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WIDE-ANGLE DIFFRACTION OF THE LASER BEAM BY A SHARP EDGE

S. P. Anokhov,[1*] R. A. Lymarenko,[1]
and A. I. Khizhnyak[2]

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Using a rigorous theory of diffraction, we explain the origin and analyze the structure of a wide-angle illuminated area observed when a limited beam is diffracted by the sharp edge of an opaque screen. It is shown that the formed plume has the structure of a cylindrical wave traveling from the screen edge and its amplitude is proportional to the beam amplitude at this edge. The observed structure is Young's boundary wave produced by diffraction of the limited beam.

1. INTRODUCTION

This work was motivated by the following unusual observation. The field arising due to diffraction of an intense laser beam by the edge of a thin screen comprises not only the bright central component whose scale is comparable with that of the initial beam, but also a wide, less pronounced plume extending with the decreasing intensity in the diffraction plane to both sides of the diffracted-beam axis (Fig. 1). The angular size of this plume exceeds the divergence of both the initial beam and the above-mentioned central field component by many times. The geometric center of the observed illuminated area coincides with the screen edge, which gives rise to the idea that it is related to the so-called Young's boundary wave [1, 2], whose actual existence has remained the subject of discussions until recently. The purpose of this paper is to establish the actual origin of this plume and the possibility of a rigorous description of its spatial structure.

The considered problem is reduced in a natural way to edge diffraction of a Gaussian beam radiated by most production-type single-mode lasers. Generally speaking, interest in the subject is not new. The edge diffraction of Gaussian beams was studied in [3–6]. However, information on the above-described wide-angle component is absent in those works. The general structure of the field formed as a result of diffraction of a three-dimensional Gaussian beam was studied approximately in [7, 8] and rigorously in [9, 10]. However, such features as the phase structure of the field and, moreover, the detailed field structure in the far zone remained outside the scope of those papers.

2. DIFFRACTION OF A GAUSSIAN BEAM BY A HALF-PLANE

Before formulating the problem, we briefly review the required results of the rigorous diffraction theory. It is based on the exact solution of the problem of diffraction of a plane wave by a perfectly conducting and, hence, perfectly reflecting half-plane (Fig. 2). This solution was obtained by Sommerfeld in 1896 and became the cornerstone for considering any kind of diffraction by two-dimensional apertures [1, 2]. In accordance with this solution, the field formed behind the half-plane due to diffraction of an arbitrarily

* anokhov@i.kiev.ua

[1] International Center "Institute of Applied Optics" of the National Academy of Sciences of Ukraine, Kiev, Ukraine; [2] Metrolaser Inc., Irvine, CA, USA. Translated from Izvestiya Vysshikh Uchebnykh Zavedenii, Radiofizika, Vol. 47, Nos. 10–11, pp. 1032–1039, October–November, 2004. Original article submitted May 27, 2004.

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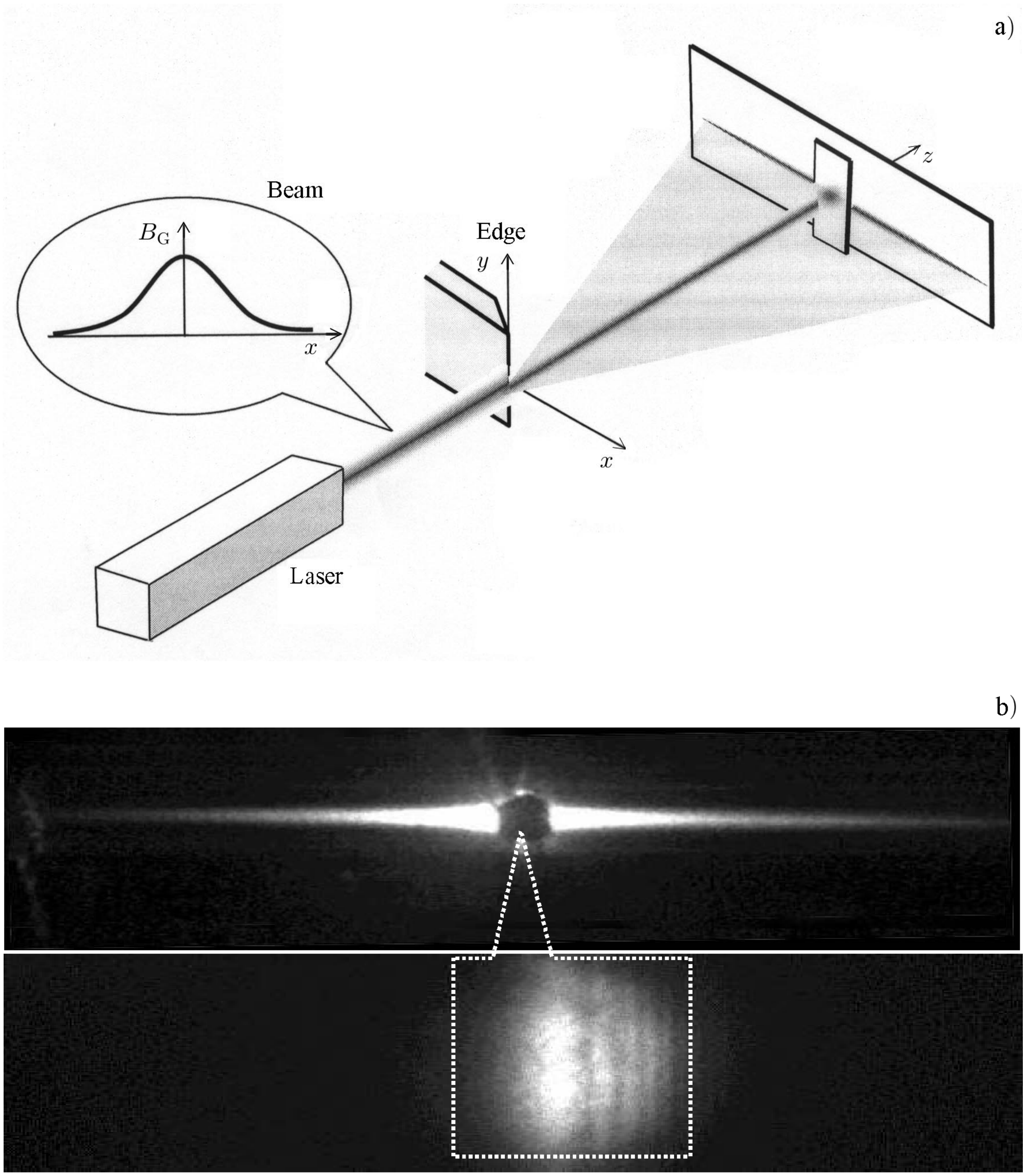

Fig. 1. Diffraction of the laser beam by the sharp edge of the screen: the optical scheme of the experiment (*a*) and the observed field distribution (*b*). Because of the significant change in the radiation intensity, the central part of the distribution is given separately with decreased intensity and increased scale.

oriented E- or H-polarized plane wave $E^{(\mathrm{pl})} = A_0 \exp[-ikr\cos(\theta - \alpha)]$ can be represented as the sum of two stable waves [1] almost everywhere behind the screen (see the discussion below) [11]:

$$E(P) = E^{(\mathrm{pl})}/2 + E^{(\mathrm{d})}, \tag{1}$$

[1] One should not confuse these waves with boundary and geometric-optical Young's components. In Sommerfeld's problem, these components have an unremovable amplitude discontinuity at the boundary of the geometrical shadow. Due to this fact, they cannot exist in free space.

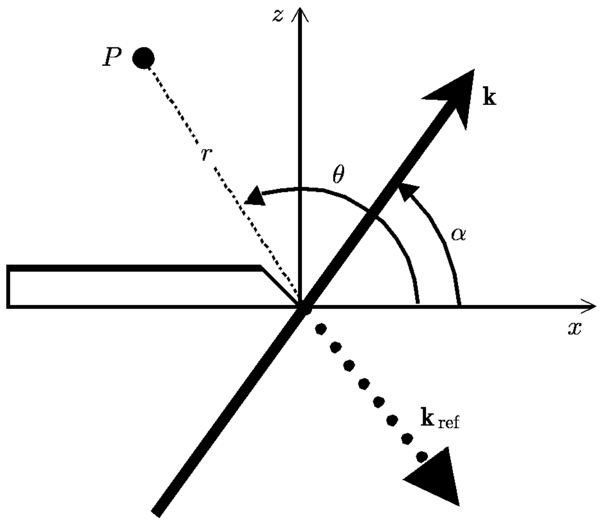

Fig. 2. Diffraction of a plane wave or Gaussian beam by the edge of a half-plane.

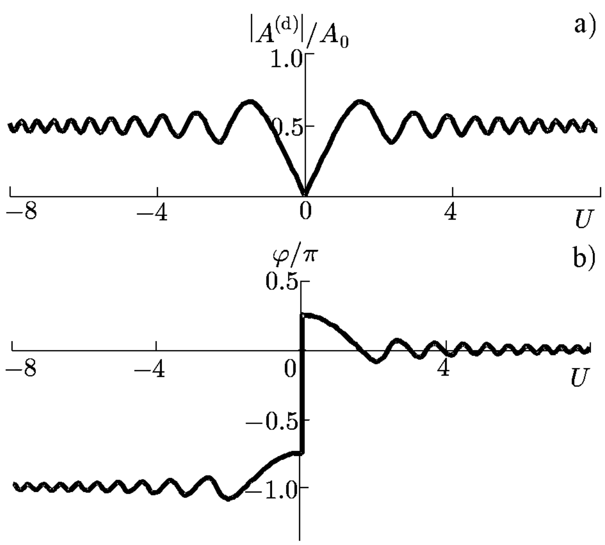

Fig. 3. Distributions of the absolute value (*a*) and the phase (*b*) of the d-wave near the boundary of the geometrical shadow.

where $E^{(\mathrm{d})} = A_0\,[(1-i)/\sqrt{2\pi}]\exp[-ikr\cos(\theta-\alpha)]F(U)$, A_0 is the incident-wave amplitude, $k = 2\pi/\lambda$ is the wave number, the meanings of the quantities r, θ, and α are easily understood from Fig. 2, and the function

$$F(U) = \int\limits_0^U \exp(i\mu^2)\,\mathrm{d}\mu \tag{2}$$

is a Fresnel integral with the upper limit

$$U = \sqrt{2kr}\,\sin\frac{\alpha-\theta}{2}\,. \tag{3}$$

Note that function (2) in graphical form represents the well-known Cornu spiral [1, 2].

Solution (1) is valid in the entire upper half-space above the half-plane (Fig. 2), except for a small region of a few tens of wavelengths in radius in close proximity to the edge and for diffraction angles exceeding a few tens of degrees. In these regions, one should take into account the field related to the component reflected from the screen [11]. The first term in Eq. (1) represents a plane wave similar to the incident wave but having a factor of 2 smaller amplitude. The second term $E^{(\mathrm{d})}$ describes a wave with a close amplitude and an infinitely extending wave front comprising the edge dislocation at the geometrical-shadow boundary (Fig. 3) [12]. All the characteristic features of edge diffraction are related to exactly this singular wave [11]. In what follows this wave (related to diffraction and dislocation) is called the d-wave.

As is known, an arbitrary wave beam can be represented as a set of plane waves with differently oriented wave vectors. The diffracted field produced by each plane wave is described by Eq. (1). As a result, the general diffracted field of the beam can be expressed as the sum of the initial beam with the halved amplitude and the field formed by the superposition of d-waves. For brevity, the latter field will be called the D-wave.

In what follows we use more illustrative Cartesian coordinates introduced by the relations $r = \sqrt{x^2+z^2}$ and $\alpha = \arccos[k_x/k]$, where k_x is the projection of the wave vector $\mathbf{k}$ onto the x axis and $\theta = \arccos[x/r] = \arccos[x/\sqrt{x^2+z^2}]$. In these notations, the dimensionless spatial parameter U has the form

$$U = \sigma\,[kr - k_x x - \sqrt{k^2-k_x^2}\,z]^{1/2}, \tag{4}$$

$$\sigma = \mathrm{sign}\big[\arccos(k_x/k) - \arccos(x/\sqrt{x^2+z^2})\big]. \tag{5}$$

The general solution for the beam field behind the screen

$$E^{(\Sigma)} = A_0 \int\limits_{-\infty}^{+\infty} f_{\mathrm{G}}(k_x)\left[\frac{1}{2} + \frac{1-i}{\sqrt{2\pi}}F(U)\right]\exp\big[i\big(k_x x + z\,\sqrt{k^2-k_x^2}\big)\big]\,\mathrm{d}k_x = \frac{E^{(\mathrm{G})}}{2} + E^{(\mathrm{D})} \tag{6}$$

is the convolution of the Fourier transform $f_{\mathrm{G}}(k_x)$ of the incident beam and solution (1) for a single plane wave.

Let a two-dimensional Gaussian beam corresponding to the geometry of the problem be incident on the screen edge. For simplicity, we consider normal incidence of the beam on the screen and place the beam constriction on the screen. The complex amplitude of such a beam is described by the expression

$$B_{\mathrm{G}}(x,z) = \exp\left(-\frac{(x-x_0)^2}{w^2}\right)\exp(ikz) = b(x)\exp(ikz), \tag{7}$$

where x_0 describes the shift of the beam axis with respect to the screen edge located at $x = 0$ and $b(x)$ is the distribution of the beam field in the screen plane. The angular spectrum of such a beam in terms of the wave-vector projection k_x onto the x axis is written as

$$f_{\mathrm{G}}(k_x) = \frac{w}{\sqrt{2}}\exp\left(-\frac{k_x^2 w^2}{4} - ik_x x_0\right). \tag{8}$$

Numerical calculation of solution (6) is difficult since its kernel comprises the complex Fresnel integral $F(U)$ which is a rapidly oscillating function of the integration variable. To remove this difficulty, the complex amplitude of the d-wave was approximated by the relation

$$A_{+}^{(\mathrm{d})} = \frac{1-i}{\sqrt{2\pi}}F(U) \approx \frac{\sigma}{2}\left\{1 - \frac{(1+i)\exp(iU^2)}{|U|\sqrt{2\pi} + \exp(-|U|^{1.04}) + i\,[1+0.4U^2]^{-0.6}}\right\} \tag{9}$$

with an explicitly expressed oscillating factor. The difference between approximating function (9) and the Fresnel integral in the entire domain of its definition ($-\infty < U < +\infty$) does not exceed 0.1% and 0.6% in absolute value and phase, respectively. As a result, for the spatial distribution of the D-wave we obtain the expression

$$E^{(\mathrm{D})} = \frac{w}{\sqrt{2}}\exp(ikr)\int\limits_{-k}^{+k}\frac{\sigma}{2}\left\{1 - \frac{1+i}{|U|\sqrt{2\pi} + \exp(-|U|^{1.04}) + i\,[1+0.4U^2]^{-0.6}}\right\} \times \exp\left(-\frac{k_x^2 w^2}{4} - ik_x x_0\right)\mathrm{d}k_x \tag{10}$$

without a rapidly oscillating factor, which simplifies significantly calculations, but leaves their accuracy intact. Let us consider the results of numerical studies of solutions (6) and (10).

3. NUMERICAL RESULTS

The spatial structures of the diffracted field and its two wave components entering Eq. (10) are shown in Figs. 4 and 5 for different distances from the screen. Figure 4 shows the distributions of the amplitudes of the fields, while Fig. 5, the distributions of their phases. In each case, the bold line shows the total field, the thin solid line, the D-wave field, and the dashed line, the field of the Gaussian beam decreased by a factor of 2. As a scale of the axial coordinate z, we use the Rayleigh length $L_{\mathrm{R}} = \pi w^2/\lambda$, which conventionally separates the zones of Fresnel and Fraunhofer diffraction.

It is seen that the total field in the zone of Fresnel diffraction ($z < L_{\mathrm{R}}$) preserves the known details of the classical distribution created by a diffracted plane wave, but the average level of the amplitude oscillations now coincides with the beam profile (Figs. 4*a* and 4*b*). With increasing distance from the screen, the frequency and the number of these oscillations decrease, so that the resulting field in the zone of Fraunhofer diffraction is completely free of such oscillations. Since the discussed evolution of the central part of this distribution is described and studied in detail in [3–8], we omit its detailed consideration and proceed to the main issue, namely, the wide-angle component of the diffracted radiation.

Its presence is evidenced by extended structureless wings which are observed in the field and are well pronounced in all the presented amplitude distributions. It is seen in these distributions that both wings belong to the D-wave which individually represents the actually observed field outside the central region of the diffracted beam. The front of the D-wave acquires a well pronounced cylindrical shape at $z_{\rm start} \approx L_{\rm R}/2$, i.e., before the field gets free of the initial amplitude oscillations. In all the cases, only the half-wave change in the D-wave phase at the boundary of the geometrical shadow remains invariant.

To explain the mechanism of formation of the wide-angle component of the field, we use an asymptotic representation of the Fresnel integral describing the d-wave complex amplitude

$$A^{(\rm d)}(U) = \frac{1-i}{\sqrt{2\pi}} F(U) \to \pm\left(\frac{1}{2} - \frac{1+i}{2U\sqrt{2\pi}} \exp(iU^2)\right), \tag{11}$$

where the minus sign corresponds to the d-wave wing in the zone of the geometrical shadow and the plus sign, to an open zone. Note that even for $|U| \approx 3$, the error of this representation does not exceed 1% and rapidly decreases with increasing distance from the shadow boundary. In accordance with Eq. (11), the d-wave in this spatial region splits into two simpler waves, namely, a plane wave with the halved amplitude and a cylindrical wave with the amplitude $(1+i)\exp(iU^2)/(2U\sqrt{2\pi})$. This allows us to analyze the structure of total field (6) in another way. This field is given by

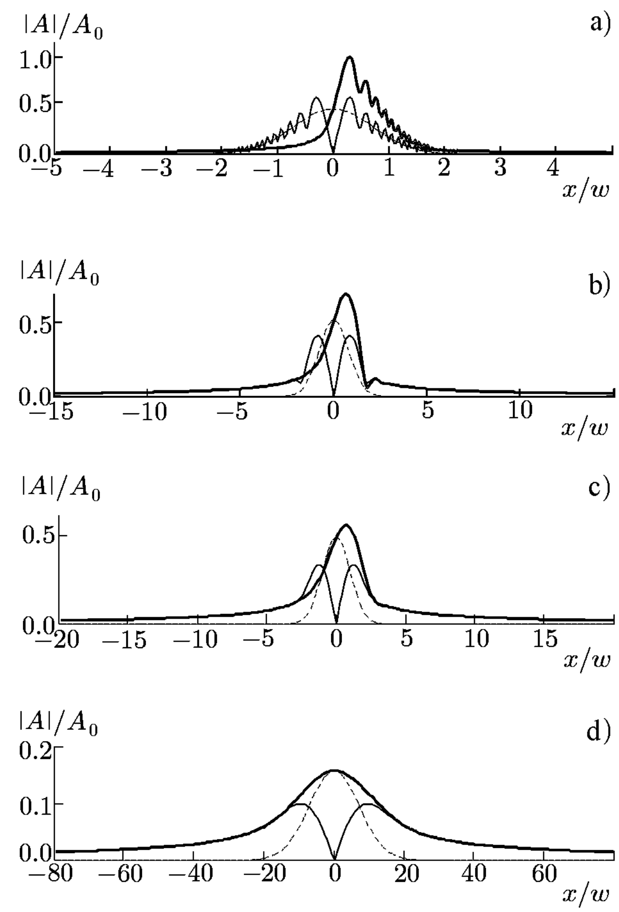

Fig. 4. Distributions of the absolute value of the complex amplitude of the total field and its basic components for different distances from the screen: *a*, $z = L_{\rm R}/25$; *b*, $z = L_{\rm R}/2$; *c*, $z = L_{\rm R}$; *d*, $z = 10L_{\rm R}$.

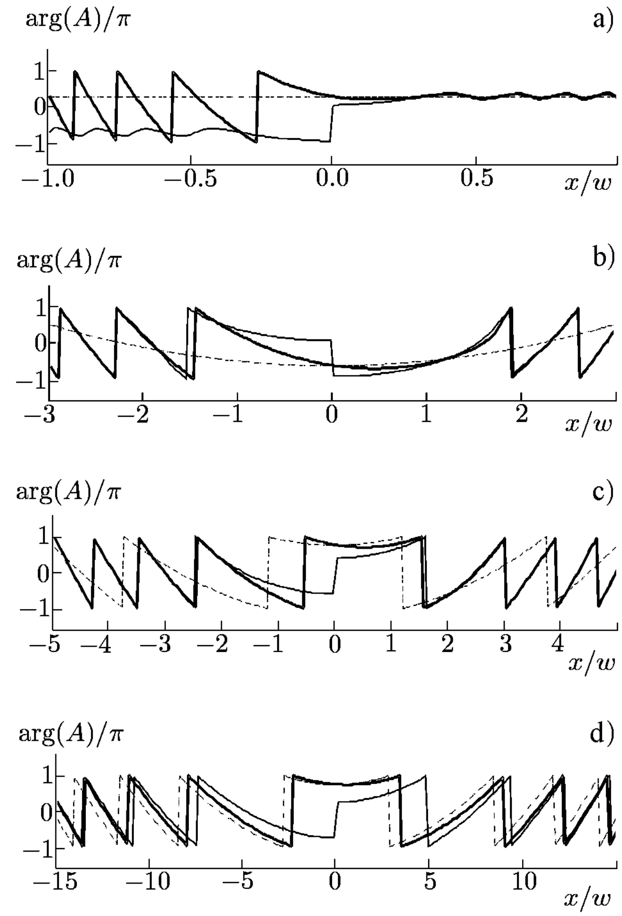

Fig. 5. Distributions of the phase of the total field and its basic components for different distances from the screen: *a*, $z = L_{\rm R}/25$; *b*, $z = L_{\rm R}/2$; *c*, $z = L_{\rm R}$; *d*, $z = 10L_{\rm R}$. The presented monotonic dependences are jump-like since the vertical-axis ranges of the plots are limited by 2π.

$$E^{(\Sigma)}(x,z) \approx \frac{E_{\mathrm{G}}}{2} + \begin{cases} -\dfrac{E_{\mathrm{G}}}{2} + \dfrac{A_0\,(1+i)}{2\sqrt{2\pi}} \exp(ikr) \displaystyle\int\limits_{-\infty}^{+\infty} \dfrac{f_{\mathrm{G}}(k_x)}{U} \exp(-ik_x x_0)\,\mathrm{d}k_x, & U < -3; \\ \dfrac{E_{\mathrm{G}}}{2} - \dfrac{A_0\,(1+i)}{2\sqrt{2\pi}} \exp(ikr) \displaystyle\int\limits_{-\infty}^{+\infty} \dfrac{f_{\mathrm{G}}(k_x)}{U} \exp(-ik_x x_0)\,\mathrm{d}k_x, & U > 3. \end{cases} \tag{12}$$

We emphasize that the first two terms compensate for each other in the shadow zone, i.e., only cylindrical components of the d-wave are important in this zone. However, in the open zone, these two terms are added to give the initial beam propagating in free space in the absence of a screen. Since this beam has a finite transverse size, the beam field decreases with distance from its axis and can be neglected beginning with a certain distance. As a result, the plume in this spatial region outside the beam is described by the same expressions as in the shadow zone (but with the reverse sign), which is indicative of the phase shift by π.

Since we consider the plume outside the region of free propagation of the Gaussian beam, integration in Eq. (12) is not difficult and leads to the following very simple representation of the plume field in this spatial region:

$$E^{(\Sigma)}(x,z) \sim \begin{cases} \dfrac{\exp(ikr)}{\sqrt{kr}}\,\dfrac{b(0)}{\cos[\theta/2]}\,, & U < -3; \\ -\dfrac{\exp(ikr)}{\sqrt{kr}}\,\dfrac{b(0)}{\cos[\theta/2]}\,, & U > 3, \end{cases} \tag{13}$$

where $b(0)$ is the amplitude of the initial beam at the screen edge and θ is the observation angle (Fig. 2).

As we see, the D-wave outside the nondiffracted component on each side of the shadow boundary is represented by a cylindrical wave traveling from the screen edge. It is important that the amplitude of the observed plume is determined by the beam amplitude at the screen edge. This means that in the case of diffraction of the beam having zero amplitude at the edge, the wide-angle plume related to the diffracted beam is absent.[2]

Thus, based on the entire set of features, the observed wide-angle illumination can be classified as Young's boundary wave produced by diffraction of a limited beam. However, in contrast to the analogous component in Sommerfeld's problem, this wave has not the amplitude discontinuity. Instead, its wave front comprises the boundary dislocation. The intensity of the discussed wave is proportional to the squared amplitude of the initial beam at the screen edge.

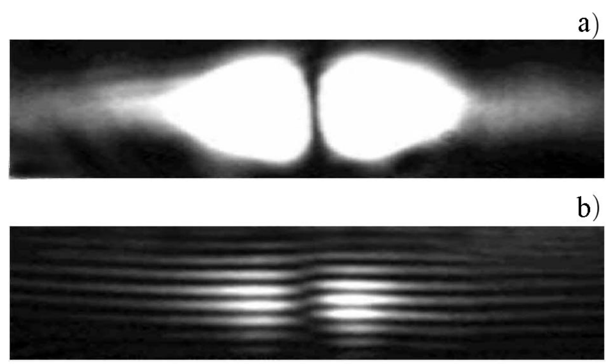

Fig. 6. The spatial distribution of the D-wave (a) and the pattern of its interference with an oblique plane wave (b).

4. ADDITIONAL EXPERIMENTAL DATA

It is easy to conclude from Eq. (6) that to observe the D-wave individually, it is sufficient to remove the second term, i.e., the Gaussian beam with the halved amplitude, from the total field. We performed such an experiment. The field of this beam was removed by means of its interference quenching with the help of an analogous beam whose phase was shifted by π. Figure 6a shows the spatial structure of the D-wave obtained in such a way. To visualize the half-wave phase shift of one part of this wave with respect to the other part, we used an additional plane wave propagating at a small angle with the initial beam. It is seen in Fig. 6b that the interference strips formed upon crossing the boundary-dislocation region are displaced by one-half of their period, which is undoubtedly indicative of the half-wave change in the phase of the studied field.

[2]In this case, however, there remains a weak component which is related to the field reflected from the screen and can rigorously be taken into account [11].

5. CONCLUSIONS

The proposed quantitative description of the wide-angle illumination observed in the case of edge diffraction of the laser beam is rigorous. In accordance with it, the field diverging from the screen edge represents the superposition of cylindrical components of the d-waves combined into the D-wave with the common front of cylindrical shape. The bright central core of radiation, which falls at the boundary of the geometrical shadow, is formed by superimposing the D-wave and the initial Gaussian beam with the halved amplitude, whereas the plume is formed by wide wings of the D-wave, which are structureless, fill almost the entire space behind the screen, and monotonically decrease in intensity with distance from the boundary of the geometrical shadow, i.e., with increasing observation angle. The intensity of the observed illumination is proportional to the squared amplitude of the wave diffracted by the sharp edge of the screen.

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