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Deterministic remote preparation of arbitrary multi-qubit equatorial states via two-qubit entangled states

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Abstract We propose an efficient scheme for remotely preparing an arbitrary n-qubit equatorial state via n two-qubit maximally entangled states. Compared to the former scheme (Wei et al. in Quantum Inf Process 16:260, 2017) that has the 50% successful probability when the amplitude factors of prepared states are $2^{-n/2}$, the probability would be increased to 100% by using of our modified proposal. The feasibility of our scheme for remote preparation arbitrary multi-qubit equatorial states is explicitly demonstrated by theoretical studies and concrete examples.

Keywords Remote state preparation · Successful probability · Arbitrary equatorial states

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1 Introduction

Quantum entanglement, an unique resource in quantum information, plays an important role in quantum communication [1–4]. One of the typical applications of quantum entanglement is remote state preparation (RSP), which is originally proposed by Lo [2] and can be used to transmit quantum states from a sender to a remote receiver with the aid of classical information and quantum entanglement. Compared with the usual teleportation [4–11], the sender in RSP knows completely the information of prepared state, while in the teleportation neither the sender nor the receiver has knowledge of the transmitted state. For the sake that RSP could be applied to remote communication, several theoretical protocols have been proposed for RSP of different input states [12–22], RSP via various quantum entanglement channels [22–26], and RSP with multi-party [27–30]. For example, Dai et al. [20] presented a scheme for remote preparation of the four-particle GHZ class state and calculated the classical communication cost for this proposal. Zhang et al. [24] explored how to realize deterministic controlled bidirectional remote state preparation via a six-qubit entangled state. Recently, some experimental implementations of RSP proposals have been presented via nuclear magnetic resonance [31] and spontaneous parametric down-conversion [32].

For single states, the pure equatorial state is restricted in the equator circle of *Bloch sphere*. Compared with general states, equatorial states contain less quantum correlation. Nevertheless, there are also many advantages for equatorial states, such as simpler method for generating quantum states [1], higher fidelity of quantum cloning [33], and less required classical communication for RSP proposal [2,34] than general states. Bruß et al. [33] presented the phase-covariant quantum cloning machine, of which the input state is from the equatorial line of *Bloch sphere*, and the fidelity is higher than that of the universal quantum cloning machine. Pati [34] demonstrated that the RSP is more economical than quantum teleportation and requires only one classical bit for equatorial states, but for general states, the RSP requires as much classical communication cost as quantum teleportation does. Li et al. [35] presented a scheme for joint remote state preparation of two-qubit equatorial states via GHZ states in a deterministic manner.

The successful probability is always viewed as one of the main performance parameters for RSP proposals. Most recently, Wei et al. [36] presented a RSP scheme for n-qubit states by using of an appropriate local $2^n \times 2^n$ unitary operation with the successful probability of 2^{-n} for general n-qubit states, and the probability for equatorial states would be increased to 1/2. We revisited this proceeding scenario and found that the successful probability for equatorial states is not optimal. In this paper, inspired by the RSP scheme in Ref. [35], we modify the measurement basis of the sender to deterministically prepare the desired state with proper unitary operations. It is noting that the receiver in our scheme can always perform a corresponding unitary operation based on the possible measurement outcome of the sender and reconstruct the prepared state with the 100% successful probability.

The rest of this paper are organized as follows: In Sect. 2, an efficient scheme for remote preparation of an arbitrary n-qubit equatorial state is presented with the aid of n maximally entangled two-qubit states. The measurement basis of the sender is given in detail. The total successful probability of our scheme is obtained, and it is equal to 100%. In Sect. 3, concrete realization processes for preparing remotely two-qubit and three-qubit states are illustrated to demonstrate explicitly the feasibility of our scheme. The paper concludes with Sect. 4.

2 RSP of *n*-qubit equatorial states

To present our protocol more clearly, let us first begin with the RSP task of arbitrary n-qubit equatorial states. Suppose there are two legitimate participants, customarily called Alice and Bob in quantum communication network, and they are spatially separated in different sites. Let Alice and Bob be the sender and receiver, respectively. Assume that Alice wants to remotely prepare multi-qubit equatorial states for Bob. Generally, an arbitrary n-qubit equatorial state has the form

$$|\psi\rangle = \frac{1}{\sqrt{2^n}} \sum_{x=0}^{2^n-1} \left(e^{i\phi_x} |d_n \dots d_2 d_1\rangle \right) \quad d_l \in \{0, 1\}; \quad x = \sum_{l=1}^{n} d_l \cdot 2^{l-1} \tag{1}$$

where x is the decimal form of the binary string $d_n \dots d_2 d_1$ and ϕ_x ($x = 0, 1 \dots 2^n - 1$) is real with the region $0 \leq \phi_x < 2\pi$. Usually, ϕ_0 is set to be zero. The maximally entangled two-qubit states previously shared along Alice and Bob can be presented as follows:

$$|\Psi\rangle_{jk} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{j,k} \quad j = 1, 3, \dots, 2n-1; \; k = j+1. \tag{2}$$

Without loss of generality, particle j belongs to the sender Alice, while particle k is hold by the receiver Bob. Alice is in possession of particle j, and Bob possesses particle k, respectively. To help Bob prepare the initial state remotely, Alice need to perform the n-qubit projective measurement on her particles $(1, 3 \dots 2n-1)$ under the 2^n mutual orthogonal measurement bases $\{|\Gamma_m\rangle \mid m = 0, 1 \dots 2^n - 1\}$ as follow

$$|\Gamma_m\rangle = \frac{1}{\sqrt{2^n}} \sum_{x=0}^{2^n-1} \left[\exp\left(\frac{i\pi x m}{2^{n-1}} - i\phi_x \right) |d_n \dots d_2 d_1\rangle \right] \tag{3}$$

Moreover, the whole system composed of n maximally entangled states can be presented as

$$\begin{aligned} &|\Psi\rangle_{1,2} \otimes |\Psi\rangle_{3,4} \cdots |\Psi\rangle_{2n-1,2n} \\ &= \frac{1}{\sqrt{2^n}} \cdot \sum_{m=0}^{2^n-1} \left\{ |\Gamma_m\rangle_{1,3,\cdots,2n-1} \otimes \sum_{x=0}^{2^n-1} \left[\exp\left(i\phi_x - \frac{i\pi x m}{2^{n-1}} \right) |d_n \dots d_2 d_1\rangle_{2,4,\cdots,2n} \right] \right\} \end{aligned} \tag{4}$$

From Eq. (4), it can be obtained that there are 2^n kinds of measurement results $\{|\Gamma_m\rangle | \, m = 0, 1 \dots 2^n - 1\}$ on Alice's particles $(1, 3 \dots 2n-1)$, of which the relative

Table 1 The measurement results of particles $(1, 3, \ldots, 2n-1)$ and the unitary operations on particles $(2, 4, \ldots, 2n)$

Measurement results of qubits $(1, 3 \ldots 2n-1)$	The state of qubits $(2, 4 \ldots 2n)$	Probability	Gates U_m^n on qubits $(2, 4 \ldots 2n)$
$\lvert\Gamma_m\rangle$	$\sum_{x=0}^{2^n-1}\left[\exp\left(i\phi_x - \frac{i\pi xm}{2^{n-1}}\right)\cdot\lvert d_n \ldots d_2 d_1\rangle\right]$	$P_m = \frac{1}{2^n}$	Eq. (5)

$m = 0, 1 \ldots 2^n - 1$

measurement probabilities P_m are all equal to $\left(1/\sqrt{2^n}\right)^2 = 1/2^n$. Note that no matter what measurement results Alice obtains, the state of Bob's particles can always be transformed into the desired form via the corresponding unitary operation

$$U_m^n = \sum_{x=0}^{2^n-1}\left[\exp\left(\frac{i\pi xm}{2^{n-1}}\right)\lvert d_n \ldots d_2 d_1\rangle\langle d_n \ldots d_2 d_1\rvert\right]$$
$$= \begin{pmatrix}1 & 0\\ 0 & e^{i\pi m}\end{pmatrix}\otimes\begin{pmatrix}1 & 0\\ 0 & e^{i\frac{\pi m}{2}}\end{pmatrix}\otimes\cdots\otimes\begin{pmatrix}1 & 0\\ 0 & e^{i\frac{\pi m}{2^n}}\end{pmatrix} \tag{5}$$

Subsequently, Bob only has to perform this unitary operation based on Alice's measurement results $\lvert\Gamma_m\rangle$ in order to obtain the original state. Table 1 shows the relation between the measurement results of particles $(1,\ 3, \ldots, 2n-1)$ with the unitary transformations on particles $(2,\ 4, \ldots, 2n)$.

The successful probability of RSP schemes is considered as one of the most important parameters. For all of the measurement results $\{\lvert\Gamma_m\rangle \mid m = 0, 1 \ldots 2^n - 1\}$, the initial state in Eq. (1) always can be prepared successfully. It could be found that each of the 2^n kinds of measurement outcomes has the same probability as $1/2^n$. Hence, the whole successful probability of our scheme is equal to

$$P_{\text{total}} = \sum_{m=0}^{2^n-1} P_m = \frac{1}{2^n}\cdot 2^n = 1 \tag{6}$$

This result is in agreement with the successful probabilities of previous RSP schemes [20,21,25] when equatorial states would be remotely prepared in the former proposals by using of quantum maximally entangled states.

3 Examples of RSP

To illustrate our proposal explicitly, we would demonstrate how remotely prepare two-qubit and three-qubit equatorial states, which are elementary resources for quantum information processing.

3.1 Two-qubit states

Suppose that the sender Alice wishes to help the receiver Bob prepare the following two-qubit equatorial state

$$|\psi\rangle = 1/2 \cdot \left(|00\rangle + \mathrm{e}^{i\phi_1}|01\rangle + \mathrm{e}^{i\phi_2}|10\rangle + \mathrm{e}^{i\phi_3}|11\rangle\right) \tag{7}$$

In order to fulfill the RSP, Alice performs the two-qubit projective measurement on particles (1, 3) in the four basis vectors $\{|\Gamma_m\rangle \mid m = 0, 1, 2, 3\}$, which are given by

$$\begin{aligned}
|\Gamma_0\rangle &= 1/2 \cdot \left(|00\rangle + \mathrm{e}^{-i\phi_1}|01\rangle + \mathrm{e}^{-i\phi_2}|10\rangle + \mathrm{e}^{-i\phi_3}|11\rangle\right)\\
|\Gamma_1\rangle &= 1/2 \cdot \left(|00\rangle + i\mathrm{e}^{-i\phi_1}|01\rangle - \mathrm{e}^{-i\phi_2}|10\rangle - i\mathrm{e}^{-i\phi_3}|11\rangle\right)\\
|\Gamma_2\rangle &= 1/2 \cdot \left(|00\rangle - \mathrm{e}^{-i\phi_1}|01\rangle + \mathrm{e}^{-i\phi_2}|10\rangle - \mathrm{e}^{-i\phi_3}|11\rangle\right)\\
|\Gamma_3\rangle &= 1/2 \cdot \left(|00\rangle - i\mathrm{e}^{-i\phi_1}|01\rangle - \mathrm{e}^{-i\phi_2}|10\rangle + i\mathrm{e}^{-i\phi_3}|11\rangle\right)
\end{aligned}$$

The maximally entangled two-qubit states between Alice with Bob can be shown as

$$\begin{aligned}
|\Psi\rangle_{1,2} &= \frac{\sqrt{2}}{2}(|00\rangle + |11\rangle)_{1,2}\\
|\Psi\rangle_{3,4} &= \frac{\sqrt{2}}{2}(|00\rangle + |11\rangle)_{3,4}
\end{aligned} \tag{8}$$

The whole system including particles (1, 2, 3, 4) can be expressed as

$$|\Psi\rangle_{1,2} \otimes |\Psi\rangle_{3,4} = \frac{1}{2}\sum_{i=0}^{2^n-1}\left(|\Gamma_i\rangle_{1,3} \otimes |\Phi_i\rangle_{2,4}\right) \tag{9}$$

where

$$\begin{aligned}
|\Phi_0\rangle &= 1/2 \cdot \left(|00\rangle + \mathrm{e}^{i\phi_1}|01\rangle + \mathrm{e}^{i\phi_2}|10\rangle + \mathrm{e}^{i\phi_3}|11\rangle\right)\\
|\Phi_1\rangle &= 1/2 \cdot \left(|00\rangle - i\mathrm{e}^{i\phi_1}|01\rangle - \mathrm{e}^{i\phi_2}|10\rangle + i\mathrm{e}^{i\phi_3}|11\rangle\right)\\
|\Phi_2\rangle &= 1/2 \cdot \left(|00\rangle - \mathrm{e}^{i\phi_1}|01\rangle + \mathrm{e}^{i\phi_2}|10\rangle - \mathrm{e}^{i\phi_3}|11\rangle\right)\\
|\Phi_3\rangle &= 1/2 \cdot \left(|00\rangle + i\mathrm{e}^{i\phi_1}|01\rangle - \mathrm{e}^{i\phi_2}|10\rangle - i\mathrm{e}^{i\phi_3}|11\rangle\right)
\end{aligned}$$

After the projective measurements $\{|\Gamma_m\rangle\} \mid (m = 0, 1 \ldots 3)$ on the qubit pair (1, 3), Alice informs Bob of her measurement results via classical communication. Then, Bob performs the following relevant unitary transformation U_m^2 on particles (2, 4) to reconstruct the original state.

$$U_m^2 = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi m} \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi m}{2}} \end{pmatrix} \tag{10}$$

Note that the successful probability for each kind of the measurement outcomes $\{|\Gamma_m\rangle \mid m = 0, 1, 2, 3\}$ is 1/4. Thus, the total probability for two-qubit equatorial states is equal to $1/4 \cdot 4 = 1$.

3.2 Three-qubit states

The three-qubit equatorial states can be presented as

$$\begin{aligned} |\psi\rangle = 1/2\sqrt{2} \cdot (&|000\rangle + e^{i\phi_1}|001\rangle + e^{i\phi_2}|010\rangle + e^{i\phi_3}|011\rangle + e^{i\phi_4}|100\rangle \\ &+ e^{i\phi_5}|101\rangle + e^{i\phi_6}|110\rangle + e^{i\phi_7}|111\rangle) \end{aligned} \tag{11}$$

The three maximally entangled two-qubit states shared previously between the sender Alice with the receiver Bob can be given by

$$|\Psi\rangle_{j,k} = \frac{\sqrt{2}}{2}(|00\rangle + |11\rangle)_{j,k} \quad j = 1, 3, 5; \quad k = j + 1. \tag{12}$$

Thus, the whole particles $(1,\ 2, \ldots, 6)$ could be rewritten as

$$|\Psi\rangle_{1,2} \otimes |\Psi\rangle_{3,4} \otimes |\Psi\rangle_{5,6} = \frac{1}{2\sqrt{2}} \sum_{i=1}^{8} \left(|\Gamma_i\rangle_{1,3,5} \otimes |\Phi_i\rangle_{2,4,6}\right) \tag{13}$$

where

$$\begin{aligned}
|\Gamma_0\rangle &= 1/2\sqrt{2} \cdot (|000\rangle + e^{-i\phi_1}|001\rangle + e^{-i\phi_2}|010\rangle + e^{-i\phi_3}|011\rangle \\
&\quad + e^{-i\phi_4}|100\rangle + e^{-i\phi_5}|101\rangle + e^{-i\phi_6}|110\rangle + e^{-i\phi_7}|111\rangle) \\
|\Gamma_1\rangle &= 1/2\sqrt{2} \cdot (|000\rangle + e^{i\frac{\pi}{4}}e^{-i\phi_1}|001\rangle + e^{i\frac{2\pi}{4}}e^{-i\phi_2}|010\rangle + e^{i\frac{3\pi}{4}}e^{-i\phi_3}|011\rangle \\
&\quad + e^{i\frac{4\pi}{4}}e^{-i\phi_4}|100\rangle + e^{i\frac{5\pi}{4}}e^{-i\phi_5}|101\rangle + e^{i\frac{6\pi}{4}}e^{-i\phi_6}|110\rangle + e^{i\frac{7\pi}{4}}e^{-i\phi_7}|111\rangle) \\
|\Gamma_2\rangle &= 1/2\sqrt{2} \cdot (|000\rangle + e^{i\frac{2\pi}{4}}e^{-i\phi_1}|001\rangle + e^{i\frac{4\pi}{4}}e^{-i\phi_2}|010\rangle + e^{i\frac{6\pi}{4}}e^{-i\phi_3}|011\rangle \\
&\quad + e^{i\frac{8\pi}{4}}e^{-i\phi_4}|100\rangle + e^{i\frac{10\pi}{4}}e^{-i\phi_5}|101\rangle + e^{i\frac{12\pi}{4}}e^{-i\phi_6}|110\rangle + e^{i\frac{14\pi}{4}}e^{-i\phi_7}|111\rangle) \\
|\Gamma_3\rangle &= 1/2\sqrt{2} \cdot (|000\rangle + e^{i\frac{3\pi}{4}}e^{-i\phi_1}|001\rangle + e^{i\frac{6\pi}{4}}e^{-i\phi_2}|010\rangle + e^{i\frac{9\pi}{4}}e^{-i\phi_3}|011\rangle \\
&\quad + e^{i\frac{12\pi}{4}}e^{-i\phi_4}|100\rangle + e^{i\frac{15\pi}{4}}e^{-i\phi_5}|101\rangle + e^{i\frac{18\pi}{4}}e^{-i\phi_6}|110\rangle + e^{i\frac{21\pi}{4}}e^{-i\phi_7}|111\rangle) \\
|\Gamma_4\rangle &= 1/2\sqrt{2} \cdot (|000\rangle + e^{i\frac{4\pi}{4}}e^{-i\phi_1}|001\rangle + e^{i\frac{8\pi}{4}}e^{-i\phi_2}|010\rangle + e^{i\frac{12\pi}{4}}e^{-i\phi_3}|011\rangle \\
&\quad + e^{i\frac{16\pi}{4}}e^{-i\phi_4}|100\rangle + e^{i\frac{20\pi}{4}}e^{-i\phi_5}|101\rangle + e^{i\frac{24\pi}{4}}e^{-i\phi_6}|110\rangle + e^{i\frac{28\pi}{4}}e^{-i\phi_7}|111\rangle) \\
|\Gamma_5\rangle &= 1/2\sqrt{2} \cdot (|000\rangle + e^{i\frac{5\pi}{4}}e^{-i\phi_1}|001\rangle + e^{i\frac{10\pi}{4}}e^{-i\phi_2}|010\rangle + e^{i\frac{15\pi}{4}}e^{-i\phi_3}|011\rangle \\
&\quad + e^{i\frac{20\pi}{4}}e^{-i\phi_4}|100\rangle + e^{i\frac{25\pi}{4}}e^{-i\phi_5}|101\rangle + e^{i\frac{30\pi}{4}}e^{-i\phi_6}|110\rangle + e^{i\frac{35\pi}{4}}e^{-i\phi_7}|111\rangle) \\
|\Gamma_6\rangle &= 1/2\sqrt{2} \cdot (|000\rangle + e^{i\frac{6\pi}{4}}e^{-i\phi_1}|001\rangle + e^{i\frac{12\pi}{4}}e^{-i\phi_2}|010\rangle + e^{i\frac{18\pi}{4}}e^{-i\phi_3}|011\rangle
\end{aligned}$$

$$
\begin{aligned}
&+\mathrm{e}^{i\frac{24\pi}{4}}\mathrm{e}^{-i\phi_4}|100\rangle+\mathrm{e}^{i\frac{30\pi}{4}}\mathrm{e}^{-i\phi_5}|101\rangle+\mathrm{e}^{i\frac{36\pi}{4}}\mathrm{e}^{-i\phi_6}|110\rangle+\mathrm{e}^{i\frac{42\pi}{4}}\mathrm{e}^{-i\phi_7}|111\rangle)\\
|\varGamma_7\rangle &= 1/2\sqrt{2}\cdot(|000\rangle+\mathrm{e}^{i\frac{7\pi}{4}}\mathrm{e}^{-i\phi_1}|001\rangle+\mathrm{e}^{i\frac{14\pi}{4}}\mathrm{e}^{-i\phi_2}|010\rangle+\mathrm{e}^{i\frac{21\pi}{4}}\mathrm{e}^{-i\phi_3}|011\rangle\\
&+\mathrm{e}^{i\frac{28\pi}{4}}\mathrm{e}^{-i\phi_4}|100\rangle+\mathrm{e}^{i\frac{35\pi}{4}}\mathrm{e}^{-i\phi_5}|101\rangle+\mathrm{e}^{i\frac{42\pi}{4}}\mathrm{e}^{-i\phi_6}|110\rangle+\mathrm{e}^{i\frac{49\pi}{4}}\mathrm{e}^{-i\phi_7}|111\rangle)\\
|\varPhi_0\rangle &= 1/2\sqrt{2}\cdot(|000\rangle+\mathrm{e}^{i\phi_1}|001\rangle+\mathrm{e}^{i\phi_2}|010\rangle+\mathrm{e}^{i\phi_3}|011\rangle\\
&+\mathrm{e}^{i\phi_4}|100\rangle+\mathrm{e}^{i\phi_5}|101\rangle+\mathrm{e}^{i\phi_6}|110\rangle+\mathrm{e}^{i\phi_7}|111\rangle)\\
|\varPhi_1\rangle &= 1/2\sqrt{2}\cdot(|000\rangle+\mathrm{e}^{i\frac{-\pi}{4}}\mathrm{e}^{i\phi_1}|001\rangle+\mathrm{e}^{-i\frac{2\pi}{4}}\mathrm{e}^{i\phi_2}|010\rangle+\mathrm{e}^{-i\frac{3\pi}{4}}\mathrm{e}^{i\phi_3}|011\rangle\\
&+\mathrm{e}^{-i\frac{4\pi}{4}}\mathrm{e}^{i\phi_4}|100\rangle+\mathrm{e}^{i\frac{-5\pi}{4}}\mathrm{e}^{i\phi_5}|101\rangle+\mathrm{e}^{-i\frac{6\pi}{4}}\mathrm{e}^{i\phi_6}|110\rangle+\mathrm{e}^{-i\frac{7\pi}{4}}\mathrm{e}^{i\phi_7}|111\rangle)\\
|\varPhi_2\rangle &= 1/2\sqrt{2}\cdot(|000\rangle+\mathrm{e}^{i\frac{-2\pi}{4}}\mathrm{e}^{i\phi_1}|001\rangle+\mathrm{e}^{-i\frac{4\pi}{4}}\mathrm{e}^{i\phi_2}|010\rangle+\mathrm{e}^{-i\frac{6\pi}{4}}\mathrm{e}^{i\phi_3}|011\rangle\\
&+\mathrm{e}^{-i\frac{8\pi}{4}}\mathrm{e}^{i\phi_4}|100\rangle+\mathrm{e}^{i\frac{-10\pi}{4}}\mathrm{e}^{i\phi_5}|101\rangle+\mathrm{e}^{-i\frac{12\pi}{4}}\mathrm{e}^{i\phi_6}|110\rangle+\mathrm{e}^{-i\frac{14\pi}{4}}\mathrm{e}^{i\phi_7}|111\rangle)\\
|\varPhi_3\rangle &= 1/2\sqrt{2}\cdot(|000\rangle+\mathrm{e}^{i\frac{-3\pi}{4}}\mathrm{e}^{i\phi_1}|001\rangle+\mathrm{e}^{-i\frac{6\pi}{4}}\mathrm{e}^{i\phi_2}|010\rangle+\mathrm{e}^{-i\frac{9\pi}{4}}\mathrm{e}^{i\phi_3}|011\rangle\\
&+\mathrm{e}^{-i\frac{12\pi}{4}}\mathrm{e}^{i\phi_4}|100\rangle+\mathrm{e}^{i\frac{-15\pi}{4}}\mathrm{e}^{i\phi_5}|101\rangle+\mathrm{e}^{-i\frac{18\pi}{4}}\mathrm{e}^{i\phi_6}|110\rangle+\mathrm{e}^{-i\frac{21\pi}{4}}\mathrm{e}^{i\phi_7}|111\rangle)\\
|\varPhi_4\rangle &= 1/2\sqrt{2}\cdot(|000\rangle+\mathrm{e}^{i\frac{-4\pi}{4}}\mathrm{e}^{i\phi_1}|001\rangle+\mathrm{e}^{-i\frac{8\pi}{4}}\mathrm{e}^{i\phi_2}|010\rangle+\mathrm{e}^{-i\frac{12\pi}{4}}\mathrm{e}^{i\phi_3}|011\rangle\\
&+\mathrm{e}^{-i\frac{16\pi}{4}}\mathrm{e}^{i\phi_4}|100\rangle+\mathrm{e}^{i\frac{-20\pi}{4}}\mathrm{e}^{i\phi_5}|101\rangle+\mathrm{e}^{-i\frac{24\pi}{4}}\mathrm{e}^{i\phi_6}|110\rangle+\mathrm{e}^{-i\frac{28\pi}{4}}\mathrm{e}^{i\phi_7}|111\rangle)\\
|\varPhi_5\rangle &= 1/2\sqrt{2}\cdot(|000\rangle+\mathrm{e}^{i\frac{-5\pi}{4}}\mathrm{e}^{i\phi_1}|001\rangle+\mathrm{e}^{-i\frac{10\pi}{4}}\mathrm{e}^{i\phi_2}|010\rangle+\mathrm{e}^{-i\frac{15\pi}{4}}\mathrm{e}^{i\phi_3}|011\rangle\\
&+\mathrm{e}^{-i\frac{20\pi}{4}}\mathrm{e}^{i\phi_4}|100\rangle+\mathrm{e}^{i\frac{-25\pi}{4}}\mathrm{e}^{i\phi_5}|101\rangle+\mathrm{e}^{-i\frac{30\pi}{4}}\mathrm{e}^{i\phi_6}|110\rangle+\mathrm{e}^{-i\frac{35\pi}{4}}\mathrm{e}^{i\phi_7}|111\rangle)\\
|\varPhi_6\rangle &= 1/2\sqrt{2}\cdot(|000\rangle+\mathrm{e}^{i\frac{-6\pi}{4}}\mathrm{e}^{i\phi_1}|001\rangle+\mathrm{e}^{-i\frac{12\pi}{4}}\mathrm{e}^{i\phi_2}|010\rangle+\mathrm{e}^{-i\frac{18\pi}{4}}\mathrm{e}^{i\phi_3}|011\rangle\\
&+\mathrm{e}^{-i\frac{24\pi}{4}}\mathrm{e}^{i\phi_4}|100\rangle+\mathrm{e}^{i\frac{-30\pi}{4}}\mathrm{e}^{i\phi_5}|101\rangle+\mathrm{e}^{-i\frac{36\pi}{4}}\mathrm{e}^{i\phi_6}|110\rangle+\mathrm{e}^{-i\frac{42\pi}{4}}\mathrm{e}^{i\phi_7}|111\rangle)\\
|\varPhi_7\rangle &= 1/2\sqrt{2}\cdot(|000\rangle+\mathrm{e}^{i\frac{-7\pi}{4}}\mathrm{e}^{i\phi_1}|001\rangle+\mathrm{e}^{-i\frac{14\pi}{4}}\mathrm{e}^{i\phi_2}|010\rangle+\mathrm{e}^{-i\frac{21\pi}{4}}\mathrm{e}^{i\phi_3}|011\rangle\\
&+\mathrm{e}^{-i\frac{28\pi}{4}}\mathrm{e}^{i\phi_4}|100\rangle+\mathrm{e}^{i\frac{-35\pi}{4}}\mathrm{e}^{i\phi_5}|101\rangle+\mathrm{e}^{-i\frac{42\pi}{4}}\mathrm{e}^{i\phi_6}|110\rangle+\mathrm{e}^{-i\frac{49\pi}{4}}\mathrm{e}^{i\phi_7}|111\rangle)
\end{aligned}
$$

From Eq. (13), it can be found that if the measurement outcome of Alice's particles $(1, 2, 3)$ is $|\varGamma_m\rangle$ $(m = 0, 1 \ldots 8)$, the particles $(2, 4, 6)$ would be transported into $|\varPhi_m\rangle$. In order to reconstruct the original state, Bob performs the relevant unitary gate U_m^3 on particles (2, 4, 6).

$$U_m^3 = \begin{pmatrix} 1 & 0 \\ 0 & \mathrm{e}^{i\pi m} \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & \mathrm{e}^{i\frac{\pi m}{2}} \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & \mathrm{e}^{i\frac{\pi m}{4}} \end{pmatrix} \tag{14}$$

It should be emphasized that the total successful probability of preparing three-qubit equatorial states is one.

4 Discussion and conclusions

In summary, we put forward an efficient scheme to prepare an arbitrary n-qubit equatorial state via n two-qubit maximally entangled states with the 100% successful probability. This is in contrast with the fact that the RSP scheme in Ref. [36] only has the 50% successful probability for equatorial states. The concrete measurement basis of the sender and the corresponding unitary operation performed by the receiver are presented in detail. The feasibility of our scheme in preparing remotely arbitrary multi-qubit equatorial states is proved by theoretical studies and concrete examples. From the point of potential applications of controlled teleportation, our scheme would be useful in the field of quantum network. Furthermore, even though the equatorial states include less quantum information than general states, some special advantages make them important for quantum computation and quantum communication, such as simplifying the remote preparation of unitary operations [37–39], reducing the required classical information of RSP [2,34], and improving the fidelity of quantum cloning [33,40]. Further research will focus on the usefulness of equatorial states on quantum information processing.

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