated for this profile were used (see Fig. 4). The estimates were made for two values of the relative error of measuring the scattered-signal amplitude A, namely, $\delta A = 0.03$ and 0.1.

TABLE 1. Relative measurement errors of electron number density.

h, km	θ	N, cm^{-3}	τ_1, s	K	δN	
					δA=0.03	δA=0.1
99	0.60	$0.68\cdot10^5$	1.5	4.70	0.13	0.42
105	0.49	$1.13\cdot10^5$	0.7	1.47	0.06	0.21
109	0.45	$1.24\cdot10^5$	0.3	1.06	0.07	0.24

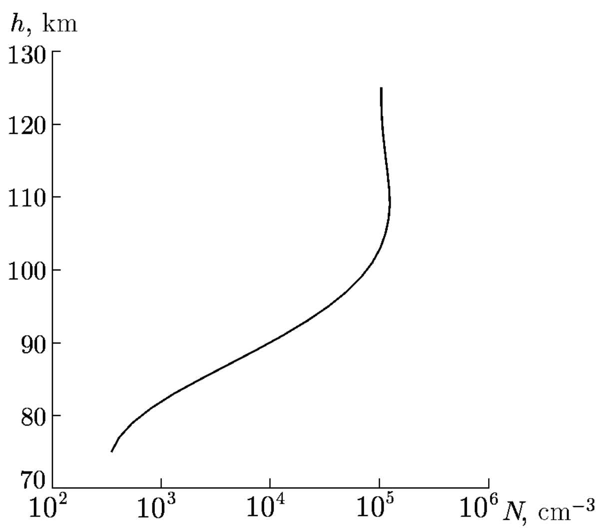

Fig. 3. Model profile of electron number density for the midday of April 7, 2004.

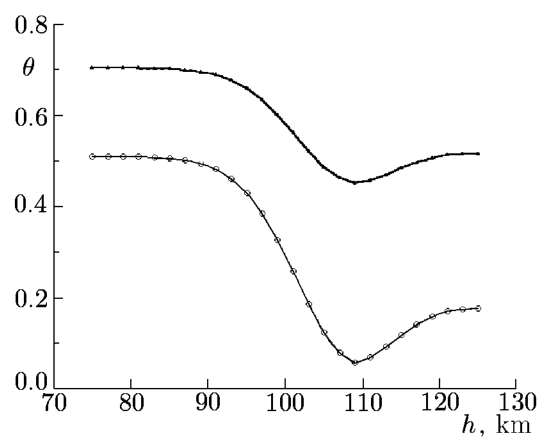

Fig. 4. Ratio of the API relaxation times as a function of altitude for the given model profile $N(h)$ shown in Fig. 3. The upper curve corresponds to $f_1 = 5.6$ MHz and $f_2 = 4.7$ MHz, and the lower curve, to $f_1 = 5.6$ MHz and $f_2 = 4.0$ MHz.

It is seen in Table 1 that as the electron number density increases and θ correspondingly decreases, the measurement errors decrease to 6–7 if the accuracy of amplitude measurements does not exceed 3%. As the error δA increases, the accuracy of determining the electron number density evidently becomes worse. It should be noted that the error of measuring θ can be decreased by averaging over an ensemble of realizations. Thus, the achievable accuracy of determining the electron number density by this method at altitudes of 105–110 km will be less than 10%.

4. CONCLUSIONS

The performed analysis shows that it is entirely possible to determine the dependence of the electron number density on the altitude in the E region of the ionosphere by measuring the API relaxation times for two frequencies. For the operating frequencies $f_1 = 5.6$ MHz and $f_2 = 4.7$ MHz, the electron number density can be determined for $N \geq 10^5$ cm^{-3}. In this case, the relative error does not exceed 10%. Measuring the electron number density along with other parameters also makes it possible to determine other parameters with greater accuracy, namely, the temperature and density of the neutral atmosphere and the velocities of the vertical motions at altitudes of 90–115 km. Monitoring of these parameters and their comparison will assist in better understanding of dynamic phenomena in this altitude interval.

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