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Convergences in cognitive science, social network analysis, pattern recognition and machine intelligence as dynamic processes in non-Euclidean space

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Abstract
Students of human cognitive and cultural processes, social networks, pattern recognition and machine intelligence often find that the coordinate systems resulting from commonly used measurement and analysis tools yield non-Euclidean configurations. Typically, researchers consider this unfortunate, and seek methods to return the spaces to Euclidean configurations. This article details all the known methods of such transformations, but presents evidence from multiple fields of inquiry that shows the non-Euclidean nature of the space is meaningful, and that all transformations to Euclidean form produce serious distortions to measured values. The article further presents methods for describing processes in the non-Euclidean spaces along with empirical examples of such uses.

Keywords Machine intelligence · Neural network · Multidimensional scaling · Social network analysis · Galileo theory · Multidimensional space · Non-Euclidean space · Artificial intelligence · Inertial reference frame

1 Introduction

Students of human cognitive and cultural processes, social networks, pattern recognition, machine intelligence and other disciplines frequently encounter square, symmetric matrices which may be interpreted as similarities, dissimilarities, distances, cosines or other measures of association. The elements of these matrices may represent words, pictures, texts, people, organizations, nations or any object whatsoever. Whatever the elements may represent, all such matrices can be represented as spatial coordinate systems by transformations to principle axes (Young and Householder 1938).[1]

[1] Not all investigators choose to make use of the spatial coordinate systems in their research. Some social network analysts, for example, prefer to use specifically network based models such as graph theory and the like, and eschew spatial representations (Lee and Tkach-Kawasaki 2018; Shapiro et al. 2018; Danowski and Park 2014). This paper does not advocate for spatial modelling over alternative approaches, but provides some cross-disciplinary findings that may be useful whenever spatial modelling is appropriate.

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Projection of original measurements onto principle axes is a distance-preserving linear transformation, so no information is gained or lost in this operation. The advantage gained, however, is that most mathematical operations are greatly simplified in an orthogonal coordinate system. Most important, given suitable rotation, translation and reflection rules, comparisons across multiple measurement sessions are greatly simplified (Hsieh 2004; Kaiser 1958; Woelfel et al. 1975, 1980, 1989). This in turn leads to the possibility of constructing inertial reference frames, in which explanatory variables such as force, mass, power and energy can be defined precisely (Barnett 1988; McIntosh and Woelfel 2017; Woelfel and Fink 1980).

This convergence of multiple independent lines of disciplinary inquiry shows an important underlying commonality, but it often reveals researchers freshly approaching old problems that have been considered carefully by earlier researchers in different disciplines. For example, often the coordinate systems resulting from commonly used measurement and analysis tools yield non-Euclidean configurations (Barnett and Rice 1985; Barnett 1989; Duin and Pekalska 2010; Duin et al. 2008; Torgerson 1958; Woelfel and Barnett 1982).

When confronted by non-Euclidean geometries, researchers often seek to transform these non-Euclidean spaces into a Euclidean form (Xu et al. 2014). These issues have been confronted before in psychometrics, factor analysis, communication research, social network analysis and now in pattern recognition and artificial intelligence. This paper will try to show clearly what non-Euclidean means, that the objections to non-Euclidean spaces are culturally based, and that there is no scientific reason not to consider cognitive and cultural processes as non-Euclidean phenomena. Based on research in sociology and communication, this paper will also show several examples of working directly with the non-Euclidean spaces as they are.

1.1 Concepts of space in social science

The concept of space in physical science has morphed from Aristotle's notion of the surface of any given body through Newton's empty, anisotropic, infinite space to Einstein's space–time continuum and the finite, heterogeneous space permeated by fields of quantum physics. Meanwhile, concepts of space in the study of cognitive and cultural process, as well as machine intelligence, have remained amorphous and ill-defined. Aristotle argued that, while sense perceptions of existing objects might have a physical referent, abstract concepts induced from those perceptions have no physical referent and are hence immaterial and non-corporeal. Abstract, immaterial concepts have no "place," i.e., *location*, in Aristotle's world, which is still, for the most part, the cultural underpinning of contemporary Western civilization, hence space as such has little meaning in psychological or cultural processes.

The sociologist Emile Durkheim's notions of *collective representations* and *social facts* are often presented in space-like terms in the late nineteenth and early twentieth centuries, but he provides no explicit descriptions of what kind of space he might imply (Durkheim 1968). Henri Poincare speaks of ideas as "…something that resembles Epicurus' hooked atoms…" that fly helter-skelter about and occasionally collide and stick together to form new ideas, but says nothing about the structure or characteristics of the space in which this process occurs (Poincaré 1946). Early factor analysts like psychologists Charles Spearman and L. L. Thurstone and their followers provide mathematical structures that are literal vector spaces, but focus their interest on the basis vectors (factors) in the space and say little or nothing about the spaces themselves, even going so far as to simply delete items which are positioned between factors rather than on or very close to them[2] (Harmon 1976).

Thurstone's well known method of attitude scaling similarly implies a space within which lie various "positions" relative to any given attitude topic. But Thurstone's goal was to identify positions that are arrayed on a line that represents an underlying attitude rather than to construct a holistic picture of the space. His famous "box problem" showed his underlying interest in spatial models—he hoped to find an algorithm that could take as input the sizes and shapes of various boxes and yield as a solution length, width and height—but never accomplished this himself and left the problem to posterity (Thurstone 1947). At this time historically, quantitative social scientists were focused closely on *variables* and *functions*. The concept of spaces and fields,[3] although well established in physics, was only beginning to emerge in the social sciences.

A correct solution to the problem of deriving a spatial coordinate system from interpoint distances was provided by the mathematician and physicist Gale Young and the mathematician A. S. Householder in 1938, and this solution was refined and widely promulgated by the psychologist Warren Torgerson in his classic text *Theory and Method of Scaling* (Torgerson 1958). For the first time, Young, Househoder and Torgerson focused attention on cognitive objects embedded in space, rather than simply on the basis vectors that spanned the space.

This model, generally referred to as "classical multidimensional scaling" or (erroneously[4]) as "metric multidimensional scaling", ran into trouble with the social science community immediately, since the outcome of using the algorithm with typical measurement procedures (such as complete paired comparisons, magnitude estimation, ten-point scales or even five-point Likert-type scales) virtually always produced spaces that were both high dimensional[5] and non-Euclidean.[6]

Social scientists' response to high dimensional non-Euclidean space was and remains irrationally hostile. Why rational scholars should react with alarm at non-Euclidean geometry when the very surface of the planet we live on is a non-Euclidean spheroid may seem hard to explain to a mathematically sophisticated scientist, but the culture of the social sciences is one in which deeply held philosophical and cultural beliefs are often considered sufficient grounds on which to overturn empirical observations (Woelfel 2016).

First, as mentioned earlier, psychological and cultural entities have been considered immaterial and non-corporeal in Western cultures since Aristotle, and hence do not occupy space, a belief which closes off efforts to construct a spatial model of cognitive and cultural

[2] The extent of Thurstone's focus on lines rather than spaces is evident from the title of his 1935 book and his presidential address to the APA: *Vectors of the Mind*.

[3] The first employment of the concept of the field was provided by Kurt Lewin in 1935, but his main work on the topic didn't emerge until 1951, and even then, his use of the term was vague and far from operational (Lewin 1951).

[4] Psychologists, sociologists and communication scientists used the term "metric" to distinguish this method from newer "non-metric" methods, but the procedure seldom met the mathematical criteria for a metric space. See method 5 below.

[5] In the social science community, "high dimensional" usually means "more than three" and often "more than two."

[6] This issue cropped up among factor analysts also in the presence of negative eigenvalues, which were virtually always treated as indicators of measurement error and a sign that further extraction of roots should cease.

processes.[7] Secondly, it is almost universally accepted among social scientists that precise measurement of these immaterial entities is not possible, so measured values are virtually always treated as subordinate to philosophical assumptions. When measurements contradict our philosophical belief that cognitive space ought to be Euclidean and 3D, we reject the measurements.

Among the methods developed by early psychometricians to reject the measurements and assure that the results met our assumptions were (1) using only the first three or so dimensions, or only the real (non-imaginary) dimensions; (2) finding an inflection point in the plot of eigenvalues (the scree line) and using only those dimensions before the inflection; (3) Attneave's additive constant, a scalar chosen such that adding it to every distance resulted in a completely Euclidean space (Attneave 1950); (4) Lambda min, a procedure which involved adding the absolute value of the largest negative eigenvalue to every eigenvalue and renormalizing to the new eigenvalues, and, most commonly, (5) non-metric scaling (Kruskal 1964; Shepard 1962) in which monotonic transformations of the measured values are performed until the resulting space was Euclidean and low (2 or 3) in dimensionality. All of these procedures resulted in sometimes massive changes in the originally measured distances. Although simple and widely available methods were well known for determining whether these changes in measured values were within the statistical confidence intervals of the original measures, with the exception of one research community, so strong were the assumptions that this was seldom if ever attempted.

1.2 Concepts of space in artificial intelligence and pattern recognition

The use of vector spaces is commonplace in machine intelligence and pattern recognition, but, in spite of the general sophistication of computer scientists in the mathematics of vector spaces, cultural biases toward Euclidean configurations still abound. As Xu et al. opine:

> Unfortunately, in many applications, the original distance measures violate the restrictive conditions required in a Euclidean space. Examples of distances which exhibit these violations include those used for problems such as shape matching in computer vision [18], [14]. Thus, dissimilarity data cannot be used to construct an embedding into a vector space without non-Euclidean distortion. The resulting loss of geometric meaning hinders the use of potentially powerful machine learning techniques such as Support Vector Machines (SVM) and Neural Networks (Xu et al. 2014).

The notion that non-Euclidean spaces lack "geometric meaning" is part of the Euclidean bias that underlies Western culture. Like the original psychometricians, computer scientists attempt to "correct" the non-Euclidean character of their data, utilizing some of the same procedures developed by the psychometricians, albeit with different terminology:

> Here the main techniques available are spectrum clip, spectrum flip and spectrum shift. Spectrum clip [6] only considers the subspace associated with the positive eigenvalues of the Gram matrix and ignores the subspace associated with the neg-

[7] This is not meant as an insult to the intelligence of social scientists. Virtually every member of Western culture believed this uncritically; cf. the physicist Erwin Schrödinger at about the same time (1950): "*For the observing mind is not a physical system, it cannot interact with any physical system*." (Schrödinger 1996, Emphasis in original).

ative eigenvalues. Distances are overestimated compared to the original distances. Spectrum flip [15], [5] uses the magnitudes of the negative eigenvalues. This has the effect of reversing the sign of the negative distance contributions from the complex dimensions of the pseudo Euclidean space. In this way, the structure of the data residing in the negative eigen-space is potentially preserved. Finally, spectrum shift [12], [11] forces the Gram matrix to be positive semi-definite by adding a suitably chosen constant $c \geq -2\lambda\text{min}$ to the off diagonal elements of the squared dissimilarity matrix, where λmin is the minimal eigenvalue of the Gram matrix. Compared to both spectrum clip and spectrum flip, spectrum shift does not change the order of similarities (Xu et al. 2014).

Notice that the first method, *spectrum clip*, is a version of methods 1 and 2 above; *spectrum flip* is a version of method 3 (lambda min), while *spectrum shift* is a version of method 4, Attneave's additive constant.

Not all authors regard the non-Euclidean character of dissimilarities data as problematic. Laub et al., found that important information is contained in the non-Euclidean components of paired comparisons data (Laub et al. 2006), and Duin and Pekalska show that important information contained in the non-Euclidean paired comparisons is lost by the standard methods of "correcting" the data to a Euclidean form (Duin and Pekalska 2010; Duin et al. 2008). As Xu et al., suggest, "These results cast doubt on imposing geometricity and emphasize that the discriminating power of the original dissimilarity measures are more important than the Euclidean property" (Xu et al. 2014).

1.3 Concepts of space in sociology and communication

As we have said, concepts of space, at least in a mathematical sense, are generally absent from models of cognitive and cultural processes in the social sciences, with few exceptions. One such exception is the research community associated with what is usually called the Galileo Model. The primitive concepts of the method are Durkheim's *collective representations*, which are indicated by the average beliefs of members of a given culture, and the *social objects* of George Herbert Mead (Mead 2015), which are the set of concepts in term of which individuals define themselves.

Unlike the Aristotelian model underlying Western culture which views concepts as immaterial and non-corporeal, the model assumes these concepts have a real physical existence as clusters of neurons in human brains. As a sociological rather than a psychological theory, Galileo assumes that the connections among neurons are mediated not only by direct synaptic junctions, but also through communication across individuals so that the set of all human brains constitutes a sparsely connected neural network of about 7.5 billion brains times 80 billion or so neurons per brain. These individual units are replaced at a rate of about 8% per year. It is this collective social network that constitutes the machine that generates concepts which are subsequently communicated to individual brains through a process of socialization.

Although concepts are generated in the cultural network, they are not universally distributed through it, but rather originate in one (or perhaps several) local neighborhoods and diffuse throughout the communication network over time. Thus, the concept of non-Euclidean geometry may have originated with Gauss and Riemann in the nineteenth century, but did not appear in social science until the mid-twentieth century.

Because the network of neurons is sparsely connected, and because synaptic connections are irrelevant unless their associated neurons are active, researchers generally attend to small "neighborhoods" of the overall space. This is consistent with Mead's concept of the *situation* (Mead 1934). The meanings of concepts are considered to be situational, and may have different meanings in different situations. A Mustang near a Camaro is a car, while a Mustang near a Palomino is a horse, and a Mustang near a Messerschmitt is an airplane.

Mead's theory proposes that the individual self is defined in relation to these social objects, which vary from one situation to another. Consequently, the self is only situationally defined, and might be quite different across situations. We take the objects in any situation to correspond to the clusters of neurons that are active in that situation.

Mead also proposes that individuals' behaviors are governed by their self-concepts. In the model, behaviors are considered social objects, and may also be arrayed in space. The theory proposes that the likelihood that a given individual will enact any given behavior is proportional to its distance from the self-concept in space.

Studies are typically conducted in one of two ways, only one of which will be discussed in the present paper.[8] A traditional study involves identifying the main concepts pertinent to the situation under study (usually 10–40 or so concepts) by interviews, text analysis or similar inductive qualitative method, followed by complete paired comparison difference judgments among the concepts identified, where each distance is estimated as a ratio to a given standard distance, usually but not always chosen from within the neighborhood under study.[9] The averages of these dissimilarities are then projected onto orthogonal coordinates using the Torgerson method (Woelfel and Fink 1980).

1.4 Cognitive space is non-Euclidean

In over 40 years of empirical research by sociologists and communication researchers, every space resulting from these procedures of measurement and analysis has been non-Euclidean, exhibiting both positive and negative eigenvalues (Evans 2017). In the beginning, researchers were every bit as alarmed by the fact that space did not turn out to be the familiar 3-dimensional Euclidean space of everyday life, and tried all of the "correction" procedures discussed earlier. The complete paired comparison magnitude estimation procedures, however, produce quite precise measurements at modest sample sizes, with average relative errors in the vicinity of 10% or less at about 100 cases, so, particularly in larger samples, transformation of the mean dissimilarities by more than a few percent is a clear violation of measurement integrity. In the end, it became clear that the only two alternatives were to accept the fact that cognitive and cultural spaces were non-Euclidean

[8] Another set of collective representations recognized by Durkheim are the artifacts created by the culture, Among the most important of these are texts, and the Galileo community usually examines these using Catpac^{tm}, an unsupervised neural network that calculates the synaptic connection weights among words in the text using a propinquity-based algorithm (Woelfel 2014). The matrix of connection weights is then projected onto orthogonal coordinates using the Galileo algorithm. Discussion of this second method of research is beyond the scope of the present paper.

[9] The magnitude estimation complete paired comparison measurement method is chosen for two reasons: first, it is the most precise psychometric technique available, and second, it is the only psychometric measurement procedure consistent with the definition of measurement in physical science and engineering: *comparison to some standard*.

Table 1 Great circle distances among five cities on earth

City	Buffalo	Melbourne	Berlin	Vladivostok	Santiago
Buffalo	0				
Melbourne	16294	0			
Berlin	6500	9921	0		
Vladivostok	9958	9086	8457	0	
Santiago	8530	12667	12514	17774	0

Table 2 Coordinates of five cities in non-Euclidean space

Coordinates	1	2	3	4	5
Buffalo	4538.3242	−6040.9688	−770.9818	84.3256	4463.502
Melbourne	−5620.9028	6685.4331	230.6651	−104.4382	3623.2007
Berlin	620.332	−1041.8146	6079.6099	11.5438	−2247.1523
Vladivostok	−7610.9912	−3985.3447	−2916.5723	−141.3346	−3101.6455
Santiago	8073.2363	4382.6953	−2622.7214	149.8811	−2737.9031
Eigenvalues	175680078	117364688	51805511.5	−60591.23	−60591.23

Table 3 Percent error ignoring imaginary eigenvectors

Percent error	Buffalo	Melbourne	Berlin	Vladivostok	Santiago
Buffalo	0				
Melbourne	0.14	0			
Berlin	43.7	16.2	0		
Vladivostok	25.6	24.42	24.03	0	
Santiago	30.9	43.7	0.08	0.03	0

and high dimensional, or to reject the accepted definition of measurement in science and engineering.[10]

1.5 What does non-Euclidean mean?

The general cultural belief is that Euclidean space is "normal" space, and non-Euclidean space is a rare, even exotic, exception. In fact, the opposite is true. Euclidean space is an idealization like points and straight lines that seldom if ever occur in everyday experience, while examples of non-Euclidean spaces abound.

A space is non-Euclidean when the axioms of Euclid do not hold. In general, this means that the space is not flat. The surface of the earth, of course, is not flat, and the axioms of

[10] Most social scientists, in fact, do reject the standard definition of measurement—comparison to some standard—and replace it with a much broader definition: assignment of numbers to observations according to some rule. Most social science measurement devices, e.g., five-point scales, rank-orders, etc., would not be recognized as measurements by scientists and engineers (Torgerson 1958).

Euclid do not hold for it. The distances among cities, for example, are not straight lines, but rather geodesics or curves. Table 1 shows the great circle distances (in kilometers) among five world cities.

Using Torgerson's procedure implemented in Galileo v 5.7 yields the following eigenstructure (Table 2).

Since the coordinates are a linear transformation of the original distances, the distances can be reproduced from the coordinates to within rounding error by the extended theorem of Pythagoras (see below). But when the imaginary coordinates are omitted (the most common method of dealing with them) considerable error is introduced, as Table 3 shows:

Ignoring the imaginary eigenvectors in an effort to "Euclideanize" the space creates errors in the original measurements ranging from 0.03% to 43.7%. This is well outside the original confidence intervals of the measured distances, which, in this instance, are accurate to a few kilometers. The other methods of adjusting the space all produce equivalent or larger errors.

2 An example

So far this paper has attempted to show that many phenomena in many disciplines exhibit non-Euclidean geometries when exposed to widely used analytic methods, and that transformations to eliminate these non-linear aspects require distortions of the originally measured values beyond statistical confidence levels. Why does this matter? The primary reason, of course, is that the defining principle of science as opposed to other systems of knowledge such as philosophy, religion, literature and the like, is that observations—measurements—are the final arbiter of the merit of any theory (Feynman 1997). Changing precisely and reliably measured values on the basis of preferences, assumptions or to simplify calculations is strictly inadmissible.

On a practical level, even for those who believe precise measurement of human cognitive processes is impossible, allowing measured values to be arbitrarily adjusted within each measurement session makes it impossible to compare results across time and across conditions, since changes might be a function of the adjustment procedure. Only when the integrity of the measurement process can be maintained across measurement sessions is it possible to detect regularities and law like processes, thus strengthening the cultural bias that human beings are special beings immune to physical and biological constraints.

2.1 Some examples

Whether it is possible to maintain a consistent metric across measurement sessions and show law like, systematic behavior, including geometric structures, is an empirical question, and we present here several examples from recent ongoing research.

In 1986, (Barnett 1988a) asked 241 undergraduates at an Eastern Polytechnic Institute to estimate the pairwise distances among several barnyard animals and their attributes. Thirty-seven years later, McIntosh and Woelfel (2017) conducted a similar study which included 11 of the same concepts. He reports:

> Comparing the 55 mean values common to both studies gave a correlation of $r=0.877$, $p<0.0001$, $df=53$, $t=13.2$. Entering both sets of means into the Galileo

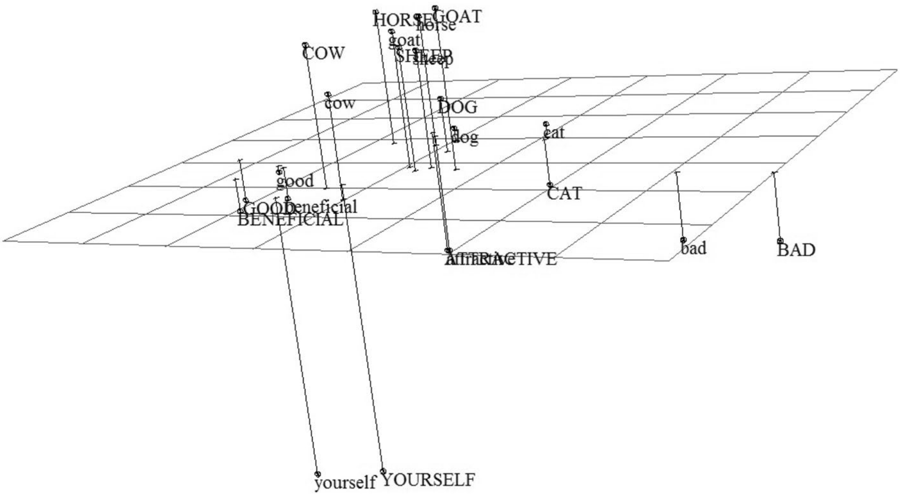

Fig. 1 Barnett's data (upper case) and replication after 37 years (lower case)

Table 4 Stability of the eigenstructure

Concept	Tl Magnitude	T2 Magnitude	Correlation	Angle
Correlations among the position vectors				
1. Bad	93.34	71.51	0.988649	8.6
2. Self	69.90	75.03	0.960027	16.3
3. Cow	47.06	42.20	0.914279	23.9
4. Beneficial	38.32	29.10	0.767806	39.8
5. Dog	39.29	38.73	0.789872	37.8
6. Horse	49.08	52.71	0.948125	18.5
7. Cat	51.29	53.91	0.922482	22.7
8. Good	42.69	25.12	0.667405	48.1
9. Sheep	41.66	42.01	0.987697	9.0
10. Attractive	40.48	37.66	0.996701	4.7
11. Goat	49.60	46.79	0.963079	15.0

version 5.7 program yielded spaces with very similar structures, as shown by the plots of the first three dimensions in Fig. 1.[11]

Equally important, the non-Euclidean character of the space is quite stable, as Table 4 shows. The first half of Table 4 shows the relations among the rows, which are the position vectors of the concepts; the second half shows the relations among the column vectors, which are the dimensions onto which the position vectors are projected. Note in particular that the over time correlation among the imaginary eigenvectors (9, 10 and 11) is 0.51, 0.92

[11] The following Figures show only the first three eigenvectors of these spaces solely for illustrative purposes. These are all multi-dimensional, non-Euclidean spaces, and all calculations are based on the entire eigenstructure.

Table 5 Dimension 8 is the null vector; dimensions 9–11 are imaginary

Dimension	T1 Magnitude	T2 Magnitude	Correlation	Angle
Correlations among the dimensions				
1	127.62	98.00	0.972411	13.5
2	113.81	103.41	0.980088	11.5
3	59.42	69.49	0.939563	20.0
4	52.56	45.21	0.856567	31.1
5	42.81	62.83	0.922666	22.7
6	34.44	42.23	0.702645	45.4
7	26.09	34.17	0.857776	30.9
8	0.19	0.26	0.873177	29.2
9	10.62	8.85	0.505827	59.6
10	40.60	40.42	0.916274	23.6
11	77.15	76.15	0.954882	17.3

Table 6 Distances among three concepts (from Serota et al. 1976)

	Me	The rich	Big business
Me	0	313	237
The rich	313	0	19
Big business	237	19	0

and 0.95 respectively. This shows that the imaginary part of the space is clearly reliable, in spite of the fact that the samples are fairly different, with Barnett's sample made up primarily of engineering undergraduates at a polytechnic institute and McIntosh's sample primarily communication undergraduates at a large state university, and the time lag is 37 years. These are clearly not random errors that may be casually transformed away (Table 5).

The statistical stability of the eigenstructure is not the only reason the non-Euclidean characteristics of the space are considered meaningful. Departures from Euclidean structure are often substantively meaningful and easily interpreted. Serota et al., for example, in a study of 55 undergraduates from a large public Midwestern university and a community college showed significant departures from Euclidean space in a study of political ideology:

> A second finding of significance is the non-Euclidean character of the ideological structure of both samples. For both groups, six of the 14 characteristic roots (eigenvalues) are negative and large, indicating substantial departures from a linear Euclidean structure.

Table 6 shows an example of a violation of the triangle inequalities that is substantively meaningful from the Serota et al. study.

No Euclidean triangle can be made from these distances, but they are substantively meaningful. The Rich and Big Business are clearly considered to be close (19 units) for obvious reasons. The average college undergraduate, however is far from the rich (313 units), but not quite so far from big business (237 units), because he or she is in contact with big business every day.

Other examples abound. In political science, for example, some theorize that the scale of liberal-conservative is not a straight line, but rather is horseshoe-shaped, with the extreme left and the extreme right closer to each other than to the center (Backes 1989). That this theory is widely disputed only enhances the need for research, but simple linear mathematics will not be able to decide the issue. Similar examples abound in everyday life, for example, many people like iced tea, and many also like hot tea, but few prefer tepid tea. Both iced and hot tea are closer to preferred than is the midpoint, tepid, so this line, too, is horseshoe shaped. Many more examples of substantively meaningful violations of the Euclidean axioms are available in Evans (2017).

2.2 Warp factor

Galileo researchers use a statistic similar to the negative eigenfraction (NEF) called the Warp Factor (w), which is given by the ratio of the sum of the positive eigenvalues to the sum of all the eigenvalues (Woelfel and Fink 1980). If there are no negative eigenvalues, the value of the Warp Factor is 1.0. If any negative eigenvalues are present, the Warp Factor rises. Some research shows that the Warp Factor increases proportionally to the amount of force impressed upon the space via external messages (McIntosh and Woelfel 2017). Barnett et al. (2013) examined the international news network on the topic of terrorism between 2000 and 2012 and found that the warp factor changed dramatically following the September 11, 2001 attack on the World Trade Center (Barnett 2013; Barnett et al. 2013). These data provide substantial evidence that the geometry of cognitive spaces is a variable that is affected by external, measurable factors.

2.3 The coordinate system as an inertial reference frame

Generally, researchers have little interest in the transformation to principle axes itself, which is well known and straightforward and dates from Jacobi in 1846, but rather are concerned with the use of the resulting coordinate system as the basis for an inertial reference system against which changes in the relationships among the concepts may be described as motions in space. The fact that the reference frame is non-Euclidean is not a problem; distances among points in the space may be calculated from the extended theorem of Pythagoras (Eq. 1):

$$\boldsymbol{S}_{ab} = \sqrt{\sum_{\mu=1}^{\boldsymbol{N}} (\boldsymbol{x}_{\mu a} - \boldsymbol{x}_{\mu b})^2} \tag{1}$$

where $\boldsymbol{S}_{ab}$, the distance between concept a and concept b; $\mathbf{x}_{\mu a}$ the coordinate of concept a on the μth dimension; $\mathbf{x}_{\mu b}$ the coordinate of concept b on the μth dimension; **N**, the number of dimensions.

Computationally, coordinates on dimensions corresponding to negative eigenvalues are imaginary, so the squares of their differences are negative; otherwise computation is the same as any Euclidean space.

A key concept in the model is the *message*. Messages are statements that relate two or more concepts in the space, e.g., *pigs are beneficial*, or *hogs are beneficial and attractive*. Theory predicts and research shows that messages behave like vectors. *Pigs are beneficial* can be represented by the vector from *pigs* to *beneficial* in cognitive space. *Hogs are beneficial and attractive* can be represented as the vector average of the vectors from *hogs* to *beneficial* and from *hogs* to *attractive.*

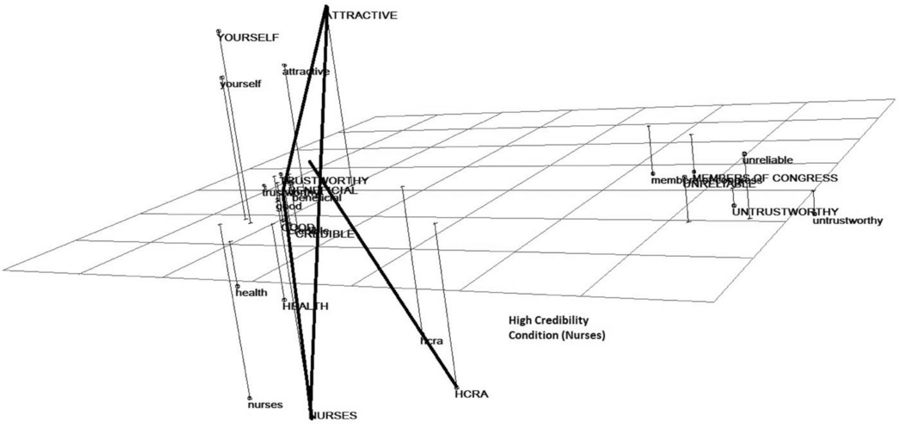

Fig. 2 Control group and nurses condition

Although the research procedure results in concepts being arrayed as points in the space, theory rather assumes that each point represents the center of a field of meaning which diminishes as a function of distance from the central point. Every point in the space is thus considered meaningful, being a combination of the meanings of nearby field points. In practice, the meaning of any point is given as the vector average of nearby points. Thus, the vector average of *beneficial* and *attractive* defines a point in space assumed to be a combination of the meanings of these two concepts.[12]

Research shows that, when messages are delivered to samples of people, the space resulting from their pairwise dissimilarities estimates changes in ways predicted by the theory. Figure 2 shows the first three dimensions of a control group and a treatment group rotated[13] to a common coordinate system (Control group is in caps, treatment group in lower case.) The treatment group read a message that said "This questionnaire will ask your opinion about the Health Care Reform Act (HCRA) which a committee of nurses said was beneficial and attractive" while the control group's message said simply "This questionnaire will ask your opinion about the Health Care Reform Act (HCRA")."

The theory predicts that the HCRA should "move" toward the average of the vectors *nurses, beneficial* and *attractive*, which lies at the center of the darkened triangle to the left of the figure in the treatment group compared to the control group. The bold vector from HCRA's position in the control group through its position in the treatment group points very close to the predicted point. The actual angle between the unweighted predicted trajectory and the observed trajectory is 45°, corresponding to a correlation coefficient of 0.703 ($p < 0.05$).

Figure 3 shows the first three dimensions of a control group and a second treatment group rotated to a common coordinate system (Control group is in caps, treatment group

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[12] Recent research with Word2Vec have shown that vector operations on the Wod2Vec space can produce substantively meaningful combinations of words (Mikolov et al. 2013).

[13] Rotation of non-Euclidean spaces is a generalization of earlier "Procrustes" procedures, modified to deal with imaginary coordinate values (Hsieh 2004; Woelfel and Fink 1980; Woelfel et al. 1980, 1989; Woelfel and Barnett 1982). Unlike the original Procrustes myth, the legs of the guests are not stretched or amputated to fit the bed, but non-Euclidean distances remain invariant under these rotations.

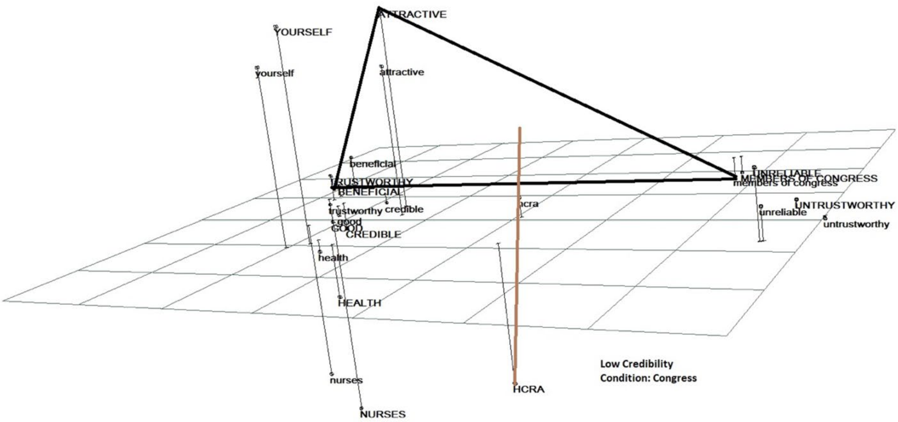

Fig. 3 Control group and congress condition

Table 7 Estimates of mass ratios beneficial/attractive

Condition	Ratio
Pig	0.516
Hog	0.724
Boar	0.963
Swine	0.608
HCRA Nurses	0.605
HCRA Congress	0.455
Mean	0.6451
SD	0.1805

in lower case.) The treatment group read a message that said “This questionnaire will ask your opinion about the Health Care Reform Act (HCRA) which a committee of members of congress said was beneficial and attractive,” while the control group is the same as shown in Fig. 2. In this case, the theory predicts that the HCRA will “move” from its position in the control group toward the middle of the darkened triangle in Fig. 3. The bold vector from HCRA’s position in the control group through its position in the treatment group points very close to the predicted point. The actual angle between the unweighted predicted trajectory and the observed trajectory is 51°, corresponding to a correlation coefficient of 0.648 ($p < 0.05$).

2.4 Inertial factors

Notice that, while the experimental concept moves (approximately) along the predicted vector, it travels only part of the way. We attribute this to the masses of the concepts. We don’t use the term mass in its formal sense, but rather consider that some concepts in the brains of respondents have been built up over many messages for a long time, have more tissue and more connections and are consequently harder to change (move) than other less substantial concepts.

In the McIntosh study discussed earlier, estimates of mass of four synonyms pig, hog, boar and swine correlated highly (r = 0.995 and 0.983) with two indices of the frequency of occurrence of those terms in English. Estimates of the relative inertial masses of the terms *beneficial* and *attractive* in six independent conditions in two different studies, for example, are given in Table 7.

The purpose of these examples is to illustrate the following points: First, it is quite possible to make measurements of human cognitive and cultural processes using the same measurement rule as the physical sciences and engineering: comparison to some standard. Second, when such measures are made, the resulting spaces virtually always exhibit non-Euclidean properties. Third, no transformation of the space that leaves the measurements invariant to within their measured precision can eliminate these non-Euclidean characteristics, Fourth, inertial properties of cognitive elements such as the masses of cognitive and cultural objects and forces of messages that are invariant across measurement sessions and experiments can be reliably established, Fifth, warpage associated with the non-Euclidean characteristics can be seen to be the result of identifiable external impacts on elements in the spaces, and Sixth, predictable, law-like processes can be measured precisely and reliably in non-Euclidean spaces.

These findings support the contention that non-Euclidean characteristics of cognitive and cultural spaces are not methodological artifacts that should be transformed away, but empirical findings that need to be respected.

3 Discussion

This paper attempts to show that there is no substantive or philosophical reason to assume that the space in which intelligent activity, individual or social network, organic or machine, takes place should be Euclidean. On the contrary, there is substantial evidence that cognitive and cultural processes, whether individual or embedded in social networks, when measured using standard measurement rules, turn out to be non-Euclidean, and processes taking place within the non-Euclidean space may well be law governed and provide a useful model of some aspects of cognition.

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