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RESEARCH NOTE

Applying a markov chain model in quality function deployment

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Abstract The relationships between customer requirements and technical measures are typically resolved by a cross-functional team with the assumption that the relationships are able to be identified objectively. However, due to the limited knowledge and experiences, determining the appropriate relationship could be difficult since the decision makers might not have enough information to evaluate the actual relationship. Moreover, the importance of technical measures is typically expressed in the current time period. It would be of interest to trace the future trends of technical measures since customer needs are fulfilled by technical measures. Under such circumstances, a Markov chain model could be an approach to model the relationship and monitor the trends of technical measures from probabilities viewpoints. With the needed probabilities, the dynamic relationships as well as the trends of technical measures can be performed by different time periods. Finally, the relationships and future trends of technical measures can be updated when the new information is available.

Keywords Markov chain model · Quality function deployment · Technical measure · House of quality · Transition matrix

1 Introduction

Quality function deployment (QFD) is one of the very practical quality systems tools commonly used to fulfill customer requirements and improve customer satisfaction for product development and production (Wu et al. 2005; Chan and Wu 2002, 2002–03). The philosophy of QFD is that specifications of quality requirements and

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deployments of quality for products and services should be started as early as possible in the life cycle (Sullivan 1986; Hasuer and Clausing 1988). The most commonly seen QFD structure consists of four phases, i.e., product planning (also known as house of quality (HOQ)), parts deployment, process planning, and production planning (Chan and Wu 1998, 2002–03). In product planning, HOQ links the voice of the customer to technical measures through which detailed processes and production plans can be developed in other phases of QFD (Chan and Wu 2002–03). Parts deployment is to translate important technical measures into parts characteristics; process planning is to translate important parts characteristics into process operations; and, finally, production planning is to translate key process operations into day to day production requirements (Chan and Wu 2005).

House of quality, depicted in Fig. 1, is composed of six major steps (Wu et al. 2005): (1) customer needs (WHATs), (2) planning matrix, (2) technical measures (HOWs), (3) relationship matrix between WHATs and HOWs, (4) technical correlation matrix, and (6) technical matrix. Chan and Wu (2005) have summarized that HOQ is first to identify customer needs of the product, incorporate the company's competitive priorities, and then fulfill customer needs by appropriate technical measures. The structures of the other three phases in QFD are essentially the same to the HOQ. Thus, most QFD studies focus mainly on the first phase of QFD. Generally, in this four-phase QFD, HOQ plays an important role to meet or even exceed customer requirements by listening to the voice of the customer, translating the voices to appropriate technical measures, and identifying important technical measures for the next stage of QFD.

Chan and Wu (1998, 2002–03) have addressed that completing the relationship between customer requirements and technical measures is a vital step since the final analyses depends heavily on the relationship matrix. To identify important technical measures, the relationship between WHATs and HOWs should be carefully examined by a cross-functional team. The relationship is typically evaluated by analyzing to what extend the technical measure could technically relate to and influence the customer requirement (Chan and Wu 2005). When the relationship between WHATs and HOWs is completed, the technical ratings of HOWs can be prioritized by a

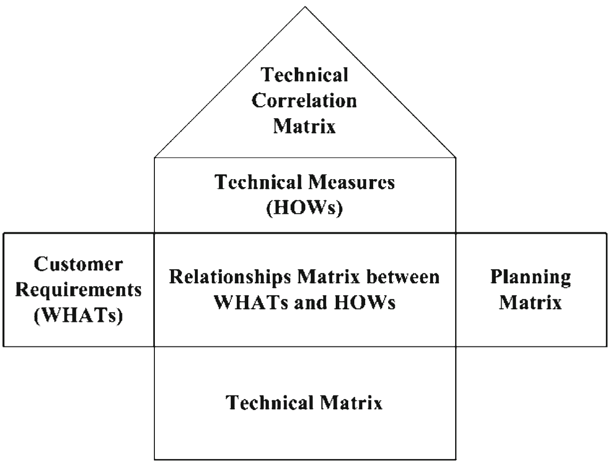

Fig. 1 The house of quality

variety of methods, such as simple additive method, technique for order preference by similarity to ideal solution, the operational competitiveness rating, grey relational analysis, and grey model (Chan and Wu 1998, 2005; Wu 2002, 2002–03, 2006). To properly determine the relationship between WHATs and HOWs, the most commonly seen approach is to use a 9–3–1 weighting system to represent strong, medium, and weak relationships (Hauser and Clausing 1988; Clausing 1994). The underlying assumption of this weighting system is that team members are clearly able to identify the relationship. However, in reality, due to the limited knowledge and experiences, determining the appropriate relationship could be difficult since the decision makers might not have enough information to evaluate the actual relationship. Under such circumstances, a Markov chain model could be an approach to model the relationship from probabilities viewpoints.

Each relationship between WHATs and HOWs could be described, for instance, by strong, medium, weak, and none with the respective probabilities. In addition, the relationship that would vary from time to time can be modeled by probabilities. For example, a particular relationship is currently considered to be strong, and the decision makers then can discuss how the relationship would stay in strong and go to medium and weak with appropriate probabilities. With enough information, the relationships between WHATs and HOWs can be further analyzed, and the trend for each technical measure can be monitored as time goes by. Thus, the decision makers would be able to identify the more important technical measures in a timely basis. To successfully satisfy customer needs, the technical measures with higher importance should be addressed earlier.

This paper is organized as follows: Section 2 reviews a Markov chain model. In Sect. 3, a framework of applying a Markov chain model in QFD is depicted. An illustrated example is provided to show how the framework works in Sect. 4. Finally, conclusions are in Sect. 5.

2 Review of a markov chain model

A Markov chain model is commonly used to study the short- and long-run behavior of certain stochastic systems, where the state of the system in any particular period is uncertain (Pfeifer and Carraway 2000; Stawicki and Lawrence 1994; Tan 1997; Betancourt 1999). Markov chain models assume that the system starts in an initial state or condition but the initial state or condition will be changed over time. Predicting these future states involves knowing the system's probability of changing from one state to another, and transition probabilities typically presented by the matrix describe the manner where the system makes transitions from one period to the next period by conditional probabilities of being in a future state given a current state.

Markov chain models are summarized as follows based on Anderson et al. (2003), Render and Stair (2000), and Taha (1997). Let a finite set $S = \{E_j | j = 1, 2, 3, \ldots, m\}$ represent the exhaustive and mutually exclusive states of a system at any time. Initially, at time t_0 the system may be in any of these states. Let $a_j^{(0)}$ be the absolute probability that the system is in state E_j at time t_0. If the system is Markovian, then define

$$p_{ij} = P\left\{X_{t_n} = j | X_{t_{n-1}} = i\right\}, \tag{1}$$

where p_{ij} is the transition probability of going from state i at time t_{n-1} to state j at time t_n, and assume these probabilities are stationary over time. X_{t_n} and $X_{t_{n-1}}$ in Eq. (1) are both random variables. The transition probabilities from state E_i to state E_j can be further expressed in a matrix form:

$$P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1m} \\ p_{21} & p_{22} & \cdots & p_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ p_{m1} & p_{m2} & \cdots & p_{mm} \end{bmatrix}, \tag{2}$$

where individual p_{ij} values are typically empirically determined, and $\sum_j p_{ij} = 1$ since all entries in matrix P are nonnegative and the entries in each row must sum to 1. Therefore, a Markov chain model is defined as a transition matrix P along with the initial probability $a_j^{(0)}$ associated with the states E_j (Taha 1997).

Let $a_j^{(n)}$ be the state probabilities of the system after n transitions at time t_n. The general expression of $a_j^{(n)}$ in terms of $a_j^{(0)}$ and P after a specified number of transitions is as follows (Taha 1997):

$$a_j^{(1)} = a_1^{(0)} p_{1j} + a_2^{(0)} p_{2j} + a_3^{(0)} p_{3j} + \cdots = \sum_i a_i^{(0)} p_{ij}. \tag{3}$$

and

$$a_j^{(2)} = \sum_i a_i^{(1)} p_{ij} = \sum_i \left(\sum_k a_k^{(0)} p_{ki} \right) p_{ij} = \sum_k a_k^{(0)} \left(\sum_i p_{ki} p_{ij} \right) = \sum_k a_k^{(0)} p_{kj}^{(2)}, \tag{4}$$

where $p_{kj}^{(2)} = \sum_i p_{ki} p_{ij}$ is the transition probability after two transitions. The transition probability $p_{kj}^{(n)}$ after n transitions is presented by the recursive formula (Taha 1997):

$$p_{kj}^{(n)} = \sum_k p_{ki}^{(n-1)} p_{ij}. \tag{5}$$

The general finite state Markov chain models can be classified into two categories in terms of irreducibility and reducibility properties. First, consider a finite state Markov chain model with transition matrix is irreducible. For the definition of irreducible, please refer to Hillier and Lieberman (2005). Note that the number of eigenvalue 1 is one and the absolute values of the other eigenvalues are less than 1. By a Jordan decomposition, it can be shown that there exists a matrix Q such that $D = Q^{-1} P Q$, where the first column of matrix Q contains the values of all ones, and the first row of Q^{-1} is a unique steady-state probability vector π presented by the following (Chen et al. 2002):

$$D = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & & & \\ \vdots & & M & \\ 0 & & & \end{bmatrix}, \tag{6}$$

where $M^n \to 0$ as n goes to infinity. Mieghem (2006) has pointed out that the chain admits a unique steady-state probability vector for any given initial distribution π^0 as follows (Sinai 1991; Chung 1967):

$$\lim_{n\to\infty} (\pi^0 P^1 + \pi^0 P^2 + \cdots + \pi^0 P^n)/n = \pi. \tag{7}$$

In particular, if the Markov chain model is aperiodic, the easy way to compute the steady-state probability vector is to raise P to powers and take the limit:

$$\lim_{n\to\infty} P^n = \lim_{n\to\infty} QD^nQ^{-1} = Q\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & & & \\ \vdots & & 0 & \\ 0 & & & \end{bmatrix} Q^{-1} = \begin{bmatrix} \pi \\ \pi \\ \vdots \\ \pi \end{bmatrix}. \tag{8}$$

The steady-state probability vector π shown in Eq. (8) can be easily computed by matlab by raising the power n until every row of P^n being the same up to the computer's accuracy.

The other case is to consider the Markov chain model being periodic with period $d > 1$. Irreducible chains with a periodic structure for each state $i \in S$, the set of possible return times to state i when starting in state i is a subset of the set $\{d, 2d, 3d, \cdots\}$, containing all but a finite set of these elements (Hillier and Liberman 2005). Then Eq. (7) becomes

$$\lim_{n\to\infty} (\pi^0 P^{n+1} + \pi^0 P^{n+2} + \cdots + \pi^0 P^{n+d})/d = \pi. \tag{9}$$

Note that all states of an irreducible finite Markov chain model have the same period. Therefore, the smallest number d with this property is the so-called period of the chain, where $d = \gcd\left\{n \in \mathrm{N} : P_{ii}^n > 0\right\}$.

Consider a reducible aperiodic finite chain with m_2 transient classes and m_1 recurrent classes. Each recurrent class acts like a small irreducible aperiodic Markov chain model and then admits a unique limit distribution denoted by π_j with $j = 1, 2, \ldots, m_1$. Suppose P be the transition matrix of a finite Markov chain model with recurrent classes $R_1, R_2, \ldots, R_{m_1}$ and transient classes $T_1, T_2, \ldots, T_{m_2}$. After renaming the states, the matrix P becomes (Hunter 1983):

$$P = \begin{pmatrix} R_1 & & & & & \\ & R_2 & & 0 & & 0 \\ & & R_3 & & & \\ & 0 & & \ddots & & \\ & & & & R_{m_1} & \\ S_1 & S_2 & S_3 & \cdots & S_{m_2} & Q \end{pmatrix}, \tag{10}$$

where Q gives probabilities of transitions from transient states to other transient states. Equation (10) can be written in a compact notation as follows:

$$P = \begin{pmatrix} R & 0 \\ S & Q \end{pmatrix}, \tag{11}$$

where R is the matrix of all recurrent classes and S is the matrix of all transitions from transient states to recurrent states. After n transitions, Eq. (11) becomes

$$P^n = \begin{pmatrix} R^n & 0 \\ R + QR + Q^2R + \cdots + Q^{n-1}R & Q^n \end{pmatrix}, \tag{12}$$

where $R+QR+Q^2R+\cdots+Q^{n-1}R$ can be computed by $(I+Q+Q^2+\cdots+Q^{n-1})R$. Note that $(I-Q)(I+Q+Q^2+\cdots+Q^{n-1}) = I-Q^n$. Then, $(I+Q+Q^2+\ldots+Q^{n-1}) = (I-Q^n)^{-1}(I-Q^n)$. Finally, $(I+Q+Q^2+\ldots+Q^{n-1})R = (I-Q^n)^{-1}(I-Q^n)R$. Each recurrent class acts as a small Markov chain model, and there exists m_1 different steady-state probability vectors $\pi_1, \pi_2, \ldots, \pi_{m_1}$ with π_k concentrated on R_k for $k = 1, 2, \ldots, m_1$. That is, $\pi_k(i) = 0$ if $i \notin R_k$. In practice, the formula of $\lim_{n\to\infty}(\pi_i^0 R_i^n) = \pi_i$ for each $i \in \{1, 2, \cdots, m_1\}$ can be used to resolve m_1 different steady-state probability vectors, i.e., $\pi_1, \pi_2, \ldots, \pi_{m_1}$. Note that the limit distribution may depend on the initial distribution.

The final case is to consider a reducible finite chain with period d_i for each recurrent class R_i, which is similar to the method used in a reducible aperiodic finite chain. In the latter case, $\lim_{n\to\infty}(\pi_i^0 R_i^n) = \pi_i$ for each $i \in \{1, 2, \ldots, m_1\}$. In the former case, $\lim_{n\to\infty}(\pi_i^0 R_i^{n+1} + \pi_i^0 R_i^{n+2} + \cdots + \pi_i^0 R_i^{n+d_i})/d_i = \pi_i$ for each $i \in \{1, 2, \ldots, m_1\}$. Note that period $d_{i_1} = 1$ for some $i_1 \in \{1, 2, \cdots, m_1\}$, and then $\lim_{n\to\infty}(\pi_{i_1}^0 R_{i_1}^{n+1}) = \lim_{n\to\infty}(\pi_{i_1}^0 R_{i_1}^n) = \pi_{i_1}$.

3 A framework of applying a Markov chain model in QFD

Table 1 provides a typical house of quality, where customer requirements and technical measures are denoted by CR and TM. The notations of w_i and $R_{ij}^{(0)}$ represent the weight for CR_i and the weight for the relationship between CR_i and TM_j in the current time period, respectively. The relationship between WHATs and HOWs is assumed to have one of the following relations: strong (S), medium (M), weak (W), and none (N) with the respective weights of s, m, w, and zero, respectively. To calculate the importance of technical measures in the current time period, the formulas are

$$\begin{aligned}
\mathrm{TM}_1(\text{current}) &= w_1R_{11}^{(0)} + w_2R_{21}^{(0)} + \cdots + w_iR_{i1}^{(0)} + \cdots + w_nR_{n1}^{(0)} \\
\mathrm{TM}_2(\text{current}) &= w_1R_{12}^{(0)} + w_2R_{22}^{(0)} + \cdots + w_iR_{i2}^{(0)} + \cdots + w_nR_{n2}^{(0)} \\
&\vdots \\
\mathrm{TM}_j(\text{current}) &= w_1R_{1j}^{(0)} + w_2R_{2j}^{(0)} + \cdots + w_iR_{ij}^{(0)} + \cdots + w_nR_{nj}^{(0)} \\
&\vdots \\
\mathrm{TM}_m(\text{current}) &= w_1R_{1m}^{(0)} + w_2R_{2m}^{(0)} + \cdots + w_iR_{im}^{(0)} + \cdots + w_nR_{nm}^{(0)}
\end{aligned}$$

Each $w_iR_{ij}^{(0)}$ expressed above can be presented in a mathematical format, where w_i is typically known and may come from customers' surveys, expert opinions, and the

Table 1 An illustrated example

		TM_1	TM_2	$\ldots$	TM_j	$\ldots$	TM_m
CR_1	w_1	$R_{11}^{(0)}$	$R_{12}^{(0)}$	$\ldots$	$R_{1j}^{(0)}$	$\ldots$	$R_{1m}^{(0)}$
CR_2	w_2	$R_{21}^{(0)}$	$R_{22}^{(0)}$	$\ldots$	$R_{2j}^{(0)}$	$\ldots$	$R_{2m}^{(0)}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ldots$	$\vdots$	$\ldots$	$\vdots$
CR_i	w_i	$R_{i1}^{(0)}$	$R_{i2}^{(0)}$	$\ldots$	$R_{ij}^{(0)}$	$\ldots$	$R_{im}^{(0)}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ldots$	$\vdots$	$\ddots$	$\vdots$
CR_n	w_n	$R_{n1}^{(0)}$	$R_{n2}^{(0)}$	$\ldots$	$R_{nj}^{(0)}$	$\ldots$	$R_{nm}^{(0)}$

like:

$$R_{ij}^{(0)} = \left[p_{ij}(s)\ p_{ij}(m)\ p_{ij}(w) \right] \begin{bmatrix} s \\ m \\ w \end{bmatrix}, \tag{13}$$

where $p_{ij}(s)$, $p_{ij}(m)$, and $p_{ij}(w)$ represent the initial probabilities of the relationships of strong, medium, and weak between CR_i and TM_j, respectively. To further consider one, two, three, and n transitions, the respective notations of $R_{ij}^{(1)}$, $R_{ij}^{(2)}$, $R_{ij}^{(3)}$, and $R_{ij}^{(n)}$ are used. The matrix presentations of $R_{ij}^{(1)}$, $R_{ij}^{(2)}$, $R_{ij}^{(3)}$, and $R_{ij}^{(n)}$ are as follows:

$$R_{ij}^{(1)} = \left[p_{ij}(s)\ p_{ij}(m)\ p_{ij}(w) \right] \begin{bmatrix} p_{ss} & p_{sm} & p_{sw} \\ p_{ms} & p_{mm} & p_{mw} \\ p_{ws} & p_{wm} & p_{ww} \end{bmatrix} \begin{bmatrix} s \\ m \\ w \end{bmatrix}, \tag{14}$$

$$R_{ij}^{(2)} = \left[p_{ij}(s)\ p_{ij}(m)\ p_{ij}(w) \right] \begin{bmatrix} p_{ss} & p_{sm} & p_{sw} \\ p_{ms} & p_{mm} & p_{mw} \\ p_{ws} & p_{wm} & p_{ww} \end{bmatrix}^2 \begin{bmatrix} s \\ m \\ w \end{bmatrix}, \tag{15}$$

$$R_{ij}^{(3)} = \left[p_{ij}(s)\ p_{ij}(m)\ p_{ij}(w) \right] \begin{bmatrix} p_{ss} & p_{sm} & p_{sw} \\ p_{ms} & p_{mm} & p_{mw} \\ p_{ws} & p_{wm} & p_{ww} \end{bmatrix}^3 \begin{bmatrix} s \\ m \\ w \end{bmatrix}, \tag{16}$$

and

$$R_{ij}^{(n)} = \left[p_{ij}(s)\ p_{ij}(m)\ p_{ij}(w) \right] \begin{bmatrix} p_{ss} & p_{sm} & p_{sw} \\ p_{ms} & p_{mm} & p_{mw} \\ p_{ws} & p_{wm} & p_{ww} \end{bmatrix}^n \begin{bmatrix} s \\ m \\ w \end{bmatrix}, \tag{17}$$

where p_{ss} is the transition probability from strong in the current time period to strong in the next period, $p_{sm}, p_{sw}, \ldots$, and p_{ww} are defined in the same way, and assume the transition matrix is regular. Therefore, the importance of TM_j can be described in a timely basis:

$$TM_j(\text{current time period}) = w_1 R_{1j}^{(0)} + w_2 R_{2j}^{(0)} + \cdots + w_i R_{ij}^{(0)} + \cdots + w_n R_{nj}^{(0)}, \tag{18}$$

$$\mathrm{TM}_j(\text{one transition}) = w_1 R_{1j}^{(1)} + w_2 R_{2j}^{(1)} + \cdots + w_i R_{ij}^{(1)} + \cdots + w_n R_{nj}^{(1)}, \tag{19}$$

$$\mathrm{TM}_j(\text{two transitions}) = w_1 R_{1j}^{(2)} + w_2 R_{2j}^{(2)} + \cdots + w_i R_{ij}^{(2)} + \cdots + w_n R_{nj}^{(2)}, \tag{20}$$

$$\mathrm{TM}_j(\text{three transitions}) = w_1 R_{1j}^{(3)} + w_2 R_{2j}^{(3)} + \cdots + w_i R_{ij}^{(3)} + \ldots + w_n R_{nj}^{(3)}, \tag{21}$$

and

$$\mathrm{TM}_j(n\text{transitions}) = w_1 R_{1j}^{(n)} + w_2 R_{2j}^{(n)} + \cdots + w_i R_{ij}^{(n)} + \cdots + w_n R_{nj}^{(n)}. \tag{22}$$

If $p_{ij}(n)$, the probability of no relationship between CR_i and TM_j, exists, Eq. (13) is revised by the following:

$$R_{ij}^{(0)}(\text{with } \ p_{ij}(n)) = \left[p_{ij}(s)\ p_{ij}(m)\ p_{ij}(w)\ p_{ij}(n) \right] \begin{bmatrix} s \\ m \\ w \\ 0 \end{bmatrix}. \tag{23}$$

Then, $R_{ij}^{(1)}$, $R_{ij}^{(2)}$, $R_{ij}^{(3)}$, and $R_{ij}^{(n)}$ by further considering $p_{ij}(n)$ become

$$R_{ij}^{(1)}(\text{with } \ p_{ij}(n)) = \left[p_{ij}(s)\ p_{ij}(m)\ p_{ij}(w)\ p_{ij}(n) \right] \begin{bmatrix} p_{ss} & p_{sm} & p_{sw} & p_{sn} \\ p_{ms} & p_{mm} & p_{mw} & p_{mn} \\ p_{ws} & p_{wm} & p_{ww} & p_{wn} \\ p_{ns} & p_{nm} & p_{nw} & p_{nn} \end{bmatrix} \begin{bmatrix} s \\ m \\ w \\ 0 \end{bmatrix}, \tag{24}$$

$$R_{ij}^{(2)}(\text{with } \ p_{ij}(n)) = \left[p_{ij}(s)\ p_{ij}(m)\ p_{ij}(w)\ p_{ij}(n) \right] \begin{bmatrix} p_{ss} & p_{sm} & p_{sw} & p_{sn} \\ p_{ms} & p_{mm} & p_{mw} & p_{mn} \\ p_{ws} & p_{wm} & p_{ww} & p_{wn} \\ p_{ns} & p_{nm} & p_{nw} & p_{nn} \end{bmatrix}^2 \begin{bmatrix} s \\ m \\ w \\ 0 \end{bmatrix}, \tag{25}$$

$$R_{ij}^{(3)}(\text{with } \ p_{ij}(n)) = \left[p_{ij}(s)\ p_{ij}(m)\ p_{ij}(w)\ p_{ij}(n) \right] \begin{bmatrix} p_{ss} & p_{sm} & p_{sw} & p_{sn} \\ p_{ms} & p_{mm} & p_{mw} & p_{mn} \\ p_{ws} & p_{wm} & p_{ww} & p_{wn} \\ p_{ns} & p_{nm} & p_{nw} & p_{nn} \end{bmatrix}^3 \begin{bmatrix} s \\ m \\ w \\ 0 \end{bmatrix}, \tag{26}$$

and

$$R_{ij}^{(n)}(\text{with } p_{ij}(n)) = \left[p_{ij}(s)\ p_{ij}(m)\ p_{ij}(w)\ p_{ij}(n) \right] \begin{bmatrix} p_{ss} & p_{sm} & p_{sw} & p_{sn} \\ p_{ms} & p_{mm} & p_{mw} & p_{mn} \\ p_{ws} & p_{wm} & p_{ww} & p_{wn} \\ p_{ns} & p_{nm} & p_{nw} & p_{nn} \end{bmatrix}^n \begin{bmatrix} s \\ m \\ w \\ 0 \end{bmatrix}, \tag{27}$$

where p_{ns}, p_{nm}, p_{nw}, and p_{nn} are the transition probabilities from none in the current time period to strong, medium, weak, and none in the next period, respectively, and assume the transition matrix is irreducible, which is $\lim_{n\to\infty}(\pi^0 P^1 + \pi^0 P^2 + \cdots + \pi^0 P^n)/n = \pi$. In addition, if the transition matrix is aperiodic, then the Markov chain model becomes regular. The above formula becomes $\lim_{n\to\infty}\pi^0 P^n = \pi$, and $\lim_{n\to\infty} P^n = \pi$. If the underlying assumption with the irreducible transition matrix does not exist, $\lim_{n\to\infty}(\pi^0 P^1 + \pi^0 P^2 + \cdots + \pi^0 P^n)/n$ does not exist. Please refer to Sect. 2.

The advantages of applying a Markov chain model to evaluate the relationships between customer requirements and technical measures are as follows: First, when the decision makers do not have enough information and experiences, using probabilities to determine the relationship provides another alternative instead of using a deterministic and subjective approach to assign a value to represent the relationship. Specifically, each numerical relationship can be expressed in terms of the expected value. Second, the relationship between WHATs and HOWs is built up in a timely basis. As time goes by, the relationships between CR_i and TM_j can be analyzed for their future trends in terms of, such as, $R_{ij}^{(0)}$, $R_{ij}^{(1)}$, and $R_{ij}^{(2)}$. Third, the importance of technical measures is dynamic and varies from time to time. Thus, the importance trends of technical measures can be traced. The decision makers can easily to identify the importance of technical measures by different time periods. Finally, the probabilities discussed above can be further updated as soon as new information is available. Therefore, the short- and medium-range predictions in the relationships between WHATs and HOWs as well as the importance trends of technical measures become practical and effective.

4 An illustrated example

A simple example is provided with only one technical measure (TM_1) and two customer requirements (CR_1 and CR_2). Assume the relationship between customer requirements and technical measures is to be one of the following relations: strong, medium, weak, and none with the respective weights of s, m, w, and zero, respectively. For the relationship between CR_1 and TM_1, assume the probabilities that the relationships would be strong, medium, and weak are 0.8, 0.1, and 0.1, respectively, and the needed information to generate transition matrix for the relationship is summarized in Table 2, where the sum in each row is equal to one. The probabilities after one, two, and three transitions as well as the probability of steady-state are as follows:

Table 2 The transition matrix for the relationship between CR_1 and TM_1

	Strong	Medium	Weak
Strong	0.9	0.1	0
Medium	0.2	0.7	0.1
Weak	0	0.2	0.8

Probability of the relationship between CR_1 and TM_1 (one transition)

$$= \begin{bmatrix} 0.8 & 0.1 & 0.1 \end{bmatrix} \begin{bmatrix} 0.9 & 0.1 & 0 \\ 0.2 & 0.7 & 0.1 \\ 0 & 0.2 & 0.8 \end{bmatrix} = \begin{bmatrix} 0.7400 & 0.1700 & 0.0900 \end{bmatrix}$$

Probability of the relationship between CR_1 and TM_1 (two transitions)

$$= \begin{bmatrix} 0.8 & 0.1 & 0.1 \end{bmatrix} \begin{bmatrix} 0.9 & 0.1 & 0 \\ 0.2 & 0.7 & 0.1 \\ 0 & 0.2 & 0.8 \end{bmatrix}^2 = \begin{bmatrix} 0.7000 & 0.2110 & 0.0890 \end{bmatrix}$$

and

Probability of the relationship between CR_1 and TM_1 (three transitions)

$$= \begin{bmatrix} 0.8 & 0.1 & 0.1 \end{bmatrix} \begin{bmatrix} 0.9 & 0.1 & 0 \\ 0.2 & 0.7 & 0.1 \\ 0 & 0.2 & 0.8 \end{bmatrix}^3 = \begin{bmatrix} 0.6722 & 0.2355 & 0.0923 \end{bmatrix}$$

The steady-state probability of the relationship between CR_1 and TM_1 can be computed by raising a large integer value on the transition matrix such that the values in columns are identically the same. Thus, the steady-state probability would become $\begin{bmatrix} 0.9 & 0.1 & 0 \\ 0.2 & 0.7 & 0.1 \\ 0 & 0.2 & 0.8 \end{bmatrix}^n = \begin{bmatrix} 0.5714 & 0.2857 & 0.1429 \\ 0.5714 & 0.2857 & 0.1429 \\ 0.5714 & 0.2857 & 0.1429 \end{bmatrix}$. The expected weights of $R_{11}^{(0)}$, $R_{11}^{(1)}$, $R_{11}^{(2)}$, $R_{11}^{(3)}$, and $R_{11}^{(\text{steady-state})}$ using Eqs. (13)-(17), are

$$\begin{aligned} R_{11}^{(0)} &= 0.8s + 0.1m + 0.1w, \\ R_{11}^{(1)} &= 0.7400s + 0.1700m + 0.0900w, \\ R_{11}^{(2)} &= 0.7000s + 0.2110m + 0.0890w, \\ R_{11}^{(3)} &= 0.6722s + 0.2355m + 0.0923w, \end{aligned}$$

and

$$R_{11}^{(\text{steady-state})} = 0.5714s + 0.2857m + 0.1429w.$$

For the relationship between CR_2 and TM_1, assume the probabilities that the relationships would be strong, medium, weak, and none are 0.2, 0.3, 0.3, and 0.2, respectively, and the needed information to generate transition matrix for the relationship is summarized in Table 3, where the sum in each row is equal to one. The probabilities after one, two, and three transitions as well as the probability of steady-state are:

Table 3 The transition matrix for the relationship between CR_2 and TM_1

	Strong	Medium	Weak	None
Strong	0.6	0.35	0.05	0
Medium	0.1	0.7	0.15	0.05
Weak	0.05	0.05	0.7	0.2
None	0	0.05	0.3	0.65

Probability of the relationship between CR_2 and TM_1 (one transition)

$$= \begin{bmatrix} 0.2 \; 0.3 \; 0.3 \; 0.2 \end{bmatrix} \begin{bmatrix} 0.60 & 0.35 & 0.05 & 0 \\ 0.10 & 0.70 & 0.15 & 0.05 \\ 0.05 & 0.05 & 0.70 & 0.20 \\ 0 & 0.05 & 0.30 & 0.65 \end{bmatrix}$$
$$= \begin{bmatrix} 0.1650 \; 0.3050 \; 0.3250 \; 0.2050 \end{bmatrix}$$

Probability of the relationship between CR_2 and TM_1 (two transitions)

$$= \begin{bmatrix} 0.2 \; 0.3 \; 0.3 \; 0.2 \end{bmatrix} \begin{bmatrix} 0.60 & 0.35 & 0.05 & 0 \\ 0.10 & 0.70 & 0.15 & 0.05 \\ 0.05 & 0.05 & 0.70 & 0.20 \\ 0 & 0.05 & 0.30 & 0.65 \end{bmatrix}^2$$
$$= \begin{bmatrix} 0.1457 \; 0.2977 \; 0.3430 \; 0.2135 \end{bmatrix}$$

and
Probability of the relationship between CR_2 and TM_1 (three transitions)

$$= \begin{bmatrix} 0.2 \; 0.3 \; 0.3 \; 0.2 \end{bmatrix} \begin{bmatrix} 0.60 & 0.35 & 0.05 & 0 \\ 0.10 & 0.70 & 0.15 & 0.05 \\ 0.05 & 0.05 & 0.70 & 0.20 \\ 0 & 0.05 & 0.30 & 0.65 \end{bmatrix}^3$$
$$= \begin{bmatrix} 0.1344 \; 0.2873 \; 0.3561 \; 0.2223 \end{bmatrix}$$

The steady-state probability is $\begin{bmatrix} 0.60 & 0.35 & 0.05 & 0 \\ 0.10 & 0.70 & 0.15 & 0.05 \\ 0.05 & 0.05 & 0.70 & 0.20 \\ 0 & 0.05 & 0.30 & 0.65 \end{bmatrix}^n = \begin{bmatrix} 0.1084 & 0.2358 & 0.3959 & 0.2599 \\ 0.1084 & 0.2358 & 0.3959 & 0.2599 \\ 0.1084 & 0.2358 & 0.3959 & 0.2599 \\ 0.1084 & 0.2358 & 0.3959 & 0.2599 \end{bmatrix}$.

The expected weights of $R_{21}^{(0)}, R_{21}^{(1)}, R_{21}^{(2)}, R_{21}^{(3)}$, and $R_{21}^{(\text{steady-state})}$, using Eqs. (23)-(27), are

$$\begin{aligned} R_{21}^{(0)} &= 0.2s + 0.3m + 0.3w, \\ R_{21}^{(1)} &= 0.1650s + 0.3050m + 0.3250w, \\ R_{21}^{(2)} &= 0.1457s + 0.2977m + 0.3430w, \\ R_{21}^{(3)} &= 0.1344s + 0.2873m + 0.3561w, \end{aligned}$$

Table 4 The numerical values for the relationships and weights

	Weight	Current TM_1	One transition TM_1	Two transitions TM_1	Three transitions TM_1	Steady-state TM_1
CR_1	0.7	7.60	7.26	7.022	6.8486	6.1426
CR_2	0.3	3	2.7250	2.5474	2.4276	2.0789

Table 5 The numerical values for the overall importance of TM_1 in different time periods

	Current TM_1	One transition TM_1	Two transitions TM_1	Three transitions TM_1	Steady-state TM_1
CR_1	5.32	5.082	4.9154	4.7940	4.2998
CR_2	0.9	0.8175	0.7642	0.7283	0.6237
Overall	6.22	5.8995	5.6796	5.5223	4.9235

and

$$R_{21}^{(\text{steady}-\text{state})} = 0.1084s + 0.2358m + 0.3959w.$$

To compute the overall importance of TM_1 for different time periods, the philosophy discussed in Eqs. (18)-(22) are summarized below.

$$\begin{aligned}
&\text{TM }_1(\text{current}) = w_1R_{11}^{(0)} + w_2R_{21}^{(0)} = w_1(0.8s+0.1m+0.1w)+w_2(0.2s+0.3m+0.3w)\\
&\text{TM }_1(\text{one transition}) = w_1R_{11}^{(1)} + w_2R_{21}^{(1)}\\
&\quad = w_1(0.7400s + 0.1700m + 0.0900w) + w_2(0.1650s + 0.3050m + 0.3250w)\\
&\text{TM }_1(\text{two transitions}) = w_1R_{11}^{(2)} + w_2R_{21}^{(2)}\\
&\quad = w_1(0.7000s + 0.2110m + 0.0890w) + w_2(0.1457s + 0.2977m + 0.3430w)\\
&\text{TM }_1(\text{three transitions}) = w_1R_{11}^{(3)} + w_2R_{21}^{(3)}\\
&\quad = w_1(0.6722s + 0.2355m + 0.0923w) + w_2(0.1344s + 0.2873m + 0.3561w)
\end{aligned}$$

and

$$\begin{aligned}
&\text{TM }_1(\text{steady-state}) = w_1R_{11}^{(\text{steady}-\text{state})} + w_2R_{21}^{(\text{steady}-\text{state})}\\
&\quad = w_1(0.5714s + 0.2857m + 0.1429w) + w_2(0.1084s + 0.2358m + 0.3959w)
\end{aligned}$$

When the numerical values of w_1, w_2, s, m, and w are given, $R_{11}^{(0)}$, $R_{11}^{(1)}$, $R_{11}^{(2)}$, $R_{11}^{(3)}$, $R_{11}^{(\text{steady-state})}$, $R_{21}^{(0)}$, $R_{21}^{(1)}$, $R_{21}^{(2)}$, $R_{21}^{(3)}$, $R_{21}^{(\text{steady-state})}$, and TM_1 for different time periods would be known. For instance, if w_1, w_2, s, m, and w are to be 0.7, 0.3, 9, 3, and 1, respectively, the above values are summarized in Tables 4 and 5 and Fig. 2.

From the illustrated example, applying a Markov chain model in QFD brings the following advantages. First, if the decision makers do not have enough information and experiences, using probabilities to determine the relationships between customer requirements and technical measures as well as the importance of technical measures provide another alternative objectively in terms of expected values. Second, the relationships and the importance of technical measures are evaluated in a timely basis. That is, the dynamic relationships as well as the trends of technical measures can be monitored by different time periods. The decision makers can easily to prioritize the

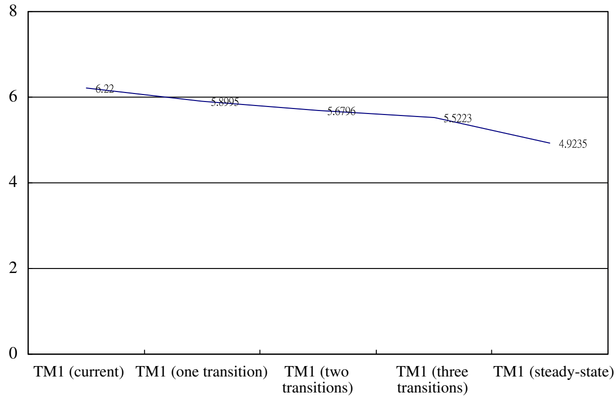

Fig. 2 The trend of the illustrated technical measure in different time periods

importance of technical measures by their respective future trends. Furthermore, the weight w_i for CR_i can be described by a Markov chain model (Wu and Shieh 2006). Thus, the computations discussed above would become time-dependent and predictions in terms of the expected values become practical. Finally, as soon as the decision makers have the most updated information, the probabilities can be adjusted, and the short- and medium-range predictions can be made practically and effectively.

5 Conclusions

This study applies a Markov chain model to evaluate the relationships between customer requirements and technical measures and the importance of technical measures from the probabilities viewpoints. Using probabilities to express the relationships in terms of the expected values provides an alternative objectively if the decision makers do not have enough information and experiences. The dynamic relationships as well as the future trends of technical measures can be plotted and traced by different time periods when the needed information is available. Finally, as soon as the new information is available, the probabilities can be updated to relentlessly monitor the relationships and the future trends of technical measures.

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