Quantum Information Processing (2019) 18:237
https://doi.org/10.1007/s11128-019-2350-2

Deterministic joint remote preparation of arbitrary multi-qubit states via three-qubit entangled states

Jiahua Wei[1] · Lei Shi[1] · Shanghong Zhao[1] · Kaihang Zhou[1] · Longqiang Yu[1] · Wei Lu[1] · Lihua Ma[1] · Boxin Zhao[2]

Received: 28 May 2018 / Accepted: 3 June 2019 / Published online: 11 June 2019

Abstract
We propose an efficient scheme for joint remote state preparation (JRSP) of arbitrary multi-qubit states from two senders to one receiver with the 100% successful probability. Quantum channel is composed of maximally three-qubit entangled states, and several special mutually orthogonal measurement basis are constructed without the introduction of auxiliary particles. We also calculate the total classical communication cost required in the JRSP processes. The concrete JRSP procedures for remotely preparing single-qubit and two-qubit states are illustrated to prove explicitly the feasibility of this JRSP protocol.

Keywords Joint RSP · Successful probability · Arbitrary multi-qubit states

1 Introduction

It is an elementary problem to safely and securely transmit quantum states in quantum network communication and quantum distributed computation. One of the most remarkable schemes for the transmission of quantum states is the so-called quantum teleportation, originally proposed by Bennett et al. [1], in which quantum states can be transmitted between remote locations via quantum channel and classical communication. Afterward, Lo [2] investigated how to send quantum information using a prior shared entanglement and the classical communication when the sender knows

✉ Jiahua Wei
weijiahua@126.com

✉ Longqiang Yu
yulq_go@163.com

1 Information and Navigation College, Air Force Engineering University, Xi'an 710077, Shanxi, People's Republic of China

2 Equipment Management and UAV Engineering College, Air Force Engineering University, Xi'an 710078, Shanxi, People's Republic of China

fully the transmitted state. This communication protocol is called remote state preparation (RSP). Compared with the usual teleportation [1,3–13], the RSP proposal can be preformed with simpler measurement and using less classical information in some special cases [14–20].

So far, lots of RSP proposals have been presented, including joint RSP [21–25], controlled RSP [26–28], optimal RSP [29] and so on. In the process of joint remote state preparation (JRSP), the senders separately own the partial classical information of quantum state they want to prepare. If and only if all of the senders cooperate with each other, the JRSP scheme can be realized. The main advantage from the RSP protocol of two-party is that each sender cannot have the final prepared state which is very useful for security. Hence, the JRSP proposal has acquired lots of attention recently. The deterministic controlled JRSP scheme [30] was presented to remotely prepare arbitrary single- and two-qubit states using partially entangled quantum channel. Chen et al. [31] propose a scheme to perform joint remote preparation of an arbitrary two-qubit state using a generalized seven-qubit brown state. Luo et al. presented that two senders can jointly prepare three-qubit states [32] and four-qubit χ-states [33] to a remote receiver via the shared GHZ states. Meanwhile, some JRSP schemes for special kinds of multi-qubit states have been explored [34–38]. For example, Wang [34] proposed a method to remotely prepare a two-qubit state via three bipartite entanglements, and generalized this method to the multi-qubit GHZ-class state case. Li et al. [35] proposed a scheme for joint remote preparation of multi-qubit equatorial states with unit successful probability. Long et al. [36] took advantages of the positive operator-valued measurement to perform multi-party joint remote preparation of an arbitrary GHZ-class state. Nowadays, some theoretical and experimental schemes of quantum information processing [39–42] have been investigated to promote the physical realization of remote state preparation.

The purpose of this paper is to present a novel JRSP proposal for arbitrary multi-qubit states from two senders to one receiver in a deterministic manner by using of maximally three-qubit entangled states. The concrete processes for our JRSP proposal are given, and some useful measurement basis are constructed without the introduction of auxiliary particles. The total classical information cost and successful probability regarding this JRSP scheme are calculated, respectively. It should be emphasized that arbitrary multi-qubit states can be remotely prepared via our protocol with the 100% successful probability. This is the most important advantage of this novel JRSP scheme.

The rest of this paper is organized as follows: In Sect. 2, an efficient scheme for remote preparation of an arbitrary n-qubit state is presented with some special measurement basis, of which the solution expressions are shown in detail. The total successful probability of this JRSP scheme can reach up to one, and the classical information cost is equal to $3n\ bits$. In Sect. 3, concrete realization processes for jointly preparing single-qubit and two-qubit states are illustrated to demonstrate explicitly the feasibility of our JRSP scheme. The paper concludes with Sect. 4.

2 Deterministic JRSP of arbitrary multi-qubit states

Suppose that the senders Alice and Charlie want to help the receiver Bob remotely prepare arbitrary n-qubit state

$$|\psi_n^0\rangle = \sum_{x=0}^{2^n-1} a_x e^{i\phi_x} |d_n \dots d_2 d_1\rangle \qquad d_i \in \{0, 1\}; \quad x = \sum_{i=1}^{n} d_i \cdot 2^{i-1}. \tag{1}$$

where $a_x, \phi_x \in \mathcal{R}$ $(x = 0, 1, \cdots, 2^n - 1)$, $\sum_{x=0}^{2^n-1} |a_x|^2 = 1$ and $\phi_0 = 0$. Usually, a_x can be considered as the amplitude factor of quantum state, and ϕ_x is known as the (relative) phase parameter. The information of a_x and ϕ_x are only available for Alice and Charlie, respectively. Note that x is the decimal form of the binary string $d_n \dots d_2 d_1$. Quantum channel is composed of n maximally entangled three-qubit states below

$$|\Psi\rangle_{A_k C_k B_k} = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{A_k C_k B_k} \quad k = 1, 2 \cdots n. \tag{2}$$

Without loss of generality, Alice, Charlie and Bob have particles A_k, C_k and B_k, respectively. The concrete processes for our deterministic JRSP protocol can be elaborated as follows:

Step 1: For the purpose to realize the JRSP, Alice needs to construct the special projective measurements basis $\{|\Gamma_m\rangle \,|m = 0, 1 \cdots 2^n - 1\}$, of which the form can be presented as

$$\left[\, |\Gamma_0\rangle, \, |\Gamma_1\rangle, \, \cdots \, |\Gamma_{2^n-2}\rangle, \, |\Gamma_{2^n-1}\rangle \,\right]^T = U[n] \begin{pmatrix} |0 \dots 00\rangle \\ |0 \dots 01\rangle \\ \vdots \\ |1 \dots 10\rangle \\ |1 \dots 11\rangle \end{pmatrix} \tag{3}$$

here

$$U[n] = \begin{pmatrix} a_0 & a_1 & \cdots & a_{2^{n-1}-1} & a_{2^{n-1}} & a_{2^{n-1}+1} & \cdots & a_{2^n-1} \\ a_1 & \ddots & \cdots & \cdots & a_{2^{n-1}+1} & \cdots & \cdots & \cdots \\ \vdots & \vdots & \ddots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{2^{n-1}-1} & \vdots & \vdots & \ddots & a_{2^n-1} & \cdots & \cdots & \cdots \\ a_{2^{n-1}} & -a_{2^{n-1}+1} & \vdots & -a_{2^n-1} & -a_0 & a_1 & \cdots & a_{2^{n-1}-1} \\ a_{2^{n-1}+1} & \vdots & \vdots & \vdots & -a_1 & \ddots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \cdots \\ a_{2^n-1} & \vdots & \vdots & \vdots & -a_{2^{n-1}-1} & \vdots & \vdots & \ddots \end{pmatrix} \tag{4}$$

Let us begin with a brief statement of the elements of this matrix $U[n]$ (For more details, see Ref. [43]). If $n = 1$, i.e., some single states need to be prepared, $U[1]$ could be presented as $[a_0, a_1;\ a_0, -a_1]$ from Eq. (4). When $n \geq 2$, the (red) elements at the upper left corner of $U[n]$ can be determined by setting $U[n](i, j) = U[n-1](i, j)$, here $2 \leq i, j \leq 2^{n-1}$. The element in the i-th row and j-th column of the complex conjugate of the matrix $U[n]$ would be denoted as the parameter $U[n](i, j)$. According to $\sum_{j=1}^{2^n} U[\Theta_n^n](i, j) \cdot U[\Theta_n^n](2^n + 1, j) = 0$ $(2 \leq i \leq 2^{n-1})$, one can obtain the (green) parameters at the upper right corner. Subsequently, the other (blue) coefficients from the $(2^{n-1} - 1)$-th row to 2^n-th row can be fulfilled in terms of $\sum_{i=1}^{2^n} U[n](i, j) \cdot U[n](i, k) = 0$ $(k = 1, 2^{n-1} + 1;\ 2 \leq j \leq 2^n;\ j \neq k)$. After that, we can obtain all of the elements of the matrix $U[n]$. Furthermore, it could be find that

$$\langle \Gamma_i | \Gamma_j \rangle = \begin{cases} 0, & i \neq j; \\ 1, & i = j. \end{cases} \qquad \sum_{i=1}^{2^n - 1} |\Gamma_i\rangle\langle\Gamma_i| = I_{2^n} \tag{5}$$

where I_{2^n} is the identical $2^n \times 2^n$ matrix. Hence, the orthogonal states $\{|\Gamma_m\rangle \ | m = 0, 1 \cdots 2^n - 1\}$ could be used as the projective measurement basis. When the result of particles $(A_1 A_2 \cdots A_n)$ is $|\Gamma_m\rangle$, particles $(C_1 C_2 \cdots C_n\ B_1 B_2 \cdots B_n)$ between Charlie and Bob could collapse into $|\Phi_i\rangle$, which can be described as follows:

$$\begin{aligned} &|\Phi_m\rangle_{C_1 B_1 C_2 B_2 \cdots C_n B_n} \\ &\quad = [\ |\Gamma_m\rangle_{A_1 A_2 \cdots A_n}]^{\dagger}\ \left[\ |\Psi\rangle_{A_1 C_1 B_1} \otimes |\Psi\rangle_{A_2 C_2 B_2} \cdots |\Psi\rangle_{A_n C_n B_n}\right] \\ &\quad = \left(\frac{1}{\sqrt{2}}\right)^n \sum_{k=0}^{2^n - 1} U[n](m, k) \cdot |l_n l_n \cdots l_2 l_2 l_1 l_1\rangle_{C_1 B_1 C_2 B_2 \cdots C_n B_n} \end{aligned} \tag{6}$$

It is should be emphasized that $l_n \cdots l_2 l_1$ is the binary string of the number k. Hence, the state $|l_n l_n \cdots l_2 l_2 l_1 l_1\rangle$ can be determined as long as k is set. From Eqs. (4) and (6), it could be found that $|\Phi_m\rangle$ can be modified to $|\Theta\rangle = \sum_{x=0}^{2^n - 1} a_x |l_n l_n \cdots l_2 l_2 l_1 l_1\rangle$ by exchanging the relative positions of the factors of $|\Phi_m\rangle$, here $x = \sum_{i=1}^{n} l_i \cdot 2^{i-1}$. For instance, $|\Phi_{2^{n-1}+1}\rangle$ can be transported into $|\Theta\rangle$ using the permutation operation $S_n^{2^{n-1}+1} = S_n[1, \overline{2^{n-1} + 1};\ \overline{2}, 2^{n-1} + 2;\ \overline{3}, 2^{n-1} + 3; \cdots;\ \overline{2^{n-1}}, 2^n]$:

$$\begin{aligned} |\Theta\rangle &\simeq S_n^{2^{n-1}+1} \cdot |\Phi_{2^{n-1}}\rangle \\ &\simeq S_n[1, \overline{2^{n-1} + 1};\ \overline{2}, 2^{n-1} + 2;\ \overline{3}, 2^{n-1} + 3; \cdots;\ \overline{2^{n-1}}, 2^n] \cdot |\Phi_{2^{n-1}}\rangle \end{aligned} \tag{7}$$

where $S_n[i, j]$ ($S_n[i, \overline{j}]$) means the i-th column and j-th column of the identical matrix $I_{2^n \times 2^n}$ have changed the positions (with a coefficient -1). Meanwhile, $S_n[i, j; \overline{k}, l]$ equals to the product of $S_n[i, j]$ and $S_n[\overline{k}, l]$.

Step 2: After the measurements $\{|\Gamma_m\rangle \ | m = 0, 1 \cdots 2^n - 1\}$, Alice informs Bob and Charlie of her measurement results via classical channel. Then, Charlie constructs the projective measurements $\{|\Omega_k^m\rangle \ | k = 0, 1 \cdots 2^n - 1\}$, which can be given by

$$\begin{pmatrix} |\Omega_0^m\rangle \\ |\Omega_1^m\rangle \\ \cdots \\ |\Omega_{2^n-2}^m\rangle \\ |\Omega_{2^n-1}^m\rangle \end{pmatrix} = \left(\frac{1}{\sqrt{2}}\right)^n S_n^m \begin{pmatrix} 1 & exp\,(-i\phi_1) & exp\,(-i\phi_2) & \cdots & exp\,(-i\phi_{2^n-1}) \\ 1 & exp\left(\frac{i\pi\cdot 1\cdot 1}{2^{n-1}} - i\phi_1\right) & exp\left(\frac{i\pi\cdot 2\cdot 1}{2^{n-1}} - i\phi_2\right) & \cdots & exp\left(\frac{i\pi\cdot(2^n-1)\cdot 1}{2^{n-1}} - i\phi_{2^n-1}\right) \\ 1 & exp\left(\frac{i\pi\cdot 1\cdot 2}{2^{n-1}} - i\phi_1\right) & exp\left(\frac{i\pi\cdot 2\cdot 2}{2^{n-1}} - i\phi_2\right) & \cdots & exp\left(\frac{i\pi\cdot(2^n-1)\cdot 2}{2^{n-1}} - i\phi_{2^n-1}\right) \\ \vdots & \cdots & \cdots & \ddots & \vdots \\ 1 & exp\left(\frac{i\pi\cdot 1\cdot(2^n-1)}{2^{n-1}} - i\phi_1\right) & exp\left(\frac{i\pi\cdot 2\cdot(2^n-1)}{2^{n-1}} - i\phi_1\right) & \cdots & exp\left(\frac{i\pi\cdot(2^n-1)\cdot(2^n-1)}{2^{n-1}} - i\phi_{2^n-1}\right) \end{pmatrix} \begin{pmatrix} |0\ldots 00\rangle \\ |0\ldots 01\rangle \\ \vdots \\ |1\ldots 10\rangle \\ |1\ldots 11\rangle \end{pmatrix} \tag{8}$$

The unitary transformation S_n^m is the corresponding permutation operation for the measurement result $|\Gamma_m\rangle$. Thus, one can find that

$$\begin{aligned} &|\Delta_k^m\rangle_{B_1B_2\ldots B_n} \\ &= [\ |\Gamma_m\rangle_{A_1A_2\ldots A_n} \otimes |\Omega_k^m\rangle_{C_1C_2\ldots C_n}\ ]^\dagger\ |\Psi\rangle_{A_1C_1B_1} \otimes |\Psi\rangle_{A_2C_2B_2} \cdots |\Psi\rangle_{A_nC_nB_n} \\ &= \frac{1}{2^n} \sum_{j=0}^{2^n-1} U[n](m,j) \cdot exp\left(i\phi_x - \frac{i\pi\cdot x\cdot k}{2^{n-1}}\right) |d_n \cdots d_2 d_1\rangle_{B_1B_2\ldots B_n} \end{aligned} \tag{9}$$

here $x = \sum_{i=1}^n d_i 2^{i-1}$. After the projective measurements $\{|\Omega_k^m\rangle\ |k = 0, 1\cdots 2^n-1\}$, Charlie tells the results to Bob via classical channel. From Eq. (9), we can find that the probability for each outcome $|\Delta_k^m\rangle$ is always equal to $(1/2^n)^2 = 4^{-n}$. Based on the quantum measurement postulate, we can get that the state of quantum channel after the projective measurements would be known exactly if the measurement outcome is obtained. Furthermore, the states of particles $(B_1,\ B_2\cdots B_n)$ after projective measurements are pure, and could be converted into the prepared states by using of some unitary operations.

Step 3: According to the result $|\Omega_k^m\rangle$ of Charlie, Bob need to introduce the unitary operation U_n^k

$$U_n^k = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi k} \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi k}{2}} \end{pmatrix} \otimes \cdots \otimes \begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi k}{2^n}} \end{pmatrix} \tag{10}$$

For the state in Eq. (9), Bob only need to perform the permutation operation S_n^m and the unitary gate U_n^k on particles $(B_1,\ B_2\cdots B_n)$ to convert the initial multi-qubit state shown as Eq. (1).

$$|\psi_n^0\rangle_{B_1B_2\ldots B_n} \simeq U_n^k \cdot S_n^m\ |\Delta_k^m\rangle_{B_1B_2\ldots B_n} \tag{11}$$

The successful probability and required classical communication cost play important roles in the JRSP schemes. Note that the probability for each measurement outcome $|\Delta_k^m\rangle$ $(m, k = 0, 1 \cdots 2^n - 1)$ is $(1/2^n)^2 = 4^{-n}$. So that the total successful probability can be presented as

$$P_{total} = \sum_{m=0}^{2^n-1} \sum_{k=0}^{2^n-1} 4^{-n} = 100\% \tag{12}$$

Based on the calculation method of classical information for RSP proposals in Refs. [44,45], this classical information required in our three-party RSP proposal can be divided into two transmitted processes. One is the classical information S_{A-BC} (including S_{A-C} and S_{A-B}) sent from Alice to Charlie and Bob, and the other is the classical information S_{C-B} from Charlie to Bob. In order to realize this JRSP proposal, Alice would perform a projective measurement on particles $A_1 A_2 \cdots A_n$ and informs them of her measurement result. It should be emphasized that the measurement basis is $\{|\Gamma_m\rangle \ |m = 0, 1 \cdots 2^n - 1\}$ shown as Eq. (3). Particles $A_1 A_2 \cdots A_n$ after this measurement will collapse into one of the 2^n kinds of outcomes. Meanwhile, it can be obtained that the 2^n kinds of results have the same probability of 2^{-n}. The classical information S_{A-BC} sent from Alice to Charlie and Bob could be presented as

$$S_{A-BC} = S_{A-C} + S_{A-B} = -\frac{1}{2^n} \sum_{m=0}^{2^n-1} \log \frac{1}{2^n} - \frac{1}{2^n} \sum_{m=0}^{2^n-1} \log \frac{1}{2^n} = 2n \quad bits \tag{13}$$

According to the measurement result of $A_1 A_2 \cdots A_n$, Charlie measures particles $C_1 C_2 \cdots C_n$ with the orthogonal states $\{|\Omega_k^m\rangle \ |k = 0, 1 \cdots 2^n - 1\}$ in Eq. (8), and informs Bob of his outcome. There are 2^n kinds of measurement results with the same probability of 2^{-n}. Hence, the classical information cost from Charlie to Bob can be given by

$$S_{C-B} = -\frac{1}{2^n} \sum_{k=0}^{2^n-1} \log \frac{1}{2^n} = n \quad bits \tag{14}$$

From the above discussions, we could find that arbitrary n-qubit states can be prepared in a deterministic manner by using our proposal. Meanwhile, the required classical communication cost of this JRSP protocol is equal to

$$S_{total} = S_{A-BC} + S_{C-B} = 2n + n = 3n \quad bits \tag{15}$$

It should be noted that n maximally entangled three-qubit states are required for remotely preparing one n-qubit state, whatever the prepared state is entangled or not.

3 Examples of JRSP

To illustrate our scheme for deterministic joint remote state preparation explicitly, we will study how to remotely prepare arbitrary single- and two-qubit states, which are elementary resources for quantum information.

3.1 JRSP of arbitrary single-qubit states

Suppose that the senders Alice and Charlie want to help the receiver Bob remotely prepare the following single-qubit state

$$|\psi\rangle = a_0|0\rangle + a_1 e^{i\phi_1}|1\rangle \tag{16}$$

where $a_0,\ a_1,\ \phi_1$ are real, and $|a_0|^2 + |a_1|^2 = 1$. Quantum channel is composed of the maximally entangled three-qubit states below

$$|\Psi\rangle_{ACB} = \frac{1}{\sqrt{2}}\,(|000\rangle + |111\rangle)_{ACB} \tag{17}$$

The subscript A, C and B denote that the particles belong to Alice, Charlie and Bob, respectively. The concrete processes can be presented as follows:

Step 1: Due to transmit the initial state given by Eq. (16), the projective measurement $\{\ |\Gamma_m\rangle \mid m = 0, 1\ \}$ need to be performed by Alice on particle A.

$$\begin{pmatrix} |\Gamma_0\rangle \\ |\Gamma_1\rangle \end{pmatrix} = \begin{pmatrix} a_0 & a_1 \\ a_1 & -a_0 \end{pmatrix} \begin{pmatrix} |0\rangle \\ |1\rangle \end{pmatrix} \tag{18}$$

After that, particles C and B would collapse into one of two kinds of outcomes below

$$|\Phi_0\rangle_{CB} \simeq [\ |\Gamma_0\rangle_A\ ]^\dagger\ |\Psi\rangle_{ACB} = \frac{1}{\sqrt{2}}(a_0|00\rangle + a_1|11\rangle)_{CB} \tag{19}$$

$$|\Phi_1\rangle_{CB} \simeq [\ |\Gamma_1\rangle_A\ ]^\dagger\ |\Psi\rangle_{ACB} = \frac{1}{\sqrt{2}}(a_1|00\rangle - a_0|11\rangle)_{CB} \tag{20}$$

Meanwhile, Alice informs Bob and Charlie of her measurement results using classical information. From the analysis of Step 1 in Sect. 2, one can obtain that

$$S_1^0 = I_2 \quad S_1^1 = S_1[1, -\overline{2}] = \sigma_z \tag{21}$$

Step 2: Charlie need to perform the following projective measurement $\{\ |\Omega_k^m\rangle \mid k = 0, 1\ \}$ on particle C in terms of Alice's measurement result $|\Gamma_m\rangle$.

$$\begin{pmatrix} |\Omega_0^0\rangle \\ |\Omega_1^0\rangle \\ |\Omega_0^1\rangle \\ |\Omega_1^1\rangle \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & e^{-i\phi_1} \\ 1 & -e^{-i\phi_1} \\ -e^{-i\phi_1} & 1 \\ e^{-i\phi_1} & 1 \end{pmatrix} \begin{pmatrix} |0\rangle \\ |1\rangle \end{pmatrix} \tag{22}$$

Table 1 The results on particles (A, C) and the unitary gates on particles B

Measurement results on		Probabilities	Operations $U_1^k S_1^m$ on particle B
Particle A	Particle C		
$\|\Gamma_0\rangle$	$\|\Omega_0^0\rangle$	1/4	I
	$\|\Omega_1^0\rangle$	1/4	σ_z
$\|\Gamma_1\rangle$	$\|\Omega_0^1\rangle$	1/4	σ_x
	$\|\Omega_1^1\rangle$	1/4	$i\sigma_y$

Meanwhile, Eqs. (17) and (18) would become

$$|\Delta_0^0\rangle_B = [\ |M_0\rangle_A |N_0^0\rangle_C\ ]^\dagger\ |Q\rangle_{ACB} = \frac{1}{2}(a_0|0\rangle + a_1 e^{i\phi_1}|1\rangle)_B \tag{23}$$

$$|\Delta_1^0\rangle_B = [\ |M_0\rangle_A |N_1^0\rangle_C\ ]^\dagger\ |Q\rangle_{ACB} = \frac{1}{2}(a_0|0\rangle - a_1 e^{i\phi_1}|1\rangle)_B \tag{24}$$

$$|\Delta_0^1\rangle_B = [\ |M_1\rangle_A |N_0^1\rangle_C\ ]^\dagger\ |Q\rangle_{ACB} = \frac{1}{2}(-a_1 e^{i\phi_1}|0\rangle + a_0|1\rangle)_B \tag{25}$$

$$|\Delta_1^1\rangle_B = [\ |M_1\rangle_A |N_1^1\rangle_C\ ]^\dagger\ |Q\rangle_{ACB} = \frac{1}{2}(a_1 e^{i\phi_1}|0\rangle + a_0|1\rangle)_B \tag{26}$$

The measurements results of particle C can be transmitted from Charlie to Bob via classical channel.

Step 3: Based on the outcomes $\{\, m, k = 0, 1 \,\}$ of Alice and Charlie, Bob performs the relative unitary operation $U_2^k S_2^m$ to prepare the original state. Table 1 indicates how to select the unitary transformation for particle B based on the measurements results of particles A and C. The unitary operation U_1^k $(k = 0, 1)$ in Table 1 can be presented as

$$U_n^k = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi k} \end{pmatrix} \tag{27}$$

here $\{\, \sigma_x, \sigma_y, \sigma_z, I\}$ are the *Pauli matrices*. From Table 1, it can be found that our scheme for joint remote state preparation of arbitrary single-qubit states can be realized with the 100% successful probability at the cost of 3 *bits* classical information. The results about successful probability are in agreement with the probabilities of Refs. [21,30].

3.2 JRSP of arbitrary two-qubit states

The two-qubit states prepared from the senders Alice and Charlie to the receiver Bob can be presented as

$$|\psi\rangle = a_0|00\rangle + a_1 e^{i\phi_1}|01\rangle + a_2 e^{i\phi_2}|10\rangle + a_3 e^{i\phi_3}|11\rangle \tag{28}$$

where a_i $(i = 0, 1 \cdots 3)$ and ϕ_j $(j = 1, 2, 3)$ are real, and $\Sigma_{x=0}^{3}|a_x|^2 = 1$. Assume that Alice has the information of a_x, and Charlie has ϕ_y. The three-qubit GHZ states shared along Alice and Charlie with Bob are presented as

$$|\Psi\rangle_{A_1C_1B_1} = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{A_1C_1B_1} \tag{29}$$

$$|\Psi\rangle_{A_2C_2B_2} = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{A_2C_2B_2} \tag{30}$$

Note that Alice, Charlie and Bob have particles A_j, C_j and B_j respectively, here $j = 1, 2$. Moreover, the detailed processes of our JRSP proposal for arbitrary two-qubit states are elaborated as follows:

Step 1: For the purpose to remotely prepare two-qubit state, Alice needs to perform the projective measurements $\{ |\Gamma_m\rangle \mid m = 0, 1, 2, 3 \}$ on her particles A_1 and A_2.

$$\begin{pmatrix} |\Gamma_0\rangle \\ |\Gamma_1\rangle \\ |\Gamma_2\rangle \\ |\Gamma_3\rangle \end{pmatrix} = \begin{pmatrix} a_0 & a_1 & a_2 & a_3 \\ a_1 & -a_0 & a_3 & -a_2 \\ a_2 & -a_3 & -a_0 & a_1 \\ a_3 & a_2 & -a_1 & -a_0 \end{pmatrix} \begin{pmatrix} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{pmatrix} \tag{31}$$

Thus, quantum channel composed of the three-qubit GHZ states could be rewritten as

$$|\Psi\rangle_{A_1C_1B_1} \otimes |\Psi\rangle_{A_2C_2B_2} = \frac{1}{2} \cdot \sum_{j=0}^{2^n-1} |\Gamma_j\rangle_{A_1A_2} \otimes |\Phi_j\rangle_{C_1B_1C_2B_2} \tag{32}$$

here

$$\begin{aligned} |\Phi_0\rangle &= a_0|0000\rangle + a_1|0011\rangle + a_2|1100\rangle + a_3|1111\rangle \\ |\Phi_1\rangle &= a_1|0000\rangle - a_0|0011\rangle + a_3|1100\rangle - a_2|1111\rangle \\ |\Phi_2\rangle &= a_2|0000\rangle - a_3|0011\rangle - a_0|1100\rangle + a_1|1111\rangle \\ |\Phi_3\rangle &= a_3|0000\rangle + a_2|0011\rangle - a_1|1100\rangle - a_0|1111\rangle \end{aligned} \tag{33}$$

Meanwhile, Alice informs Bob and Charlie of her measurement results using classical information. From the analysis of Step 1 in Sect. 1 and Eq. (29), it can be find that

$$\begin{array}{ll} S_2^0 = I_4 & S_2^1 = S_n[1, \overline{2};\ 3, \overline{4}] = I_2 \otimes i\sigma_y \\ S_2^2 = S_n[1, \overline{3};\ \overline{2}, 4] = i\sigma_y \otimes \sigma_z & S_2^3 = S_n[1, \overline{4};\ 2, \overline{3}] = i\sigma_y \otimes \sigma_x \end{array} \tag{34}$$

Step 2: Charlie need to perform the following projective measurement $\{ |\Omega_k^m\rangle \mid k = 0, 1 \}$ on particles C_1 and C_2 in terms of Alice's measurement result $|\Gamma_m\rangle$.

$$\begin{pmatrix} |\Omega_0^m\rangle \\ |\Omega_1^m\rangle \\ |\Omega_2^m\rangle \\ |\Omega_3^m\rangle \end{pmatrix} = \frac{1}{2} S_2^m \begin{pmatrix} 1 & e^{-i\phi_1} & e^{-i\phi_2} & e^{-i\phi_3} \\ 1 & ie^{-i\phi_1} & -e^{-i\phi_2} & -ie^{-i\phi_3} \\ 1 & -e^{-i\phi_1} & e^{-i\phi_2} & -e^{-i\phi_3} \\ 1 & -ie^{-i\phi_1} & -e^{-i\phi_2} & ie^{-i\phi_3} \end{pmatrix} \begin{pmatrix} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{pmatrix} \tag{35}$$

Thus, one can find that

$$
\begin{aligned}
|\Delta_0^0\rangle_{B_1B_2} &= 1/4\cdot(a_0|00\rangle + a_1e^{i\phi_1}|01\rangle + a_2e^{i\phi_2}|10\rangle + a_3e^{i\phi_3}|11\rangle)_{B_1B_2}\\
|\Delta_1^0\rangle_{B_1B_2} &= 1/4\cdot(a_0|00\rangle + a_1ie^{i\phi_1}|01\rangle - a_2e^{i\phi_2}|10\rangle - a_3ie^{i\phi_3}|11\rangle)_{B_1B_2}\\
|\Delta_2^0\rangle_{B_1B_2} &= 1/4\cdot(a_0|00\rangle - a_1e^{i\phi_1}|01\rangle + a_2e^{i\phi_2}|10\rangle - a_3e^{i\phi_3}|11\rangle)_{B_1B_2}\\
|\Delta_3^0\rangle_{B_1B_2} &= 1/4\cdot(a_0|00\rangle - a_1ie^{i\phi_1}|01\rangle - a_2e^{i\phi_2}|10\rangle + ia_3e^{i\phi_3}|11\rangle)_{B_1B_2}\\
|\Delta_0^1\rangle_{B_1B_2} &= 1/4\cdot(-a_0|01\rangle + a_1e^{i\phi_1}|00\rangle - a_2e^{i\phi_2}|11\rangle - a_3e^{i\phi_3}|10\rangle)_{B_1B_2}\\
|\Delta_1^1\rangle_{B_1B_2} &= 1/4\cdot(-a_0|01\rangle + a_1ie^{i\phi_1}|00\rangle + a_2e^{i\phi_2}|11\rangle + a_3ie^{i\phi_3}|10\rangle)_{B_1B_2}\\
|\Delta_2^1\rangle_{B_1B_2} &= 1/4\cdot(-a_0|01\rangle - a_1e^{i\phi_1}|00\rangle - a_2e^{i\phi_2}|11\rangle + a_3e^{i\phi_3}|10\rangle)_{B_1B_2}\\
|\Delta_3^1\rangle_{B_1B_2} &= 1/4\cdot(-a_0|01\rangle - a_1ie^{i\phi_1}|00\rangle + a_2e^{i\phi_2}|11\rangle - ia_3e^{i\phi_3}|10\rangle)_{B_1B_2}\\
|\Delta_0^2\rangle_{B_1B_2} &= 1/4\cdot(-a_0|10\rangle + a_1e^{i\phi_1}|11\rangle + a_2e^{i\phi_2}|00\rangle - a_3e^{i\phi_3}|01\rangle)_{B_1B_2}\\
|\Delta_1^2\rangle_{B_1B_2} &= 1/4\cdot(-a_0|10\rangle + a_1ie^{i\phi_1}|11\rangle - a_2e^{i\phi_2}|00\rangle + a_3ie^{i\phi_3}|01\rangle)_{B_1B_2}\\
|\Delta_2^2\rangle_{B_1B_2} &= 1/4\cdot(-a_0|10\rangle - a_1e^{i\phi_1}|11\rangle + a_2e^{i\phi_2}|00\rangle + a_3e^{i\phi_3}|01\rangle)_{B_1B_2}\\
|\Delta_3^2\rangle_{B_1B_2} &= 1/4\cdot(-a_0|10\rangle - a_1ie^{i\phi_1}|11\rangle - a_2e^{i\phi_2}|00\rangle - ia_3e^{i\phi_3}|01\rangle)_{B_1B_2}\\
|\Delta_0^3\rangle_{B_1B_2} &= 1/4\cdot(-a_0|11\rangle - a_1e^{i\phi_1}|10\rangle + a_2e^{i\phi_2}|01\rangle + a_3e^{i\phi_3}|00\rangle)_{B_1B_2}\\
|\Delta_1^3\rangle_{B_1B_2} &= 1/4\cdot(-a_0|11\rangle - a_1ie^{i\phi_1}|10\rangle - a_2e^{i\phi_2}|01\rangle - a_3ie^{i\phi_3}|00\rangle)_{B_1B_2}\\
|\Delta_2^3\rangle_{B_1B_2} &= 1/4\cdot(-a_0|11\rangle + a_1e^{i\phi_1}|10\rangle + a_2e^{i\phi_2}|01\rangle - a_3e^{i\phi_3}|00\rangle)_{B_1B_2}\\
|\Delta_3^3\rangle_{B_1B_2} &= 1/4\cdot(-a_0|11\rangle + a_1ie^{i\phi_1}|10\rangle - a_2e^{i\phi_2}|01\rangle + ia_3e^{i\phi_3}|00\rangle)_{B_1B_2}
\end{aligned}
\tag{36}
$$

Meanwhile, Table 2 shows the relationship between the measurement results $\{\ m, k \mid m, k = 0, 1, 2, 3\ \}$ with the unitary transformation $U_2^k \cdot S_2^m$ on particles B_1 and B_2.

here, $\{\ S_2^m\ |m = 0, 1, 2, 3\ \}$ are given by Eq. (30), and $\{\ U_2^k\ |k = 0, 1, 2, 3\ \}$ can be presented as

$$
\begin{aligned}
U_2^0 &= I_4 \qquad & U_2^1 &= \sigma_z \otimes \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}\\
U_2^2 &= I_2 \otimes \sigma_z \qquad & U_2^3 &= \sigma_z \otimes \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix}
\end{aligned}
\tag{37}
$$

From the above discussions, we can obtain that arbitrary two-qubit states can be transmitted deterministically by using our scheme. It should be emphasized that the successful probability is equal to one. In this JRSP scheme, Alice needs 4 *bits* classical information to tell the measurement outcomes to Charlie and Bob. Meanwhile, 2 *bits* classical communication cost is required for Charlie to inform Bob of his measurement results. Thus, the total classical information is equal to 6 *bits*. We would like to point out that an efficient three-party RSP scheme for entangled two-qubit states from a sender to either of two receivers is presented by Dai et al. [44]. It is shown that total classical communication costs of such a RSP scheme in a general case and two particular cases via the maximally entangled channel are 2.5 *bits* and 5 *bits*, respectively. The classical information costs are less than our RSP protocol for

Table 2 The results $\{m, k\}$ and the unitary gates $U_2^k \cdot S_2^m$ on particles (B_1, B_2)

Measurement results on		State of particles B_1B_2	Probabilities	Operations $U_2^k \cdot S_2^m$ on particles B_1B_2
Particles A_1A_2	Particles C_1C_2			
$\|\Psi_0\rangle$	$\|\Omega_0^0\rangle$	$\|\Delta_0^0\rangle$	1/4	$U_2^0 \cdot S_2^0$
	$\|\Omega_1^0\rangle$	$\|\Delta_1^0\rangle$	1/4	$U_2^1 \cdot S_2^0$
	$\|\Omega_2^0\rangle$	$\|\Delta_2^0\rangle$	1/4	$U_2^2 \cdot S_2^0$
	$\|\Omega_3^0\rangle$	$\|\Delta_3^0\rangle$	1/4	$U_2^3 \cdot S_2^0$
$\|\Psi_1\rangle$	$\|\Omega_0^1\rangle$	$\|\Delta_0^1\rangle$	1/4	$U_2^0 \cdot S_2^1$
	$\|\Omega_1^1\rangle$	$\|\Delta_1^1\rangle$	1/4	$U_2^1 \cdot S_2^1$
	$\|\Omega_2^1\rangle$	$\|\Delta_2^1\rangle$	1/4	$U_2^2 \cdot S_2^1$
	$\|\Omega_3^1\rangle$	$\|\Delta_3^1\rangle$	1/4	$U_2^3 \cdot S_2^1$
$\|\Psi_2\rangle$	$\|\Omega_0^2\rangle$	$\|\Delta_0^2\rangle$	1/4	$U_2^0 \cdot S_2^2$
	$\|\Omega_1^2\rangle$	$\|\Delta_1^2\rangle$	1/4	$U_2^1 \cdot S_2^2$
	$\|\Omega_2^2\rangle$	$\|\Delta_2^2\rangle$	1/4	$U_2^2 \cdot S_2^2$
	$\|\Omega_3^2\rangle$	$\|\Delta_3^2\rangle$	1/4	$U_2^3 \cdot S_2^2$
$\|\Psi_3\rangle$	$\|\Omega_0^3\rangle$	$\|\Delta_0^3\rangle$	1/4	$U_2^0 \cdot S_2^3$
	$\|\Omega_1^3\rangle$	$\|\Delta_1^3\rangle$	1/4	$U_2^1 \cdot S_2^3$
	$\|\Omega_2^3\rangle$	$\|\Delta_2^3\rangle$	1/4	$U_2^2 \cdot S_2^3$
	$\|\Omega_3^3\rangle$	$\|\Delta_3^3\rangle$	1/4	$U_2^3 \cdot S_2^3$

two-qubit states. Actually, the classical message required in RSP proposals is primarily determined by the initial condition, communication task, entanglement sources, implement steps and so on. Compared with our novel protocol, the sender of former scheme has the whole information of quantum state they want to prepare, while the senders of current JRSP scheme separately have partial information. The communication task in Ref. [44] is to prepare entangled two-qubit states from one sender to either of two receiver, and arbitrary two-qubit states from two senders to one receiver can be prepared via our RSP protocol, whatever the prepared state is entangled or not. Quantum channel in Ref. [44] is the combination of a non-maximally entangled two-qubit state and a partially entangled three-qubit state, and two maximally entangled three-qubit states are required for remotely preparing one two-qubit state in this novel proposal. The implement steps of the former scheme include one single-qubit measurement and one two-qubit measurement, only the two-qubit measurement is relative with the information of the prepared state, and two kinds of two-qubit quantum measurements, both of which are corresponding to the prepared two-qubit state, would be performed by the two senders in our scheme, respectively. Additionally, the projective measurements of previous scheme are mutually independent, meanwhile they are simpler than ones of our JRSP scheme. These features are useful for the physical realization. The new method in this paper and the former proposal complement each other.

4 Discussion and conclusions

In summary, we put forward a general proposal to deterministically prepare arbitrary multi-qubit states by using of maximally three-qubit entangled states. Two special kinds of mutually orthogonal measurement basis, of which the analytical expressions are presented in the form of iterative process, are constructed without the introduction of auxiliary particles. Furthermore, the realization procedures of this novel protocol are elaborated in detail, and the total classical communication cost required in our JRSP scheme is also calculated. In contrast to previous methods, the significant advantage is that the successful probability of this JRSP proposal for arbitrary multi-qubit states can reach up to 100%. Frankly speaking, it is a prerequisite of this advantage that quantum channel is composed of maximally three-qubit entangled state in our scheme. Actually, there are several protocols presented for RSP via various quantum entanglement channels. Further research will focus on the schemes for preparing arbitrary multi-qubit states by using partially entangled states.

Acknowledgements The authors thank J. Jiang, J.W. Luo and Y. Zhu for helpful discussions. This work is supported by the Program for National Natural Science Foundation of China (Grant Nos. 61803382, 61703428, and 61703420), Natural Science Basic Research Plan in Shaanxi Province of China (Program No. 2018JQ6020) and China Postdoctoral Science Foundation Funded Project (Project No. 2018M643869).

References

1. Bennett, C.H., Brassard, G., Grepeau, C., Jozsa, R., Peres, A., Wootters, W.K.: Teleporting an unknown quantum state via dual classical and Einstein–Podolsky–Rosen channels. Phys. Rev. Lett. **70**, 1895 (1993)
2. Lo, H.K.: Classical-communication cost in distributed quantum-information processing: a generalization of quantum-communication complexity. Phys. Rev. A **62**, 012313 (2000)
3. Li, W.L., Li, C.F., Guo, G.C.: Probabilistic teleportation and entanglement matching. Phys. Rev. A **61**, 034301 (2000)
4. Dai, H.Y., Chen, P.X., Liang, L.M., Li, C.Z.: Classical communication cost and remote preparation of the four-particle GHZ class state. Phys. Lett. A **355**, 285 (2006)
5. Zhang, Z.J.: Controlled teleportation of an arbitrary n-qubit quantum information using quantum secret sharing of classical message. Phys. Lett. A **352**, 55 (2006)
6. Dai, H.Y., Chen, P.X., Li, C.Z.: Probabilistic teleportation of an arbitrary two-particle state by a partially entangled three-particle GHZ state and W state. Opt. Commun. **231**, 281 (2004)
7. Dai, H.Y., Chen, P.X., Li, C.Z.: Probabilistic teleportation of an arbitrary two-particle state by two partial three-particle entangled W states. J. Opt. B **6**, 106 (2004)
8. Zhou, P., Li, X.H., Deng, F.G., Zhou, H.Y.: Multiparty-controlled teleportation of an arbitrary m-qudit state with a pure entangled quantum channel. J. Phys. A **40**, 13121 (2007)
9. Dai, H.Y., Li, C.Z., Chen, P.X.: Probabilistic teleportation of the three-particle entangled state by the partial three-particle entangled state and the threeparticle entangled W state. Chin. Phys. Lett. **20**, 1196 (2003)
10. Li, X.H., Ghose, S.: Analysis of N-qubit perfectly controlled teleportation schemes from the controllers point of view. Phys. Rev. A **91**, 052305 (2015)
11. Wei, J.H., Shi, L., Han, C., Xu, Z.Y., et al.: Probabilistic teleportation via multi-parameter measurements and partially entangled states. Quantum Inf. Process **17**, 82 (2018)
12. Long, Y.X., Qiu, D.W., Long, D.Y.: Perfect teleportation between arbitrary split of six partites by a maximally genuinely entangled six-qubit state. Int. J. Quantum Inf. **8**, 821 (2010)
13. Li, L.Z., Qiu, D.W.: The states of W-class as shared resources for perfect teleportation and superdense coding. J. Phys. A **40**, 10871 (2007)

14. Bennett, C.H., DiVincenzo, D.P., Shor, P.W., Smolin, J.A., Terhal, B.M., Wootters, W.K.: Remote state preparation. Phys. Rev. Lett. **87**, 077902 (2001)
15. Pati, A.K.: Minimum classical bit for remote preparation and measurement of a qubit. Phys. Rev. A **63**, 014302 (2001)
16. Dai, H.Y., Zhang, M., Zhang, Z.R., Xi, Z.R.: Probabilistic remote preparation of a four-particle entangled W state for the general case and for all kinds of the special cases. Commun. Theor. Phys. **60**, 313 (2013)
17. Jiang, M., Dong, D.Y.: A recursive two-phase general protocol on deterministic remote preparation of a class of multi-qubit states. J. Phys. B **45**, 205506 (2012)
18. Wei, J.H., Shi, L., Ma, L.H., Xue, Y., Zhuang, X.C., et al.: Remote preparation of an arbitrary multi-qubit state via two-qubit entangled states. Quantum Inf. Process. **16**, 260 (2017)
19. Xia, Y., Lu, P.M., Song, J., Song, H.S.: Deterministic remote preparation of electrons states in coupled quantum dots by stimulated raman adiabatic passage. Int. J. Theor. Phys. **49**, 2045 (2010)
20. Wei, J.H., Shi, L., Zhu, Y., Xue, Y., et al.: Deterministic remote preparation of arbitrary multi-qubit equatorial states via two-qubit entangled states. Quantum Inf. Process **17**, 70 (2018)
21. Xia, Y., Song, J., Song, H.S.: Multiparty remote state preparation. J. Phys. B **40**, 3719 (2007)
22. Yang, K.Y., Xia, Y.: Joint remote preparation of a general three-qubit state via non-maximally GHZ states. Int. J. Theor. Phys. **51**, 1647 (2011)
23. Zhang, Y.B., Ma, P.C.: Deterministic joint remote preparation of arbitrary two- and three-qubit entangled states. Quantum Inf. Process. **12**, 997 (2013)
24. Peng, J.Y., Luo, M.X., Mo, Z.W.: Joint remote state preparation of arbitrary two-particle states via GHZ-type states. Quantum Inf. Process. **12**, 2325 (2013)
25. Zhou, P., Li, H.W., Long, L.R.: Probabilistic multiparty joint remote preparation of an arbitrary m-qubit state with a pure entangled channel against collective noise. Int. J. Theor. Phys. **52**, 849 (2013)
26. Wang, Z.Y., Liu, Y.M., Zuo, X.Q., Zhang, Z.J.: Controlled remote state preparation. Commun. Theor. Phys. **52**, 235 (2009)
27. Zhang, D., Zha, X.W., Duan, Y.J., Yang, Y.Q.: Deterministic controlled bidirectional remote state preparation via a six-qubit maximally entangled state. Quantum Inf. Process. **15**, 2169 (2016)
28. Chen, N., Quan, D.X., Yang, H., Pei, C.X.: Deterministic controlled remote state preparation using partially entangled quantum channel. Quantum Inf. Process. **15**, 1719 (2016)
29. Berry, D.W., Sanders, B.C.: Optimal remote state preparation. Phys. Rev. Lett. **90**, 027901 (2003)
30. Chen, N., Quan, D.X., Zhu, C.H., et al.: Deterministic joint remote state preparation via partially entangled quantum channel. Int. J. Quantum Inf. **14**, 1650015 (2016)
31. Chen, Q.Q., Xia, Y.: Joint remote preparation of an arbitrary two-qubit state via a generalized seven-qubit brown state. Laser Phys. **26**, 015203 (2016)
32. Luo, M.X., Chen, X.B., Ma, S.Y., Niu, X.X., Yang, Y.X.: Joint remote preparation of an arbitrary three-qubit state. Opt. Commun. **283**, 4796 (2010)
33. Luo, M.X., Deng, Y.: Joint remote preparation of an arbitrary 4-qubit χ-State. Int. J. Theor. Phys. **51**, 3027 (2012)
34. Wang, Z.Y.: Joint remote preparation of a multi-qubit GHZ-class state via bipartite entanglements. Int. J. Quantum Inf. **9**, 809 (2011)
35. Li, X.H., Ghose, S.: Optimal joint remote state preparation of equatorial states. Quantum Inf. Process. **14**, 4585 (2015)
36. Long, L.R., Zhou, P., Li, Z., Yin, C.L.: Multiparty joint remote preparation of an Arbitrary GHZ-class state via positive operator-valued measurement. Int. J. Theor. Phys. **51**, 2438 (2012)
37. Zhou, P.: Joint remote preparation of an arbitrary m-qudit state with a pure entangled quantum channel via positive operator-valued measurement. J. Phys. A **45**, 215305 (2012)
38. Jiang, M., Zhou, L.L., Chen, X.P., You, S.H.: Deterministic joint refdemote preparation of general multi-qubit states. Opt. Commun. **301**, 39 (2013)
39. Bouwmeester, D., Pan, J.W., Mattle, K., Eibl, M., Weinfuter, H., Zeilinger, A.: Experimental quantum teleportation. Nature **390**, 575 (1997)
40. Situ, H.Z., Qiu, D.W.: Investigating the implementation of restricted sets of multiqubit operations on distant qubits: a communication complexity perspective. Quantum Inf. Process **10**, 609 (2011)
41. Pittman, T.B., Jacobs, B.C., Franson, J.D.: Demonstration of nondeterministic quantum logic operations using linear optical elements. Phys. Rev. Lett. **88**, 257902 (2002)
42. Mattle, K., Weinfurter, H., Kwiat, P.G., Zeilinger, A.: Dense coding in experimental quantum communication. Phys. Rev. Lett. **76**, 4656 (1996)

43. Wei, J.H., Shi, L., Luo, J.W., Zhu, Y., Kang, Q.Y., et al.: Optimal remote preparation of arbitrary multi-qubit real-parameter states via two-qubit entangled states. Quantum Inf. Process **17**, 141 (2018)
44. Dai, H.Y., Chen, P.X., Zhang, M., Li, C.Z.: Remote preparation of an entangled two-qubit state with three parties. Chin. Phys. B **17**, 27 (2008)
45. Dai, H.Y., Zhang, M., Kuang, L.M.: Classical communication cost and remote preparation of multi-qubit with three-party. Commun. Theor. Phys. **50**, 73 (2008)