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A proposed method to evaluate the quality of services using Fuzzy sets theory

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Abstract This paper seeks to provide the services sector with a focus on the assessment of quality and for this purpose, a technique that may able a quantitative approach to evaluating quality is proposed. The use of the fuzzy sets theory to process data was used, thus allowing a more flexible and suitable insight into the characteristics of the service sector. An extension of the technique for order performance by similarity to the ideal solution was used. This informs managers of the distance from the company/s current level of quality, if compared to a company of perfect quality by means of an overall evaluation. The same technique was used to detect changes in the level of quality during the period surveyed by using a stratified assessment. Finally, a practical application of the approach proposed is presented.

Keywords Quality · Fuzzy sets · Service · TOPSIS

1 Introduction

The competition among companies to remain in the market is increasingly fierce and as a result there arises the need for them to evolve procedures with a view to seeking greater efficiency and quality. As with any activity, this evolution will only be achieved when the current situation is assessed. It is by doing this which enables a distinction to be made between different criteria in order to learn about those which have, or those which have not, contributed to quality.

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In an evaluation, it is important to make a thorough analysis of the quality of service rendered in the past, which may enable defects to be corrected in accordance with current specifications so as achieve better results in the future.

However, for this evaluation to be effective, it must consider the characteristics of service activities that distinguish them from others and prevent the models used with regard to the manufactured goods. According to Macur (2011), measuring quality has therefore become a methodological challenge, which necessarily requires a detailed assessment that, first of all, determines what the quality of the current service is.

Another point that needs to be raised, and one which is also related to the characteristics of service activities, is that several models available in the literature and computer programs, in order to analyze data, use techniques that do not consider the consequences of such particularities in the evaluation, such as, the client's subjective assessment, and the ambiguity and imprecision of the answers obtained. These are characteristics that are more intense in assessing the quality of the service sector, when compared to assessing the quality of goods.

This paper proposes a method to evaluate the service quality by applying Fuzzy set theory and TOPSIS methodology. The choice in this work of the Fuzzy set theory proposed by Zadeh (1965) is founded on this being a way of dealing with data in a different manner than does classical set theory, in which the elements are classified according to their pertinence.

Since it considers data in fuzzy environment, quality assessment approaches of human judgment which in its individual rationality is characterized by vagueness and imprecision, and yet this presents other benefits such as the ability to use linguistic terms during the evaluation, which facilitates the expression of human feelings and processing the data. Hsieh et al. (2010) state that Fuzzy set theory is the computer technology tool best suited to assessing in detail issues related to vagueness and uncertainty.

Finally, by conducting an analysis using the TOPSIS—technique for order performance by similarity to ideal solution, proposed by Hwang and Yoon (1981), which seeks to solve the problem of the comparison with the perfect alternative, i.e., to obtain information on the distance of the quality of service rendered in relation to a perfect service using the criteria considered for evaluating the quality (based on ordering of the alternatives made by TOPSIS), and it also seeks to detect changes in the level of quality.

This paper is organized as follows: In Sect. 2, the basic theory of Fuzzy sets and the TOPSIS method are set out. Section 3 describes the proposed model. Section 4, by way of illustration, gives the details of a case study and discusses the results. Conclusions from the study are drawn in Sect 5.

2 Basic theory

2.1 Fuzzy set theory

Fuzzy set theory was proposed by Zadeh (1965) followed by a study of decision making in fuzzy environments by Bellman and Zadeh (1970). Since then, several studies have used Fuzzy Set Theory, such as those by Benítez et al. (2007); Hu et al. (2010), and Chintha and Vinodh (2011). As evidenced by citations from the literature, Fuzzy set theory is a research discipline that continues to grow (Lin and Hsu 2011).

This theory can be formally defined, according to Zadeh (1965), as a class of objects with continuous degrees of pertinence, which assigns each object to a degree of pertinence, which is between the interval [0,1]. Fuzzy sets apply the notion that for each element there is an associated value in the range of 0–1, indicating its degree of pertinence to the set, where

Table 1 Linguistic terms and fuzzy numbers

Linguistic term	Fuzzy number
Poor	(0, 4, 5)
Satisfactory	(4, 5.5, 7.5)
Good	(7, 7.5, 8.5)
Very good	(8, 8.5, 9.5)
Excellent	(9, 9.5, 10)

the value 0 (zero) indicates complete non-pertinence to the set and a value (a) indicates the certainty of pertinence.

In the case of a satisfaction survey, linguistic terms such as "good", "satisfactory" and "poor" can be used to assess the service, which is what this study does. These terms represent individual consumers' evaluation of a given criterion.

In these and in other cases in which the data are susceptible to subjective judgment, the use of Fuzzy sets contributes a flexible assessment, i.e., for each value input into the system, a degree of continuous pertinence is assigned.

According to Tsaur et al. (2002), this feature sees to it that Fuzzy sets are recommended for capturing decision makers' preferences, because they use everyday terms, which are easy to interpret. According to the authors, it is suitable for situations in which the ambiguity of the concepts is associated with human subjective judgment.

2.1.1 Triangular fuzzy numbers

This is the pertinence function that will characterize the linguistic terms to be analyzed. Such numbers interpret the classification assigned to given linguistic terms by subjective assessment. Thus, for each linguistic term used to assess the variable, there is a fuzzy number.

The fuzzy numbers most used to assess quality are triangular. Such use is due to their greater suitability for aspects inherent in the process of quality assessment, for example, the process of data collection and the need to aggregate opinions to obtain an overall evaluation.

One way of representing a *fuzzy* number is by using equations which consider a triangular number (a, b, c), as in Eq. 1.

$$\mu(x) = \begin{cases} \frac{x-a}{b-a} & se\ a \leq x \leq b \\ \frac{x-c}{b-c} & se\ b \leq x \leq c \\ 0 & se\ x \leq a\ ou\ x \geq c \end{cases} \quad (1)$$

In the case of the triangular *fuzzy* number that will be used in this study, each point of the scale of a linguistic assessment characterized by vagueness and ambiguity is represented by a trio of numbers in the reference interval (X) defined for the set.

In this paper it is proposed that that for each linguistic term available for expressing an opinion as to the quality of service in each aspect, a triangular fuzzy number, or its variation, be assigned to it as this will permit a qualitative assessment to be made.

In relation to the assessment, five options of linguistic terms to express the judgment in relation to the service quality are used. Note the overlap of the values of the ratings which enables the imprecision that characterizes subjective evaluations to be reduced. The five linguistic terms and their parameters are shown in Table 1 as is the description of their values.

Having defined the fuzzy numbers for each linguistic term, what is then carried out is the fuzzification of the data, i.e. the data are transformed into linguistic terms—poor, satisfactory, good, very good, excellent—in the numbers previously determined.

2.1.2 Defuzzification

Defuzzification is the transformation of numbers in a fuzzy format to the space of real numbers, because some applications of fuzzy sets require this processing after the data have been treated.

After having assigned a fuzzy number to each term—and, consequently, a pertinence function to each linguistic term—the fuzzy environment is characterized for the treatment of the data. With the data, the individual results can be aggregated using the fuzzy operator, thus obtaining an assessment of the group, i.e. a fuzzy number for each term.

Having obtained the opinion of the group of evaluators, the fuzzy sets must then be transformed into a real number in order to proceed to the comparison between the alternatives in each term assessed. For this process of defuzzification, the literature describes several methods, some of which were described by Zhao and Govind (1991) such as the mean of maxima, the centre of gravity and the alpha cut method.

However, an extension of the ordering technique will be used which will be presented below, the main advantage of which is the non-transformation of fuzzy numbers. Consequently, there is a gain in performance of the method proposed, because, no matter how accurate the transformation methods are, they not able to portray, accurately, assessment in the fuzzy environment.

2.2 TOPSIS method

The TOPSIS technique was proposed by Hwang and Yoon (1981). It aims at establishing a rating of the alternatives from pre-established multiple criteria, where the best alternative is the one that maximizes the distance to the worst solution while minimizing the distance to the best solution.

According to Kim et al. (1997), the three main advantages of TOPSIS are: the search for human rationality in choices; providing a value that considers the distance to both the best and worst alternative; computational simplification which enables spreadsheets to be used. Moreover, it offers the possibility of working with multiple criteria simultaneously.

These advantages have been observed in several studies in a great variety of areas. For example, the TOPSIS technique has already been used successfully in allocation analysis by Yoon and Hwang (1985); quality control by Yang and Chou (2005); selecting logistics information technology by Kahraman et al. (2007); the construction of projects by Tan et al. (2010); in critical success factors in the manufacturing industry by Khanna et al. (2011); and in determining regions for investment in Turkey by Eraslan (2011). In addition, there are articles by Tsaur et al. (2002) and Benítez et al. (2007) in the area of quality in services.

In the TOPSIS technique, the ideal solution is considered as being the best assessment in each criterion established in the method of assessment used by the group of professionals. For example, for each criterion, the highest values of the evaluation group (EG) are used which, as seen, is the aggregate value of each professional's individual assessment. On the other hand, the negative situation is configured with the EG's lowest values.

The best alternative is one that is closest to the ideal solution and farthest from the negative solution in the simultaneous evaluation. Figure 1 graphically represents the idea developed in TOPSIS.

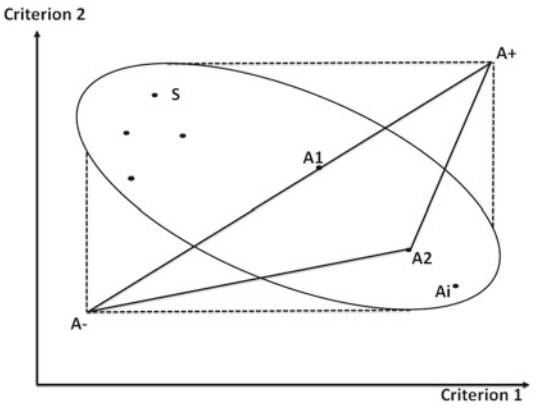

Fig. 1 The objective space of the two criteria—distances for ideal and negative solutions. Hwang and Yoon (1981)

If A^+ is considered the ideal solution and A^- the negative solution and there is the problem of choosing between the alternatives A_1 and $A_{2,}$, the alternative A_1 is closer to the ideal solution A^+ and at the same time, to the negative solution A^-, when compared with the alternative A_2.

Therefore, solution A_1cannot be considered superior on the justification that it is closer to the ideal solution, because the upper and lower distances of these two points (A_1 and A_2) in comparison with the ideal and negative solutions, respectively, cannot be compared separately, since there is a trade-off between the upper and lower distances.

The TOPSIS technique helps to solve this problem, since in the calculation for ordering the alternatives, relative closeness is considered, i.e. the two distances are correlated, both for the ideal solution and for the negative solution.

3 Proposed model

The extension used to tackle the TOPSIS technique in a fuzzy environment has a mathematical reasoning for ordering the alternatives, close to the one used by Chen (2000). However, the reason that justifies this extension differs from the one described in his paper.

The aim of the approach taken is to propose a technique for, besides ordering the alternatives, to evaluate them independently. That is, to provide an idea of how far the alternatives are from an optimal solution, even if this optimal solution is utopian.

The possibility of a utopian solution is mentioned since the variables considered for assessing the quality of the service may characterize conflicts and trade-offs, in which advances or improvements in one variable imply a lower investment in others.

Step 1: Aggregate the fuzzy numbers— the fuzzy aggregation operator was used to aggregate the evaluations of each parameter of the questionnaire, such as hospitals, clinics, laboratories, coverage and general aspects.

$$\tilde{A} = (a_1,\ b_1,\ c_1) = (1/n) \oplus \left(\tilde{A}_1 \oplus \tilde{A}_2 \oplus \ldots \tilde{A}_n\right) = \left\{ \frac{\sum_{i=1}^{n} a_1^{(i)}, \sum_{i=1}^{n} b_1^{(i)}, \sum_{i=1}^{n} c_1^{(i)}}{n} \right\} \tag{2}$$

The sets $\tilde{A}$ represent the global value of each fuzzy set evaluated by customers. By using these operators, the individual evaluations of each client in each dimension are transcribed into a global evaluation aggregation.

Step 2: Selecting the ideal solution and the negative ideal solution— as to the mathematical reasoning for ordering, the difference lies in the definition of the optimal and negative solutions to be considered in ordering the alternatives. The linguistic term with the highest value—'excellent ', for the optimal solution A^+, will be assigned to all variables, and the parameters previously defined for the definition of the fuzzy numbers will be maintained, while the variable of least value, i.e. 'poor' is assigned to the negative solution A^-. To calculate the rating of the alternatives, the following equations are used.

$$A^+ = \left\{\left(\max V_{ij} | j \in J\right), \left(min V_{ij} | j \in J'\right), i = 1, 2, 3, \cdots m\right\} \tag{3}$$

$$A^- = \left\{\left(\min V_{ij} | j \in J\right), \left(\max V_{ij} | j \in J'\right), i = 1, 2, 3, \cdots m\right\} \tag{4}$$

Step 3: Determining the Euclidean distance in fuzzy environment— having determined the values of A^+ and A^- for all criteria, the continuity of the calculations is similar, both in the extension of Wang and Lee (2007), as in that of Chen (2000). The Euclidean distance in a fuzzy environment is defined as being:

$$d^+\left(V_{ij}, A_j^+\right) = \sqrt{\frac{1}{3}\left[\left(a_1^+ - a_1\right)^2 + \left(a_2^+ - a_2\right)^2 + \left(a_3^+ - a_3\right)^2\right]} \tag{5}$$

$$d^-\left(V_{ij}, A_j^-\right) = \sqrt{\frac{1}{3}\left[\left(a_1^- - a_1\right)^2 + \left(a_2^- - a_2\right)^2 + \left(a_3^- - a_3\right)^2\right]} \tag{6}$$

Step 4: Sum of the Euclidean distances with respect to the values of the linguistic terms of the alternative— the sum of the distances of the linguistic terms or criteria j for each alternative is calculated, thus determining the values of S_i^+ and S_i^-.

$$S_i^+ = \sum_{j=1}^{n} d^+\left(V_{ij}, A_j^+\right) \tag{7}$$

$$S_i^- = \sum_{j=1}^{n} d^-\left(V_{ij}, A_j^-\right) \tag{8}$$

Step 5: I—Determining the closeness relative to the ideal solution for each alternative— the placement in the ordering C_i of each alternative is determined by the result from the following equation.

$$C_i = \frac{S_i^-}{S_i^+ + S_i^-} \tag{9}$$

The values of C_i are in the range [0; 1] and the closer to 1 that C_i is, the closer the criterion will be to the positive ideal solution. C_i gets close to 1 when S_i^+ tends to zero. This occurs when the alternative i is evaluated for all variables V_{ij} with values that are equal to A_j^+, i.e. their observations are the best in all criteria, when compared with the other alternatives considered in the database.

The inverse situation is also easily observed, C_i gets close to zero when S_i^- tends to zero. This occurs when the alternative i is evaluated for all variables V_{ij} with values that are equal

to A_j^-, i.e. their observations are the worst in all criteria, when compared with the other alternatives considered in the database.

Thus, it is possible to establish a ordering for decision making based on two aspects: (I) proximity to the ideal situation and (II) to the worst situation; considering the multiple criteria taken together.

If used for ordering and defining the parameters $A^+$and A^-, respectively, the best and the worst assessments from customers, instead of the maximum and minimum values of the linguistic terms adopted, would see to it that the results would present a comparison with current practices, and thus possibly provide unrealistic information on the magnitude of the maximum improvement that can be achieved in the quality of the services.

Step 5: II—Use when assessing the service is performed in shorter time intervals, i.e., subintervals of the interval used in assessing the overall quality—Wang and Lee (2007) criticize the proposals of other researchers and propose a new way to use the TOPSIS technique in a fuzzy environment. In their proposal, an operator R is defined in the set of fuzzy numbers which seeks to determine the highest A^+ and lowest A^-. As there are two triangular *fuzzy* numbers 'Y' and 'Z', with the format (a_1, a_2, a_3), 'Y' is deemed to be greater than or equal to 'Z', if R(Y) > R(Z), where:

$$Rs(N) = \frac{a_1 + a_2 - 2 \times L(s)}{a_1 - a_3 + 2 \times (U(s) - L(s))} \tag{10}$$

In this operator, to calculate each R, what are considered are the values: $L\ (s)$ which is the minimum a_1 among the fuzzy numbers of set s and $U\ (s)$ which is the maximum a_3 among the fuzzy numbers of set s.

The method proposed by Wang and Lee (2007) is used to evaluate the quality of the service rendered in shorter intervals of time, i.e. subintervals of those used in evaluating the overall quality.

4 A case study

4.1 Survey

The method proposed is to evaluate the quality of services in a health insurance company. To do so, a questionnaire was applied which was divided into 5 parts so as to have a basis on which to rate : (1) the quality of the hospitals, (2) the quality of the clinics, (3) the quality of the laboratories, (4) the quality of the coverage of the plan, and (5) the quality of general aspects. In all, there were 23 questions: seven relating to the hospitals; six to assess the clinics; six to assess the laboratories; three on the coverage; and one related to the general aspects. This questionnaire was authored by the company.

The parts relating to the hospitals, clinics and laboratories were subdivided into 'attending to the clients' and 'facilities'. In the first part, the items assessed were 'promptness in being attended', 'quality' and 'reception'. As to 'facilities', assessments were made of their 'appearance', and level of 'comfort' and 'cleanliness'. Only in the part concerning 'hospitals', in the subdivision of 'facilities', was a question on 'hospital admissions /accommodation' added.

In the part on the 'coverage' of the plan, what were evaluated were the 'number of clinics, laboratories and consulting rooms for conducting exams', 'the number of doctors offered by the health insurance plan' and an evaluation of the service of 'scheduling appointments'. Four

Table 2 Assessment of fuzzy aggregate

Evaluation items	Fuzzy number
Hospital	(7.52, 8.23, 9.15)
Clinics	(7.62, 8.29, 9.19)
Laboratories	(7.86, 8.49, 9.33)
Coverage	(7.15, 7.94, 8.95)
General aspects	(7.55, 8.21, 9.15)

hundred and sixty-five clients were interviewed so as to evaluate the quality of the services rendered within a 5-week period.

4.2 Findings

Step 1. Fuzzy aggregation

This step shows the fuzzy aggregation for each of the five parts of the questionnaire (hospitals, clinics, laboratories, coverage and general aspects). The aggregations fuzzy of the 5 week period were calculated using Eq. 2.

Table 2 presents the exact values of the evaluation in the fuzzy environment of each part of the questionnaire. As can be seen, the highest rating was given to the Laboratories, with a fuzzy number (7.86, 8.49, 9.33). On the other hand, the lowest rating was for Coverage, with the fuzzy number (7.15, 7.94, 8.95). Despite the difference between the numbers, all the services were evaluated as being between the terms 'good' and 'excellent'.

The information from the evaluation through Fuzzy set theory explained that, in all five parts of the questionnaire, the parameter on the extreme left of parameter 'a', which gives the smallest value of the reference range that belongs to that set (or linguistic term), is within the range of the term 'good' and, of the five parts, only 'coverage' did not obtain this parameter exclusively in the range of the term 'good' because the value of 7.15 is in the transition of the term 'satisfactory', which ends at 7.5, and 'good', which starts at 7.0.

The central parameter 'b' for all items, which represents the value of the reference range that belongs completely to the set/linguistic term, positioned within the range of the term 'very good' except for 'Coverage' by a difference 0.051, demonstrates the proximity of the assessments to the term 'very good'. At the other extreme, parameter 'c' represents the value of the most optimistic reference range for the set/linguistic term. All parts of the questionnaire obtained a value within the range of the variable 'excellent', except for 'Coverage' by a difference of 0.047.

The analysis of these parameters points to the quality of the service rendered by the health insurance plan as nearing a rating considered 'excellent' in all aspects considered in the evaluation. However, the actual distance from the service rendered to an assessment of 'excellent ' cannot be measured.

Step 2. Ideal solution and negative ideal solution

The ideal solution A^+ is considered to be a trio of identical parameters allocated to the trio of the linguistic term 'excellent' (9, 9.5, 10) and for the negative solution A^-, it is a trio with parameters identical to the trio assigned to the linguistic term 'poor' (0, 4, 5).

Step 3. Euclidean distances in a fuzzy environment

In this step, the values of the Euclidean distances of the evaluations in a fuzzy number to the ideal solution were calculated using Eq. 5, and for the negative solution Eq. 6, in each criterion (j) that was used to assess the quality of service.

Table 3 Ordering of the assessment of the quality of service

Hospitals		Clinics		Laboratories		Coverage	General aspects
Attending to client	Facilities	Attending to client	Facilities	Attending to client	Facilities		
0.813	0.824	0.823	0.836	0.868	0.849	0.776	0.820
0.819		0.830		0.858		0.776	0.820

The values of the Euclidean distances d^+ and d^-obtained for the question 'number of doctors offered by the plan' was 1.57 and 5.18 respectively. While the minimum, in whatever case, is 0 (zero) and the maximum possible, in this case, was 6.73.

Step 4. Sum of Euclidean distances with respect to the values of the linguistic terms of the alternative

The values of these distances in the various factors considered in the survey were added up so that there could be an overall assessment of the service in its different characteristics. Eqs. 7 and 8 were used so as define the ordering.

Step 5. Calculation of the ordering of the alternatives

This is the calculation of the ordering, by Eq. 9, which will determine the real situation of the service measured in relation to a service of optimal performance.

For the question 'number of doctors offered by the plan' the ordering (Ci) obtained a value of 0.76. On grouping the three aspects that make up this part of the questionnaire, the 'coverage' had a value of 0.77, with the values of the rating varying in the interval [0,1] in which the closer to 1 (one), the closer the service assessed will be to the ideal or perfect service.

Table 3 shows the ordering, that is, the distances to the optimal service, which vary from 0 to 1.

It is seen that the health plan, among all the criteria evaluated, had the worst assessment in the aspect of 'Coverage' which was rated as 0.77. The best aspect evaluated refers to the 'Labs', both in serving clients and facilities, and was evaluated as having superior quality to that of the clinics and hospitals.

4.2.1 Stratified assessment of quality

On continuing the evaluation of the services rendered by the health insurance plan, the next step is to seek to determine possible variations in the level of quality during the evaluation period.

Using the same data collected by the survey, but stratified by week, the extension of TOPSIS proposed by Wang and Lee (2007) was applied to detect variations in the level of quality within the assessment period.

By this method, for all the criteria analysed, what will be determined is the week in which each criterion analysed received its best and worst rating, indicated by the value 1 and 0 on the rating scale. This evaluation is only a comparative one between the results of the 5 weeks; positioning at 1 or 0 does not indicate, therefore, that the criterion was rated as 'poor' or 'excellent', respectively.

Table 4 Assessment of hospitals by period

Hospitals			
Period	($\mathbf{d}^+$)	($\mathbf{d}^-$)	Ordering
Week 1	1 e 6*	5*	0.58
Week 2	–	6 e 7*	0.28
Week 3	2, 4 e 5*	–	0.84
Week 4	3*	–	0.66
Week 5	7*	1, 2, 3, 4*	0.29

* Criteria: (1) promptness in attending to clients, (2) quality of care, (3) receiving the service, (4) the facilities, (5) comfort of the facilities, (6) cleanliness of the premises, (7) hospital admissions/accommodation

Hospitals by period

It was observed that there was considerable variation in the level of quality during the evaluation period, which can be seen in Table 4, where the criteria considered were 'promptness in attending to clients', 'quality of care', 'receiving care', the 'appearance of the facilities', 'comfort facilities', 'cleanliness of the premises', and 'hospital admissions/ accommodation'.

The second column of the table shows in which period of the survey, the best assessment (d^+), in all periods, occurred. The third column shows in which period the worst assessment (d^-) occurred. And the last column, shows the ordering of the periods as to the level of quality, where the closer to 1 (one), the better the ordering is.

In the, the items were evaluated as satisfactory (0.58), and even had a positive evaluation, above 0.80, in three of the seven aspects assessed, but in the other aspects, the evaluation varied from satisfactory to poor, below 0.70, with one aspect—comfort of the facilities—having the worst rating in that week of all the weeks.

Similarly, the criteria that received the same rating, 1 or 0, did not necessarily have the same level of quality, but it was, during that week that they obtained the best and worst rating of all the 5 weeks.

In the Week 2, the clients rated the services offered in the hospitals as being of poorer quality, when compared with the previous week. There was a drop in all the aspects, except in the comfort of the facilities. However, even this had low quality (0.15).

In the Week 3 we can see a great evolution in the quality of services relating to the seven criteria analyzed in relation to the previous week. All aspects evaluated in this part of the research increased and for three of the seven criteria, the rating was the best one given over the 5 weeks. Therefore, the week obtained the best ratings in the whole period, in the part related to the hospitals, which received 0.84 in the rating scale, as seen in Table 4.

In the Week 4, only two points received a better rating, 'promptness in attending to the client' and 'reception', while the others had a drop in quality. Note the reduction in 'attending to the client' and 'appearance' of the facilities which saw the largest falls. Consequently, the rating for this period dropped to 0.66, as per Table 4.

In the last week of evaluation, the Week 5, the reduction in the quality level of services rendered in the hospitals continued, and came close to the ratings in the Week 2, which was the worst of the five. In the Week 5, four of the seven aspects received their worst rating of all in the whole period. Only the criterion of 'hospital admissions' was regarded as having a good level of quality, and achieved its best rating in that week of all the 5 weeks.

Table 5 Assessment of the clinic by period

Period	(d^+)	(d^-)	Ordering
Week 1	1, 2, 3, 5 and 6*	–	0.98
Week 2	–	–	0.62
Week 3	–	–	0.51
Week 4	4*	–	0.58
Week 5	–	1, 2, 3, 4, 5,6*	0.00

* Criteria: (1) flexibility in attendance, (2) quality of care, (3) receiving the service, (4) the facility (5) the comfort of the facility (6) the cleanliness of the premises

Clinics by period

In relation to services provided in clinics, there was a gradual reduction in the level of quality, as shown in Table 5.

The Week 1 has the best result. ff the six criteria, five had their highest rating in that week, and therefore, the rating was close to the optimum, as it was 0.98 on the rating scale.

In the Week 2, there was a very sharp fall, which reduced the overall assessment to 0.62. In this week, there was no rating which stood out as being positive or negative. All that is pointed out is the aspect of 'Reception' which, from being the best assessment of the entire study period, fell to 0.37, the second worst rating of this criterion over the 5 weeks.

The reduction in the quality level continued in the Week 3. Again no one factor stood out as the best or worst ordering. Overall the week in question obtained 0.51 points in the overall rating of the six criteria evaluated.

The Week 4 had the best assessment in the category of 'appearance of the facilities'. Thus, the overall rating of the week saw a slight increase to 0.58. The other criteria underwent insignificant changes which did not have an important effect on the overall assessment.

During the last week, the Week 5, the worst assessment was for the clinics of the health insurance plan. All six items were rated as being of lower quality in that week. Consequently, the overall assessment of the week received 0 (zero) in the rating scale of the weeks.

Laboratories by period

In the evaluation of the aspects related to the laboratories accredited by the health plan, the results showed a reduction in the level of quality similar to that presented in the clinics, with the four worst results of the six issues under evaluation, these being concentrated in the last week, as shown in Table 6.

The Week 1, had the best result as it received 0.88 in the rating scale. This result was driven by the following aspects: quality and reception in attending to the client and the cleanliness of the premises. These criteria received their highest ratings in this week.

In the Week 2, only 'Promptness' in attending to clients received a better assessment, when compared with the previous week, which was when it had its best evaluation. There was a downward variation in the other criteria evaluated with regard to the level of quality. Those that stand out are : 'comfort' which received its worst rating of the 5 weeks and 'cleanliness', which fell from receiving the best rating, i.e. 1 in the rating scale by TOPSIS, to only 0,1.

The evaluations of the Week 3 showed an improvement in the level of quality of the aspects related to the facilities in the laboratories. However, there was a reduction in aspects related to attending to the client. 'appearance' and 'comfort' stand out as having had the best ratings, which for 'comfort' was an improvement compared to its worst assessment which it had received in the previous week. Thus, the week improved its rating from 0.54 to 0.68.

Table 6 Assessment of the laboratories by period

Period	(d^+)	(d^-)	Rating
Week 1	2, 3, 6*	–	0.88
Week 2	1*	5*	0.54
Week 3	4,5*	–	0.68
Week 4	–	4*	0.32
Week 5	–	1, 2, 3, 6*	0.12

* Criteria: (1) flexibility in attendance, (2) quality of care, (3) receiving the service, (4) the facilities (5) the comfort of the facility (6) the cleanliness of the premises

Table 7 Assessment of coverage by period

Period	(d^+)	(d^-)	Rating
Week 1	–	–	0.49
Week 2	2*	1*	0.56
Week 3	–	2, 3*	0.03
Week 4	3*	–	0.84
Week 5	1*	–	0.56

* Criteria: (1) Number of clinical laboratories and offices for examinations, (2) Number of physicians offered by the plan, (3) How do you rate our service schedule

However, this growth was not sustained. On the contrary, there was a reduction in the Week 4 in all aspects evaluated. The 'appearance of the facilities' was evaluated as the worst among the 5 weeks, although it had been evaluated as the best in the previous week. Thus, the rating of the week fell to 0.32 in the TOPSIS rating.

In the Week 5, once again, the aspects displayed a reduction in quality, except 'appearance', since, in the previous week, it had received its worst rating. All aspects used to evaluate the 'service received in the laboratory', plus the 'cleanliness' of the premises, received their worst ratings of the 5 weeks. Consequently, the rating of the week was only 0.12 in the interval [0,1] used by TOPSIS for ordering.

Coverage by period

In the fourth part of the questionnaire on the coverage of the health plan, there was a variation in the quality of services, mainly in the Week 3. However, this result was improved on, in the weeks immediately following, as seen in Table 7.

The Week 1, did not emphasize anything as to the best and worst ratings among the periods assessed. It got a score of 0.49 in the TOPSIS rating. However, it is pointed out that in the aspect of the 'number of doctors offered by the plan', the evaluation indicated good quality of service with a rating of 0.83 being awarded.

In the Week 2 two aspects stood out: the 'number of clinics, laboratories and consulting rooms for conducting tests' on the negative side; while the' number of physicians offered by the plan 'was the best of the period. The third aspect obtained a median value, 0.63. Therefore, the result of the week was 0.56.

In the Week 3, there was a sharp drop in the level of quality. The evaluation by TOPSIS gave a rating of just 0.03. With two of the three aspects being the worst of all the 5 weeks, and the third aspect evaluated being very close to the worst, with a rating of only 0.12.

An increase in quality above the deficit of the previous week was observed in Week 4, resulting in 0.84 in the joint rating of the three aspects, as shown in Table 7. All three aspects obtained gains in quality, according to the clients' assessment. Note that the aspect of 'how do you evaluate our scheduling of appointments service' which had the worst result in the

Table 8 Assessment of general aspects by period

Period	(d^+)	(d^-)	Rating
Week 1	–	–	0.84
Week 2	1*	–	1
Week 3	–	–	0.24
Week 4	–	–	0.43
Week 5	–	1	0

* Criteria: (a) a generally as (a) M. (a) classifies our plan?

Table 9 Overall assessment of service by period

Period	Rating
Week 1	0.79
Week 2	0.50
Week 3	0.59
Week 4	0.58
Week 5	0.17

previous week, rose in the evaluation to the best result within the 5 weeks. This improvement saw to it that the rating of 'coverage' maintained, in general, an average result.

The last week of assessment, Week 5, despite having the best rating in one aspect and its not having any that were the worst in the period, had a result less than that of the immediately preceding week with 0.56 in the rating. This result is due to a sharp drop in the evaluation of quality in the aspect relating to scheduling appointments which obtained only 0.26 in the rating interval [0, 1].

General aspects by period

As to the question on 'general aspects', a negative change was observed between the first and last weeks of assessment, as per Table 8. The best ratings in this regard occurred in the second week, while the worst was in the Week 5.

Overall assessment by period

On aggregating the five items for an overall evaluation of the service, per week, a decrease in the level of quality between the first and last week can be observed, as per Table 9.

With this evaluation stratified per week, large variations can be observed in quality between successive weeks during the evaluation interval, despite the general ratings of the period having indicated that clients approve the services rendered by the health insurance plan. The importance of this identification is emphasized, even in cases where the client approves the quality of service, because the service should be provided more homogeneously.

The desired homogeneity in the quality of services implies a greater number of clients has been satisfied with them as well as that the provision of these services to clients who have already tried them repeatedly leads to loyalty. Therefore, being able to reproduce the level of quality is fundamental to business success, and this method of assessment has proved accurate in identifying possible variations in the aspects considered.

5 Conclusions

The method proposed in this paper proved to be adequate in the various stages of the evaluation. Initially, the possibility of formulating questions in which the evaluator chooses linguistic terms, according to Tsaur et al. (2002), facilitates the response, a feature that, according to

the authors, indicates the applicability of fuzzy sets to capturing the evaluators' preferences, in addition to measuring the ambiguity of the concepts associated with subjective judgment made by humans.

The linguistic terms linked to fuzzy numbers enable the inaccuracy of responses to be diminished since, by definition, these numbers are related to a reference set wider than that tackled by classical set theory, besides the overlap of the numbers which enables a smoother transition between adjacent evaluations.

The current distance of the quality of service rendered by the health insurance company in relation to an ideal service was calculated by means of the extension to the TOPSIS technique proposed. The data showed a level of quality of the various aspects discussed which were close to the term 'very good'

The item that was farthest was the coverage offered by the insurance plan, its rating being less than 0.8 in the rating range [0, 1]. It is noteworthy that in this part, three items were covered: the number of doctors; the number of clinics, laboratories and consulting rooms for examinations; and the scheduling service. It is seen that the three items have an impact on the other parts, especially on hospitals, clinics and laboratories.

The remaining items obtained a rating, as to the evaluation of quality, which were close to each other. However, they were still far from being a service that was considered perfect, and therefore, there are opportunities for improving the service. These opportunities also instigated an internal review in order to identify issues that prevented the client from making a better judgment of the quality of services.

The internal evaluation was undertaken by identifying variations in the quality level during the research period with the method proposed by Wang and Lee (2007). It was found that in all criteria measured in the different parts of questionnaire there were changes in the level of quality, i.e. the customer perceived changes in the quality of service.

These changes occurred both in the positive sense, raising the level of quality and in the negative sense, reduction in the quality of service. And yet, the overall assessment of the whole period judged the service as being one of good quality. However, this inconsistency in quality, besides causing some clients to be dissatisfied, impedes making them loyal, since clients are not sure that that will get the same quality of service when they are next attended to, despite the overall assessment indicating a satisfactory quality.

As to the analysis of the part of 'coverage', it was found that the assessments related to 'scheduling appointments' and the 'number of clinics, laboratories and medical offices for conducting exams' in four of the 5 weeks had a rating of less than 0.70 in the interval [0, 1]. Therefore, despite the number of doctors having been considered satisfactory during the assessment, the scheduling system and the number of establishments for the examinations were found wanting. Such instances of failure had a much sharper effect than the approval of the former, which implies the quality of the coverage of the health insurance plan was low.

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