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Subjective perception of time and decision inconsistency in interval effect

Viviana Ventre[1] · Roberta Martino[1] · Fabrizio Maturo[1]

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Abstract
The interval effect refers to the phenomenon in which the discount rate decreases as the interval considered increases. It represents one of the many anomalies of the decision-making process in the context of intertemporal choices. This paper suggests that the latter anomaly is due to the perceived time and emotional drives involved in the moment of choice and their interaction. The study is developed through a direct comparison between empirical preferences and those predicted by the normative model, respectively determined by proper time, i.e., empirical time and normative time, which are different from objective time. Although it was known in the literature that the perception of time has a substantial impact on preferences and the phenomenon of temporal inconsistency, our study presents a measure that quantifies the decision-making bias caused by the subjective perception of time and contributes to the normalisation of choices defined as irrational. By the term normalisation, we mean to clarify the extent to which the cognitive structures of the decision-maker respect the principles of economic rationality. From an operational point of view, the present work's originality lies in proving that the same description of subjective time is not constant in the context of the interval effect. The experimental implementation provides empirical evidence of the latter considerations. The contribution of this work refers mainly to the field of behavioural finance as it aims to describe anomalies as inevitable consequences of individual cognitive processes.

Keywords Financial anomalies · Hyperbolic discounting · Impatience · Intertemporal choice · Subjective time perception

✉ Fabrizio Maturo
fabrizio.maturo@unicampania.it

Viviana Ventre
viviana.ventre@unicampania.it

Roberta Martino
roberta.martino@unicampania.it

[1] Department of Mathematics and Physics, University of Campania Luigi Vanvitelli, Caserta, Italy

1 Introduction

Intertemporal choices are decisions in which alternatives are distributed over time and greatly influence daily life. For example, buying a car, booking a trip, and investing are intertemporal choices (Lu and Yang 2015). The complexity of the topic has involved numerous disciplines to understand how decision-making develops when alternatives are spread over several periods (Prelec 2004). The Discounted Utility Model (Samuelson 1952) is the main reference for studying an individual's behaviour when faced with an inter-temporal choice. There are mainly two reasons why the decision-making process that develops when time-separated outcomes are involved is so complex. First, preferring a delayed outcome is equivalent to foregoing one in the present (Noor 2011); Second, it is difficult to keep one's preferences constant over time (Sayman and Öncüler 2009) in the sense that an individual's preferred alternative today may not be the same preferred alternative tomorrow (Rohde 2010).

The Samuelson's normative model assumes that the value of alternatives decreases over time because it is calculated as the product of the cardinal utility of the outcome and the discount function. The discount function determines a reduction in cardinal utility based on the time distance between the choice's instant and the outcome's correspondence. The rate at which this decrease occurs determines the pattern of preferences and reflects the indeterminacy of the future as perceived by the individual. The main elements that describe the discount function represented by an individual are the discount factor and impatience: the discount factor represents the proportional change in the discount function over a standard period (Read 2004); impatience, on the other hand, represents the amount of money one is willing to lose to receive a unit of money immediately (Cruz Rambaud and Muñoz Torrecillas 2016).

Empirical evidence has shown that when Samuelson's model is compared with investor behaviour some discrepancies relate to the phenomenon of decision inconsistency, that is, preferences are not constant over time. The interest in understanding the extent to which empirical behaviour can comply with the normative principles of financial models has enabled numerous advances by combining cognitive and psychological models with behavioural models of finance (Barber and Odean 2013; Hirshleifer 2014). Cognitive psychology has led to the understanding that financial anomalies, which are those attitudes that cannot be rationalised from a normative point of view, are due to human beings. Simon (1957) was the first to consider that the cognitive capacity of individuals is limited, and therefore, in a very rich decision-making environment, it is not possible to process all the information needed for the proper assessment (Kahneman 2003). In addition, there are mechanisms to distort the decision-making process called behavioural biases (Kahneman 2017), which intervene during the evaluation and the selection of alternatives. From a financial point of view, a hyperbolic formulation of the discount function turns out to be more in line with investors' attitudes than the exponential formulation predicted by the normative model (Prelec 2004; Rohde 2010; Attema et al. 2010; Muñoz Torrecillas et al. 2018). Hyperbolic discounting is characterised by a variable discount rate and a degree of impatience that decreases over time (Prelec 2004). These elements are strongly associated with the influence of emotional drives in decision-making (Ventre et al. 2022; Andrade and Ariely 2009). Recent research has considered time perception as the third key element in understanding the psychological mechanisms underlying hyperbolic discounting (Van Boven et al. 2010; Zauberman et al. 2009; Kim and Zauberman 2019; Agostino et al. 2021). What is derived from the relationship between subjective perception of time and intertemporal preferences is that decreasing impatience and a varying discount rate are determined by how time is perceived. Moreover, time preferences can be altered by changing the perception of time (Kim and Zauberman 2019; Kim et al. 2012).

To further investigate the relationship between decreasing impatience, decreasing discount rate and subjective perception of time, this paper compares consistent and inconsistent preferences with the assumption that they are dictated by their own and other than objective times. The inconsistency we consider is related to the interval effect, an anomaly of intertemporal choices (Read 2004; Read and Roelofsma 2003) that represents the financial phenomenon for which the discount rate reduces as the interval considered increases. The main results we will prove are twofold. First, the interval effect is a phenomenon that behaves differently when studied with the normative model or independently of it. In particular, the empirical time, that is the time perceived by the hyperbolic discount function, is first smaller and then larger than the normative time, i.e., the time perceived by the exponential discount function. Secondly, at the same time, empirical time is always larger than objective one. From a psychological perspective, the comparison with the exponential function confirms the proven result by Van Boven et al. (2010), confirming that the discrepancy between empirical behaviour and expected financial behaviour is due to the failure to include the emotional intensity of the decision-maker in the Discounted Utility Model. Comparison versus objective time, on the other hand, is mathematical evidence of the tendency of those with declining cognitive abilities (Pouya et al. 2015) or in depressive states (Kent et al. 2019; Kitamura and Kumar 1982; Stanghellini et al. 2017; Thönes and Oberfeld 2015) to overestimate time. In this regard, we point out that time preferences are strongly related to cognitive abilities (James et al. 2015) and are linked to attitudes such as drug use, obesity, and gambling (Petry 2001). In addition to contributing new properties of the hyperbolic discount function to the present literature, this paper offers a new measure that could be used to quantify decision inconsistency from the normative model in the intertemporal choice. In fact, other financial anomalies can also be described by starting with a degree of impatience that decreases over time and a decreasing discount rate (Ventre et al. 2022).

Our work has a strong impact on the field of organisational behaviour and psychology. Quantifying the inconsistency of decision-making makes it possible to quantify the cognitive and emotional biases involved in the evaluation and selection of alternatives and classify decision-makers according to the extent of their inconsistency (Pompian 2012, 2008; Pompian and Longo 2005). The present study could open a new avenue for strategic personalisation and customer profiling involving any sector, not just financial.

The paper is subdivided as follows: the next section presents contributions to subjective time and its influence on decision-making; the financial description of the interval effect for objective, normative and empirical time are formalised; in the end, there is an experimental implementation that confirms the results obtained. The paper ends with a discussion and conclusions.

2 The subjective process of temporal perception

Time is the representation of how each event follows one another and concretises the intuitive perception of the relationship between different occurrences. The complexity involved in this definition highlights how much the concept of time has been studied by multiple disciplines, such as philosophy, religion, psychology, and physics, but it still represents a

Table 1 Summary of the temporal profiles described by the psychological theory of the temporal approach (Zimbardo and Boyd 2008)

Positive past	Individuals oriented toward good memories of the past
Negative past	Individuals addressing the past with a negative attitude
Hedonistic present	Individuals living in the present and enjoying the moment
Fatalistic present	Individuals who live life as if everything were already written
Future	Individuals aiming at their projects and actions
Transcendent future	Individuals more focused on what comes after life

great mystery to be interpreted. Over the years, there have been numerous references to a subjective dimension of time in which the concept of time is identified with a necessary human intuition. As early as the 4th century, St Augustine stated that time cannot be quantified except in the soul of the perceiver. In particular, he stated: "*What is time?[…] If no one questions me I know. If I want to explain it to those who question me I do not know*" (Augustine 1876). This conception is formalised by Kant's prestigious philosophy, which describes time as the form of the internal sense, understood as the intuition of ourselves and our internal state. In this regard, it is important to note that although time is universally recognised as an objective quantity, the perception of time is a subjective process and indicates how the individual experiences it. Therefore, objective time denotes chronological time, while subjective time denotes the inner time that concretises how an individual perceives the passage of chronological time. Time perception is a field of study in psychology, cognitive linguistics, and neuroscience whose aim is to analyse subjective time. From a psychological perspective, time can be seen as the quintessential individual dimension because it marks the uniqueness of individual perceptions. According to the psychological theory of the Temporal Approach (Zimbardo and Boyd 2008), how an individual perceives time determines her cognitive, emotional, and motivational style, resulting in a distinct profile. The dimensions and profiles to which Temporal Approach Theory refers are briefly described in Table 1.

The descriptions in Table 1 show that subjective time is determined by the individual, but also cultural and social characteristics (Eldor et al. 2017; Wilson et al. 2009). The context in which the individual is formed defines a strong impact on temporal perception. In fact, accelerated social rhythms produce a cult of urgency that pushes individuals to constantly confront the indeterminacy of the future, generating emotions linked to the ungovernability of the uncertain. Another interesting theory that clarifies the mechanisms of the perception of the future is provided by the concept of temporal focus (Shipp et al. 2009). According to temporal focus, how individuals pay attention to the past, present and future is decisive for perceiving past, present and future experiences.

Turning to a neurocognitive perspective, although there are yet no ways to measure or experience the subjective perception of time directly, there are many debates about the neural mechanisms involved in the subjective perception of time (Eagleman et al. 2005; Wittmann and van Wassenhove 2009). Rao et al. (2001) argued that human beings are endowed with a complementary system (or systems) that manages temporal perception (Rao et al. 2001). They proved that the cerebral cortex, cerebellum, and basal ganglia are involved in time perception, while other studies (Heron et al. 2012, 2013) have proven that there are neurons dedicated to processing very short durations. From a perceptual point of view, recent studies show that the duration of an event can be deflected by experimental techniques (Wittman et al. 2010), and the duration of an interval depends on the physical characteristics of the stimuli (Fraisse 1984) and their weight on the individual's emotional sphere (Noulhiane et al. 2007; Droit-Volet and Gil 2009). In particular, the duration of stimuli used to alter the duration of events is systematically overestimated (Tse et al. 2004; van Wassenhove et al. 2008).

2.1 Subjective time in decision-making

By definition of intertemporal choice, the decision-making process involved in evaluating intertemporal perspectives is strongly influenced by the perception of time (Agostino et al. 2021; Zauberman et al. 2009). Initially, time was considered an objective fact of the decision-making context, but developments in the fields of psychology and neuroscience have invited researchers to consider the subjective nature of time perception in intertemporal choices. Since the perception of time has a neurological basis and is affected by emotional conditions (Van Boven et al. 2010), the decision-making process understood as a combination of rationality and emotion of the individual is strongly influenced by it. Analysing the subjective perception of time amounts to investigating one of the deepest dynamics determining the perception of the context in which a choice is made. In fact, if context interacts with experience (Glicksohn 1987), then it also interacts with temporal experience (Glicksohn 1991).

The impact of the subjective perception of time on the development of intertemporal preferences has a significant psychological significance because it is linked to the decision-makers concept of impatience. The first consequence of this relationship is that a degree of impatience that decreases over time is a phenomenon related to a non-linear perception of time (Nyberg et al. 2010). In this regard, Zauberman et al. (2009) proved that the hyperbolic discount is due to the impact of the subjective time bias, and therefore, the integration of the subjective time perception into the economic model reduces the hyperbolic discount. This result emphasises that the time inconsistency in intertemporal choices is a process directly related to individual internal structures. As a result, subjective perception of time can alter time preferences. In this regard, research has sought to determine how elements of the decision-making context can influence the perception of time by altering time preferences (Kim and Zauberman 2019). Furthermore, in the context of intertemporal choices where outcomes are distributed over time, the mechanism of projection into the future (Loewenstein 1996; Loewenstein et al. 2003, 1998; Wheeler et al. 1997) is essential to predict which outcome will satisfy us most. Under conditions of uncertainty, it is even more challenging to imagine how our emotions vary when time correction is applied to the atemporal representation (Gilbert et al. 2002).

Therefore, because the subjective perception of time is closely related to the degree of impatience and satisfaction the decision-maker expects from a future outcome, integrating the psychological mechanisms of subjective time into mathematical models of intertemporal choices can help understand the cognitive determinants of decision inconsistency.

3 Financial description of the interval effect

This section presents the psychological principles underlying the Discounted Utility Model (Samuelson 1952). To investigate how temporal information is processed and internalised by the decision-maker, an original description of the interval effect is presented and contextualised in terms of subjective perception of time. The interval effect is an anomaly of the Discounted Utility Model, i.e., an attitude that is difficult to rationalise by the classical model (Thaler 1993; Kahneman and Riepe 1998; Shefrin 1999; Shiller 2000; Waeneryd 2001; Loewenstein and Prelec 1992). Anomalies are due to a distortion of the decision-making process (Kahneman and Tversky 1979, 2000) and refer to the phenomenon of decision inconsistency, that is, when preferences vary over time (Ventre et al. 2022). The description we offer makes it possible to quantify the extent to which the stages of evaluation and selection of alternatives are distorted by a subjective perception of time (Blavatskyy 2017). The mathematical description of the interval effect provides insight into how psychological mechanisms influence the evaluation of intertemporal prospects. Therefore, this section aims to obtain a measure of temporal inconsistency from the observation of the phenomenon.

3.1 The psychological foundations of the mathematical model

The Discounted Utility Model states that, given $\left(x_0, t_0; \dots ;x_n, t_n\right)$ an intertemporal prospect with $t_0 < \cdots < t_n$, the associated utility is calculated as $U\left(x_0, t_0; \dots ;x_n, t_n\right) = \sum_{i=0}^{n} x_i f\left(t_i\right)$ in which $f(t)$ represents the discount function. The discount function is defined as positive, monotonically decreasing, such that $f(0) = 1$ and has the task of determining a reduction in the value of the outcome based on the time distance from the selection of the alternative to the actual transposition. The psychological factors underlying decision-making are encapsulated in two key elements of this economic model. The first element is the discount rate, defined as $\rho(t) = -\frac{f'(t)}{f(t)}$, which represents "*the proportional variation of f in a standard period*" (Read 2004). The second element is impatience,[1] i.e. "*the amount of money that the agent is willing to lose in exchange for anticipating the availability of a \$1 reward*" (Cruz Rambaud and Muñoz Torrecillas 2016), defined in an interval $[t_i, t_j]$ as $1 - \frac{f(t_j)}{f(t_i)}$. Initially, the discount function was formalised with an exponential trend that, in line with the profile of a rational investor assumed by classical economics, determines consistent preferences over time. In the exponential case, both impatience and discount rate are constant over time. However, empirical evidence (Green and Myerson 1996; Dasgupta and Maskin 2005; Lu and Yang 2015) of temporally inconsistent preferences, that is, inconsistency over time (Ventre et al. 2022), has urged the formalisation of alternative discount function. Prelec (2004) was the first to prove that the irrationality underlying temporally inconsistent preferences is reflected in a degree of impatience that decreases over time. By further investigating the features of decision-making that result in a hyperbolic pattern of the discount function, Prelec (2004) also proved that the degree to which impatience decreases, defined as $-\frac{(lnf(t))''}{(lnf(t))'}$, quantifies the difference between the individual's impatience and the concept of temporal preference. Rohde (2010) introduced the second type of inconsistency, called relative impatience, in which the extent of impatience depends on the payment made in advance. The degree to which relative impatience decreases is defined by the ratio $-\frac{f''(t)}{f'(t)}$ and is related to the other quantities according to the following relationship $DRI(t) = \rho(t) + DI(t)$. The term anomalies refer to those empirical results that do not meet the expectations dictated by the model and determine inconsistent preferences, i.e., a

[1] Given the intertemporal prospects (100,0) and (150, tomorrow), a decision-maker who prefers (100,0) is more impatient than one who prefers (150, tomorrow).

hyperbolic discounting. In the context of intertemporal choices, there are different anomalies, e.g. delay effect (Tahler 1981), magnitude effect (Loewenstein and Thaler 1989), and sign effect (Loewenstein and Thaler 1989). We want to focus on the interval effect since it turns out to be explicitly related to the concept of subjective time (Zauberman et al. 2009). The interval effect refers to the phenomenon where the discount factor decreases as the interval considered increases (Read and Roelofsma 2003).

3.2 Analysis of time inconsistency in the interval effect

The purpose is to analyse the inconsistency referred to as the interval effect by directly comparing the discount applied by the inconsistent and normative preferences. Let's consider two situations: the first consists of three intertemporal prospects in which x_0, x_1, x_2 $(x_0 < x_1 < x_2)$ spread over three different times t_0, t_1, t_2 in such a way that $t_{i+1} = t_i + h, h > 0$; the second consisting of two intertemporal prospects x_0, x_2 $(x_0 < x_2)$ spread over two different times t_0, t_2 in such a way that $t_{i+2} = t_i + 2h, h > 0$. From a normative point of view, according to the assumptions of the mathematical model of Discounted Utility, one should have that if $\left(t_0, x_0\right) \sim \left(t_1, x_1\right)$ and $\left(t_1, x_1\right) \sim \left(t_2, x_2\right)$ then $\left(t_0, x_0\right) \sim \left(t_2, x_2\right)$.

Let $f(t)$ a normative discount function according to which $\left(t_0, x_0\right) \sim \left(t_1, x_1\right)$. Thus, if $f(t) = e^{-kt}, k > 0$, then $u\left(x_0\right) = u\left(x_1\right)e^{-k\left(t_1 - t_0\right)} = e^{-kh}$ where $h := t_1 - t_0$. Obviusly, if $t_2 := t_1 + h = t_0 + 2h$ then $\left(t_1, x_1\right) \sim \left(t_2, x_2\right)$ and $\left(t_0, x_0\right) \sim \left(t_2, x_2\right)$, that is to say, transitivity is guaranteed.

Let $\tilde{f}(t)$ be a discount function exhibiting the interval effect such that, according to this function, $\left(t_0, x_0\right) \sim \left(t_1, x_1\right)$. For example, if $\tilde{f}(t)$ is the hyperbolic discount function $\tilde{f}(t) = \frac{1}{1+it}, i > 0$, there is i_0 such that $\left(t_0, x_0\right) \sim \left(t_1, x_1\right)$, according to $\tilde{f}(t)$.

Analogously, for every h such that $0 < h < t_0$, the avarege discount rate in $[0, h)$ is greter than the average discount rate in $\left[0, t_0\right]$, that is to say, the average discount rate in $[0, h]$ is greater than k: $\tilde{\delta}(0) > \delta(0) = k$. As $\tilde{f}(0) = f(0) = 1$, one has $\tilde{f}'(0) < f'(0)$.

The interval effect occurs when $\left(t_0, x_0\right) \sim \left(t_1, x_1\right)$ and $\left(t_1, x_1\right) \sim \left(t_2, x_2\right)$ imply $\left(t_0, x_0\right) < \left(t_2, x_2\right)$. This is equivalent to stating that if $\tilde{f}(t)$ is a discount function that verifies the interval effect and $f(t)$ is a normative discount function then (Read 2004):

$$\begin{aligned} f\left(t_0\right)U\left(x_0\right) &= f\left(t_1\right)U\left(x_1\right) = f\left(t_2\right)U\left(x_2\right) \\ \tilde{f}\left(t_0\right)U\left(x_0\right) &< \tilde{f}\left(t_2\right)U\left(x_2\right) \end{aligned} \tag{1}$$

Since the aim is to understand the psychological mechanisms that characterise the discrepancy respect the decision maker's attitude between prospect that involve the short term or the long term, the monotonicity of the discount function makes it possible to assume $t_0 = 0$ without restricting the assumptions of the study. In particular, the assumption $t_0 = 0$ refers to the fact that the behavioral anomalies that generate the financial anomaly affects periods closer to the present. In fact, because in Ventre et al. (2022) it has been proven that the interval effect is determined by a decrease in impatience, as time increases the behavior of the anomaly stabilises and the study becomes meaningless. Therefore, $\tilde{f}(0) = f(0) = 1$ and $U\left(x_0\right) = f\left(t_2\right)U\left(x_2\right) < \tilde{f}\left(t_2\right)U\left(x_2\right)$ implies $\tilde{f}\left(t_2\right) > f\left(t_2\right)$. The decision-making process during the valuation of an intertemporal prospectus is marked by how the discount function decreases and the speed at which the discount function decreases is decisive for preference trends.

Proposition 1 *Let $f(t)$ an exponential discount function and $\tilde{f}(t)$ an hyperbolic discount function that verifies the interval effect in the sense of Eq.* (1). *Then $\exists\,\bar{t}, T : \bar{t} < T$ for which the following occurs*:

1. $\tilde{f}(t) < f(t)$ *in* $(0, T)$ *and* $\tilde{f}(t) > f(t)$ *in* $(T, +\infty)$;
2. $\tilde{I} > I$ *in* $(0, \bar{t})$ *and* $\tilde{I} < I$ *in* $(\bar{t}, +\infty)$;
3. $\widetilde{f'}(t) < f'(t)$ *in* $(0, \bar{t})$, $\widetilde{f'}(t) > f'(t)$ *in* $(\bar{t}, T)$;
4. $\rho(t) < \tilde{\rho}(t)$ *in* $[0, \bar{t} + h)$ *and* $\rho(t) > \tilde{\rho}(t)$ *in* $(\bar{t} + h, +\infty)$ *for some* $h > 0$.

Proof

1. Discount functions resulting in inconsistent preferences are characterised by having a higher slope in the initial instants for the property of decreasing impatience (Prelec 2004). This implies that $\tilde{f}'(0) < f'(0)$ and for continuity of discount functions $\forall t \in [0, h]$ $\tilde{f}'(t) \le f'(t)$. It also follows from this observation that $\rho(0) < \tilde{\rho}(0)$. Combining that $0 > f'(0) > \tilde{f}'(0)$ and $\tilde{f}(t_2) > f(t_2)$, it is possible to state that the function $\tilde{f}$ decreases faster in the first period by intersecting f at a certain point $0 < T < t_2$.
2. A steeper trend in the discount function equals greater impatience (Cruz Rambaud and Muñoz Torrecillas 2016) and the impatience of the function f is constant over time by hypothesis. In fact, for each t such that $0 < t < T$ and $\tilde{f}(t) < f(t)$:

$$I(0,t) = 1 - \frac{f(t)}{f(0)} < 1 - \frac{\tilde{f}(t)}{\tilde{f}(0)} = \widetilde{I(0,t)} \tag{2}$$

   On the other hand, for each t such that $T < t < +\infty$ and $\tilde{f}(t) > f(t)$:

$$I(T,t) = 1 - \frac{f(t)}{f(T)} > 1 - \frac{\tilde{f}(t)}{\tilde{f}(T)} = \widetilde{I(T,t)} \tag{3}$$

   Let be $g(t) := \tilde{I}(t, t+h) - I(t, t+h)$. By definition $g(t)$ is a countinuos function and $\forall t : T < t < \infty$ it's possible to verify that $g(a)g(b) < 0\ \forall[0, t]$ in which $a = 0$ and $b = t$. By the Bolzano theorem $\exists\ \bar{t}$ in which $g(\bar{t}) = 0$. By construction of $g(t)$, $\bar{t} < T$ since $g(T) < 0$.
3. By Eq. (3):

$$\begin{aligned} &\frac{f(T) - f(T+h)}{h} > \frac{\tilde{f}(T) - \tilde{f}(T+h)}{h} \\ &\lim_{h\to 0}\frac{f(T) - f(T+h)}{h} > \lim_{h\to 0}\frac{\tilde{f}(T) - \tilde{f}(T+h)}{h} \\ &f'(T) < \tilde{f}'(T) \end{aligned} \tag{4}$$

   The thesis is obtained as discussed in the point 2).
4. By point 2) of Proposition1:

$$\widetilde{I(0,t)} > I(0,t) = I(T,t) > \widetilde{I(T,t)} \tag{5}$$

Let's also observe that the decrease in impatience for the function $\tilde{f}$ is positive and the above means that $\widetilde{DRI}(t) > \tilde{\rho}(t)$ (Rohde 2010), i.e.:

$$
\begin{aligned}
-\left[ln\tilde{f}'(t)\right]' &> -\left[ln\tilde{f}(t)\right]' \\
ln\tilde{f}'(t) - ln\tilde{f}'(t+h) &> ln\tilde{f}(t) - ln\tilde{f}(t+h) \\
\frac{\tilde{f}'(t)}{\tilde{f}'(t+h)} &> \frac{\tilde{f}(t)}{\tilde{f}(t+h)} \\
\frac{\tilde{f}'(t)}{\tilde{f}(t)} &< \frac{\tilde{f}'(t+h)}{\tilde{f}(t+h)} \\
\tilde{\rho}(t) &> \tilde{\rho}(t+h)
\end{aligned} \tag{6}
$$

However, for the interval effect to occur, hyperbolic preferences become more patient than exponential preferences from a certain $\bar{t} < T$. The Eq. (5) means by Prelec (2004, p. 7, definition 2) that $\forall 0 \le s < t, \sigma, \rho$:

$$
\begin{cases}
(x, s) \sim_{\tilde{f}} (y, t) \\
(\mathrm{x}, \mathrm{s}+\sigma) \sim_{\tilde{f}} (\mathrm{y}, \mathrm{t}+\sigma+\mathrm{p}) \\
(\mathrm{x}, \mathrm{s}) \sim_{\mathrm{f}} (\mathrm{y}, \mathrm{t})
\end{cases}
\Rightarrow (\mathrm{x}, \mathrm{s}+\sigma) \ge_{\mathrm{f}} (\mathrm{y}, \mathrm{t}+\sigma+\mathrm{p}) \tag{7}
$$

$$
\frac{\tilde{f}(\mathrm{s}+\sigma)}{\tilde{f}(\mathrm{t}+\sigma+\mathrm{p})} \le \frac{f(s+\sigma)}{f(t+\sigma+p)} \tag{8}
$$

Let's say $\mathrm{s}+\sigma = \mathrm{S}, \mathrm{t}+\sigma+\mathrm{p} = \mathrm{S}+\mathrm{h}$:

$$
\begin{aligned}
\frac{\tilde{f}(\mathrm{S})}{\tilde{f}(\mathrm{S}+\mathrm{h})} &\le \frac{f(\mathrm{S})}{\mathrm{f}(\mathrm{S}+\mathrm{h})} \\
\ln\tilde{f}(\mathrm{S}) - \ln\tilde{f}(\mathrm{S}+\mathrm{h}) &\le \ln \mathrm{f}(\mathrm{S}) - \ln f(\mathrm{S}+\mathrm{h}) \\
\lim_{\mathrm{h}\to 0} \frac{\ln\tilde{f}(\mathrm{S}+\mathrm{h}) - \ln\tilde{f}(\mathrm{S})}{\mathrm{h}} &\ge \lim_{\mathrm{h}\to 0} \frac{\ln \mathrm{f}(\mathrm{S}+\mathrm{h}) - \ln \mathrm{f}(\mathrm{S})}{\mathrm{h}} \\
&+ \left[\ln\tilde{f}\right]' \ge +[\ln \mathrm{f}]' \\
\tilde{\rho}(t) &< \rho(t)
\end{aligned} \tag{9}
$$

The thesis follows by considering that $\tilde{\rho}(0) > \rho(0), \tilde{\rho}\left(\bar{t}\right) > \rho\left(\bar{t}\right), \tilde{\rho}(T) < \rho(T)$ and Eq. (6). □

The analysis adds essential considerations to a result already proven by Ventre et al. (2022) regarding this phenomenon. Specifically, the position $t_0 = 0$ allowed us to prove that the interval effect occurs only if the time instants of the prospectuses considered belong to one of the two intervals bounded by T, i.e., $[0, T]$ or $[T, +\infty)$. The instant T represents the moment when the discount function that promotes temporal inconsistency equals the temporally consistent discount function in line with the mathematical model. Thus, at first, the impatience of the hyperbolic discount function is greater than that of the exponential discount function. Before the two curves intersect, this relationship is reversed, also affecting the discount rate. Figure 1 graphically represents the proven result. The highlighted points represent the intersections between the two curves. At the light grey point, the curves intersect at the origin. At the dark-grey point, the curves intersect at T.

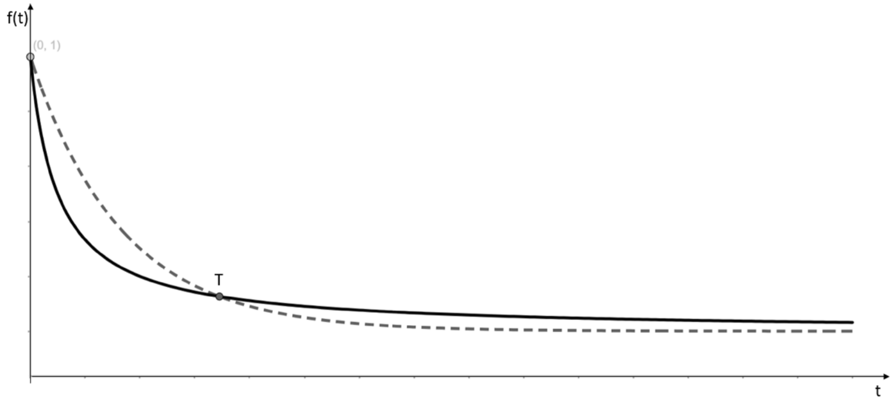

Fig. 1 Relationship between a discount function that promotes the interval effect (represented by the non-deshed line) and one that meets the expectations of the economic model (represented by the deshed line). Source: own elaboration

Table 2 Summary of proven results regarding the interval effect. Source: Own elaboration

Interval	$(0, \bar{t})$	$(\bar{t}, T)$	$(\mathrm{T}, \bar{\bar{t}})$	$\left(\bar{\bar{t}}, +\infty\right)$
Impatience	$\tilde{I} > I$	$\tilde{I} < I$	$\tilde{I} < I$	$\tilde{I} < I$
Discount Function	$\tilde{f}(t) < f(t)$	$\tilde{f}(t) < f(t)$	$\tilde{f}(t) > f(t)$	$\tilde{f}(t) > f(t)$
Decreasing impatience	$\widetilde{DI} > DI$	$\widetilde{DI} > DI$	$\widetilde{DI} > DI$	$\widetilde{DI} > DI$
First derivative	$\tilde{f}'(t) < f'(t)$	$\tilde{f}'(t) > f'(t)$	$\tilde{f}'(t) > f'(t)$	$\tilde{f}'(t) < f'(t)$

Observation 1 What is the relationship between $\tilde{f}'(t)$ and $f'(t)$ in $[T, +\infty)$?

By Proposition 1, $\tilde{f}(t) < f(t)$ in $[0, T)$ and $\tilde{f}(t) > f(t)$ in $[\mathrm{T}, +\infty)$. Therefore, the function $f(t) - \tilde{f}(t)$ is positive in $[0, T)$ and negative in $[\mathrm{T}, +\infty)$. By definition of discount functions $\lim_{t\to\infty} f(t) = \lim_{t\to\infty} \tilde{f}(t) = 0$. It means that $f(t) - \tilde{f}(t)$ reaches a minimum value in $\bar{\bar{t}} \in (\mathrm{T}, +\infty)$ for which $f'\left(\bar{\bar{t}}\right) = \tilde{f}'\left(\bar{\bar{t}}\right)$. By the monotonicity of the functions, the uniqueness of the minimum assures that $f'(t) > \tilde{f}'(t)$ in $\left(\bar{\bar{t}}, +\infty\right)$.

Table 2 summarises the conclusions.

The relationships in Table 2 describe the inconsistency as the vertical distance between the hyperbolic and exponential discount functions. For each instant, the quantity $f(t) - \tilde{f}(t)$ describes how far the function $\tilde{f}(t)$ deviates from the normative trend.

4 Relativistic interpretation of the interval effect

To understand how the decision-making context influences the evaluation and selection phases of alternatives in intertemporal choices, this section aims to integrate psychological theories related to the subjective perception of time with the mathematical description of the interval effect just proved.

4.1 Relationship between exponential and hyperbolic time in interval effect

First, starting from the definition of the interval effect, this paper investigates the relationship between the time instants considered. By comparing the preferences determined by an exponential and a hyperbolic discount function, the present study aims to prove that the interval effect results in a dilated perception of the time interval involved and vice versa on the assumption that $t_2 > T$.

Proposition 2 *Let $f(t)$ an exponential discount function and $\tilde{f}(t)$ an hyperbolic discount function that verifies the interval effect in the sense of Eq. (1) on the assumption that $t_2 > T$ where T is the point defined in Proposition 1. If the time perception of $\tilde{f}(t)$ is dilated respect the time perception of $f(t)$, the interval effect occurs and viceversa.*

Proof We begin by proving that the interval effect implies a time dilation. Let's consider $t_2 > t_1 > t_0$ such that $t_2 - t_1 = t_1 - t_0 = h$ and $t_2 > T$.

By the condition $f(t_2) < \tilde{f}(t_2)$ and by definition of discount factor, a point $\widetilde{t_2}$ exists such that $f(t_2) = \tilde{f}(\widetilde{t_2})$, with $t_2 < \widetilde{t_2}$. So:

$$\begin{aligned} & t_2 - t_0 < \widetilde{t_2} - t_0 \\ & 2h = t_2 - t_1 + t_1 - t_0 < \widetilde{t_2} - t_0 = 2\tilde{h} \end{aligned} \tag{10}$$

The above means that the interval effect implies that the individual has an altered perception of time, i.e., the interval of length h is perceived with length $\tilde{h}$, $h < \tilde{h}$.

Let us now prove the opposite implication, i.e., that subjective time dilation implies the interval effect. Similarly to the previous demonstration, suppose that the individual has an altered perception of the length of the interval, instead of the length h, $\tilde{h}$ is perceived. Let's consider $t_2 > t_1 > t_0$ such that $t_2 - t_1 = t_1 - t_0 = h$ and $\widetilde{t_2} > \widetilde{t_1} > \widetilde{t_0}$ such that $\widetilde{t_2} - \widetilde{t_1} = \widetilde{t_1} - \widetilde{t_0} = \tilde{h}$ with $h < \tilde{h}$ and $t_0 = \widetilde{t_0} = 0$ under the condition that $f(t_0)U(x_0) = f(t_1)U(x_1) = f(t_2)U(x_2)$. Then:

$$\begin{aligned} & h = t_2 - t_1 + t_1 - t_0 < \widetilde{t_2} - \widetilde{t_1} + \widetilde{t_1} - \widetilde{t_0} = \tilde{h} \\ & t_2 < \widetilde{t_2} \\ & f(t_2) > f(\widetilde{t_2}) \end{aligned} \tag{11}$$

Consider a function $\widetilde{f(\tilde{t})} = f(\tilde{t}) + (f(t) - f(\tilde{t}))$ such that $\widetilde{f(\tilde{0})} = f(\tilde{0}) + (f(0) - f(\tilde{0})) = 1$ and $\widetilde{f'(\tilde{t})} = f'(\tilde{t}) + \underset{\smile}{(f'(t)} - f'(\tilde{t})) = f'(t)$. The function $\widetilde{f(\tilde{t})}$ is a discount function that satisfies the condition $\widetilde{f(\tilde{t}_2)} = f(\tilde{t}_2) + (f(t_2) - f(\tilde{t}_2)) = f(t_2)$.

Since $t_2 < \widetilde{t_2}$:

$$\begin{aligned} \widetilde{f(t_2)} &> \widetilde{f(\tilde{t}_2)} = f(t_2) \\ U(x_2)\widetilde{f(t_2)} &> U(x_2)f(t_2) = U(x_0) \\ (t_2, x_2) &> (t_0, x_0) \end{aligned} \tag{12}$$

□

In case that $t_2 > T$, it was therefore verified that the interval effect causes dilation of subjective time and vice versa. The perception of time is, in any case, not uniform. In fact, considering instants $t < T$ it has already been mentioned that $f(t) > \tilde{f}(t)$.

Table 3 Relationship between real and perceived time based on point T. Source: own elaboration

Interval	Relationship between exponential and hyperbolic time
(0,T)	Exponential perception time > hyperbolic perception time
(T, + ∞)	Exponential perception time < hyperbolic perception time

The last inequality means that a point $\tilde{t} < t$ exists such that $f(t) = \tilde{f}(\tilde{t})$. We can immediately observe that $\tilde{h} = \tilde{t} - t_0 < t - t_0 = h$ implying $\tilde{h} < h$, i.e., there is a contraction of the perceived time. In addition, if $f(t) > \tilde{f}(t)$ it's possible to state that for every $T > t_2 > t_1 > t_0$ under the condition $t_2 - t_1 = t_1 - t_0 = h$ and for every x_2, x_1, x_0 such that $f(t_0)U(x_0) = f(t_1)U(x_1) = f(t_2)U(x_2)$ we have that $f(t_2)U(x_2) > \tilde{f}(t_2)U(x_2)$. Placing $t_0 = 0$, $f(t_0)U(x_0) > \tilde{f}(t_2)U(x_2)$ means $(0, x_0) \sim (t_1, x_1)$ and $(t_1, x_1) \sim (t_2, x_2)$ then $(0, x_0) > (t_2, x_2)$. In this case, is not possible to have an interval effect. Table 3 summarises the results evaluated regarding the subjective perception of time.

We can therefore conclude that point T, which is equivalent to the instant at which the anomalous behaviour aligns with the description of the Discounted Utility model, represents the point at which the time of consistent and inconsistent preferences coincide.

4.2 Objective, empirical, and normative time

Since it is impossible to speak of time without considering its subjective nature, this section supplements the results obtained before with the hypothesis that exponential preferences are also marked by subjective time. We will therefore speak of three times: objective time that runs absolutely, normative time, which is the subjective time perceived by a rational decision-maker, and empirical time that is instead the subjective time perceived by a decision-maker whose preferences respond to the phenomenon of temporal inconsistency.

At this point of the paper, it is important to note that the exponential discount function is also determined with respect to a subjective perception of time. In fact, given $f_1(t)$ and $f_2(t)$ two exponential discount functions and suppose without loss of generality that $f_1(t) < f_2(t)$. Therefore, for the monotonicity of discount functions, a point $\breve{t} < t$ exists such that $f_1\left(\breve{t}\right) = f_2(t)$. The time perceived by $f_1(t)$ is more contracted than that perceived by $f_2(t)$. In conclusion, even functions that respect the principles of economic rationality base preferences on a subjective perception of time.

Proposition 3 *An exponential discount function has a constant time misperception for intervals of equal width.*

Proof By definition, $f(t)$ has a constant degree of impatience over time. This is equivalent to stating that, given two intervals $\left[t_i, t_j\right]$ and $[t_{i+h}, t_{j+h}]$, for an exponential discount function, the condition $\delta(t) = \delta$ holds (Cruz Rambaud and Muñoz Torrecillas 2016) and:

$$
\begin{aligned}
1 - e^{-\int_{t_i}^{t_j} \delta(t)dt} &= 1 - e^{-\int_{t_{i+h}}^{t_{j+h}} \delta(t)dt} \\
e^{-\int_{t_i}^{t_j} \delta(t)dt} &= e^{-\int_{t_{i+h}}^{t_{j+h}} \delta(t)dt} \\
\int_{t_i}^{t_j} \delta(t)dt &= \int_{t_{i+h}}^{t_{j+h}} \delta(t)dt \\
\left[t_i, t_j\right] &= \left[t_{i+h}, t_{j+h}\right]
\end{aligned}
\tag{13}
$$

Indeed, since the ratio $-\frac{f'(t)}{f(t)}$ is constant, each exponential function has a constant perception of time intervals of the same length, although its perception varied from function to function. □

Proposition 4 *Let $\tilde{f}$ a hyperbolic discount function than the interval effect respect to objective time occurs irrespective of the instant considered.*

Proof For the discount function $\tilde{f}$, the ratio $-\frac{\tilde{f}'(t)}{\tilde{f}(t)}$ decreases over time, i.e., for each $t > s$:

$$
\begin{aligned}
-\frac{\tilde{f}'(s)}{\tilde{f}(s)} &> -\frac{\tilde{f}'(t)}{\tilde{f}(t)} \\
-\frac{\tilde{f}'(s)}{\tilde{f}(s)} &> -\frac{\tilde{f}'(t)}{\tilde{f}(t)} > -\frac{\tilde{f}'(t)}{\tilde{f}(s)} \\
-\tilde{f}'(s) &> -\tilde{f}'(t) \\
\lim_{h\to 0} -\frac{\tilde{f}(s+h) - \tilde{f}(s)}{h} &> \lim_{h\to 0} -\frac{\tilde{f}(t+h) - \tilde{f}(t)}{h} \\
-\tilde{f}(s+h) + \tilde{f}(s) &> -\tilde{f}(t+h) + \tilde{f}(t) \\
\frac{\tilde{f}(s+h) - \tilde{f}(s)}{\tilde{f}(s)} &< \frac{\tilde{f}(s+h) - \tilde{f}(s)}{\tilde{f}(t)} < \frac{\tilde{f}(t+h) - \tilde{f}(t)}{\tilde{f}(t)} \\
\frac{-\tilde{f}(s+h) + \tilde{f}(s)}{\tilde{f}(s)} &> \frac{-\tilde{f}(t+h) + \tilde{f}(t)}{\tilde{f}(t)} \\
1 - \frac{\tilde{f}(s+h)}{\tilde{f}(s)} &> 1 - \frac{\tilde{f}(t+h)}{\tilde{f}(t)}
\end{aligned}
\tag{14}
$$

We have proved that the patience exhibited from s to $s + h$ is less than that exhibited from t to $t + h$.

Therefore, given two intervals of equal amplitude $\left[t_i, t_j\right]$ and $[t_{i+h}, t_{j+h}]$ then:

$$
\int_{t_i}^{t_j} \delta(t)dt < \int_{t_{i+h}}^{t_{j+h}} \delta(t)dt \tag{15}
$$

So, for each $t_i < t_j$ and $t_s = \frac{t_j - t_i}{2}$

$$
\begin{aligned}
&e^{\int_{t_i}^{t_j}\delta(t)dt} < e^{2\int_{t_i}^{t_s}\delta(t)dt} = e^{\int_{t_i}^{t_s}\delta(t)dt}\, e^{\int_{t_s}^{t_j}\delta(t)dt} \\
&e^{-\int_{t_i}^{t_j}\delta(t)dt} > e^{-\int_{t_i}^{t_s}\delta(t)dt}\, e^{-\int_{t_s}^{t_j}\delta(t)dt} \\
&I(t_i,t_j) = 1 - e^{-\int_{t_i}^{t_j}\delta(t)dt} < 1 - e^{-\int_{t_i}^{t_s}\delta(t)dt}\, e^{-\int_{t_s}^{t_j}\delta(t)dt} = I(t_i,t_s;t_s,t_j) \\
&I(t_i,t_j) < I(t_i,t_s;t_s,t_j)
\end{aligned}
\tag{16}
$$

The above is tantamount to proving that the decision-maker's impatience calculated over broken intervals is greater than that calculated over the entire interval, an individual characteristic of hyperbolic preferences. □

The interval effect, therefore, responds to different laws depending on whether hyperbolic time is compared with exponential or objective time.

Proposition 5 *Let $\tilde{f}$ a hyperbolic discount function than the hyperbolic time is greater than objective time.*

Proof Follows by Propositions 2 and 4. □

The main difference in terms of time perception between empirical and normative preferences lies in the fact that while normative perceived time is constant relative to objective time over intervals of the same length, empirical perceived time decreases relative to objective time over intervals of the same length. From a behavioural perspective, this means that classical theory assumes constant emotional flow in decision-making, whereas, empirically, emotional intensity decreases over time.

5 Experimental phase

The experimental part consists of two phases referring to two different results tested previously. Specifically, the first part refers to the phenomenon for which $I(t_i,t_j) < I(t_i,t_s;t_s,t_j)$, discussed in Sect. 4.2; the second part aims to compare the subjective hyperbolic perception of time with exponential normative time to prove the results described in Tables 2 and 3.

For both sections, questionnaires were constructed and administered through an online implementation, a website connected to a database that collects responses and user information (gender, age, region). Respondents had only 20 seconds to answer each question. The structural choice to constrain the response time is due to wanting to simulate as much as possible the agitation and lack of complete awareness of the decision-making context, as well as the uncertainty that characterises intertemporal choices. Fifty people aged between 18 and 60 were interviewed, of whom 58.14% are men.

5.1 Interval effect in the system of a hyperbolic discount

This experimental part aims to verify how impatience varies with respect to if the considered interval breaks at its midpoint. From a practical point of view, the experiment aims to

Table 4 Intervals and subintervals considered in the first experimental phase

Intervals	Sub-intervals
[0,4]	[0,2] [2,4]
[0,14]	[0,7] [7,14]
[0,20]	[0,10] [10,20]
[0,60]	[0,30] [30,60]
[0,90]	[0,45] [45,90]

Table 5 Variability of the discount function F(t). Source: own elaboration

Interval	$F(t)$	Minimum	Maximum
[0,4]	$F(4)$	0.03	2.50
[0,14]	$F(14)$	0.02	33.3
[0,20]	$F(20)$	0.04	33.33
[0,60]	$F(60)$	0.016	14.29
[0,90]	$F(90)$	0.00	3.33

investigate, considering $\left[0, t_j\right]$, the impatience expressed by the preference for the prospect $\left(x_i, 0\right) \sim \left(x_j, t_j\right)$ and $\left(x_i, 0\right) \sim \left(x_h, t_h\right) \sim \left(x_j, t_j\right)$ with the condition $t_h = \frac{t_j}{2}$.

The intervals considered are five and Table 4 shows their midpoint splits.

The inhomogeneity between intervals considered was necessary in the construction of the experiment to preserve homogeneity in the perception of the future. For each respondent, the values of $f(t)$ in the case of intervals and subintervals were calculated. In both cases, a fixed initial figure of €100 was set. Specifically, in the case of intervals, it was asked: *"You have to receive 100 euros today, how much do you want to receive in t days to consider the offer equivalent?".*

In this way:

$$F(t) = \frac{F(0) * 100}{U(x(t))} \tag{17}$$

The subinterval discount function was constructed by iteration according to the following succession and fixing an initial figure at the value of €100:

$$f(t) = \begin{cases} f(0) = 1 \\ f\left(t_{i+1}\right) = \frac{f(t_i) * U(x(t_i))}{U(x(t_{i+1}))} \end{cases} \tag{18}$$

It was asked *"You have to receive $U(x\left(t_i\right))$ euros t_i days, how much do you want to receive in t_{i+1} days to consider the offer equivalent?".*

The great variability of the results, evident from the values shown in Table 5, made the median the best statistical index for the study.

In Table 6, impatience in the intervals $[0, t]$ was calculated according to the definition as $1 - \frac{F(t)}{F(0)}$ and total impatience of the subintervals was instead calculated according to the observations made before about it, as $1 - \frac{f(s)}{f(0)} \frac{f(t)}{f(s)}$, in the Sect. 4.2.

As impatience decreases with time, it is found that $I\left(t_i, t_s; t_s, t_j\right) > I\left(t_i, t_j\right)$. Since the decision-maker is less impatient, her preference will fall on the higher and less imminent

Table 6 For each line, the interval, the impatience calculated on the interval broken at its midpoint, and the impatience calculated on the entire interval are given in order. Source: own elaboration

Interval	$I(t_i, t_s; t_s, t_j)$	$I(t_i, t_j)$
[0,4]	0.500	0.333
[0,14]	0.818	0.474
[0,20]	0.841	0.688
[0,60]	0.958	0.800
[0,90]	0.947	0.889

figure. This shows how emotional impulses reflected in the degree of impatience play a key role in decision-making and the perception of time. The anomaly of the interval effect, which implies a dilation of objective time, is a consequence of the decrease in the degree of impatience. This result is empirical evidence of the link between emotion and perception of the decision-making context during the evaluation and selection of alternatives. In fact, from a psychological point of view, to estimate the value of an event that has not yet been realised, one must imagine that it will be realised by projecting into the future (Loewenstein 1996; Loewenstein et al. 2003, 1998; Wheeler et al. 1997). Imagining more distant, and therefore more uncertain, events require greater cognitive effort by defining a more complex decision-making context. In a complex environment, decision-making develops through a trade-off between effort and solution (Kahneman 2017), and emotional drives may be less understood (Gilbert et al. 2002).

5.2 Comparison of the hyperbolic and exponential discount system

In the second part of the experiment, the discount function of all respondents was constructed by iteration. Specifically, having set an initial figure of €100, each candidate answered the following question *"You have to receive $U(x(t_i))$ in t_i days, how much do you want to receive in t_{i+1} days to consider the offer equivalent?"*.

Thus, the discount function was obtained by interpolating the values of $f(t)$ calculated as:

$$f(t) = \begin{cases} f(0) = 1 \\ f(t_{i+1}) = \frac{f(t_i) * U(x(t_i))}{U(x(t_{i+1}))} \end{cases} \tag{19}$$

for $t = 0, 2, 4, 7, 10, 14, 20, 30, 45, 60, 90$.

As in Sect. 5.1, the inhomogeneity between intervals is part of the experimental structure and the high variability of the data indicated the median as the best statistical index for constructing the discount function. The values obtained are shown in Table 7 and the interpolated function is shown in Fig. 2.

Since the aim is to investigate the relationship between exponential and hyperbolic preferences, the exponential function that best interpolates the values in Table 7 was constructed. Figure 3 shows the comparison between the hyperbolic trend of the exponential function and the most similar exponential function. In particular, $y(t) = \partial^t, \partial = \left(\frac{1}{e}\right)^{0.045}$.

As can be seen from Fig. 4, the two curves meet at a precise point as proven in Table 2. To investigate this relationship further, the difference between $y(t)$ and $f(t)$ was calculated as shown in Table 8 and represented in Fig. 4.

Table 7 Median of the values of f(t). Source: own elaboration

t	f(t)
0	1.0000
2	0.7143
4	0.5000
7	0.4000
10	0.2000
14	0.1818
20	0.1587
30	0.1000
45	0.0870
60	0.0667
90	0.0532

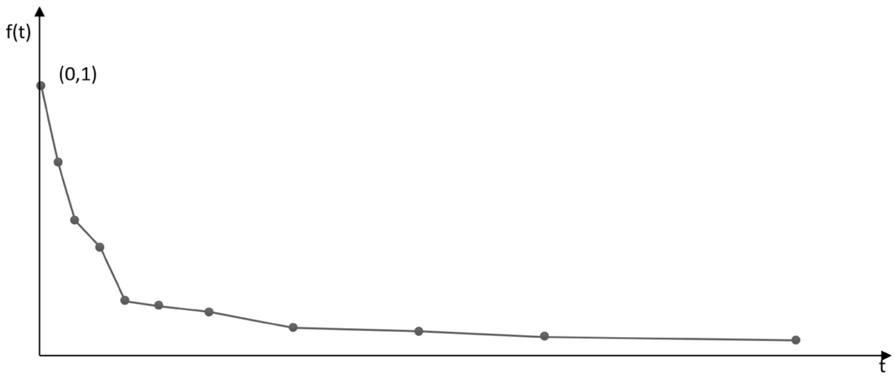

Fig. 2 Discount function constructed by interpolating the data in Table 7. Source: own elaboration

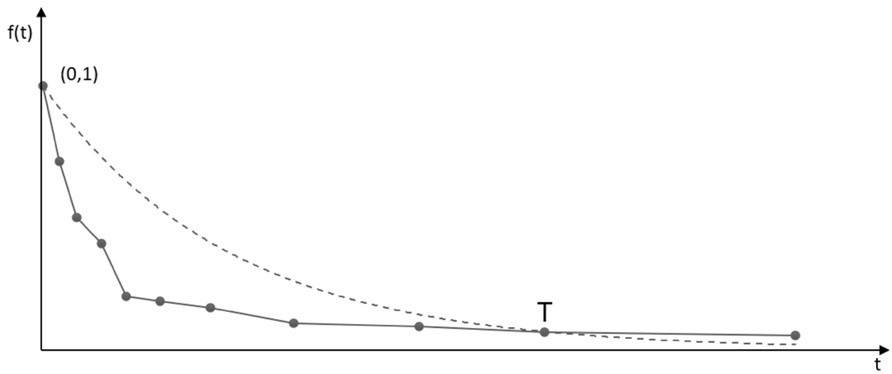

Fig. 3 Comparison of exponential and hyperbolic trends. Source: own elaboration

It can be observed that the difference $y(t) - f(t)$ reaches a maximum value before cancelling around $t = 60$. Furthermore, comparing the value of impatience as shown in Table 9, it can be observed that the impatience of the hyperbolic function becomes greater than that shown by the exponential function at a point prior to $t = 60$, as shown in Table 2.

Table 8 Value of the difference function between exponential and hyperbolic preferences. Source: own elaboration

t	2	4	7	10	14	20	30	45	60	90
$y(t) - f(t)$	0.200	0.335	0.330	0.438	0.351	0.248	0.159	0.045	0.000	-0.0367

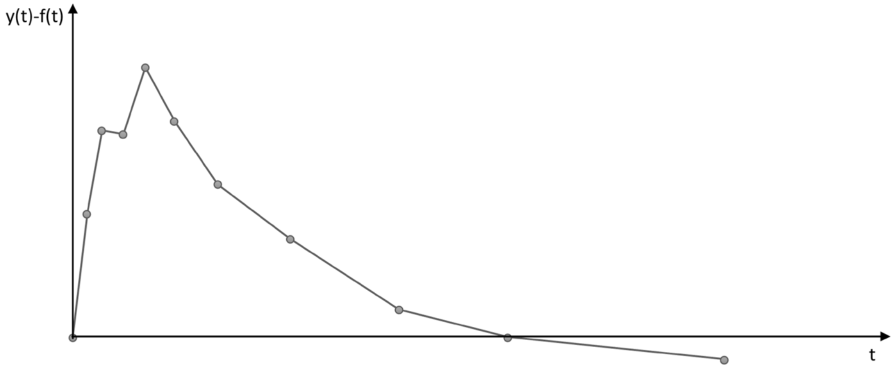

Fig. 4 Graphical representation of the difference function between exponential and hyperbolic preferences. Source: own elaboration

Table 9 Comparison of hyperbolic and exponential function impatience. Source: Own elaboration

$[t_i, t_j]$	$I_{f[t_i,t_j]}$	$I_{y[t_i,t_j]}$	$I_{f[t_i,t_j]} - I_{y[t_i,t_j]}$
[0,2]	0.29	0.09	0.20
[2,4]	0.30	0.09	0.21
[4,7]	0.20	0.13	0.07
[7,10]	0.50	0.13	0.37
[10,14]	0.09	0.16	–0.07
[14,20]	0.13	0.24	–0.11
[20,30]	0.37	0.36	0.01
[30,45]	0.13	0.49	–0.36
[45,60]	0.23	0.49	–0.26
[60,90]	0.20	0.74	–0.54

Moreover, comparing $x(t_i)$ of hyperbolic preferences and exponential preferences, it is possible to provide empirical evidence that interval effect occurs only if $t_2 > T$. Table 10 also confirms the relationship between the impatience experienced in Table 2 by studying the relationship of two successive outcomes. In fact, as can be seen from Table 10, after $t = 60$ the exponential preference figure is higher than the hyperbolic one. Despite being experimentally proven only for $t = 90$, the continuity of the discount functions allows us to conclude that from $t = 60$ an individual with hyperbolic preferences will be inclined to accept lower figures than an individual with behavior conforming to the normative description, as shown in Fig. 5.

Table 10 Comparison of the figures required by hyperbolic and exponential indifferences confirms that compared to the normative model, the interval effect only occurs from a certain point onwards. Source: own elaboration

t	x_f(t)	x_y(t)	$x\_f(t_{i+1})/x\_f(t_i)$	$x\_y(t_{i+1})/x\_y(t_i)$
0	100	100.0	1.40	1.09
2	140	109.4	1.43	1.09
4	200	119.7	1.25	1.14
7	250	137.0	2.00	1.14
10	500	156.8	1.10	1.20
14	550	187.8	1.15	1.31
20	630	246.0	1.59	1.57
30	1000	385.7	1.15	1.96
45	1150	757.6	1.30	1.96
60	1500	1488.0	1.25	3.86
90	1880	5739.7		

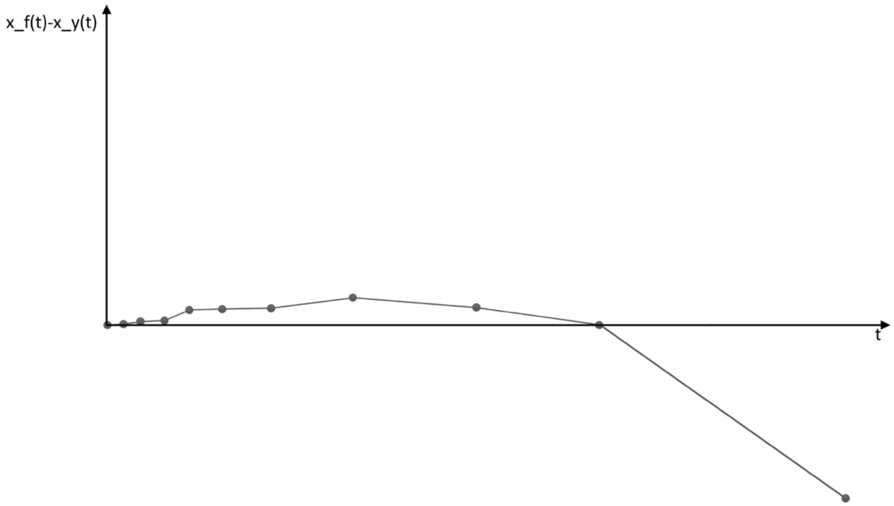

Fig. 5 Graphical representation of the comparison of the figures required by hyperbolic and exponential indifferences. Source: own elaboration

The 0.01 value of the interval [20,30] could be associated with the limits of the experimental structure since the intervals over which impatience is calculated have two by two coincident extremes.

Finally, to assess the relationship between objective and perceived time, the inverse function of the exponential that best approximates the hyperbolic trend was derived.

$$\boldsymbol{t} = -\frac{\ln(f(t))}{0.045} \tag{20}$$

In this way, point by point, t represents the time perceived by respondents and $\boldsymbol{t}$ represents the exponential time value with respect to the time perceived by the empirical discount function. Table 11 and Fig. 6 confirm the description in Table 3.

Table 11 Relationship between exponential and hyperbolic time in interval effect. Source: own elaboration

Hyperbolic time	Exponential time	Exponential-hyperbolic
0	0.000	0.000
2	7.477	5.477
4	15.403	11.403
7	20.362	13.362
10	35.765	25.765
14	37.883	23.883
20	40.901	20.901
30	51.169	21.169
45	54.274	9.274
60	60.179	0.179
90	65.197	−24.803

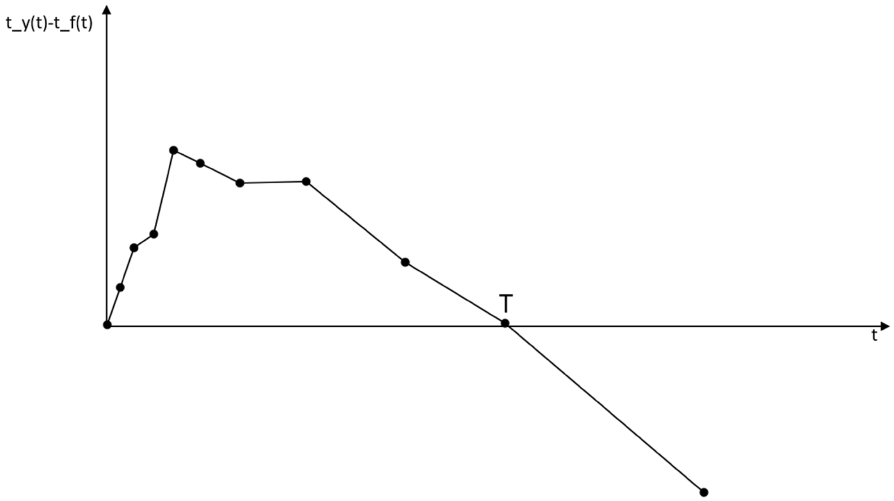

Fig. 6 Graphical representation of the difference between exponential and hyperbolic time. Source: own elaboration

This result proves that the time perceived by the empirical evidence is less than that assumed by the mathematical model until the point where the preferences determined by exponential and hyperbolic discounting coincide. Table 8 describes the deviation from consistency as time changes and quantifies instant by instant the distance from perfect financial rationality, as discussed in Table 2. Thus, we have experienced that the maximum distance between empirical and normative behaviour is reached when the distance between the normative and empirical time is maximum.

6 Discussion and conclusion

In the context of intertemporal choices, empirical evidence has shown that individuals exhibit hyperbolic discounting related to a lack of self-control or the inability to evaluate the best alternative (Prelec 2004) correctly. The direct consequence of these mechanisms is that investors manage their money with limited capacity and plans are not always optimal. Financial anomalies have been investigated and associated with the hypothesis of a perfectly rational investor (Shiller 1981). From a mathematical point of view, financial anomalies have been associated with non-constant impatience over time and a decreasing degree of impatience (Green and Myerson 1996; Lu and Yang 2015). These elements define a hyperbolic trend of the discount function and have been described concerning the emotional drives involved in decision-making (Ventre et al. 2022).

The present paper enriches the psychological mechanisms underlying hyperbolic discounting by considering the relationship between intertemporal preferences and subjective perception of time. How the individual relates to the inevitable passage of time has assumed an essential role with respect to the decision-making process of individuals (Shipp et al. 2009). Although time is unidirectional, individuals mentally move between the past, present and future (Bluedorn, 2002; Nuttin, 2014). How the present is perceived varies from individual to individual, and delving into its dynamics allows us to investigate organisational attitudes, motivation and performance (Shipp et al. 2009), as well as preference trends (Lucci 2013). Our study refers to the interval effect, a particular anomaly of the Discounted Utility Model. The relationships between the main quantities of a discount function (impatience, rate of decrease, discount rate) when it defines temporally consistent or inconsistent preferences with respect to the interval effect were investigated. The results obtained and summarised in Table 2 highlight the existence of two crucial points for the interaction of an exponential discount function with a hyperbolic one: $\bar{t}$ istant that defines the point at which impatience, decreasing and the discount rate of the exponential discount function exceeds those manifested by the hyperbolic discount function; a T instant ($T > \bar{t}$) at which the hyperbolic discount function becomes greater than the exponential discount function. From a mathematical point of view, determines conditions of existence for the interval effect. In fact, if we compare the preferences predicted by the financial model and those defined by hyperbolic discounting, it is evident from what has been said that the anomaly exists only if the inversion of the relationships shown in Table 2 occurs. From a behavioural point of view, this result defines time intervals in which the decision-maker defined by the classical "irrational" model differs from the normative attitude. This step is critical because it represents a new measure to quantify the distortion of the decision-making process for the preferences expected by the theory.

Section 4.2 investigated the relationship between the interval effect and subjective perception of time. The study begins with the relationship between time perceived by hyperbolic preferences and time perceived by exponential preferences. The instant T at which these preferences coincide represents the instant at which the two-time perceptions equal each other. Before T, the time perceived by hyperbolic preferences is contracted. This phenomenon can be explained from a psychological point of view by the work of Van Boven et al. (2010), in which emotional intensity is responsible for the reduction in perceived distance. Thus, if this contraction is correlated with emotional factors, which are specific to the decision-maker, our result highlights from a financial point of view that the contraction manifested by the individual for the uncertainty of the future is a behavioural and emotional factor that the empirical model did not consider in modelling.

To better investigate the relationship between the normative and the empirical preferences, on the other hand, we proved that the behaviour idealised by the financial model is also subject to a subjective perception of time. Then, since the normative preferences are determined by the subjective perception of time, we investigated the interval effect by disengaging from the relationship with the normative financial model. The investigation showed that in this case the interval effect always occurs for an investor with a time-varying discount rate because the perceived impatience on one interval is less than that perceived on two subintervals of the same length. The phenomenon for which empirical time is greater than objective time can be explained by the effect of the decision-making context in which the choice is made. Several studies have proven that the perception of time depends on the characteristics of foreign stimuli (Fraisse 1984) and that intervals are generally overestimated for them (Tse et al. 2004; van Wassenhove et al. 2008). This shows that the perception of time and the framing effect are related.

The implementation of the experimental part confirms the results described in the section about the financial description of the interval effect.

The contribution of this paper refers to different contexts. The first context concerns the financial behaviour of individuals. Investigating how perceptions of time affect intertemporal preferences can contribute to understanding behaviour in the face of debt and investment dynamics (Hoang and Hoxha 2016). Furthermore, since the perception of time is influenced by emotional factors (McLoughlin 2019; Droit-Volet and Meck 2007), clarifying how it affects decision-making can help to investigate the relationship between financial assets and investor sentiment (Schmeling 2009). Since risk can be seen as a feeling (Loewenstein et al. 2001) , the integration of subjective time in the study of financial behaviour can improve the description of risk propensity or aversion (Wang et al. 2011; Broihanne et al. 2014; Sun and Li 2010). In this regard, our contribution would make it possible to quantify the extent to which hyperbolic time deviates from empirical time and can provide a new criterion for the classification of the investor, in addition to personality traits (Conlin et al. 2015; Tauni et al. 2015). Classifying individuals would make strategic customisation easier, applicable not only in the financial but also in the social and therapeutic context. In fact, hyperbolic preference trends are also linked to cognitive degeneration (Milenkova et al. 2011), procrastination mechanisms, depressive attitudes, drug use (Petry 2001; Prelec 2004) and gambling addiction (Calluso et al. 2020).

Another possible development of the present paper refers to neurofinance. Since the perception of time has neurological origins, a further context to which this paper refers is neuroscience. We suggest that intertemporal choice theory, together with the study of the subjective perception of time, can help clarify the difference between the concept of impulsivity and the concept of impatience, often used as synonyms in the decision-making context. This is because impulsive behaviour is described as a mechanism of lack of self-control (Monterosso and Ainslie 1999) and can therefore be confused with impatience (Berlin et al. 2004; McLeish and Oxoby 2007). However, while impulsivity is the inability to restrain oneself, impatience is a feeling of restlessness concerning waiting. We suggest looking at impulsiveness as an attitude and impatience as a feeling: the subjective perception of time could be a point of contact to establish and formalise the difference between impatience and impulsiveness.

Further development of this work could involve generalising the results obtained for the hyperbolic discount function without considering the single anomaly of the interval effect.

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Declarations

Conflict of interest The authors have no relevant financial or non-financial interests to disclose.

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