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A novel scalable three-level hierarchical quantum information splitting scheme for two-qubit unknown states

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Abstract
This paper proposes a scalable three-level hierarchical quantum information splitting (HQIS) protocol with double quantum unknown states. Generally, the traditional HQIS is to distribute the secret to two levels of receivers for purpose of achieving the secret recovery with the help of some receivers to recover the secret. Our proposed protocol uses the product states of two four-particle cluster states and two three-particle GHZ state as quantum channels to transmit the information of double-quantum unknown states, which can face more complex application scenarios and realize the extension of the agent level. By using multi-qubit GHZ state, the proposed protocol can be modified and extended to achieve the change of the number of agents, and it can make the scheme scalable. Finally, we analyzed the efficiency and safety of the scheme and gave a comparison of its similar schemes.

Keywords Hierarchical quantum information splitting · Cluster state · GHZ states

1 Introduction

The security of classical channel communication depends on complex mathematical calculations, but the difficulty of these calculations decreases as technology improves,

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so messages become easier to avoid eavesdropping on. Unlike classical channel communication, the security of quantum communication does not depend on complex mathematical calculations, but on the basic knowledge and properties of quantum mechanics. At present, the research directions of quantum communication include quantum teleportation [1–6], quantum secure communication [7–11], quantum dense coding [12–14] and quantum secret sharing (QIS) [15–24] and so on.

Quantum entanglement is a very magical phenomenon. With its help, quantum information splitting (QIS) protocol has been developed very well since it was proposed by Hillary et al. [15] in 1999. In the QIS protocol, the sender divides the secret into two or more parts and sends it to different agents for storage. No agent can recover the secret alone, and it must cooperate with other agents to recover the secret. Zheng [16] in their scheme uses the w state as a quantum channel to split a quantum information into two or more, and is robust against decoherence. Zhang [17] proposed two schemes for QIS using tripartite entanglement and demonstrated their security under certain eavesdropping. Yang [18] proposed a (t, n) threshold multi-party QIS protocol, the protocol idea is derived from standard teleportation. The above QIS protocols focus on sharing secrets equally, but this is often not the case in practice. For example, the key of a confidential document is distributed to different people, whose identity levels are often different. For trustworthy people, it needs less help to restore the secret, while for untrustworthy people, it needs more help. Gottesman [20] pointed out that a more extensive QIS protocol is needed, and the power of recovering secrets among different agents is unbalanced, but he did not give a specific implementation method. In 2010, Wang et al. [21] first proposed a layered quantum information splitting (HQIS) scheme to deal with the situation of classification according to the agent's own need for secret recovery. In HQIS, the sender can decide which agents have high and low authority based on the position and credit score of the recipient (agent). This hierarchy of rights is realized through the distribution method of entangled channel particles. Before the communication begins, each agent is informed of his level of authority. If an agent decides to restore the target state after the secret delivery, the other agents will assist him accordingly. According to the quantum non-cloning theorem [25], only the specified receiver can restore the target state, because the recovery operation depends on the measurements of other agents, and these messages are only delivered to the specified receiver. After that, wang proposed a variety of HQIS protocols, including the implementation of HQIS using cluster state [22]. Using six-photon cluster states, he divided agents into two levels, with more than one high-level and more than one low-level. Xu et al. [23] proposed a deterministic HQIS scheme that realizes arbitrary double qubit states through cluster states, and extended the scheme to multiple parties by means of the symmetry of cluster states. In 2022, Tang et al. [24] proposed a general HQIS protocol. Although this protocol extended the unknown state qubit to multiple, it still studied the two-level agent HQIS protocol. According to previous studies, agents are almost divided into two levels, high agent and low agent. Faced with the actual situation, the two levels of agents often have limitations, and the number of different levels is often limited.

In this paper, we use the product states of two 4-particle cluster states and two 3-particle GHZ state as quantum channels to realize hierarchical quantum information splitting of two-particle unknown states. In this scheme, one secret sender and several secret receivers are designed. The authority level of the secret receiver is divided into three levels, and different authority levels of the receivers have different ability to recover the secret. First, the sender makes two Bell measurements of the double quantum unknown state and the assigned two particles, and sends the measurements to the receiver over the classical channel. Due to the different authority levels of different agents, the operations are also different, part of the CNOT gate operation, part of the single bit measurement or Hadamard gate operation. After the Secret Restorer measures its own particles, with the help of some or all agents, the Restorer makes corresponding unitary transformations to the particles in its possession to recover the original quantum information. Not only that, considering the actual situation, the agent of high-level corresponds to the super administrator, often only one, the agent of mid-level and the number of low-level agents is often more. Finally, we extend the protocol partially, which can realize the theoretically infinite expansion of the number of low-level agents.

The rest of this article is organized as follows: Sect. 2 introduces some quantum knowledge used in this article. Section 3 gives our HQIS protocol. Section 4 is our extension of Sect. 3 HQIS protocol. Section 5 is about the analysis of the protocol in this paper. Finally, we conclude in Sect. 6.

2 Quantum techniques

In a two-dimensional Hilbert space, there are three possible states for a quantum state:$|0\rangle$, $|1\rangle$, and the superposition of these two states $\alpha|0\rangle+\beta|1\rangle$. But what state this quantum state is in can only be known by measurement. There exists such a set of orthogonal bases $|+\rangle={}^{1}\!\big/\!\sqrt{2}(|0\rangle+|1\rangle)$ and $|-\rangle={}^{1}\!\big/\!\sqrt{2}(|0\rangle-|1\rangle)$. In the case of a single photon measurement there is a probability of $\left|{}^{1}\!\big/\!\sqrt{2}\right|^2$ getting $|0\rangle$, there is a probability of $\left|{}^{1}\!\big/\!\sqrt{2}\right|^2$ getting $|1\rangle$.

A quantum system consists of two parts: subsystem A and subsystem B. The state of subsystem A is represented by the quantum state $|\psi_A\rangle$, and the state of subsystem B is represented by the quantum state $|\psi_B\rangle$. At this time, the state of the whole system is represented by $|\psi_A\rangle \otimes |\psi_B\rangle$.

The GHZ state is a common three-particle entangled state, and the specific form is $|GHZ\rangle_{123} = {}^{1}\!\big/\!\sqrt{2}(|000\rangle + |111\rangle)_{123}$. At present, the GHZ state has been successfully extended to the case of particle number n, that is, the entangled state of n particles is $|GHZ\rangle_{12...n} = {}^{1}\!\big/\!\sqrt{2}(|00...0\rangle + |11...1\rangle)_{12...n}$.

Quantum logic gate plays a very important role in quantum secure communication. It can be classified according to the number of quantum bits processed. The following are the single-qubit logic gate and two-qubit logic gate we use.

2.1 Quantum logic gate

1. Gate I, gate I acting on the qubit will not cause any change in the qubit.
2. Gate X, gate X will reverse the qubit, $|0\rangle \rightarrow |1\rangle$ and $|1\rangle \rightarrow |0\rangle$.
3. Gate Z, gate Z scoped qubit causes the qubit to undergo a phase reversal, where 0 remains unchanged and $|1\rangle \rightarrow -|1\rangle$.
4. Gate Y, iY is usually applied to the corresponding qubit, resulting in phase reversals and bit reversals, $|0\rangle \rightarrow -|1\rangle$ and $|1\rangle \rightarrow |0\rangle$
5. Hadamard Gate, H gate for short. The H gate converts $|0\rangle$ to $|+\rangle$, $|1\rangle$ to $|-\rangle$, and vice versa, $|+\rangle$ to $|0\rangle$ and $|-\rangle$ to $|1\rangle$.
6. Quantum controlled not gate, or CNOT gate. The CNOT gate operates on two qubits, one as a control qubit that does not change, and the other as a target particle that changes according to the state of the control particle. If the control particle is $|0\rangle$, the target particle does not change; if the control particle is $|1\rangle$, the target particle performs bit reversal.

3 Three-level of HQIS protocol

Before introducing this Agreement, take a look at Fig. 1, which shows the architecture of this Agreement. After obtaining the double quantum unknown state, the secret sender makes the Bell measurement with some entangled resources, and then distributes the remaining entangled resources to the secret agent. Next, the HQIS protocol is designed by double four-particle bit cluster state and double three-particle GHZ state. Here, we extend the low-level agents so that three low-level agents can be replaced by n low-level agents, as detailed in Sect. 4.

Note that the sender is Alice, and the permission levels are Bob, Charlie, and David in descending order. The numeric subscript of the agent name indicates that there are multiple agents at the layer, and the number is used to distinguish agents at the same layer.

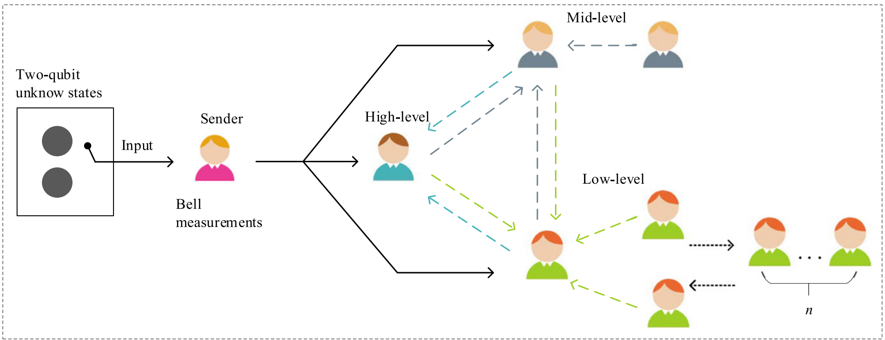

Fig. 1 The solid lines represent quantum channels where the sender sends secret information to different levels of agents. The dotted lines between High-level, Mid-level, and Low-level represent the classical channels, and also represent the help the secret restorer needs to recover the secret

Suppose Alice has a two-qubit state,

$$|\varphi\rangle_{xy} = (\alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle)_{xy} \tag{1}$$

Here, $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$ As the sender, Alice sends a secret state $|\varphi\rangle_{xy}$ to the six receivers. The quantum channel between them is composed of 4-qubit cluster states and 3-qubit GHZ state.

$$\begin{aligned}|C_1\rangle_{1234} &= \frac{1}{2}(|0000\rangle + |0011\rangle + |1100\rangle - |1111\rangle)_{1234} \\ &= \frac{1}{2}\left(|0\rangle\left|\varphi^0\right\rangle + |1\rangle\left|\varphi^1\right\rangle\right)_{1234} \\ |C_2\rangle_{abcd} &= \frac{1}{2}(|0000\rangle + |0011\rangle + |1100\rangle - |1111\rangle)_{abcd} \\ &= \frac{1}{2}\left(|0\rangle\left|\varphi^0\right\rangle + |1\rangle\left|\varphi^1\right\rangle\right)_{abcd}\end{aligned} \tag{2}$$

$$\begin{aligned}|G_1\rangle_{567} &= \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{567} \\ |G_2\rangle_{efg} &= \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{efg}\end{aligned} \tag{3}$$

Here

$$\begin{aligned}\left|\varphi^0\right\rangle &= |000\rangle + |011\rangle \\ \left|\varphi^1\right\rangle &= |100\rangle - |111\rangle\end{aligned} \tag{4}$$

Among them, particles x, y, 1 and a belong to Alice, particles 2 and b belong to Bob, particles 3 and c, 4 and d belong to Charlie1 and Charlie2 respectively, particles 5 and e belong to David1, particles 6 and f belong to David2, and particles 7 and g belong to David3.

While the whole system is written as

$$\begin{aligned}|\Theta\rangle &= |\varphi\rangle_{xy} \otimes |C_1\rangle_{1234} \otimes |G_1\rangle_{567} \otimes |C_2\rangle_{abcd} \otimes |G_2\rangle_{efg} \\ &= \frac{1}{8}\big(\alpha|0000\rangle\left|\varphi^0\varphi^0\right\rangle + \alpha|0001\rangle\left|\varphi^0\varphi^1\right\rangle + \alpha|0010\rangle\left|\varphi^1\varphi^0\right\rangle + \alpha|0011\rangle\left|\varphi^1\varphi^1\right\rangle \\ &+\beta|0100\rangle\left|\varphi^0\varphi^0\right\rangle + \beta|0101\rangle\left|\varphi^0\varphi^1\right\rangle + \beta|0110\rangle\left|\varphi^1\varphi^0\right\rangle + \beta|0111\rangle\left|\varphi^1\varphi^1\right\rangle \\ &+\gamma|1000\rangle\left|\varphi^0\varphi^0\right\rangle + \gamma|1001\rangle\left|\varphi^0\varphi^1\right\rangle + \gamma|1010\rangle\left|\varphi^1\varphi^0\right\rangle + \gamma|1011\rangle\left|\varphi^1\varphi^1\right\rangle \\ &+\delta|1100\rangle\left|\varphi^0\varphi^0\right\rangle + \delta|1101\rangle\left|\varphi^0\varphi^1\right\rangle + \delta|1110\rangle\left|\varphi^1\varphi^0\right\rangle + \delta|1111\rangle\left|\varphi^1\varphi^1\right\rangle\end{aligned} \tag{5}$$

Here $|\Theta\rangle$ uses the fewest quantum resources of the three-layer HQIS protocol, namely $|C_1\rangle$,$|C_2\rangle$,$|G_1\rangle$ and $|G_2\rangle$. In Sect. 4, we will extend this protocol, and the quantum resources used by the extended protocol will be determined according to the number of low-level agents.

Alice performs two Bell measurements on her qubit $(x, 1)$ and (y, a), respectively. The four Bell states are

$$\begin{aligned}\left|\Phi^{\pm}\right\rangle &= \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)\\ \left|\Psi^{\pm}\right\rangle &= \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)\end{aligned} \tag{6}$$

She sends the measurement to the agent over the classical channel. What we know is that Alice can get one of the 16 outcomes with equal probability, and the other qubits collapse into one of the 16 states $\left|\theta^{i}\right\rangle_{234567bcdefg}(i = 0, 1, ..., 14, 15)$. Please refer to Table 1 for specific Alice measurements and collapse results of other agents.

Without losing generality, let us give an example of how this agreement works. Let's say Alice's measurement is $\left|\Phi^{+}\Phi^{+}\right\rangle_{x1y5}$, the other six agents' state will collapse into the state

$$\begin{aligned}\left|\theta^{0}\right\rangle_{234bcd567efg} &= \frac{1}{4}\left(\alpha\left|\varphi^{0}\varphi^{0}\right\rangle + \beta\left|\varphi^{0}\varphi^{1}\right\rangle + \gamma\left|\varphi^{1}\varphi^{0}\right\rangle + \delta\left|\varphi^{1}\varphi^{1}\right\rangle\right)|G_1G_2\rangle\\
&= \frac{1}{4}[\alpha(|000000000000\rangle + |000000000111\rangle + |000000111000\rangle + |000000111111\rangle\\
&+|000011000000\rangle + |000011000111\rangle + |000011111000\rangle + |000011111111\rangle\\
&+|011000000000\rangle + |011000000111\rangle + |011000111000\rangle + |011000111111\rangle\\
&+|011011000000\rangle + |011011000111\rangle + |011011111000\rangle + |011011111111\rangle)\\
&+\beta(|000100000000\rangle + |000100000111\rangle + |000100111000\rangle + |000100111111\rangle\\
&-|000111000000\rangle - |000111000111\rangle - |000111111000\rangle - |000111111111\rangle\\
&+|011100000000\rangle + |011100000111\rangle + |011100111000\rangle + |011100111111\rangle\\
&-|011111000000\rangle - |011111000111\rangle - |011111111000\rangle - |011111111111\rangle)\\
&+\gamma(|100000000000\rangle + |100000000111\rangle + |100000111000\rangle + |100000111111\rangle\\
&+|100011000000\rangle + |100011000111\rangle + |100011111000\rangle + |100011111111\rangle\\
&-|111000000000\rangle - |111000000111\rangle - |111000111000\rangle - |111000111111\rangle\\
&-|111011000000\rangle - |111011000111\rangle - |111011111000\rangle - |111011111111\rangle)\\
&+\delta(|100100000000\rangle + |100100000111\rangle + |100100111000\rangle + |100100111111\rangle\\
&-|100111000000\rangle - |100111000111\rangle - |100111111000\rangle - |100111111111\rangle\\
&-|111100000000\rangle - |111100000111\rangle - |111100111000\rangle - |111100111111\rangle\\
&+|111111000000\rangle + |111111000111\rangle + |111111111000\rangle + |111111111111\rangle)_{234bcd567efg}\end{aligned} \tag{7}$$

Case 1 Agent in high-level recovers the secret state.

See Fig. 2 for a diagram of the operational structure associated with recovering secrets by a high-level agent. Alice chooses Bob to recovery the secret state. First, Bob and David1 jointly take particles 5 and e as control particles, and particles 2 and b as target particles to execute CNOT gate, namely $(5, 2)$ and (e, b) respectively. The system collapses into the following form:

Table 1 Alice's measurements and the collapse state of the system

Alice's measurement	System collapse state
$\left\|\Phi^+\Phi^+\right\rangle$	$\left\|\theta^0\right\rangle = {}^1/_4\left(\alpha\left\|\varphi^0\varphi^0\right\rangle + \beta\left\|\varphi^0\varphi^1\right\rangle + \gamma\left\|\varphi^1\varphi^0\right\rangle + \delta\left\|\varphi^1\varphi^1\right\rangle\right)\|G_1G_2\rangle$
$\left\|\Phi^+\Phi^-\right\rangle$	$\left\|\theta^1\right\rangle = {}^1/_4\left(\alpha\left\|\varphi^0\varphi^0\right\rangle - \beta\left\|\varphi^0\varphi^1\right\rangle + \gamma\left\|\varphi^1\varphi^0\right\rangle - \delta\left\|\varphi^1\varphi^1\right\rangle\right)\|G_1G_2\rangle$
$\left\|\Phi^+\Psi^+\right\rangle$	$\left\|\theta^2\right\rangle = {}^1/_4\left(\beta\left\|\varphi^0\varphi^0\right\rangle + \alpha\left\|\varphi^0\varphi^1\right\rangle + \delta\left\|\varphi^1\varphi^0\right\rangle + \gamma\left\|\varphi^1\varphi^1\right\rangle\right)\|G_1G_2\rangle$
$\left\|\Phi^+\Psi^-\right\rangle$	$\left\|\theta^3\right\rangle = {}^1/_4\left(-\beta\left\|\varphi^0\varphi^0\right\rangle + \alpha\left\|\varphi^0\varphi^1\right\rangle - \delta\left\|\varphi^1\varphi^0\right\rangle + \gamma\left\|\varphi^1\varphi^1\right\rangle\right)\|G_1G_2\rangle$
$\left\|\Phi^-\Phi^+\right\rangle$	$\left\|\theta^4\right\rangle = {}^1/_4\left(\alpha\left\|\varphi^0\varphi^0\right\rangle + \beta\left\|\varphi^0\varphi^1\right\rangle - \gamma\left\|\varphi^1\varphi^0\right\rangle - \delta\left\|\varphi^1\varphi^1\right\rangle\right)\|G_1G_2\rangle$
$\left\|\Phi^-\Phi^-\right\rangle$	$\left\|\theta^5\right\rangle = {}^1/_4\left(\alpha\left\|\varphi^0\varphi^0\right\rangle - \beta\left\|\varphi^0\varphi^1\right\rangle - \gamma\left\|\varphi^1\varphi^0\right\rangle + \delta\left\|\varphi^1\varphi^1\right\rangle\right)\|G_1G_2\rangle$
$\left\|\Phi^-\Psi^+\right\rangle$	$\left\|\theta^6\right\rangle = {}^1/_4\left(\beta\left\|\varphi^0\varphi^0\right\rangle + \alpha\left\|\varphi^0\varphi^1\right\rangle - \delta\left\|\varphi^1\varphi^0\right\rangle - \gamma\left\|\varphi^1\varphi^1\right\rangle\right)\|G_1G_2\rangle$
$\left\|\Phi^-\Psi^-\right\rangle$	$\left\|\theta^7\right\rangle = {}^1/_4\left(-\beta\left\|\varphi^0\varphi^0\right\rangle + \alpha\left\|\varphi^0\varphi^1\right\rangle + \delta\left\|\varphi^1\varphi^0\right\rangle - \gamma\left\|\varphi^1\varphi^1\right\rangle\right)\|G_1G_2\rangle$
$\left\|\Psi^+\Phi^+\right\rangle$	$\left\|\theta^8\right\rangle = {}^1/_4\left(\gamma\left\|\varphi^0\varphi^0\right\rangle + \delta\left\|\varphi^0\varphi^1\right\rangle + \alpha\left\|\varphi^1\varphi^0\right\rangle + \beta\left\|\varphi^1\varphi^1\right\rangle\right)\|G_1G_2\rangle$
$\left\|\Psi^+\Phi^-\right\rangle$	$\left\|\theta^9\right\rangle = {}^1/_4\left(\gamma\left\|\varphi^0\varphi^0\right\rangle - \delta\left\|\varphi^0\varphi^1\right\rangle + \alpha\left\|\varphi^1\varphi^0\right\rangle - \beta\left\|\varphi^1\varphi^1\right\rangle\right)\|G_1G_2\rangle$
$\left\|\Psi^+\Psi^+\right\rangle$	$\left\|\theta^{10}\right\rangle = {}^1/_4\left(\delta\left\|\varphi^0\varphi^0\right\rangle + \gamma\left\|\varphi^0\varphi^1\right\rangle + \beta\left\|\varphi^1\varphi^0\right\rangle + \alpha\left\|\varphi^1\varphi^1\right\rangle\right)\|G_1G_2\rangle$
$\left\|\Psi^+\Psi^-\right\rangle$	$\left\|\theta^{11}\right\rangle = {}^1/_4\left(-\delta\left\|\varphi^0\varphi^0\right\rangle + \gamma\left\|\varphi^0\varphi^1\right\rangle - \beta\left\|\varphi^1\varphi^0\right\rangle + \alpha\left\|\varphi^1\varphi^1\right\rangle\right)\|G_1G_2\rangle$
$\left\|\Psi^-\Phi^+\right\rangle$	$\left\|\theta^{12}\right\rangle = {}^1/_4\left(\gamma\left\|\varphi^0\varphi^0\right\rangle + \delta\left\|\varphi^0\varphi^1\right\rangle - \alpha\left\|\varphi^1\varphi^0\right\rangle - \beta\left\|\varphi^1\varphi^1\right\rangle\right)\|G_1G_2\rangle$
$\left\|\Psi^-\Phi^-\right\rangle$	$\left\|\theta^{13}\right\rangle = {}^1/_4\left(\gamma\left\|\varphi^0\varphi^0\right\rangle - \delta\left\|\varphi^0\varphi^1\right\rangle - \alpha\left\|\varphi^1\varphi^0\right\rangle + \beta\left\|\varphi^1\varphi^1\right\rangle\right)\|G_1G_2\rangle$
$\left\|\Psi^-\Psi^+\right\rangle$	$\left\|\theta^{14}\right\rangle = {}^1/_4\left(\delta\left\|\varphi^0\varphi^0\right\rangle + \gamma\left\|\varphi^0\varphi^1\right\rangle - \beta\left\|\varphi^1\varphi^0\right\rangle - \alpha\left\|\varphi^1\varphi^1\right\rangle\right)\|G_1G_2\rangle$
$\left\|\Psi^-\Psi^-\right\rangle$	$\left\|\theta^{15}\right\rangle = {}^1/_4\left(-\delta\left\|\varphi^0\varphi^0\right\rangle + \gamma\left\|\varphi^0\varphi^1\right\rangle + \beta\left\|\varphi^1\varphi^0\right\rangle - \alpha\left\|\varphi^1\varphi^1\right\rangle\right)\|G_1G_2\rangle$

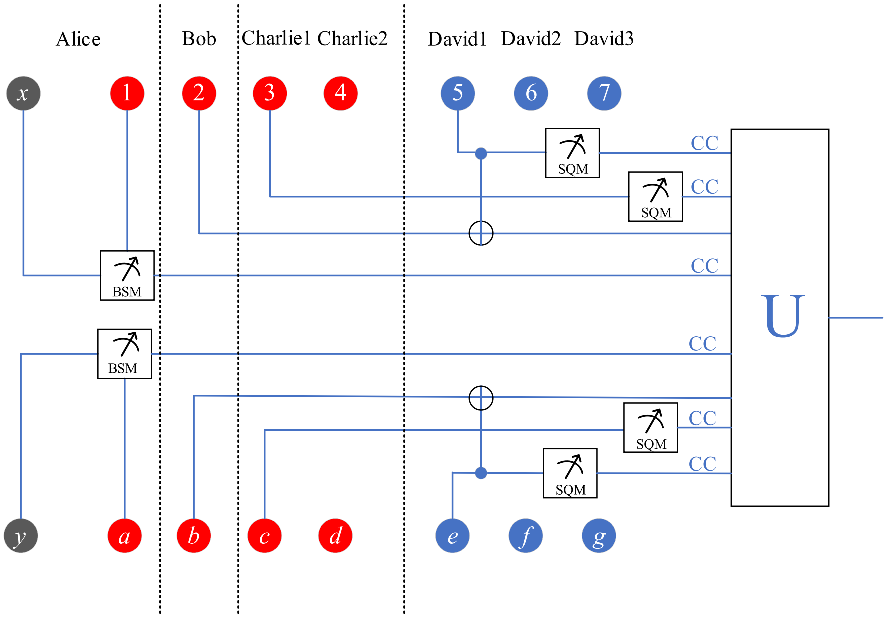

Fig. 2 The black sphere is the unknown state double qubit, the red sphere is two four-qubit clusters, and the blue sphere is two three-particle GHZ state. BSM is a Bell measurement, SQM is a single bit measurement, CC is a classical channel, and U is a unitary operation. Particle 5 is the control particle, particle 2 is the target particle to form the CNOT gate, so is (e, b)

$$
\begin{aligned}
\left|\theta^0\right\rangle &= \frac{1}{4}\left(\alpha\left|\varphi^0\varphi^0\right\rangle+\beta\left|\varphi^0\varphi^1\right\rangle+\gamma\left|\varphi^1\varphi^0\right\rangle+\delta\left|\varphi^1\varphi^1\right\rangle\right)|G_1G_2\rangle \\
&= \frac{1}{4}\big[|0000000000\rangle(\alpha|00\rangle+\beta|01\rangle+\gamma|10\rangle+\delta|11\rangle) \\
&\quad +|0000010101\rangle(\alpha|01\rangle+\beta|00\rangle+\gamma|11\rangle+\delta|10\rangle) \\
&\quad +|0000101010\rangle(\alpha|10\rangle+\beta|11\rangle+\gamma|00\rangle+\delta|01\rangle) \\
&\quad +|0000111111\rangle(\alpha|11\rangle+\beta|10\rangle+\gamma|01\rangle+\delta|00\rangle) \\
&\quad +|0101000000\rangle(\alpha|00\rangle-\beta|01\rangle+\gamma|10\rangle-\delta|11\rangle) \\
&\quad +|0101010101\rangle(\alpha|01\rangle-\beta|00\rangle+\gamma|11\rangle-\delta|10\rangle) \\
&\quad +|0101101010\rangle(\alpha|10\rangle-\beta|11\rangle+\gamma|00\rangle-\delta|01\rangle) \\
&\quad +|0101111111\rangle(\alpha|11\rangle-\beta|10\rangle+\gamma|01\rangle-\delta|00\rangle) \\
&\quad +|1010000000\rangle(\alpha|00\rangle+\beta|01\rangle-\gamma|10\rangle-\delta|11\rangle) \\
&\quad +|1010010101\rangle(\alpha|01\rangle+\beta|00\rangle-\gamma|11\rangle-\delta|10\rangle) \\
&\quad +|1010101010\rangle(\alpha|10\rangle+\beta|11\rangle-\gamma|00\rangle-\delta|01\rangle) \\
&\quad +|1010111111\rangle(\alpha|11\rangle+\beta|10\rangle-\gamma|01\rangle-\delta|00\rangle) \\
&\quad +|1111000000\rangle(\alpha|00\rangle-\beta|01\rangle-\gamma|10\rangle+\delta|11\rangle) \\
&\quad +|1111010101\rangle(\alpha|01\rangle-\beta|00\rangle-\gamma|11\rangle+\delta|10\rangle) \\
&\quad +|1111101010\rangle(\alpha|10\rangle-\beta|11\rangle-\gamma|00\rangle+\delta|01\rangle) \\
&\quad +|1111111111\rangle(\alpha|11\rangle-\beta|10\rangle-\gamma|01\rangle+\delta|00\rangle)\big]_{3c4d5e6f7g2b}
\end{aligned}
\tag{8}
$$

To help Bob reconstruct the original secret state, the other receivers take measurements of their own particles with appropriate measuring bases and publish their measurements. If Charlie1 and Charlie2 choose $\{|0\rangle, |1\rangle\}$ to measure their particles, their measurements will always be the same. Similarly, David1, David2 and David3 also choose $\{|0\rangle, |1\rangle\}$ to measure their own particles, and their results are the same. Therefore, it is only necessary to measure any of Charlie1 and Charlie2 and any of David1, David2 and David3 and send the result to Bob. Bob can reconstruct the unknown state through corresponding unitary operation, so as to recover the secret and complete the communication.

Without loss of generality, assuming that the measurement results of Alice, Charlie1 and David1 are $\left|\Phi^+\Phi^+\right\rangle_{x1y5}$, $|01\rangle_{3c}$ and $|01\rangle_{5e}$ respectively, then the measurement results obtained by Bob are

$$|\varphi\rangle_{2b} = \alpha|01\rangle - \beta|00\rangle + \gamma|11\rangle - \delta|10\rangle \tag{9}$$

To recovery the original secret state $|\varphi\rangle_{xy}$, Bob does unitary operation $U = I \otimes \sigma_X\sigma_Z$ on his particle to get formula (1). When the measurement result of Alice is $\left|\Phi^+\Phi^+\right\rangle_{x1y5}$, all the measurement results and unitary operations are shown in Table 2.

Case 2 Agent in mid-level recovers the secret state.

See Fig. 3 for a diagram of the operational structure associated with recovering secrets by a mid-level agent. Here again, formula (7) is chosen as an example. Alice chooses Charlie1 to recovery the secret state. First, David1 and Charlie1 jointly take particles 5 and e as control particles, and particles 3 and c as target particles to execute

Table 2 Charlie1's and David1's measurements, unitary operation of Bob

Charlie1's and David1's measurements	Bob's operations
$\|0000\rangle$	$I \otimes I$
$\|0001\rangle$	$I \otimes \sigma_X$
$\|0010\rangle$	$\sigma_X \otimes I$
$\|0011\rangle$	$\sigma_X \otimes \sigma_X$
$\|0100\rangle$	$I \otimes \sigma_Z$
$\|0101\rangle$	$I \otimes \sigma_Y$
$\|0110\rangle$	$\sigma_X \otimes \sigma_Z$
$\|0111\rangle$	$\sigma_X \otimes \sigma_Y$
$\|1000\rangle$	$\sigma_Z \otimes I$
$\|1001\rangle$	$\sigma_Z \otimes \sigma_X$
$\|1010\rangle$	$\sigma_Y \otimes I$
$\|1011\rangle$	$\sigma_Y \otimes \sigma_X$
$\|1100\rangle$	$\sigma_Z \otimes \sigma_Z$
$\|1101\rangle$	$\sigma_Z \otimes \sigma_Y$
$\|1110\rangle$	$\sigma_Y \otimes \sigma_Z$
$\|1111\rangle$	$\sigma_Y \otimes \sigma_Y$

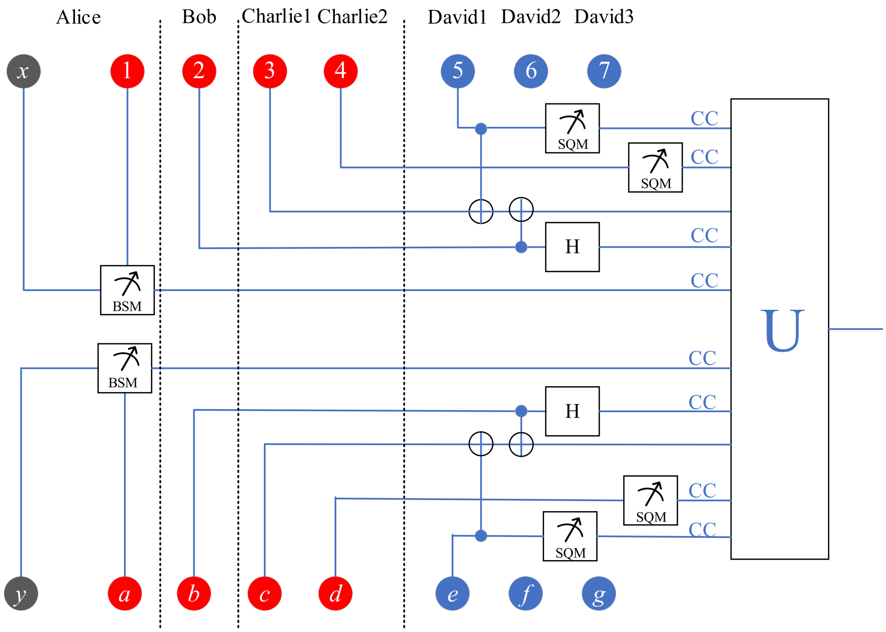

Fig. 3 The black sphere is the unknown state double qubit, the red sphere is two four-qubit clusters, and the blue sphere is two three-particle GHZ state. BSM is a Bell measurement, SQM is a single bit measurement, CC is a classical channel, H is a Hadamard gate, and U is a unitary operation. Particle 5 is the control particle, particle 3 is the target particle to form the CNOT gate, so is (2, 3), (e, c) and (b, c)

CNOT gate, namely (5, 3) and (e, c) respectively. Then, Bob and Charlie1 jointly take particles 2 and b as control particles, and particles 3 and c as target particles to execute CNOT gate, namely (2, 3) and (b, c) respectively. Finally, Bob's particles 2 and b perform the Hadamard gate transformation. The system collapses into the following form:

$$
\begin{aligned}
|\theta^0\rangle &= \frac{1}{4}\left(\alpha|\varphi^0\varphi^0\rangle + \beta|\varphi^0\varphi^1\rangle + \gamma|\varphi^1\varphi^0\rangle + \delta|\varphi^1\varphi^1\rangle\right)|G_1G_2\rangle \\
&= \frac{1}{8}\left\{|00000000\rangle\begin{bmatrix}|00\rangle(\alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle) + |01\rangle(\alpha|00\rangle - \beta|01\rangle + \gamma|10\rangle - \delta|11\rangle) \\ +|10\rangle(\alpha|00\rangle + \beta|01\rangle - \gamma|10\rangle - \delta|11\rangle) + |11\rangle(\alpha|00\rangle - \beta|01\rangle - \gamma|10\rangle + \delta|11\rangle)\end{bmatrix}\right. \\
&+ |00010101\rangle\begin{bmatrix}|00\rangle(\alpha|01\rangle + \beta|00\rangle + \gamma|11\rangle + \delta|10\rangle) + |01\rangle(\alpha|01\rangle - \beta|00\rangle + \gamma|11\rangle - \delta|10\rangle) \\ +|10\rangle(\alpha|01\rangle + \beta|00\rangle - \gamma|11\rangle - \delta|10\rangle) + |11\rangle(\alpha|01\rangle - \beta|00\rangle - \gamma|11\rangle + \delta|10\rangle)\end{bmatrix} \\
&+ |00101010\rangle\begin{bmatrix}|00\rangle(\alpha|10\rangle + \beta|11\rangle + \gamma|00\rangle + \delta|01\rangle) + |01\rangle(\alpha|10\rangle - \beta|11\rangle + \gamma|00\rangle - \delta|01\rangle) \\ +|10\rangle(\alpha|10\rangle + \beta|11\rangle - \gamma|00\rangle - \delta|01\rangle) + |11\rangle(\alpha|10\rangle - \beta|11\rangle - \gamma|00\rangle + \delta|01\rangle)\end{bmatrix} \\
&+ |00111111\rangle\begin{bmatrix}|00\rangle(\alpha|11\rangle + \beta|10\rangle + \gamma|01\rangle + \delta|00\rangle) + |01\rangle(\alpha|11\rangle - \beta|10\rangle + \gamma|01\rangle - \delta|00\rangle) \\ +|10\rangle(\alpha|11\rangle + \beta|10\rangle - \gamma|01\rangle - \delta|00\rangle) + |11\rangle(\alpha|11\rangle - \beta|10\rangle - \gamma|01\rangle + \delta|00\rangle)\end{bmatrix} \\
&+ |01000000\rangle\begin{bmatrix}|00\rangle(\alpha|01\rangle - \beta|00\rangle + \gamma|11\rangle - \delta|10\rangle) + |01\rangle(\alpha|01\rangle + \beta|00\rangle + \gamma|11\rangle + \delta|10\rangle) \\ +|10\rangle(\alpha|01\rangle - \beta|00\rangle - \gamma|11\rangle + \delta|10\rangle) + |11\rangle(\alpha|01\rangle + \beta|00\rangle - \gamma|11\rangle - \delta|10\rangle)\end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
&+|01010101\rangle\begin{bmatrix}|00\rangle(\alpha|00\rangle-\beta|01\rangle+\gamma|10\rangle-\delta|11\rangle)+|01\rangle(\alpha|00\rangle+\beta|01\rangle+\gamma|10\rangle+\delta|11\rangle)\\+|10\rangle(\alpha|00\rangle-\beta|01\rangle-\gamma|10\rangle+\delta|11\rangle)+|11\rangle(\alpha|00\rangle+\beta|01\rangle-\gamma|10\rangle-\delta|11\rangle)\end{bmatrix}\\
&+|01101010\rangle\begin{bmatrix}|00\rangle(\alpha|11\rangle-\beta|10\rangle+\gamma|01\rangle-\delta|00\rangle)+|01\rangle(\alpha|11\rangle+\beta|10\rangle+\gamma|01\rangle+\delta|00\rangle)\\+|10\rangle(\alpha|11\rangle-\beta|10\rangle-\gamma|01\rangle+\delta|00\rangle)+|11\rangle(\alpha|11\rangle+\beta|10\rangle-\gamma|01\rangle-\delta|00\rangle)\end{bmatrix}\\
&+|01111111\rangle\begin{bmatrix}|00\rangle(\alpha|10\rangle-\beta|11\rangle+\gamma|00\rangle-\delta|01\rangle)+|01\rangle(\alpha|10\rangle+\beta|11\rangle+\gamma|00\rangle+\delta|01\rangle)\\+|10\rangle(\alpha|10\rangle-\beta|11\rangle-\gamma|00\rangle+\delta|01\rangle)+|11\rangle(\alpha|10\rangle+\beta|11\rangle-\gamma|00\rangle-\delta|01\rangle)\end{bmatrix}\\
&+|10000000\rangle\begin{bmatrix}|00\rangle(\alpha|10\rangle+\beta|11\rangle-\gamma|00\rangle-\delta|01\rangle)+|01\rangle(\alpha|10\rangle-\beta|11\rangle-\gamma|00\rangle+\delta|01\rangle)\\+|10\rangle(\alpha|10\rangle+\beta|11\rangle+\gamma|00\rangle+\delta|01\rangle)+|11\rangle(\alpha|10\rangle-\beta|11\rangle+\gamma|00\rangle-\delta|01\rangle)\end{bmatrix}\\
&+|10010101\rangle\begin{bmatrix}|00\rangle(\alpha|11\rangle+\beta|10\rangle-\gamma|01\rangle-\delta|00\rangle)+|01\rangle(\alpha|11\rangle-\beta|10\rangle-\gamma|01\rangle+\delta|00\rangle)\\+|10\rangle(\alpha|11\rangle+\beta|10\rangle+\gamma|01\rangle+\delta|00\rangle)+|11\rangle(\alpha|11\rangle-\beta|10\rangle+\gamma|01\rangle-\delta|00\rangle)\end{bmatrix}\\
&+|10101010\rangle\begin{bmatrix}|00\rangle(\alpha|00\rangle+\beta|01\rangle-\gamma|10\rangle-\delta|11\rangle)+|01\rangle(\alpha|00\rangle-\beta|01\rangle-\gamma|10\rangle+\delta|11\rangle)\\+|10\rangle(\alpha|00\rangle+\beta|01\rangle+\gamma|10\rangle+\delta|11\rangle)+|11\rangle(\alpha|00\rangle-\beta|01\rangle+\gamma|10\rangle-\delta|11\rangle)\end{bmatrix}\\
&+|10111111\rangle\begin{bmatrix}|00\rangle(\alpha|01\rangle+\beta|00\rangle-\gamma|11\rangle-\delta|10\rangle)+|01\rangle(\alpha|01\rangle-\beta|00\rangle-\gamma|11\rangle+\delta|10\rangle)\\+|10\rangle(\alpha|01\rangle+\beta|00\rangle+\gamma|11\rangle+\delta|10\rangle)+|11\rangle(\alpha|01\rangle-\beta|00\rangle+\gamma|11\rangle-\delta|10\rangle)\end{bmatrix}\\
&+|11000000\rangle\begin{bmatrix}|00\rangle(\alpha|11\rangle-\beta|10\rangle-\gamma|01\rangle+\delta|00\rangle)+|01\rangle(\alpha|11\rangle+\beta|10\rangle-\gamma|01\rangle-\delta|00\rangle)\\+|10\rangle(\alpha|11\rangle-\beta|10\rangle+\gamma|01\rangle-\delta|00\rangle)+|11\rangle(\alpha|11\rangle+\beta|10\rangle+\gamma|01\rangle+\delta|00\rangle)\end{bmatrix}\\
&+|11010101\rangle\begin{bmatrix}|00\rangle(\alpha|10\rangle-\beta|11\rangle-\gamma|00\rangle+\delta|01\rangle)+|01\rangle(\alpha|10\rangle+\beta|11\rangle-\gamma|00\rangle-\delta|01\rangle)\\+|10\rangle(\alpha|10\rangle-\beta|11\rangle+\gamma|00\rangle-\delta|01\rangle)+|11\rangle(\alpha|10\rangle+\beta|11\rangle+\gamma|00\rangle+\delta|01\rangle)\end{bmatrix}\\
&+|11101010\rangle\begin{bmatrix}|00\rangle(\alpha|01\rangle-\beta|00\rangle-\gamma|11\rangle+\delta|10\rangle)+|01\rangle(\alpha|01\rangle+\beta|00\rangle-\gamma|11\rangle-\delta|10\rangle)\\+|10\rangle(\alpha|01\rangle-\beta|00\rangle+\gamma|11\rangle-\delta|10\rangle)+|11\rangle(\alpha|01\rangle+\beta|00\rangle+\gamma|11\rangle+\delta|10\rangle)\end{bmatrix}\\
&+|11111111\rangle\left.\begin{bmatrix}|00\rangle(\alpha|00\rangle-\beta|01\rangle-\gamma|10\rangle+\delta|11\rangle)+|01\rangle(\alpha|00\rangle+\beta|01\rangle-\gamma|10\rangle-\delta|11\rangle)\\+|10\rangle(\alpha|00\rangle-\beta|01\rangle+\gamma|10\rangle-\delta|11\rangle)+|11\rangle(\alpha|00\rangle+\beta|01\rangle+\gamma|10\rangle+\delta|11\rangle)\end{bmatrix}\right\}_{4d5e6f7g2b3c}
\end{aligned}
\tag{10}
$$

According to formula (11), if $\{|0\rangle, |1\rangle\}$ is selected for measurement, the measurement results of David1, David2 and David3 are always the same. Therefore, we choose the measurement results of David1 as the representative to help Charlie1 carry out secret recovery. In addition, we also need to know the measurement results of Bob and Charlie2 to help Charlie1 perform unitary operation to recover the secret.

Without loss of generality, assuming that the measurement results of Alice, Bob, Charlie2 and David1 are $\left|\Phi^+\Phi^+\right\rangle_{x1y5}$,$|01\rangle_{2b}$,$|00\rangle_{4d}$ and $|10\rangle_{5e}$ respectively, then the measurement results obtained by Charlie1 are

$$|\varphi\rangle_{3c}=\alpha|10\rangle-\beta|11\rangle+\gamma|00\rangle-\delta|01\rangle \tag{11}$$

To recovery the original secret state $|\varphi\rangle_{xy}$, Bob does the unitary operation $U=\sigma_X\otimes\sigma_Z$ on his particle to obtain formula (1). When the measurement result of Alice is $\left|\Phi^+\Phi^+\right\rangle_{x1y5}$, all the measurement results and unitary operations are shown in Table 3.

Case 3 Agent in low-level recovers the secret state.

Figure 4 is a diagram of the operational structure associated with recovering a secret by a low-level agent. Here, Formula (7) is still taken as an example. Alice chooses low-level agent David3 to recover the secret. Firstly, Bob, David2 and David3 jointly take particles 2 and b as control particles and particles 6 and$f$, 7 and g as target particles

Table 3 Bob's, Charlie2's and David1's measurements, unitary operation of Charlie1

Bob's, Charlie2's and David1's measurements	Charlie1's operations
$\vert 000000\rangle$,$\vert 010101\rangle$,$\vert 101010\rangle$,$\vert 111111\rangle$	$I \otimes I$
$\vert 010000\rangle$,$\vert 000101\rangle$,$\vert 111010\rangle$,$\vert 101111\rangle$	$I \otimes \sigma_Z$
$\vert 100000\rangle$,$\vert 110101\rangle$,$\vert 001010\rangle$,$\vert 011111\rangle$	$\sigma_Z \otimes I$
$\vert 110000\rangle$,$\vert 100101\rangle$,$\vert 011010\rangle$,$\vert 001111\rangle$	$\sigma_Z \otimes \sigma_Z$
$\vert 000001\rangle$,$\vert 010100\rangle$,$\vert 101011\rangle$,$\vert 111110\rangle$	$I \otimes \sigma_X$
$\vert 010001\rangle$,$\vert 000100\rangle$,$\vert 111011\rangle$,$\vert 101110\rangle$	$I \otimes \sigma_Y$
$\vert 100001\rangle$,$\vert 110100\rangle$,$\vert 001011\rangle$,$\vert 011110\rangle$	$\sigma_Z \otimes \sigma_X$
$\vert 110001\rangle$,$\vert 100100\rangle$,$\vert 011011\rangle$,$\vert 001110\rangle$	$\sigma_Z \otimes \sigma_Y$
$\vert 000010\rangle$,$\vert 010111\rangle$,$\vert 101000\rangle$,$\vert 111101\rangle$	$\sigma_X \otimes I$
$\vert 010010\rangle$,$\vert 000111\rangle$,$\vert 111000\rangle$,$\vert 101101\rangle$	$\sigma_X \otimes \sigma_Z$
$\vert 100010\rangle$,$\vert 110111\rangle$,$\vert 001000\rangle$,$\vert 011101\rangle$	$\sigma_Y \otimes I$
$\vert 110010\rangle$,$\vert 100111\rangle$,$\vert 011000\rangle$,$\vert 001101\rangle$	$\sigma_Y \otimes \sigma_Z$
$\vert 000011\rangle$,$\vert 010110\rangle$,$\vert 101001\rangle$,$\vert 111100\rangle$	$\sigma_X \otimes \sigma_X$
$\vert 010011\rangle$,$\vert 000110\rangle$,$\vert 111001\rangle$,$\vert 101100\rangle$	$\sigma_X \otimes \sigma_Y$
$\vert 100011\rangle$,$\vert 110110\rangle$,$\vert 001001\rangle$,$\vert 011100\rangle$	$\sigma_Y \otimes \sigma_X$
$\vert 110011\rangle$,$\vert 100110\rangle$,$\vert 011001\rangle$,$\vert 001100\rangle$	$\sigma_Y \otimes \sigma_Y$

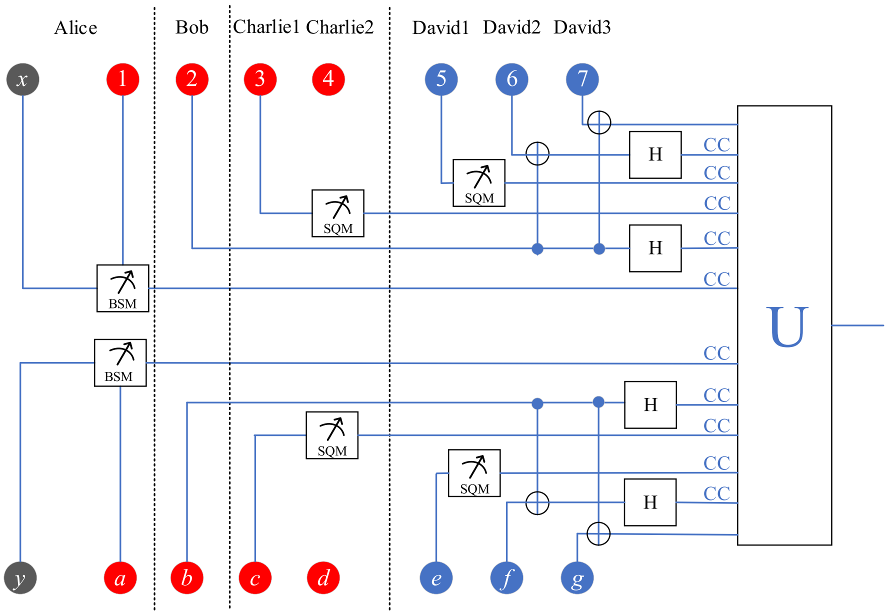

Fig. 4 The black sphere is the unknown state double qubit, the red sphere is two four-qubit clusters, and the blue sphere is two three-particle GHZ state. BSM is a Bell measurement, SQM is a single bit measurement, CC is a classical channel, H is a Hadamard gate, and U is a unitary operation. Particle 2 is the control particle, particle 6 is the target particle to form the CNOT gate, so is (2, 7), (b, f) and (b, g)

to execute CNOT gate, namely (2, 6), (2, 7) and (*b*,*f*), (*b*, *g*) respectively. Finally, Bob's particles 2 and *b* and David2's particles 6 and *f* undergo the Hadamard gate transformation. The system collapses into the following forms:

$$\begin{aligned}\left|\theta^0\right\rangle_{234bcd567efg} &= \frac{1}{4}\left(\alpha\left|\varphi^0\varphi^0\right\rangle + \beta\left|\varphi^0\varphi^1\right\rangle + \gamma\left|\varphi^1\varphi^0\right\rangle + \delta\left|\varphi^1\varphi^1\right\rangle\right)|G_1G_2\rangle \\ &= \frac{1}{4}\{\alpha|{+}{+}\rangle[|{+}{+}\rangle|00\rangle(|000000\rangle + |001100\rangle + |110000\rangle + |111100\rangle) \\ &\quad + |{+}{-}\rangle|01\rangle(|000001\rangle + |001101\rangle + |110001\rangle + |111101\rangle) \\ &\quad + |{-}{+}\rangle|10\rangle(|000010\rangle + |001110\rangle + |110010\rangle + |111110\rangle) \\ &\quad + |{-}{-}\rangle|11\rangle(|000011\rangle + |001111\rangle + |110011\rangle + |111111\rangle)] \\ &\quad + \beta|{+}{-}\rangle[|{+}{-}\rangle|00\rangle(|000001\rangle - |001101\rangle + |110001\rangle - |111101\rangle) \\ &\quad + |{+}{+}\rangle|01\rangle(|000000\rangle - |001100\rangle + |110000\rangle - |111100\rangle) \\ &\quad + |{-}{-}\rangle|10\rangle(|000011\rangle - |001111\rangle + |110011\rangle - |111111\rangle) \\ &\quad + |{-}{+}\rangle|11\rangle(|000010\rangle - |001110\rangle + |110010\rangle - |111110\rangle)] \\ &\quad + \gamma|{-}{+}\rangle[|{-}{+}\rangle|00\rangle(|000010\rangle + |001110\rangle - |110010\rangle - |111110\rangle) \\ &\quad + |{-}{-}\rangle|01\rangle(|000011\rangle + |001111\rangle - |110011\rangle - |111111\rangle) \\ &\quad + |{+}{+}\rangle|10\rangle(|000000\rangle + |001100\rangle - |110000\rangle - |111100\rangle) \\ &\quad + |{+}{-}\rangle|11\rangle(|000001\rangle + |001101\rangle - |110001\rangle - |111101\rangle)] \\ &\quad + \delta|{-}{-}\rangle[|{-}{-}\rangle|00\rangle(|000011\rangle - |001111\rangle - |110011\rangle + |111111\rangle) \\ &\quad + |{-}{+}\rangle|01\rangle(|000010\rangle - |001110\rangle - |110010\rangle + |111110\rangle) \\ &\quad + |{+}{-}\rangle|10\rangle(|000001\rangle - |001101\rangle - |110001\rangle + |111101\rangle) \\ &\quad + |{+}{+}\rangle|11\rangle(|000000\rangle - |001100\rangle - |110000\rangle + |111100\rangle)]\}_{2b6f5e34cd7g}\end{aligned} \tag{12}$$

David3's reconstruction of Alice's secret state requires other receivers to make single-particle measurements of their own particles respectively and send the measurement results to David3 through classical channels. As can be seen from formula (12), when $\{|0\rangle, |1\rangle\}$ is selected as the measurement basis, the measurement results of Charlie1 and Charlie2 are always the same, so only one of them is needed to help, and Charlie1 is chosen here. That is to say, David3 needs the help of Bob, Charlie1, David1 and David2 to restore the secret state.

When the measurement result of Alice is $\left|\Phi^+\Phi^+\right\rangle_{x1y5}$, all the measurement results and unitary operations are shown in Table 4.

Higher-level agents need the least help to recover secrets, and lower-level agents need more help. Therefore, the measurement results and corresponding operations required by lower-level agents to recover secrets are more complex.

4 Scalable three-level of HQIS protocol

Generally speaking, agents of the same level often have multiple agent nodes, which requires better universality of the protocol. Here we propose a universal three-level

Table 4 David1's, Charlie1's, Bob's and David2's measurements, unitary operation of David3. The measurements in parentheses correspond to the actions in parentheses

David1	Charlie1	Bob and David2	David3's operations
$\|00\rangle$	$\|00\rangle$	$\|0000\rangle, \|0101\rangle, \|1010\rangle, \|1111\rangle$	$I \otimes I$
	$\|01\rangle$	$\|0001\rangle, \|0100\rangle, \|1011\rangle, \|1110\rangle$	
	$\|10\rangle$	$\|0010\rangle, \|0111\rangle, \|1000\rangle, \|1101\rangle$	
	$\|11\rangle$	$\|0011\rangle, \|0110\rangle, \|1001\rangle, \|1100\rangle$	
	$\|00\rangle$	$\|0001\rangle, \|0100\rangle, \|1011\rangle, \|1110\rangle$	$I \otimes \sigma_Z$
	$\|01\rangle$	$\|0000\rangle, \|0101\rangle, \|1010\rangle, \|1111\rangle$	
	$\|10\rangle$	$\|0011\rangle, \|0110\rangle, \|1001\rangle, \|1100\rangle$	
	$\|11\rangle$	$\|0010\rangle, \|0111\rangle, \|1000\rangle, \|1101\rangle$	
	$\|00\rangle$	$\|0010\rangle, \|0111\rangle, \|1000\rangle, \|1101\rangle$	$\sigma_Z \otimes I$
	$\|01\rangle$	$\|0011\rangle, \|0110\rangle, \|1001\rangle, \|1100\rangle$	
	$\|10\rangle$	$\|0000\rangle, \|0101\rangle, \|1010\rangle, \|1111\rangle$	
	$\|11\rangle$	$\|0001\rangle, \|1011\rangle, \|1110\rangle$	
	$\|00\rangle$	$\|0011\rangle, \|0110\rangle, \|1001\rangle, \|1100\rangle$	$\sigma_Z \otimes \sigma_Z$
	$\|01\rangle$	$\|0010\rangle, \|0111\rangle, \|1000\rangle, \|1101\rangle$	
	$\|10\rangle$	$\|0001\rangle, \|0100\rangle, \|1011\rangle, \|1110\rangle$	
	$\|11\rangle$	$\|0000\rangle, \|0101\rangle, \|1010\rangle, \|1111\rangle$	
$\|01\rangle$	$\|00\rangle$	$\|0000\rangle, \|1010\rangle, (\|0101\rangle, \|1111\rangle)$	$I \otimes \sigma_X, (I \otimes \sigma_Z \sigma_Y)$
	$\|01\rangle$	$\|0100\rangle, \|1110\rangle, (\|0001\rangle, \|1011\rangle)$	
	$\|10\rangle$	$\|0010\rangle, \|1000\rangle, (\|0111\rangle, \|1101\rangle)$	
	$\|11\rangle$	$\|0110\rangle, \|1100\rangle, (\|0011\rangle, \|1001\rangle)$	
	$\|00\rangle$	$\|0100\rangle, \|1110\rangle, (\|0001\rangle, \|1011\rangle)$	$I \otimes \sigma_Y, (I \otimes \sigma_Z \sigma_X)$
	$\|01\rangle$	$\|0000\rangle, \|1010\rangle, (\|0101\rangle, \|1111\rangle)$	
	$\|10\rangle$	$\|0110\rangle, \|1100\rangle, (\|0011\rangle, \|1001\rangle)$	
	$\|11\rangle$	$\|0010\rangle, \|1000\rangle, (\|0111\rangle, \|1101\rangle)$	
	$\|00\rangle$	$\|0010\rangle, \|1000\rangle, (\|0111\rangle, \|1101\rangle)$	$\sigma_Z \otimes \sigma_X, (\sigma_Y \sigma_X \otimes \sigma_X)$
	$\|01\rangle$	$\|0110\rangle, \|1100\rangle, (\|0011\rangle, \|1001\rangle)$	
	$\|10\rangle$	$\|0000\rangle, \|1010\rangle, (\|0101\rangle, \|1111\rangle)$	
	$\|11\rangle$	$\|0100\rangle, \|1110\rangle, (\|0001\rangle, \|1011\rangle)$	
	$\|00\rangle$	$\|0110\rangle, \|1100\rangle, (\|0011\rangle, \|1001\rangle)$	$\sigma_Z \otimes \sigma_Y, (\sigma_Z \otimes \sigma_Z \sigma_X)$
	$\|01\rangle$	$\|0010\rangle, \|1000\rangle, (\|0111\rangle, \|1101\rangle)$	
	$\|10\rangle$	$\|0100\rangle, \|1110\rangle, (\|0001\rangle, \|1011\rangle)$	
	$\|11\rangle$	$\|0000\rangle, \|1010\rangle, (\|0101\rangle, \|1111\rangle)$	
$\|10\rangle$	$\|00\rangle$	$\|0000\rangle, \|0101\rangle, (\|1010\rangle, \|1111\rangle)$	$\sigma_X \otimes I, (\sigma_Z \sigma_Y \otimes I)$
	$\|01\rangle$	$\|0001\rangle, \|0100\rangle, (\|1011\rangle, \|1110\rangle)$	
	$\|10\rangle$	$\|1000\rangle, \|1101\rangle, (\|0010\rangle, \|0111\rangle)$	
	$\|11\rangle$	$\|1001\rangle, \|1100\rangle, (\|0011\rangle, \|0110\rangle)$	

Table 4 (continued)

David1	Charlie1	Bob and David2	David3's operations
	$\|00\rangle$	$\|0001\rangle, \|0100\rangle, (\|1011\rangle, \|1110\rangle)$	$\sigma_X \otimes \sigma_Z, (\sigma_Z\sigma_Y \otimes \sigma_Z)$
	$\|01\rangle$	$\|0000\rangle, \|0101\rangle, (\|1010\rangle, \|1111\rangle)$	
	$\|10\rangle$	$\|1001\rangle, \|1100\rangle, (\|0011\rangle, \|0110\rangle)$	
	$\|11\rangle$	$\|1000\rangle, \|1101\rangle, (\|0010\rangle, \|0111\rangle)$	
	$\|00\rangle$	$\|0010\rangle, \|0111\rangle, (\|1000\rangle, \|1101\rangle)$	$\sigma_Z\sigma_X \otimes I, (\sigma_X\sigma_Z \otimes I)$
	$\|01\rangle$	$\|0011\rangle, \|0110\rangle, (\|1001\rangle, \|1100\rangle)$	
	$\|10\rangle$	$\|1010\rangle, \|1111\rangle, (\|0000\rangle, \|0101\rangle)$	
	$\|11\rangle$	$\|1011\rangle, \|1110\rangle, (\|0001\rangle, \|0100\rangle)$	
	$\|00\rangle$	$\|0011\rangle, \|0110\rangle, (\|1001\rangle, \|1100\rangle)$	$\sigma_Z\sigma_X \otimes \sigma_Z, (\sigma_Y \otimes \sigma_Z)$
	$\|01\rangle$	$\|0010\rangle, \|0111\rangle, (\|1000\rangle, \|1101\rangle)$	
	$\|10\rangle$	$\|1011\rangle, \|1110\rangle, (\|0001\rangle, \|0100\rangle)$	
	$\|11\rangle$	$\|1010\rangle, \|1111\rangle, (\|0000\rangle, \|0101\rangle)$	
$\|11\rangle$	$\|00\rangle$	$\|0000\rangle, \|1111\rangle, (\|0101\rangle, \|1010\rangle)$	$\sigma_X \otimes \sigma_X, (\sigma_X \otimes \sigma_Z\sigma_Y)$
	$\|01\rangle$	$\|0100\rangle, \|1011\rangle, (\|0001\rangle, \|1110\rangle)$	
	$\|10\rangle$	$\|0111\rangle, \|1000\rangle, (\|0010\rangle, \|1101\rangle)$	
	$\|11\rangle$	$\|0011\rangle, \|1100\rangle, (\|0110\rangle, \|1001\rangle)$	
	$\|00\rangle$	$\|0001\rangle, \|1110\rangle, (\|0100\rangle, \|1011\rangle)$	$\sigma_X \otimes \sigma_Z\sigma_X, (\sigma_X \otimes \sigma_Y)$
	$\|01\rangle$	$\|0101\rangle, \|1010\rangle, (\|0000\rangle, \|1111\rangle)$	
	$\|10\rangle$	$\|0110\rangle, \|1001\rangle, (\|0011\rangle, \|1100\rangle)$	
	$\|11\rangle$	$\|0010\rangle, \|1101\rangle, (\|0111\rangle, \|1000\rangle)$	
	$\|00\rangle$	$\|0010\rangle, \|1101\rangle, (\|0111\rangle, \|1000\rangle)$	$\sigma_Z\sigma_X \otimes \sigma_X, (\sigma_Y \otimes \sigma_X)$
	$\|01\rangle$	$\|0110\rangle, \|1001\rangle, (\|0011\rangle, \|1100\rangle)$	
	$\|10\rangle$	$\|0101\rangle, \|1010\rangle, (\|0000\rangle, \|1111\rangle)$	
	$\|11\rangle$	$\|0001\rangle, \|1110\rangle, (\|0100\rangle, \|1011\rangle)$	
	$\|00\rangle$	$\|0011\rangle, \|1100\rangle, (\|0110\rangle, \|1001\rangle)$	$\sigma_X\sigma_Z \otimes \sigma_Y, (\sigma_Z\sigma_X \otimes \sigma_Y)$
	$\|01\rangle$	$\|0111\rangle, \|1000\rangle, (\|0010\rangle, \|1101\rangle)$	
	$\|10\rangle$	$\|0100\rangle, \|1011\rangle, (\|0001\rangle, \|1110\rangle)$	
	$\|11\rangle$	$\|0000\rangle, \|1111\rangle, (\|0101\rangle, \|1010\rangle)$	

agent scheme. It can be observed from formula (5) that we usually take out the first particle of the entangled state and do Bell measurement with the particle of the unknown state, and the remaining particles are distributed to agents of different levels respectively. Our general protocol can be written as follows:

$$
\begin{aligned}
|\Theta\rangle &= |\varphi\rangle_{xy} \otimes |\Omega_1\rangle \otimes |Z_1\rangle \otimes |\Omega_2\rangle \otimes |Z_2\rangle \\
&= \frac{1}{8}\Big(\alpha|0000\rangle\left|\varphi^0\varphi^0\right\rangle + \alpha|0001\rangle\left|\varphi^0\varphi^1\right\rangle + \alpha|0010\rangle\left|\varphi^1\varphi^0\right\rangle + \alpha|0011\rangle\left|\varphi^1\varphi^1\right\rangle \\
&\quad + \beta|0100\rangle\left|\varphi^0\varphi^0\right\rangle + \beta|0101\rangle\left|\varphi^0\varphi^1\right\rangle + \beta|0110\rangle\left|\varphi^1\varphi^0\right\rangle + \beta|0111\rangle\left|\varphi^1\varphi^1\right\rangle
\end{aligned}
$$

$$+\gamma|1000\rangle\left|\varphi^0\varphi^0\right\rangle+\gamma|1001\rangle\left|\varphi^0\varphi^1\right\rangle+\gamma|1010\rangle\left|\varphi^1\varphi^0\right\rangle+\gamma|1011\rangle\left|\varphi^1\varphi^1\right\rangle$$
$$+\delta|1100\rangle\left|\varphi^0\varphi^0\right\rangle+\delta|1101\rangle\left|\varphi^0\varphi^1\right\rangle+\delta|1110\rangle\left|\varphi^1\varphi^0\right\rangle+\delta|1111\rangle\left|\varphi^1\varphi^1\right\rangle\Big)|Z_1Z_2\rangle \tag{13}$$

Here, $|\Omega\rangle$ is that we use a four-particle cluster state, in which the first particle is owned by Alice, and the rest particles are divided into high-level and mid-level proxies. $|Z\rangle$ is a multi-qubit GHZ state, which can realize the expansion of the number of low-grade agents.

$$|Z\rangle=\frac{1}{\sqrt{2}}(|0...0\rangle+|1...1\rangle)_n \tag{14}$$

In order to verify the universality of our protocol, formula (5) is substituted into formula (13). $|\Omega\rangle$ and $|Z\rangle$ are product states, so only $|Z\rangle$ changes in the system state. We observe from Case 1 and Case 2 in Sect. 2 that the states of low-level agents are correlated, when the state of one is known, the state of the other agents is known. Therefore, when we want to verify the correctness of the universal protocol, we only need to verify the low-level proxy recovery secret.

Case 3' Agent in low-level recovers the secret state.

Here we still assume that Alice's Bell measurement result is $\left|\Phi^+\Phi^+\right\rangle_{x1y5}$, and the corresponding collapse result of the system can be written as:

$$|\theta\rangle=\frac{1}{4}\Big(\alpha\left|\varphi^0\varphi^0\right\rangle+\beta\left|\varphi^0\varphi^1\right\rangle+\gamma\left|\varphi^1\varphi^0\right\rangle+\delta\left|\varphi^1\varphi^1\right\rangle\Big)|Z_1Z_2\rangle \tag{15}$$

After the expansion of formula (15), the particles of the low-level agents have correlations. Low-level agent recovery secret requires the help of a high-level agent, a mid-level agent, and all other low-level agents. We chose to use the CNOT gate and the corresponding Hadamard gate operation to break the correlation between low-level agents.

Here we take the N-particle GHZ state as an example,

$$=\frac{1}{4}\Bigg\{\alpha|{+}{+}\rangle\Bigg[\,|{+}{+}\rangle\Big|\underbrace{++\ldots++}_{n-2}\Big\rangle(|000000\rangle+|001100\rangle+|110000\rangle+|111100\rangle)$$
$$+|{+}{-}\rangle\Big|\underbrace{+-\ldots+-}_{n-2}\Big\rangle(|000001\rangle+|001101\rangle+|110001\rangle+|111101\rangle)$$
$$+|{-}{+}\rangle\Big|\underbrace{-+\ldots-+}_{n-2}\Big\rangle(|000010\rangle+|001110\rangle+|110010\rangle+|111110\rangle)$$
$$+|{-}{-}\rangle\Big|\underbrace{--\ldots--}_{n-2}\Big\rangle(|000011\rangle+|001111\rangle+|110011\rangle+|111111\rangle)\Bigg]$$

$$
\begin{aligned}
&+\beta|+-\rangle\Bigg[\,|+-\rangle\Bigg|\underbrace{++\ldots++}_{n-2}\Bigg\rangle(|000001\rangle-|001101\rangle+|110001\rangle-|111101\rangle)\\
&+|++\rangle\Bigg|\underbrace{+-\ldots+-}_{n-2}\Bigg\rangle(|000000\rangle-|001100\rangle+|110000\rangle-|111100\rangle)\\
&+|--\rangle\Bigg|\underbrace{-+\ldots-+}_{n-2}\Bigg\rangle(|000011\rangle-|001111\rangle+|110011\rangle-|111111\rangle)\\
&+|-+\rangle\Bigg|\underbrace{--\ldots--}_{n-2}\Bigg\rangle(|000010\rangle-|001110\rangle+|110010\rangle-|111110\rangle)\Bigg]\\
&+\gamma|-+\rangle\Bigg[\,|-+\rangle\Bigg|\underbrace{++\ldots++}_{n-2}\Bigg\rangle(|000010\rangle+|001110\rangle-|110010\rangle-|111110\rangle)\\
&+|--\rangle\Bigg|\underbrace{+-\ldots+-}_{n-2}\Bigg\rangle(|000011\rangle+|001111\rangle-|110011\rangle-|111111\rangle)\\
&+|++\rangle\Bigg|\underbrace{-+\ldots-+}_{n-2}\Bigg\rangle(|000000\rangle+|001100\rangle-|110000\rangle-|111100\rangle)\\
&+|+-\rangle\Bigg|\underbrace{--\ldots--}_{n-2}\Bigg\rangle(|000001\rangle+|001101\rangle-|110001\rangle-|111101\rangle)\Bigg]\\
&+\delta|--\rangle\Bigg[\,|--\rangle\Bigg|\underbrace{++\ldots++}_{n-2}\Bigg\rangle(|000011\rangle-|001111\rangle-|110011\rangle+|111111\rangle)\\
&+|-+\rangle\Bigg|\underbrace{+-\ldots+-}_{n-2}\Bigg\rangle(|000010\rangle-|001110\rangle-|110010\rangle+|111110\rangle)\\
&+|+-\rangle\Bigg|\underbrace{-+\ldots-+}_{n-2}\Bigg\rangle(|000001\rangle-|001101\rangle-|110001\rangle+|111101\rangle)\\
&+|++\rangle\Bigg|\underbrace{--\ldots--}_{n-2}\Bigg\rangle(|000000\rangle-|001100\rangle-|110000\rangle+|111100\rangle)\Bigg]\Bigg\}_{2b6f(n-2)34cd7g}
\end{aligned}
\tag{16}
$$

In the double n particle GHZ state, two particles are responsible for recovering the secret, they and a pair of particles as the target particles to perform the CNOT gate, the rest of the low-level particles and a pair of particles are Hadamard gate transformation. We take $\{|0\rangle,|1\rangle\}$ as the measuring basis for single-bit measurement, but part of the base vector of the collapsed state after Hadamard gate transformation is $\{|+\rangle,|-\rangle\}$, so we need to use $|+\rangle = {}^{1}\!\big/\!\sqrt{2}(|0\rangle+|1\rangle)$ and $|-\rangle = {}^{1}\!\big/\!\sqrt{2}(|0\rangle-|1\rangle)$ to convert the Z base to the X base, and then get the corresponding relationship between the measurement results of the agent helping to recover the secret and the operation needed to recover the secret.

5 Analysis

5.1 Outsider attack

The common attack mode in quantum communication is the attack from the external eavesdrop Eve. By preparing the quantum state $|E\rangle$, Eve performs unitary operation on the $|E\rangle$ particle and the particles sent in the quantum channel to generate entanglement between the particles. After that, Eve obtains information by measuring $|E\rangle$ and then recovers the secret messages. Specific operations are as follows:

$$\begin{aligned} U|0\rangle|E\rangle &= \alpha|0\rangle|e_1\rangle + \beta|1\rangle|e_2\rangle \\ U|1\rangle|E\rangle &= \gamma|0\rangle|e_3\rangle + \delta|1\rangle|e_4\rangle \end{aligned} \tag{17}$$

Among them, $\beta|e_2\rangle = \gamma|e_3\rangle = 0$. Eve can infer a single quantum of information, but we're transmitting two qubits. When Eve gets both quanta at the same time, he cannot get the arrangement of the two quanta, so his eavesdropping will fail.

Eve wants to steal the quantum information $|\varphi\rangle_{xy}$ of an unknown state. Let's take case1 in Sect. 3 as an example to illustrate. The job of recovering unknown quantum state information is done by Bob, and Eve has no way of knowing who the agent is (Bob, Charlie, or David). Eve should first determine what level the three-tier agent that restores the secret belongs to, assuming that this process is known to the attacker. Second, Alice's different measurements result in Bob's different measurements. In addition, the different measurement results of Charlie1 and David1 will also cause Bob to perform different operations. Suppose these measurements were stolen by the attacker Eve. At the last unitary operation, Eve, the unknown quantum state $|\varphi\rangle_{xy}$ is known, but Bob's measurements are not available. Because this process is done by Bob himself, and there is no need to transmit information. All of the above assumptions require Eve to steal in order to obtain the correct information, but this process has been shown in the previous paragraph is not feasible. When the agent to restore the secret is Charlie or David, the protocol is more secure because Eve has more information to steal.

5.2 Insider attack

A dishonest agent wants to recover the secret alone, so he may resort to a forgery pester attack. Forged entanglement attack refers to the agent of secret recovery. By intercepting the particles sent by the sender to other agents, and creating fake entangled particles to send to other agents, the attacker tests the particles needed for secret recovery, and then completes the recovery of double qubit information. In this agreement, Alice does not specify which is the secret restorer and which is the helper when she sends the particle to the agent. After Eve sends the fake particle to the corresponding

Table 5 Internal attack security analysis

Hierarchy of insider attackers	Needed to intercept particles	Particle sequence	Exposure or not
High-level	$\|p\rangle_{3c}, \|p\rangle_{5e}$	Unknown	Yes
Mid-level	$\|p\rangle_{2b}, \|p\rangle_{4\mathrm{d}}, \|p\rangle_{5\mathrm{e}}$	Unknown	Yes
Low-level	$\|p\rangle_{2b}, \|p\rangle_{3c}, \|p\rangle_{5\mathrm{e}}, \|p\rangle_{6f}$	Unknown	Yes

agent, the agent assisting the secret recovery performs the corresponding Hadamard gate transform or CNOT gate operation, which is associated with the level of the secret recovery agent. When the high-level agent recovers the secret, Bob acts as the target particle of the CNOT operation; When the low-level agent recovers the secret, Bob is the controlling particle of the CNOT operation and performs the Hadamard transform; As a result, the unitary operations required vary. In addition, Eve has acquired two particles from an agent, but he does not know the order in which the two particles are arranged. All of this causes Eve to fail to recover the secrets properly, and other agents to discover that they have lost their connection to each other when performing CNOT gate and Hadamard operations. Table 5 shows the information stolen and the exposure that agents at different levels want to recover secrets individually.

5.3 Efficiency

Previously, Cabello [26] proposed a formula for calculating the communication efficiency of the quantum key distribution protocol. Since there is no unified standard for calculating the efficiency of quantum information splitting communication protocol, Cabello's scheme is adopted here.

$$\varepsilon = \frac{q_s}{q_t + b_t} \tag{18}$$

q_s represents the number of unknown state qubits to be shared in the protocol, q_t represents the number of qubits transmitted in the protocol, and b_t represents the number of classical bits transmitted in the protocol. Table 6 shows the efficiency of the protocol proposed in this paper.

5.4 Comparison

Since 2010, when Wang [21] et al. first proposed HQIS protocol, many researchers have studied HQIS and used different entangled resources to achieve different quantum information resolution for quantum channels. This paper compares with other quantum

Table 6 HQIS protocol efficiency

Category	Recovery level	HQIS protocol efficiency ε
HQIS of 3 + 3 receivers	Bob (High-level)	$\varepsilon = \frac{2}{12+(4+2+2)} = 10\%$
	Charlie1 (Mid-level)	$\varepsilon = \frac{2}{12+(4+2+2+2)} = 9.1\%$
	David3 (Low-level)	$\varepsilon = \frac{2}{12+(4+2+2+2+2)} = 8.3\%$
HQIS of 3 + n receivers	Bob (High-level)	$\varepsilon = \frac{2}{2(3+n)+(4+2+2)} = \frac{2}{2n+14}$
	Charlie1 (Mid-level)	$\varepsilon = \frac{2}{2(3+n)+(4+2+2+2)} = \frac{2}{2n+16}$
	David3 (Low-level)	$\varepsilon = \frac{2}{2(3+n)+(4+2+2)+2(n-1)} = \frac{2}{4n+12}$

information splitting protocols in terms of hierarchical structure, entangled resources, split quantum information, scalability and recovery operations. See Table 7 for details.

As can be seen from Table 7, three-level of two-particle unknown state HQIS schemes are proposed for the first time. We expanded the number of low-level agents, which is to take into account that in real life, the number of high-level agents is often very limited, while the number of low-level agents tends to be very large and commonly used. For example, the key to the bank vault is secret information, the bank president is the senior agent, the vice president is the middle agent, and the manager is the junior agent. Look at Fig. 5.

6 Conclusion

We propose an extensible three-level HQIS protocol with two quantum unknown states. First, we give a HQIS protocol for double quantum unknown states based on the product states of double four-particle cluster states and double three-particle GHZ states. The secret sender shares the double quantum unknown state with six agents. The sender needs to first make Bell measurements with the particles and the unknown state particles in her hand, and then send her measurement results to the receiver through the classical channel. When the sender decides that a high-level agent will recover the secret, the high-level agent needs the help of a mid-level agent and a low-level agent to recover the secret. When a mid-level agent restores the secret, he needs the help of a high-level agent, a mid-level agent, and a low-level agent to restore the secret. When the low-level agent recovers the secret, he needs the help of a high-level agent, a mid-level agent, and all remaining low-level agents. We then extended the protocol. In real scenarios, the number of agents of high-level and mid-level is often small, while the number of agents of low-level is often large. Therefore, we extend the three-particle GHZ state to the multi-particle GHZ state, and make the product

Table 7 HQIS protocol efficiency

Protocol	Hierarchical structure	Entangled resources	Split quantum information	Scalability	Recovery operations
Wang [22]	(1, 2)	Four-particle $\vert\chi\rangle$ state	Single qubit state $\vert\xi\rangle = \alpha\vert 0\rangle + \beta\vert 1\rangle$	No scalability	Pauli operation
Wang [23]	(2, 3)	Six-particle cluster state			
Xu [24]	(1, 2)	Four-particle cluster state	Double qubit state $\vert\varphi\rangle = \alpha\vert 00\rangle + \beta\vert 01\rangle + \gamma\vert 10\rangle + \delta\vert 11\rangle$	(m, n)	
Tang [25]	(1, 2)	Four-particle cluster state	Three qubit states $\vert\zeta\rangle = \alpha_1\vert 000\rangle + \alpha_2\vert 001\rangle + \alpha_3\vert 010\rangle + \alpha_4\vert 011\rangle + \alpha_5\vert 100\rangle + \alpha_6\vert 101\rangle + \alpha_7\vert 110\rangle + \alpha_8\vert 111\rangle$	(x,y)	
This protocol	(1, 2, 3)	Four-particle cluster state & GHZ state	Double qubit state $\vert\varphi\rangle = \alpha\vert 00\rangle + \beta\vert 01\rangle + \gamma\vert 10\rangle + \delta\vert 11\rangle$	$(1, 2, n)$	

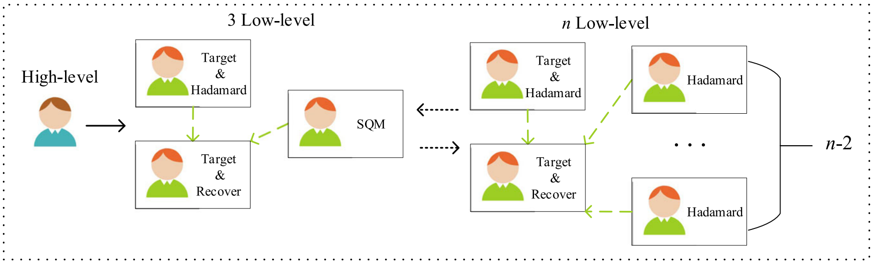

Fig. 5 High-level provides help from control particles, and Low-level performs CNOT gate or performs single bit measurement or Hadmard gate operation with the help of control particles. Three low-level agents can be replaced by *n* low-level agents

state of the multi-particle GHZ state and the four-particle cluster state constitute the quantum channel to realize the extension of the protocol.

We compared our protocol with previous studies, and the tertiary HQIS protocol was not involved by previous researchers. In the future, we will continue to study the HQIS protocol that transmits more information, is more efficient and has more universality.

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Data Availability The authors confirm that the data supporting the findings of this study are available within the article.

Declarations

Conflict of interest The authors declared that they have no conflicts of interest to this work.

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