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Effect of partial-collapse measurement on quantum entanglement in noninertial frames

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Abstract
An efficient method is proposed to improve two-qubit entanglement under the action of noise channel in noninertial frames by using partial-collapse measurement. We focus on the influence of partial-collapse measurement on entanglement for different noise channels in noninertial frames. It is shown that entanglement can be enhanced greatly for phase-flip, phase-damping, depolarizing and amplitude-damping channels. We obtain the optimal concurrence for the four noise channels, respectively. Moreover, 'entanglement sudden death' can be avoided for amplitude-damping environment. Our work provides a novel method to improve quantum entanglement under both noise environment and Unruh effect and exhibits the ability of partial-collapse measurement as an important technique in relativistic quantum information.

Keywords Quantum entanglement · Partial-collapse measurement · Noninertial frames · Noise channels

1 Introduction

The study of entanglement in the relativistic framework is important not only from quantum information perspective but also to understand deeply the black hole thermodynamics [1,2] and the black hole information paradox [3,4]. In a realistic regime, quantum systems are unavoidably subjected to the exotic environments. Decoherence appears when a system interacts with its exotic environment in an irreversible way. Both dissipative environment and Unruh effect triggered by particle's acceleration can give rise to decoherence. Many authors [5–11] have investigated entanglement in the relativistic framework, in particular on how the Unruh effect will change the degree

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of entanglement. The classical work is Alsing' research in 2006 [12]. They analyzed the entanglement between two modes of a free Dirac field as seen by two relatively accelerated parties. After that, some interesting works have been carried out, e.g., the recent study along with this topic related to GHZ, W, W-class states and others done by Qiang and his co-authors [13–16]. Particularly, they found that the entanglement of the Greenberger–Horne–Zeilinger-like state is degraded by the Unruh effect, but this entangled system always remains entangled to a degree and can be used in quantum teleportation between parties in relatively uniform acceleration [13]. However, Khan et.al [10] investigated the effects of decoherence on the entanglement generated by Unruh effect in noninertial frames under the action of noise channel. The entanglement sudden death can happen irrespective of the acceleration of the noninertial frame for phase-flip and phase-damping channels. So, it is important to discover some methods to protect entanglement in noninertial frames. Because if we want to realize a distant quantum teleportation by quantum satellite [17] or others [18,19], the earth can no longer be seen as an inertial system.

Partial-collapse measurement which is also known as weak measurement (note the difference with weak value) is a generalization of von Neumann measurement. In 1988, partial-collapse measurement was introduced by Aharonov, Albert and Vaidman (AAV) [20]. Partial-collapse measurement is very useful and can help understand many counterintuitive quantum phenomena, for example, Hardy's paradoxes [21–23]. Recently, partial-collapse measurement has been applied as a practically implementable method for protecting entanglement and quantum fidelity of quantum states undergoing decoherence through the amplitude-damping channel [24–30]. In Ref. [27], by using partial-collapse measurement and measurement reversal, Kim et al. experimentally demonstrated a scheme for protecting entanglement from amplitude-damping decoherence. Moreover, Some previous works have verified that partial-collapse measurement and measurement reversal can recover a quantum state suffering from noises [24,25,31–33]. However, these investigations are mainly limited to the studies of entanglement protection in inertial frames. Recently, it is shown that the Unruh effect can be completely eliminated by the technique of partial-collapse measurement [34]. However, the impact of noise environment was not considered. Ye et al. [35] have shown that the local nonunitary operation can enhance multipartite entanglement when the systems suffer from amplitude-damping noise and one subsystem is under noninertial frames.

Motivated by recent studies of Unruh-effect decoherence on quantum entanglement and the application of partial-collapse measurement, we study the influence of partial-collapse measurement on two-qubit entanglement for different noise channels in noninertial frames. The rest of this paper is organized as follows. In Sect. 2, the theoretical framework for this paper is given. We show how two-qubit entanglement could be enhanced under the action of phase-flip, phase-damping, depolarizing and amplitude-damping channels by using partial-collapse measurement in Sects. 3, 4, 5 and 6, respectively. Both analytical and numerical results are given. We offer our conclusions in Sect. 7.

2 Theoretical framework

For a single qubit with computational basis $|0\rangle$ and $|1\rangle$, partial-collapse measurement and measurement reversal can be written, respectively, as a nonunitary quantum operation [36]

$$M = \begin{bmatrix} 1 & 0 \\ 0 & m \end{bmatrix} \qquad M_r = \begin{bmatrix} n & 0 \\ 0 & 1 \end{bmatrix} \tag{1}$$

where the measurement strengths $m, n \in [0, \infty)$. M is the projective measurement when $m = 0$. And when $m \in (0, 1)$, M is a measurement partially collapsing the quantum system to the ground state. When $m \in (1, \infty)$, M partially collapses the quantum system to the excited state. Usually, a partial-collapse measurement can be parameterized as $M = diag\{1, \sqrt{1-p}\}$, and the measurement reversal operator is written as $M_r = diag\{\sqrt{1-p_r}, 1\}$. In our article, for convenience and generality, we use $M = diag\{1, m\}$ and $M_r = diag\{n, 1\}$ with $m, n \in [0, \infty)$. Equation (1) will not be modified in noninertial frames because the devices we used to perform a weak measurement have an acceleration same with Rob (noninertial observers). It means that Rob and the devices are relatively static. And the devices come to thermal equilibrium with Rob. So, the operations we perform in noninertial frames are same with in inertial frames.

We consider two observers, Alice and Rob, that share a maximally entangled initial state of two qubits at a point in flat Minkowski spacetime. Then, Rob moves with a uniform acceleration and Alice stays stationary. Let the two modes of Minkowski spacetime that correspond to Alice and Rob are, respectively, given by $|n\rangle_A$ and $|n\rangle_B$. Moreover, we assume that the observers are equipped with detectors that are sensitive only to their respective modes and share the following maximally entangled initial state

$$|\psi_1\rangle_{A,R} = \frac{1}{\sqrt{2}}(|00\rangle_{A,R} + |11\rangle_{A,R}) \tag{2}$$

Rob performs a partial-collapse measurement of Eq. (1) on his own particle. According to the formula $|\psi_2\rangle = \frac{(I\otimes M)|\psi_1\rangle}{\sqrt{\langle\psi_1|(I\otimes M^\dagger)(I\otimes M)|\psi_1\rangle}}$, if partial-collapse measurement is successfully carried out, the state of Eq. (2) becomes to

$$|\psi_2\rangle = \frac{1}{\sqrt{N_1}}\left[\frac{1}{\sqrt{2}}|00\rangle_{A,R} + \frac{1}{\sqrt{2}}m|11\rangle_{A,R}\right] \tag{3}$$

where $N_1 = \frac{1}{2}m^2 + \frac{1}{2}$ is the normalization factor.

Then, Rob moves with a uniform acceleration and Alice stays stationary. From the accelerated Rob's frame, the Minkowski vacuum state is found to be a two-mode squeezed state [12]

$$|0\rangle_M = \cos(r)|0\rangle_I|0\rangle_{II} + \sin(r)|1\rangle_I|1\rangle_{II} \tag{4}$$

where $\cos(r) = (e^{-2\pi\omega c/a} + 1)^{-1/2}$. Note that the acceleration parameter r is in the range $0 \le r \le \pi/4$ corresponding to $0 \le a \le \infty$ in this case. The constant ω, c and a, in the exponential stand, respectively, for Dirac particle's frequency, light's speed in vacuum and Rob's acceleration. In Eq. (4), the subscripts I and II of the kets represent the Rindler modes in region I and II, respectively, in the Rindler spacetime diagram. The excited state in Minkowski spacetime is related to Rindler modes as follows: [12]

$$|1\rangle_M = |1\rangle_I |0\rangle_{II} \tag{5}$$

So, the state of Rob should be expanded as Eqs. (4) and (5). Thus, the state of Eq. (3) changes to

$$\begin{aligned} |\psi_3\rangle = \frac{1}{\sqrt{N_1}} \Bigg[& \frac{1}{\sqrt{2}} (\cos(r)|0\rangle_A|0\rangle_I|0\rangle_{II} + \sin(r)|0\rangle_A|1\rangle_I|1\rangle_{II}) \\ & + \frac{1}{\sqrt{2}} m|1\rangle_A|1\rangle_I|0\rangle_{II} \Bigg] \end{aligned} \tag{6}$$

Since Rob is causally disconnected from region II, we must trace over the mode II, which results in a mixed state between Alice and Rob

$$\rho_{A,I} = \frac{1}{N_1} \begin{bmatrix} \frac{1}{2}\cos(r)^2 & 0 & 0 & \frac{1}{2}m\cos(r) \\ 0 & \frac{1}{2}\sin(r)^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{2}m\cos(r) & 0 & 0 & \frac{1}{2}m^2 \end{bmatrix} \tag{7}$$

We consider that only the Rob's qubit is coupled to a noisy environment. The density matrix of the system in the Kraus operators representation becomes

$$\rho_f = \sum_i (I \otimes E_i)\rho_{A,I}(I \otimes E_i^\dagger) \tag{8}$$

where $\rho_{A,I}$ is the density matrix of the system given by Eq. (7), I is the single-qubit identity matrix, and E_i are single-qubit Kraus operators of the channel under consideration.

In order to remove or weaken the Unruh effect and noise decoherence, a partial-collapse measurement reversal is performed by Rob in the region I. After successful performance of partial measurement reversal, we obtain

$$\rho_F = \frac{(I \otimes M_r)\rho_f(I \otimes M_r^\dagger)}{Tr[(I \otimes M_r)\rho_f(I \otimes M_r^\dagger)]} \tag{9}$$

The degree of entanglement in the two-qubit mixed state can be quantified conveniently by concurrence C, which is given as [37,38]

$$C = max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\} \tag{10}$$

where $\lambda_1, \ldots, \lambda_4$ are the eigenvalues of the non-Hermitian matrix $\tilde{\rho} = \rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$. ρ is the density matrix which represents the quantum state. The matrix elements are taken with respect to the standard eigenbasis $|1\rangle_1 \otimes |1\rangle_2$, $|1\rangle_1 \otimes |0\rangle_2$, $|0\rangle_1 \otimes |1\rangle_2$, $|0\rangle_1 \otimes |0\rangle_2$. The concurrence varies from $C = 0$ for unentangled qubits to $C = 1$ for the maximally entangled qubits.

3 Improving entanglement for phase-flip channel

Single-qubit Kraus operators for phase-flip (PF) channel are

$$E_0 = \sqrt{1-p}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad E_1 = \sqrt{p}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{11}$$

where p is the decoherence parameter.

According to Eq. (9), after successful performance of partial measurement reversal, we obtain the final density matrix

$$\rho_F = \frac{1}{A_1}\begin{pmatrix} n^2\cos(r)^2 & 0 & 0 & -m\,n\,\cos(r)\,(2\,p-1) \\ 0 & \sin(r)^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -m\,n\,\cos(r)\,(2\,p-1) & 0 & 0 & m^2 \end{pmatrix} \tag{12}$$

where $A_1 = m^2 + n^2\cos(r)^2 - \cos(r)^2 + 1$.

The eigenvalues of the non-Hermitian matrix $\tilde{\rho}_F$ are the following:

$$\begin{aligned} \lambda_1 &= \frac{m^2\,n^2\cos(r)^2\left(4\,p^2 - 8\,p + 4\right)}{m^4 + 2\,m^2\,n^2\cos(r)^2 + 2\,m^2\sin(r)^2 + n^4\cos(r)^4 + 2\,n^2\cos(r)^2\sin(r)^2 + \sin(r)^4} \\ \lambda_2 &= \frac{4\,m^2\,n^2\,p^2\cos(r)^2}{m^4 + 2\,m^2\,n^2\cos(r)^2 + 2\,m^2\sin(r)^2 + n^4\cos(r)^4 + 2\,n^2\cos(r)^2\sin(r)^2 + \sin(r)^4} \\ \lambda_3 &= 0 \\ \lambda_4 &= 0 \end{aligned} \tag{13}$$

To gain the optimal concurrence, we let

$$\frac{\partial C^{PF}}{\partial n} = 0 \tag{14}$$

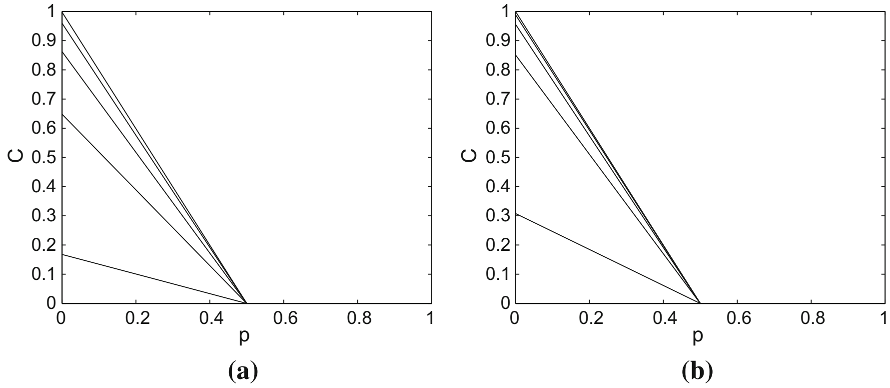

Fig. 1 The concurrence C under the action of phase-flip channel is plotted against decoherence parameter p for $m = 0.1, 0.5, 1, 2, 8$ (from below to above). **a** $r = \pi/5$; **b** $r = \pi/10$

From the above equation, we get $n = \frac{\sqrt{m^2+\sin(r)^2}}{\cos(r)}$. Substituting it into Eq. (10), the optimal concurrence becomes

$$C = \left(\sqrt{4\,m^4\,p^2 - 8\,m^4\,p + 4\,m^4 + 4\,m^2\,p^2\sin(r)^2 - 8\,m^2\,p\sin(r)^2 + 4\,m^2\sin(r)^2} - 2\sqrt{\frac{1}{\cos(r)^2}}\sqrt{\cos(r)^2}\sqrt{m^2+\sin(r)^2}\sqrt{m^2}\sqrt{p^2}\right)\sqrt{\frac{1}{4\,m^4 + 8\,m^2\sin(r)^2 + 4\sin(r)^4}} = \frac{m(1-2p)}{\sqrt{m^2+\sin(r)^2}} \tag{15}$$

In Fig. 1, we plot the dependence of entanglement for the phase-flip channel on the parameter p and the measurement strength m with $r = \pi/5$ and $r = \pi/10$. From below to above, the lines correspond to $m = 0.1, m = 0.5, m = 1, m = 2, m = 8$. It is shown that entanglement can be improved greatly with by using partial-collapse measurement both for $r = \pi/5$ and $r = \pi/10$. The bigger the measurement strength m is, the bigger the concurrence is. Especially, from Eq. (15), we obtain $\lim_{m\to\infty} C = \lim_{m\to\infty} \frac{m(1-2p)}{\sqrt{m^2+\sin(r)^2}} = 1-2p$. In other words, whatever the value of acceleration parameter r, the entanglement approaches the same maximum value $C_{max} = 1-2p$ when $m >> \sin(r)$. This result is the same as that in Ref. [10] when Rob's acceleration $a = 0$, i.e., the acceleration parameter $r = 0$, which means that the Unruh effect can be completely eliminated by the technique of partial-collapse measurement. However, the 'entanglement sudden death' cannot be avoided at the point $p = 0.5$. This is because that from Eq. (15), we get $C = 0$ when $p = 0.5$.

4 Improving entanglement for phase-damping channel

Single-qubit Kraus operators for phase-damping (PD) channel are

$$E_0 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{bmatrix} \quad E_1 = \begin{bmatrix} 0 & 0 \\ 0 & \sqrt{p} \end{bmatrix} \tag{16}$$

where p is the decoherence parameter.

According to Eq. (9), we obtain the final density matrix

$$\rho_F = \frac{1}{A_2} \begin{pmatrix} n^2 \cos(r)^2 & 0 & 0 & m\,n\,\cos(r)\,\sqrt{1-p} \\ 0 & \sin(r)^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ m\,n\,\cos(r)\,\sqrt{1-p} & 0 & 0 & m^2 \end{pmatrix} \tag{17}$$

where $A_2 = m^2 + n^2\cos(r)^2 - \cos(r)^2 + 1$.

The eigenvalues of the non-Hermitian matrix $\tilde{\rho}_F$ are the following:

$$\begin{aligned} \lambda_1 &= \frac{m^2 n^2 \cos(r)^2 \left(2\sqrt{1-p} - p + 2\right)}{m^4 + 2m^2 n^2 \cos(r)^2 + 2m^2 \sin(r)^2 + n^4 \cos(r)^4 + 2n^2 \cos(r)^2 \sin(r)^2 + \sin(r)^4} \\ \lambda_2 &= -\frac{m^2 n^2 \cos(r)^2 \left(p + 2\sqrt{1-p} - 2\right)}{m^4 + 2m^2 n^2 \cos(r)^2 + 2m^2 \sin(r)^2 + n^4 \cos(r)^4 + 2n^2 \cos(r)^2 \sin(r)^2 + \sin(r)^4} \\ \lambda_3 &= 0 \\ \lambda_4 &= 0 \end{aligned} \tag{18}$$

To gain the optimal concurrence, we let

$$\frac{\partial C^{PD}}{\partial n} = 0 \tag{19}$$

From the above equation, we get $n = \frac{\sqrt{m^2+\sin(r)^2}}{\cos(r)}$. Substituting it into Eq. (10), the maximum concurrence becomes

$$\begin{aligned} C = &-\Big(\sqrt{2m^2\sin(r)^2 - m^4 p - 2m^4\sqrt{1-p} + 2m^4 - m^2 p \sin(r)^2 - 2m^2 \sin(r)^2 \sqrt{1-p}} \\ &-\sqrt{2m^2\sin(r)^2 - m^4 p + 2m^4\sqrt{1-p} + 2m^4 - m^2 p \sin(r)^2 + 2m^2 \sin(r)^2 \sqrt{1-p}}\Big) \\ &\sqrt{\frac{1}{4m^4 + 8m^2\sin(r)^2 + 4\sin(r)^4}} \\ = &\frac{m\sqrt{1-p}}{\sqrt{m^2+\sin(r)^2}} \end{aligned} \tag{20}$$

In Fig. 2, we plot the dependence of entanglement for the phase-damping channel on the parameter p and the measurement strength m with $r = \pi/5$. From below to above,

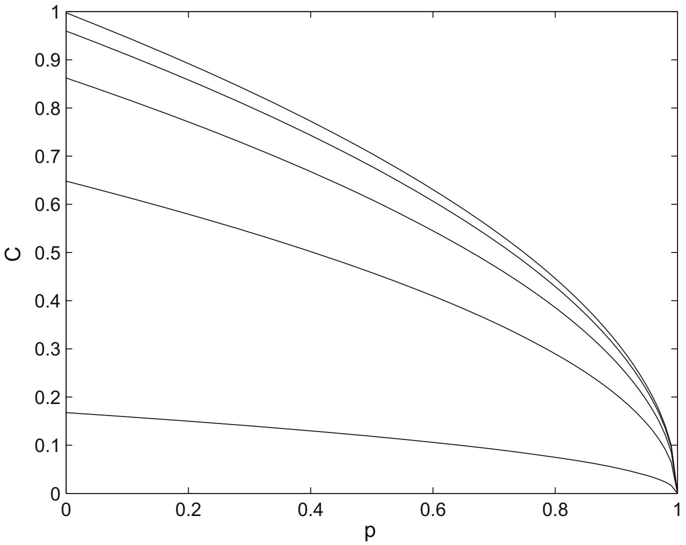

Fig. 2 The concurrence C under the action of phase-damping channel is plotted against decoherence parameter p for $r = \pi/5$, $m = 0.1, 0.5, 1, 2, 8$ (from below to above)

the lines correspond to $m = 0.1, m = 0.5, m = 1, m = 2, m = 8$. As in the case of phase-flip channel, the results show that entanglement can be enhanced greatly with partial-collapse measurement. The bigger the measurement strength m is, the bigger the concurrence is. Moreover, we find that, whatever the value of acceleration parameter r, the entanglement approaches the same maximum value $C_{max} = \sqrt{1-p}$ when $m >> \sin(r)$. This is because that from Eq. (20), we obtain $\lim_{m\to\infty} C = \lim_{m\to\infty} \frac{m\sqrt{1-p}}{\sqrt{m^2+\sin(r)^2}} = \sqrt{1-p}$. However, the 'entanglement sudden death' cannot be avoided at the point $p = 1$. The result can be explained from Eq. (20). From the equation, we get $C = 0$ when $p = 1$.

5 Improving entanglement for depolarizing channel

Single-qubit Kraus operators for depolarizing (DP) channel are

$$E_0 = \sqrt{1-p}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad E_1 = \sqrt{\frac{p}{3}}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad E_2 = \sqrt{\frac{p}{3}}\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$
$$E_3 = \sqrt{\frac{p}{3}}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{21}$$

where p is the decoherence parameter.

According to Eq. (9), we obtain the final density matrix

$$\rho_F = \frac{1}{A_3} \begin{pmatrix} n^2\cos(r)^2 & 0 & 0 & \frac{m\,n\,\cos(r)\,(4\,p-3)}{2\,p-3} \\ 0 & \sin(r)^2 & -\frac{2\,m\,n\,p\,\cos(r)}{2\,p-3} & 0 \\ 0 & -\frac{2\,m\,n\,p\,\cos(r)}{2\,p-3} & 0 & 0 \\ \frac{m\,n\,\cos(r)\,(4\,p-3)}{2\,p-3} & 0 & 0 & \frac{m^2\,(2\,p-3)\,(3\,m^2+3)\,(m^2-n^2\,\sin(r)^2+n^2+\sin(r)^2)}{3\,(m^2+1)\,(2\,p\,\sin(r)^2-3\,\sin(r)^2+2\,m^2\,p-3\,m^2+3\,n^2\,(\sin(r)^2-1)-2\,n^2\,p\,(\sin(r)^2-1))} \end{pmatrix} \tag{22}$$

where $A_3 = \sin(r)^2 + m^2 - n^2\,\left(\sin(r)^2 - 1\right)$.

The eigenvalues of the non-Hermitian matrix $\tilde{\rho}_F$ are the following:

$$\begin{aligned} \lambda_1 &= \frac{m^2\,n^2\cos(r)^2\,\left(2\sqrt{1-p}-p+2\right)}{m^4\,n^4\cos(r)^4 + 2\,m^4\,n^2\cos(r)^2\sin(r)^2 + m^4\sin(r)^4 + 2\,m^2\,n^2\cos(r)^2 + 2\,m^2\sin(r)^2 + 1} \\ \lambda_2 &= -\frac{m^2\,n^2\cos(r)^2\,\left(p+2\sqrt{1-p}-2\right)}{m^4\,n^4\cos(r)^4 + 2\,m^4\,n^2\cos(r)^2\sin(r)^2 + m^4\sin(r)^4 + 2\,m^2\,n^2\cos(r)^2 + 2\,m^2\sin(r)^2 + 1} \\ \lambda_3 &= 0 \\ \lambda_4 &= 0 \end{aligned} \tag{23}$$

To gain the optimal concurrence, we let

$$\frac{\partial C^{DP}}{\partial n} = 0 \tag{24}$$

From the above equation, we get $n = \frac{\sqrt{m^2+\sin(r)^2}}{\cos(r)}$. Substituting it into Eq. (10), the maximum concurrence becomes

$$\begin{aligned} C &= 2\sqrt{\frac{16\,m^2\,p^2 - 24\,m^2\,p + 9\,m^2}{16\,m^2\,p^2 - 48\,m^2\,p + 36\,m^2 + 16\,p^2\sin(r)^2 - 48\,p\sin(r)^2 + 36\sin(r)^2}} \\ &= \left|\frac{4p-3}{3-2p}\right| \frac{m}{\sqrt{m^2+\sin(r)^2}} \end{aligned} \tag{25}$$

In Fig. 3, we plot the dependence of entanglement for the depolarizing channel on the parameter p and the measurement strength m with $r = \pi/5$. From below to above, the lines correspond to $m = 0.1, m = 0.5, m = 1, m = 2, m = 8$. As in the case of phase-flip and phase-damping channels, the results show that entanglement can be improved significantly by using partial-collapse measurement. The bigger the measurement strength m is, the bigger the entanglement is. Moreover, we find that, whatever the value of r, the entanglement approaches the same maximum value $C_{max} = |\frac{4p-3}{3-2p}|$ when $m >> \sin(r)$. This is because that from Eq. (25), we obtain $\lim_{m\to\infty} C = \lim_{m\to\infty} |\frac{4p-3}{3-2p}| \frac{m}{\sqrt{m^2+\sin(r)^2}} = |\frac{4p-3}{3-2p}|$. However, the 'entanglement sudden death' cannot be avoided at the point $p = 0.75$. This result can be explained from Eq. (25). From the equation, we get $C = 0$ when $p = 0.75$.

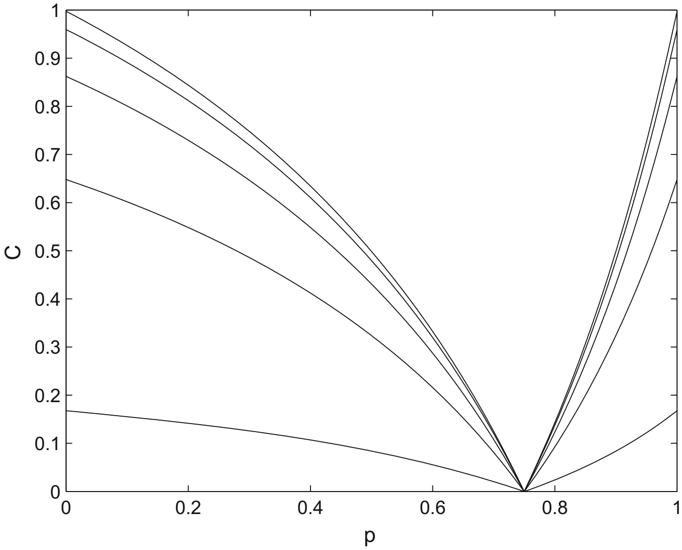

Fig. 3 The concurrence C under the action of depolarizing channel is plotted against decoherence parameter p for $r = \pi/5$, $m = 0.1, 0.5, 1, 2, 8$ (from below to above)

6 Improving entanglement for amplitude-damping channel

Single-qubit Kraus operators for amplitude-damping (AD) channel are as follows:

$$E_0 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{bmatrix} \quad E_1 = \begin{bmatrix} 0 & \sqrt{p} \\ 0 & 0 \end{bmatrix} \tag{26}$$

where p is the decoherence parameter.

According to Eq. (9), we obtain the final density matrix

$$\rho_F = \frac{1}{A_4} \begin{pmatrix} n^2 \left(\cos(r)^2 - p\left(\cos(r)^2 - 1\right)\right) & 0 & 0 & m\,n\,\cos(r)\,\sqrt{1-p} \\ 0 & -\sin(r)^2\,(p-1) & 0 & 0 \\ 0 & 0 & m^2\,n^2\,p & 0 \\ m\,n\,\cos(r)\,\sqrt{1-p} & 0 & 0 & -m^2\,(p-1) \end{pmatrix} \tag{27}$$

where $A_4 = \sin(r)^2 - p\sin(r)^2 - m^2\,p + m^2 - n^2\left(\sin(r)^2 - 1\right) + m^2\,n^2\,p + n^2\,p\sin(r)^2$.

The eigenvalues of the non-Hermitian matrix $\tilde{\rho}_F$ are the following:

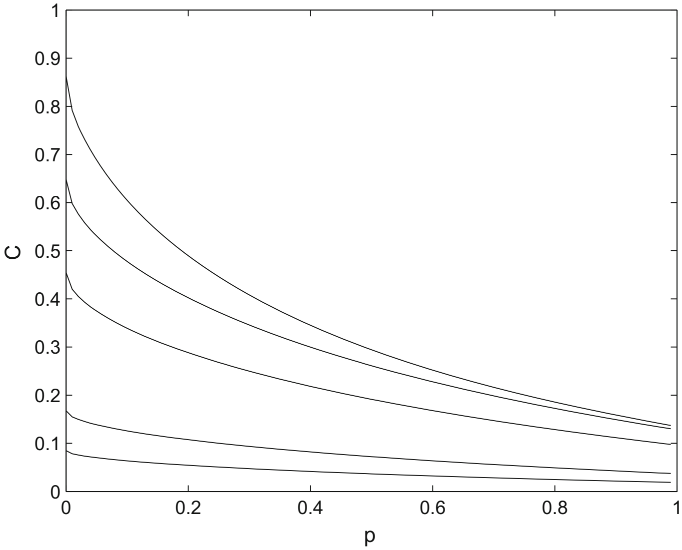

Fig. 4 The concurrence C under the action of amplitude-damping channel is plotted against decoherence parameter p for $r = \pi/5$, $m = 0.05, 0.1, 0.3, 0.5, 1$ (from below to above)

$$\begin{aligned}
\lambda_1 &= \frac{m^2\,n^2\cos(r)^2\,\left(2\sqrt{1-p}-p+2\right)}{m^4\,n^4\cos(r)^4+2\,m^4\,n^2\cos(r)^2\sin(r)^2+m^4\sin(r)^4+2\,m^2\,n^2\cos(r)^2+2\,m^2\sin(r)^2+1}\\
\lambda_2 &= -\frac{m^2\,n^2\cos(r)^2\,\left(p+2\sqrt{1-p}-2\right)}{m^4\,n^4\cos(r)^4+2\,m^4\,n^2\cos(r)^2\sin(r)^2+m^4\sin(r)^4+2\,m^2\,n^2\cos(r)^2+2\,m^2\sin(r)^2+1}\\
\lambda_3 &= 0\\
\lambda_4 &= 0
\end{aligned} \tag{28}$$

To gain the optimal concurrence, we let

$$\frac{\partial C^{AD}}{\partial n} = 0 \tag{29}$$

From above equation, we get $n = \frac{\sqrt{-\left(m^2+\sin(r)^2\right)(p-1)\left(p\,m^2+\cos(r)^2+p\sin(r)^2\right)}}{p\,m^2+\cos(r)^2+p\sin(r)^2}$. Substituting it into Eq. (10), the maximum concurrence becomes

$$\begin{aligned}
C = \frac{1}{1+m^2}\Bigg[&\sqrt{\frac{1}{p\,m^4+m^2\cos(r)^2+p\,m^2\sin(r)^2\,2+\cos(r)^2\sin(r)^2+p\sin(r)^4}}\\
&\sqrt{m^6\cos(r)^2+2\,m^4\cos(r)^2+m^2\cos(r)^2}-\\
&\sqrt{\frac{p\,m^6\sin(r)^2+p\,m^4\sin(r)^2\,2+p\,m^2\sin(r)^2}{p\,m^4+m^2\cos(r)^2+p\,m^2\sin(r)^2\,2+\cos(r)^2\sin(r)^2+p\sin(r)^4}}\Bigg]
\end{aligned}$$

$$= \frac{m(\cos(r) - \sin(r)\sqrt{p})}{\sqrt{(m^2 + \sin(r)^2)(pm^2 + p\sin(r)^2 + \cos(r)^2)}} \tag{30}$$

In Fig. 4, we plot the dependence of entanglement for the amplitude-damping channel on the parameter p and the measurement strength m with $r = \pi/5$. From below to above, the lines correspond to $m = 0.05$, $m = 0.1$, $m = 0.3$, $m = 0.5$, $m = 1$. As in the case of the previous three noise channels, the results show that the bigger the measurement strength m, the higher the degree of entanglement. Especially, we are able to enhance entanglement greatly and to avoid 'entanglement sudden death' with partial-collapse measurement for amplitude-damping channel. This result is different from the previous three noise channels. From Eq. (30), we get $C > 0$ when $\cos(r) > \sin(r)\sqrt{p}$ (i.e., $\tan(r) < \frac{1}{\sqrt{p}}$). Note that the acceleration parameter r is in the range $0 \le r \le \pi/4$; thus, with the exception of $r = \pi/4$, 'entanglement sudden death' can be avoided for $0 \le r < \pi/4$.

7 Conclusion

An efficient method is proposed to improve two-qubit entanglement under the action of noise channel in noninertial frames by using partial-collapse measurement in this paper. We focus on the influence of partial-collapse measurement on entanglement for different noise channels in noninertial frames. It is shown that entanglement can be enhanced greatly for phase-flip, phase-damping, depolarizing and amplitude-damping channels. We obtain the optimal concurrence for the four noise channels, respectively. Moreover, 'entanglement sudden death' can be avoided for amplitude-damping environment. Our work provides a novel method to improve quantum entanglement under both noise environment and Unruh effect and exhibits the ability of partial-collapse measurement as an important technique in relativistic quantum information.

The above discussion can be generalized to the situations that Alice is also uniformly accelerated and both qubits of the two observers interact with noisy environment. Furthermore, it can be extended to the most general situation that two observers undergo relative nonuniform accelerations. And, we believe that partial-collapse measurement scheme can be used to improve entanglement under some more general channels in noninertial frames. These topics will be investigated in our future researches.

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