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Local Monotonicity of Power: Axiom or just a Property?

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Abstract. This paper discusses whether Local Monotonicity (LM) should be regarded as a property of the power distribution of a specific voting game under consideration, indicated by a power measure, or as a characteristic of power *per se*. The latter would require reasonable power measures to satisfy a corresponding LM axiom. The former suggests that measures which do not allow for a violation of LM fail to account for dimensions of power which can cause nonmonotonicity in voting weight. Only if a measure is able to indicate nonmonotonicity, it can help design voting games for which power turns out to be monotonic. The argument is discussed in the light of recent extensions of traditional power indices.

Key words: Power measures, monotonicity, voting

1. Introduction

Assume that there are three parties, A, B and C, which have a share of parliament seats of 45%, 35%, and 20%, respectively. Given that decisions are made by simple majority it seems not very likely that the distribution of power, however defined, coincides with the distribution of votes. Power indices have been developed to discuss issues of assigning power values to the resources (e.g., votes) of decision makers and to explain how these values change if the resource distribution changes or a new decision rule is applied. They seem to be valuable instruments to analyze institutional changes and potential effects of alternative institutional design. The two volumes, "Power, Voting, and Voting Power" (Holler 1982a) and "Power Indices and Coalition Formation" (Holler & Owen 2001) not only contain original contributions to this discussion but also illustrate the development in this field over the last twenty years. A recent monograph by Felsenthal and Machover (1998), "The Measurement of Voting Power", gives a comprehensive formal treatment.

There is a growing interest in power measures such as the Shapley–Shubik index and the Banzhaf index, to name the two most popular measures. Their application to political institutions, in particular to the analysis of the European Union,[1] has thrived. There are also new theoretical instruments and perspectives

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that support these applications. Of prime importance is the probabilistic model of coalition formation which has been made operational by the multilinear extension of coalition games introduced by Owen (1972). This instrument triggered off a reinterpretation of existing power indices and the formulation of new ones. It has further been generalized by Laruelle and Valenciano (2002) and Napel and Widgren (2002, 2004). The latter framework has allowed to analyze not only power derived from easily identifiable resources such as voting weights but decision makers' entire set of feasible strategies.

This development has been accompanied by an intensive discussion of the concept of power in general – asking the question: what do we measure when we apply power measures? – and the properties that an adequate measure of power has to satisfy. The question about which index is the right one is not answered. Selection criteria have been proposed derived from stories which accompany the indices but are only implicit to the formal measure concept. Other selection criteria refer to plausible properties – such as monotonicty – which are however (a) sometimes not unambiguously defined and (b) not necessarily capturing all aspects of power.

In this paper we will challenge the strategy to create power measures which are immune to violating monotonicity: Only if the measure is able to indicate nonmonotonicity, it can serve to answer how a voting game has to be designed such that power values are monotonic, e.g. indicate more power for decision makers with greater voting weight.

In the following section we will specify monotonicity and derive implications from interpreting local monotonicity as a property of a particular power relation on the one hand, and as an axiom for all valid power measures on the other. Section 3 discusses these two perspectives with reference to Napel and Widgrén's (2002, 2004) unifying approach to power measurement. Alternative views on properties of power are presented in Section 4. The discussion shows that it is not always plausible to expect monotonicity. In Section 5 we look at basic relations between the Banzhaf index and the Public Good Index and interpret them with respect to the differences in monotonicity properties of these measures. Section 6 concludes the paper.

2. Two perspectives on local monotonicity and related properties

Given the great number of power measures proposed in the last 60 years, a possible strategy is to choose a favorite power measure and to try to convince others to share this choice. An alternative is to accept the diversity of measures and their differing interpretations, and select an appropriate measure in concurrence with the intuition possibly based on the accompanying stories. A third alternative is to postulate discriminating properties, possibly phrased as axioms, which a power measure has to satisfy in order to qualify as an appropriate measure.

Local Monotonicity (*LM*) has been proposed as such a compulsory property. Felsenthal and Machover (1998: 221ff) are very explicit that any a priori measure of power that violates LM is 'pathological' and should be disqualified as serving as a valid yardstick to measuring power. For weighted voting games, LM requires that a voter i who controls a larger share of vote cannot have a smaller share of power than a voter j with a smaller voting weight.[2]

LM is an implication of *desirability* as proposed by Isbell (1958). A voter i is called 'at least as desirable as' a voter j if for any coalition S such that the union of S and $\{j\}$ is a winning coalition, the union of S and $\{i\}$ is also winning. Freixas and Gambarelli (1997) use desirability as a yardstick which defines reasonable power measures. Since both the Deegan–Packel index and Public Good Index violate LM, they also violate desirability (Deegan & Packel 1978; Holler 1982b; Holler & Packel 1983). For example, given the vote distribution $w = (35, 20, 15, 15, 15)$ for five decision makers and a decision rule $d = 51$, the corresponding values of Public Good Index are equal to:

$$h(d, w) = \left(\frac{16}{60}, \frac{8}{60}, \frac{12}{60}, \frac{12}{60}, \frac{12}{60}\right).$$

A comparison of w and $h(d, w)$ shows a violation of LM for the second player. Because of the *basic principles* underlying the Public Good Index, which derive from the notion of pure public good (i.e. nonrivalry in consumption and exclusion of free-riding), only minimal winning coalitions are considered when it comes to measuring power. All other coalitions are either nonwinning or contain at least one excess member which does not contribute to winning. If coalitions of the second type are formed, then it is by luck, similarity of preferences, tradition, etc. – but not because of power. It should be noted that the Public Good Index does not maintain that only minimal winning coalitions will be formed.

In Holler et al. (2001), the authors analyse alternative constraints on the number of players and other properties of the decision situations with respect to their consequences for local monotonicity of the Public Good Index. For example, it is obvious that local monotonicity will not be violated by any of the known power measures, including the Public Good Index and the Deegan–Packel index, if there are n voters and n-2 voters are dummies. It is, however, less obvious that local monotonicity is also satisfied for the Public Good Index if one constrains the set of games so that there are only n-4 dummies.

Braham and Steffen (2002) demonstrate that applications of Straffin's (1977) partial homogeneity approach do not always produce results which are consistent with LM either. This is because partial homogeneity does not treat players symmetrically so that coalitions are not of equal weight. The power of a voter i depends not only upon the number of coalitions for which i is critical but also upon the probabilities by which the various coalitions arise. This points to important *a priori* knowledge about voters' likely behavior and institutional features such as agenda setting which can cause asymmetries among players (see Section 3).

Straffin's partial homogeneity approach can be interpreted as a combination of Banzhaf index and the Shapley–Shubik. These two indices happen to satisfy

LM, although their original axiomatization does not include LM. It is important to ask whether LM is a property which the power distribution of the voting game under consideration, as expressed by a suitable power measure, can have or if it is an essential characteristic of power *per se* (and therefore a reasonable power measure has to satisfy a LM axiom). If we accept the property perspective, then any measure which does not allow for a violation of LM is inappropriate to account for this dimension of power. Obviously, only if the power measure is able to indicate nonmonotonicity it can serve to answer how the voting game has to be designed such that power values will be monotonic. Otherwise a situation is unstable as, for instance, in the above five-voters example, the voter 2 suffering from the violation of LM might try to form a bloc with one of other voters.

In addition to LM, Felsenthal and Machover (1995, 1998) propose three postulates, namely, the *transfer principle*, the *bloc principle* and the *dominance principle*, as major desiderata for a measure of voting power. The dominance principle maintains that the power value of j is larger than the power value of i if for every coalition S such that j is not in S, the union of S and $\{j\}$ is a winning coalition given that the union of S and $\{i\}$ is a winning coalition. The transfer principle implies that the power of a voter i, should not increase if i donates a part of his voting rights to another voter j and i is the sole donor. If this principle is violated then the corresponding power measure suffers from the *donation paradox*. The bloc principle requires that the power of the merged entity $\{i, j\}$ will be larger than the power of player i if j is not a dummy.

Obviously, the transfer principle and the bloc principle presuppose that votes are transferable, at least, to some extent. However, if vote transfers are voluntarily, then in fact we do not need these principles (in the form of axioms) because i will not form a bloc with j if the power of $\{i, j\}$ is smaller than the power of $\{i\}$, unless i wants to give up power. But we need a measure which tells i that he should not merge with j in this case, i.e. is a desirable property if a power measure is able to point out this "dilemma". It is good to know that the Banzhaf index violates the bloc postulate and the transfer postulate, but not the dominance principle. The Deegan–Packel index and the Public Good Index might be somewhat "over-qualified" as they violate all three principles (and LM) while the Shapley–Shubik index obeys all three (and LM).[3]

3. A priori power as the expected sensitivity of outcomes

The Deegan–Packel index, the Public Good Index, indices based on partial homogeneity, and, in fact, all established indices including the Banzhaf index and the Shapley–Shubik index make assumptions, i.e. impose restrictions, about players' voting behavior or the power-relevant aspects of coalition formation. They study the impact that a given player's change of action or coalition participation (a 'no' vote instead of a 'yes' vote or being outside instead of inside a coalition) would have on the outcome for a given voting configuration or coalition. The respective (ex ante) power index is a weighted average of these hypothetical marginal impacts also known as *marginal contributions*, *swing positions* or *pivot positions*.

Which weights are applied to the respective marginal contributions in all possible voting configurations or coalitions is precisely what distinguishes the mentioned indices. The Banzhaf index, assuming that players vote 'yes' or join a randomly formed coalition with probability 0.5, applies an equal weight of $1/2^{n-1}$ to every feasible coalition. The Shapley–Shubik index, though it was introduced axiomatically without reference to any explicit behavioral assumptions, weights each coalition S proportional to the number of possible player orderings σ such that the first $|S| - 1$ players in σ are the coalition members of S bar the player i whose marginal contribution is to be weighted. The Deegan–Packel index and the Public Good Index only give minimal winning coalitions a positive weight (proportional to the inverse of the number of members for the Deegan–Packel index and equal for all for the Public Good Index). Weights for partial homogeneity-based indices are calculated from the assumed partially dependent acceptance rates.

This probabilistic view on power indices reveals that all established indices can be interpreted as the expected *sensitivity* of the outcome of voting or coalition formation to the actions of individual players. The view of power measurement as sensitivity analysis has been elaborated and extended from the simple game framework to a more general context by Napel and Widgrén (2002, 2004).[4] The extension allows for analysis of the power derived by players not only from their voting weights but also the procedural details of decision-making, e.g. agenda-setting power.[5]

Particular indices are explicitly or implicitly based on particular assumptions on how the expected value of the sensitivity of the outcome to the actions of individual players is to be formed, i.e. different probability models for the power-relevant states of the world, reflecting different power-relevant average behavior of the players. A priori – that is in the absence of information about players' goals and the (perfectly, boundedly, or not-at-all rational) way in which they pursue them – there is no 'wrong' or 'right' probability model, no intrinsically correct or incorrect way to aggregate marginal contributions across feasible coalitions or voting configurations.

If the probabilistic assumptions that are explicitly underlying an index (Banzhaf index and partial homogeneity-based indices) or can be regarded as hidden in its axioms when the respective index is interpreted probabilistically (Shapley–Shubik index, Deegan–Packel index, or Public Good Index) make it non-monotonic, this is useful information. Namely, for the considered behavior of players, power is not simply a monotonic transformation of weight. Rather, it depends specifically on the simple game at hand. The interaction between the general decision behavior of players and the decision situation under consideration is thus taken seriously. From a given vote distribution and the valid behavioral assumptions that players only derive power from minimal winning coalitions, it can logically follow that a player with less voting weight than another enjoys more (ex ante) power.

This stresses that power is not only depending on all players' resources in a given decision situation (voting weights, in the framework of simple game-analysis), which is the rationale for calculating power indices rather than being satisfied with voting weights, but also on how players are assumed to employ them. Restricting one's tool-box to indices that are locally monotonic rules out plausible ways of using resources. Because none of the established indices is based on the explicit assumption of particular *preferences* that are rationally pursued by players, the possible argument that players who have more weight but less power than others would 'rationally' forgo part of their resources is misleading.

If power of a player is measured by looking at the expected sensitivity of collective decisions to that player's behavior – as all established indices implicitly do – it can be argued that one finds too little rather than too much non-monotonicity in the literature on power indices. Different expected behavior of players for a given decision situation can imply monotonicity or non-monotonicity in players' voting weight. In addition, also the decision situation itself can imply either monotonicty or non-monotonicity in weights if other – often highly relevant – aspects of the decision-situation are taken into account. For example, the outcome of collective decision-making is known to depend very much on the agenda – and hence on the decision-maker who sets it. This can, unfortunately, not be analyzed in a satisfactory way in the traditional framework of simple games. It requires generalized versions of established indices (Napel and Widgrén 2002, 2004) which feature a possible non-monotonicity in the weight dimension of power as an essential characteristic. If sensitivity of outcomes to *all* dimensions of players' resources (including strategic ones) is to be accounted for, the resulting a priori measure cannot be expected to be monotonic in any particular single one (e.g. votes).

4. Alternative views on monotonicity properties

Holler (1998) argues that when it comes to monotonicity of power with respect to voting weights, it is important to note that none of the existing measures guarantees that the power measure of player i will *not* decrease if his or her voting weight increases. Fischer and Schotter (1978) demonstrate this result (i.e., the paradox of redistribution) for the Shapley–Shubik index and the normalized Banzhaf index. This paradox stresses the fact that power is a social concept: if we discuss the power of an individual member of a group in isolation from his or her social context, i.e. related only to his or her individual resources, we may experience all sorts of paradoxical results.[6]

It seems that sociologists are quite aware of this problem and nonmonotonicity of an individual's power with respect to his or her individual resources does not come as a surprise to them (Caplow 1968). Political scientists, however, often see the nonmonotonicity of power as a threat to the principle of democracy. To them it is hard to accept that increasing the number of votes which a group has could decrease its power, although it seems that there is ample empirical evidence for it. (See Brams and Fishburn (1995) for references.) In general, economists also assume that controlling more resources is more likely to mean more power than less. However, they also deal with concepts like monopoly power, bargaining, and exploitation which stress the social context of power and the social value of resources (assets, money, property, etc.). The dilemma of a durable-good monopolist facing a continuum of heterogeneous buyers illustrates this: The availability of more options (in particular that of lowering prices after some time) *decreases* the seller's market power – in the limit to zero according to the Coase conjecture.

Of course, the larger the probability of a coalition S for which i is critical, the larger is the aggregate power or rather ex ante power which i derives from this coalition. However, in the two border cases of partial homogeneity – that is in case of strictly independent voters (Banzhaf index) and the case of homogeneous voters (Shapley–Shubik index) – LM is guaranteed.

Needless to say that this result depends on acceptance of the probability interpretation of power and power measures. Based on it, Braham and Steffen (2002) argue that Straffin's homogeneity approach is not less a priori than the Banzhaf index and the Shapley–Shubik index.[7] If we generalize the partial homogeneity approach to the Public Good Index and apply zero probabilities to winning coalitions with surplus players then this a priori argument should also be available for the Public Good Index. Brueckner (2001) demonstrates that we can extend the probabilistic characterization so that the Public Good Index follows from the homogeneity assumption if we consider minimal winning coalitions only. That is, there is an axiomatic approach and probability interpretation for the Public Good Index. From this point of view there is no difference to the Banzhaf index or the Shapley–Shubik index – and the question of *a priori*-ness cannot be answered on this basis.

From the analysis of Straffin's partial homogeneity approach we can conclude that there is the possibility of a violation of LM whenever coalitions (or permutations) are not taken into consideration for the calculation of the power measure with equal probability. This, of course, will be the default case for all *a posteriori measures* which derive probabilities for coalitions from empirical (or historical) data.[8] For example, the extensions of the Shapley–Shubik index and Banzhaf index, proposed in Owen (1977) and Owen (1982), do not satisfy LM. (See Alonso-Meijide and Bowles (2003).)

5. Relations between indices

The relations between the various available power measures are as yet not completely clarified. However, there is work which tries to contribute to this program. Allingham (1975) has shown that the Dahl measure is simply the Shapley value without weights. More recently, Widgrén (2001) analyzed the probabilistic relationship of the Public Good Index (h_i) and the normalized Banzhaf index (β_i) and demonstrates that β_i can be written as a linear function of h_i such that

$\beta_i = (1 - \pi)h_i + \pi\epsilon_i$. Here $(1 - \pi)$ represents the share of minimal winning coalitions, compared to all decisive coalitions, i.e. coalitions that contain at least one swinger; and ϵ_i expresses the share of decisive coalitions which have i as a member, but are not minimal, compared to the number of all decisive coalitions which are not minimal. Obviously, the larger these two shares the more β_i and h_i deviate from each other.

Widgrén interprets the part of the function that is independent of the Public Good Index, $\pi\epsilon_i$, as an expression of a special type of luck in the sense of Barry (1980). If the institutional setting is such that decisive coalitions form which are not necessarily minimal, and the corresponding coalition goods are produced, then the normalized Banzhaf index seems an appropriate measure. In this case, local monotonicity is guaranteed. This implies that the institutions are such that the fundamental free-rider problem, of which the Public Good Index takes care (Holler 1982b), does not arise.

At the first glance, a comparison of the axioms underlying the various power measures seems to be a suitable approach to clarify the relationship between the various power measures. However, since most axiom sets differ by only one axiom, one has to conclude that it is their hard-to-fathom combinations which brings about the variance reflected by the observed multitude of measures.

Another source of variance is the difference in the notion of power which the authors of the various measure propose. Is power a probability, capacity, or potential – or merely a theoretical concept? Does power depend on preferences — and, if so, on which preferences? Unfortunately, the power index community is far from finding a consensus answer to these questions.

6. Conclusion

This is a first step to a responsivity test of the various power measures. It seems that indices which produce values which are close to voting weights, irrespective of the vote distribution and the decision rule, and do not indicate any possible reversal of order (nonmonotonicity) are rather useless instruments. A power measure should clarify the properties of the game so that, for example, a disfavoured player (or unhappy designer) can change the game.

Power indices that detect rather than postulate monotonicity can also be of help for a more abstract analysis of decision situations with respect to power. Myerson (1999: 1080) argues that "the task for economic theorists in the generations after Nash has been to identify the game models that yield the most useful insights into economic problems. The ultimate goal of this work will be to build a canon of some dozens of game models, such that a student who has worked through the analysis of these canonical examples should be prepared to understand the subtleties of competitive forces in the widest variety of real social situations". What can be said of understanding the subtleties of competitive forces also applies to power. The analysis of alternative social situations with respect to power by means of power measures complements more direct approaches to enhance the understanding of power (Braham and Holler 2003).

Notes

1. For an overview of game-theoretical approaches to the EU institutions, see Nurmi (2000). Recent contributions include Baldwin et al. (2001), Felsenthal and Machover (2001), and Leech (2002).
2. This coincides with axiom A.4 in Allingham (1975).
3. See Felsenthal and Machover (1995) for this result.
4. The view of *power measurement as sensitivity analysis* has been elaborated and extended from the simple game framework, to which established indices confine themselves, to a more general context by Napel and Widgrén (2002, 2004). The extension allows for analysis of the power derived by players not only from their voting weights but also the procedural details of decision-making, e.g. agenda setting power. Optionally, players' behavior can be derived from the strategic pursuit of well-defined (distributions of) preferences.
5. Optionally, players' behavior can be derived from the strategic pursuit of well-defined (distributions of) preferences.
6. A similar point is made by Napel and Widgrén (2001) in the context of the possible additivity of power, investigated as an axiom for power indices by Dubey and Shapley (1979).
7. Since, originally, the Deegan–Packel index and the Public Good Index derive from an axiomatic approach, probability arguments do not necessarily bite for them. However, applying the probability model to these measures, Braham and Steffen conclude that the argument in Deegan and Packel (1978), Holler (1997, 1998) and Brams and Fishburn (1995) that the power must be accepted to be not locally monotonic "is not entirely correct either".
8. For example, see Stenlund et al. (1985).

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