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# Remote preparation of an arbitrary multi-qubit state via two-qubit entangled states

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**Abstract** We propose a novel scheme for remote preparation of an arbitrary $n$-qubit state with the aid of an appropriate local $2^n \times 2^n$ unitary operation and $n$ maximally entangled two-qubit states. The analytical expression of local unitary operation, which is constructed in the form of iterative process, is presented for the preparation of $n$-qubit state in detail. We obtain the total successful probabilities of the scheme in the general and special cases, respectively. The feasibility of our scheme in preparing remotely multi-qubit states is explicitly demonstrated by theoretical studies and concrete examples, and our results show that the novel proposal could enlarge the applied range of remote state preparation.

**Keywords** Quantum information · Remote state preparation · Successful probability · Arbitrary multi-qubit states

## 1 Introduction

Remote state preparation (RSP), originally proposed by Lo [1], is the communication process that transmits quantum states from a sender to a remote receiver using a prior shared entanglement and some classical information. Compared with the usual

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teleportation [2–9], the sender in RSP knows completely the transmitted state to be prepared by the receiver, while in the teleportation neither the sender nor the receiver has knowledge of the transmitted state.

For the sake of that the RSP could be applied to quantum information, the RSP has acquired lots of attention recently [10–25]. Pati [26] demonstrated that the RSP is more economical than quantum teleportation and requires only one classical bit for special states chosen from equatorial or polar great circles on a Bloch sphere, but for general states, the RSP requires as much classical communication cost as quantum teleportation does. Dai et al. [27] presented a scheme for probabilistic remote preparation of an entangled two-qubit state with three parties from a sender to either of two receivers via one partially entangled state. Wei et al. [28] explored how to realize the RSP when the information of the partially entangled state is only available for the sender. Meantime, some RSP schemes have been implemented experimentally via nuclear magnetic resonance [29] and spontaneous parametric down-conversion [30]. Nevertheless, there are also many important and open subjects to be taken into account for the RSP. One of them is the preparation of an arbitrary multi-qubit states in the general and special cases.

The purpose of this paper is to give a new scheme to prepare an arbitrary $n$-qubit state by using of an appropriate local $2^n \times 2^n$ unitary operation, of which the solution procedure is presented in the form of iterative process. Quantum channel is composed of $n$ maximally entangled two-qubit states, which can be obtained experimentally with photons, electrons and so on [31–36]. The total successful probability of the RSP is considered as one of the most important parameters; we calculated the successful probabilities in the general and particular case, respectively. It is noting that the successful probability for a general $n$-qubit state is only equal to $2^{-n}$ via maximally entangled two-qubit states. Moreover, when the relative phase parameters of transmitted states are all zero, the successful probability would be improved to $2^{1-n}$, twice as much as the probability for general states. Besides, the successful probability would be equal to $1/2$ when the amplitudes of prepared states are $\sqrt{2^{-n}}$. The feasibility of the proposed scheme is proved by theoretical studies and concrete examples. With our proposal, one can prepare two-qubit states and three-qubit states in the general and particular cases, respectively. Our results show that the novel proposal could enlarge the applied range of remote state preparation.

The rest of this paper are organized as follows: In Sect. 2, a novel scheme for remote preparation of an arbitrary $n$-qubit state is presented via an appropriate local $2^n \times 2^n$ unitary operation, of which the solution process is given in detail. The total successful probability of the scheme is obtained, and it is equal to $2^{-n}$. In Sect. 3, we discuss how to realize the RSP of special states. If the factors of transmitted states are all real number, the successful probability is twice as much as the probability in the general case, and equals to $2^{1-n}$. Moreover, the successful probability would be improved to $1/2$ when the amplitude of transmitted states equal to $\sqrt{2^{-n}}$. In Sect. 4, concrete realization processes for preparing remotely two-qubit and three-qubit states in the general and special cases are illustrated to demonstrate explicitly the feasibility of our scheme. The paper concludes with Sect. 5.

## 2 RSP of general $n$-qubit states

Suppose that the sender Alice wants to help the receiver Bob remotely prepare the follow quantum state

$$
\begin{aligned}
|\psi\rangle &= c_1|0\ldots 00\rangle + c_2|0\ldots 01\rangle + \cdots + c_{2^n-1}|1\ldots 10\rangle + c_{2^n}|1\ldots 11\rangle \\
&= [c_1,\ c_2,\ \ldots\ c_{2^n-1},\ c_{2^n}] \cdot [|\Gamma\rangle]^{\mathrm{T}}
\end{aligned}
\tag{1}
$$

where the real number $c_1$ and complex ones $c_2, c_3, \ldots, c_{2^n}$ satisfy $\sum_{i=1}^{2^n} |c_i|^2 = 1$, and $[|\Gamma\rangle] = [|\Gamma_1\rangle \cdots |\Gamma_{2^n-1}\rangle,\ |\Gamma_{2^n}\rangle] = [|0\ldots 00\rangle \cdots |1\ldots 10\rangle,\ |1\ldots 11\rangle]$. If and only if the original state is not entangled, it could be presented as $|\psi\rangle_1 \otimes |\psi\rangle_2 \otimes \cdots \otimes |\psi\rangle_n$, here $|\psi\rangle_i$ $(i = 1, 2\ldots n)$ is the state of particle $i$. The unentangled states would be transmitted through the $n$-time processes of preparing an arbitrary single-qubit state [26]. Quantum channel is composed of $n$ maximally entangled two-qubit states below

$$
|\Psi\rangle_{jk} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{jk} \quad j = 1, 3, \ldots, 2n-1;\ k = j+1.
\tag{2}
$$

Without loss of generality, particle $j$ belongs to the sender Alice, while particle $k$ is hold by the receiver Bob. Alice need to construct a special $2^n \times 2^n$ unitary operation $U[\Theta_n^n]$ to realize the RSP. Actually, $U[\Theta_m^n]$ $(m = 1, 2, \ldots, n)$ takes the form of the following $2^m \times 2^m$ matrix

$$
U[\Theta_m^n] = \begin{cases}
\begin{pmatrix} c_{d+2}^* & -c_{d+1}^* \\ c_{d+1} & c_{d+2} \end{pmatrix} & m = 1; \\
\begin{pmatrix} U[\Theta_{m-1}^n]|_{d_m=0} & U[\Theta_{m-1}^n]|_{d_m=1} \\ \widetilde{\eta_m^d} U[\Theta_{m-1}^n]|_{d_m=0} & -\widetilde{\eta_m^d}^{-1} U[\Theta_{m-1}^n]|_{d_m=1} \end{pmatrix} & m \geq 2.
\end{cases}
\tag{3}
$$

here

$$
\Theta_m^n = (n;\ m;\ c_{d+1}, c_{d+2}, \ldots, c_{d+2^m};\ d_n \ldots d_2 d_1)
\tag{4}
$$

$$
d = 2^{n-1} \cdot d_n + 2^{n-2} \cdot d_{n-1} + \cdots + 2d_2
\tag{5}
$$

$$
\eta_m^d\ |_{d_m=0} = \frac{\sqrt{\sum_{i=1}^{2^{m-1}} |c_{d+i}|^2}\ \ |_{d_m=1}}{\sqrt{\sum_{i=1}^{2^{m-1}} |c_{d+i}|^2}\ \ |_{d_m=0}}
\tag{6}
$$

$$
\widetilde{\eta_m^d} = \mathbf{I}_{2^{m-1}} \cdot \eta_m^d\ |_{d_m=0}
\tag{7}
$$

It should be emphasized that the binary number $d_n \cdots d_2 d_1$ has initial state $d_n = d_{n-1} = \cdots = d_1 = 0$, and this number would be changed with the development of the iterative process, given by Eq. (3). Note that $U[\Theta_n^n]$ can be presented as

$$\left(\begin{array}{cccccccc} c_2^* & -c_1^* & c_4^* & -c_3^* & \cdots & c_{2^n}^* & -c_{2^n-1}^* \\ c_1 & c_2 & c_3 & c_4 & \cdots & c_{2^n-1} & c_{2^n} \\ \eta_2^0 c_2^* & -\eta_2^0 c_1^* & -(\eta_2^0)^{-1} c_4^* & (\eta_2^0)^{-1} c_3^* & \cdots & -(\eta_2^{2^n-4})^{-1} c_{2^n}^* & (\eta_2^{2^n-4})^{-1} c_{2^n-1}^* \\ \eta_2^0 c_1 & \eta_2^0 c_2 & -(\eta_2^0)^{-1} c_3 & -(\eta_2^0)^{-1} c_4 & \cdots & -(\eta_2^{2^n-4})^{-1} c_{2^n-1} & -(\eta_2^{2^n-4})^{-1} c_{2^n} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \prod_{i=2}^n \eta_i^0\ c_2^* & -\prod_{i=2}^n \eta_i^0\ c_1^* & -\prod_{i=3}^n \eta_i^0\ (\eta_2^0)^{-1} c_4^* & \prod_{i=3}^n \eta_i^0\ (\eta_2^0)^{-1} c_3^* & \cdots & \left(\prod_{i=2}^n -\eta_i^{2^n-2^i}\right)^{-1} c_{2^n}^* & -\left(\prod_{i=2}^n -\eta_i^{2^n-2^i}\right)^{-1} c_{2^n-1}^* \\ \prod_{i=2}^n \eta_i^0\ c_1 & \prod_{i=2}^n \eta_i^0\ c_2 & -\prod_{i=3}^n \eta_i^0\ (\eta_2^0)^{-1} c_3 & -\prod_{i=3}^n \eta_i^0\ (\eta_2^0)^{-1} c_4 & \cdots & \left(\prod_{i=2}^n -\eta_i^{2^n-2^i}\right)^{-1} c_{2^n-1} & \left(\prod_{i=2}^n -\eta_i^{2^n-2^i}\right)^{-1} c_{2^n} \end{array}\right) \tag{8}$$

In order to fulfill the RSP, Alice carries out the $n$-qubit projective measurement on her particles $(1, 3 \ldots 2n-1)$ in a set of mutually orthonormal basis vectors $\{|\lambda_1\rangle, |\lambda_2\rangle, \ldots, |\lambda_{2^n}\rangle\}$, which are given by

$$[|\lambda\rangle]^{\mathrm{T}} = [|\lambda_1\rangle, |\lambda_2\rangle \ldots |\lambda_{2^n}\rangle]^{\mathrm{T}} = U[\Theta_n^n]\,[|\Gamma\rangle]^{\mathrm{T}} \tag{9}$$

Meanwhile, the joint state of the whole system, formed by $n$ maximally entangled states, can be rewritten as

$$\begin{aligned} &|\Psi\rangle_{12} \otimes |\Psi\rangle_{34} \cdots |\Psi\rangle_{(2n-1)2n} \\ &\quad = \left(\frac{1}{\sqrt{2}}\right)^n \cdot \sum_{i=1}^{2^n} |\lambda_i\rangle_{13\cdots 2n-1} \cdot \left(\sum_{j=1}^{2^n} U^*[\Theta_n^n](i,j)|\Gamma_j\rangle_{24\cdots 2n}\right) \end{aligned} \tag{10}$$

The parameter $U^*[\Theta_n^n](i,j)$ presents the element in the $i$-th row and $j$-th column of the complex conjugate of the matrix $U[\Theta_n^n]$. After performing the measurement given by Eq. (9), Alice informs Bob of her measurement results via classical channel. If particles of Alice is $|\lambda_1\rangle$ with the probability of $2^{-n}$, particles $(2, 4 \ldots 2n)$ would be

$$c_2|\Gamma_1\rangle - c_1|\Gamma_2\rangle + c_4|\Gamma_3\rangle - c_3|\Gamma_4\rangle + \cdots + c_{2^n}|\Gamma_{2^n-1}\rangle - c_{2^n-1}|\Gamma_{2^n}\rangle \tag{11}$$

Hence, Bob performs $\sigma_x \cdot \sigma_z$ on particle $2n$, and then Bob's particles would be on the original state, given by Eq. (1). From Eq. (10), one can find that if Alice obtains the result $|\lambda_k\rangle$ $(k = 2, 3 \ldots 2^n)$, particles $(2, 4 \ldots 2n)$ will collapse into $\sum_{j=1}^{2^n} U^*[\Theta_n^n](i,j)|\Gamma_j\rangle_{24\cdots 2n}$. Because of that Bob has no information of this parameters $c_i^*$ and $\eta_n^d$, given by Eq. (8), Bob's particles can not be unitarily converted into the original state from $|\lambda_k\rangle$.

From the above discussions, one can find that the RSP could be realized successfully when the state of Alice's particles is $|\lambda_1\rangle$ with the probability of $2^{-n}$. The results are in agreement with the probabilities of Refs. [12,13,27,28] when quantum channel is a maximally entangled state.

## 3 RSP of special *n*-qubit states

In this sequel, the successful probability of the preparation for special $n$-qubit states, in which $c_i = c_i^*$ $(i = 2, \ldots, 2^n)$ or $|c_i| = \sqrt{2^{-n}}$, would be calculated based on the analyses in Sect. 2.

For the states shown as Eq. (10), it can be found that if Alice's measurement outcome is $|\lambda_2\rangle$ and $c_i$ are all real, i.e., $|c_i| = c_i^*$, particles $(2, 4, \ldots, 2n)$ should be prepared remotely to

$$c_1|\Gamma_1\rangle + c_2^*|\Gamma_2\rangle + \cdots + c_{2^n}^*|\Gamma_{2^n}\rangle = c_1|\Gamma_1\rangle + c_2|\Gamma_2\rangle + \cdots + c_{2^n}|\Gamma_{2^n}\rangle$$

The above state is just equal to the original multi-qubit state without any unitary operations. Because of that $|\lambda_1\rangle$ and $|\lambda_2\rangle$ are both advisable for the RSP when $c_i = c_i^*$, the successful probability of RSP would be twice as much as the one of the general case, and equals $2^{1-n}$.

On the other hands, if $|c_j| = \sqrt{2^{-n}}$, the parameters given by Eq. (6) would always be $\eta_m^d = 1$, and the unitary transformation $U[\Theta_n^n]$ could be reduced to

$$U[\Theta_n^n]|_{|c_j|=\sqrt{2^{-n}}} = \begin{pmatrix} c_2^* & -c_1 & c_4^* & -c_3^* & \cdots & c_{2^n}^* & -c_{2^n-1}^* \\ c_1 & c_2 & c_3 & c_4 & \cdots & c_{2^n-1} & c_{2^n} \\ c_2^* & -c_1 & -c_4^* & c_3^* & \cdots & -c_{2^n}^* & c_{2^n-1}^* \\ c_1 & c_2 & -c_3 & -c_4 & \cdots & -c_{2^n-1} & -c_{2^n} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ c_2^* & -c_1 & -c_4^* & c_3^* & \cdots & (-1)^{n-1}c_{2^n}^* & (-1)^n c_{2^n-1}^* \\ c_1 & c_2 & -c_3 & -c_4 & \cdots & (-1)^{n-1}c_{2^n-1} & (-1)^{n-1}c_{2^n} \end{pmatrix}$$

From Eq. (10), one can find that

$$|\lambda_{2l-1}\rangle_{13\cdots 2n-1} \quad \rightarrow \quad U^*[\Theta_n^n]|_{|c_j|=\sqrt{2^{-n}}}(2l-1, j)|\Gamma_j\rangle_{24\cdots 2n} \tag{12}$$

Furthermore, for $l \in \{\mathbf{Z}^+ \mid 2l - 1 \le 2^n\}$, the elements in the $(2l-1)$-th row of $U^*[\Theta_n^n]|_{|c_{2l-1}|=\sqrt{2^{-n}}}$ would belong to $\{c_j\}$. Hence, the RSP could be realized successfully when the measurement result of Alice's particles belongs to $\{|\lambda_{2l-1}\rangle\}$ with the probability of $1/2$.

## 4 Examples of RSP

To illustrate the above scheme explicitly, we will demonstrate how to prepare two-qubit states and three-qubit states, which are elementary and important resources for quantum information.

### 4.1 Two-qubit states

Suppose that one wants to prepare the follow two-qubit state

$$|\psi\rangle = c_1|00\rangle + c_2|01\rangle + c_3|10\rangle + c_4|11\rangle \tag{13}$$

From Eq. (6), one can find that $U[\Theta_2^2]$ can be expressed as

$$U[\Theta_2^2] = \begin{pmatrix} c_2^* & -c_1^* & c_4^* & -c_3^* \\ c_1 & c_2 & c_3 & c_4 \\ \eta_2^0 c_2^* & -\eta_2^0 c_1 & -(\eta_2^0)^{-1} c_4^* & (\eta_2^0)^{-1} c_3^* \\ \eta_2^0 c_1 & \eta_2^0 c_2 & -(\eta_2^0)^{-1} c_3 & -(\eta_2^0)^{-1} c_4 \end{pmatrix} \tag{14}$$

Quantum channel is composed of two maximally entangled states below

$$|\Psi\rangle_{12} = \frac{\sqrt{2}}{2}(|00\rangle + |11\rangle)_{12} \qquad |\Psi\rangle_{34} = \frac{\sqrt{2}}{2}(|00\rangle + |11\rangle)_{34} \tag{15}$$

The whole particles can be rewritten as

$$|\Psi\rangle_{12} \otimes |\Psi\rangle_{34} = \frac{1}{2}\sum_{i=1}^{4}(|\lambda_i\rangle_{13}|\Phi_i\rangle_{24}) \tag{16}$$

where

$$|\lambda_1\rangle = c_2^*|00\rangle - c_1|01\rangle + c_4^*|10\rangle - c_3^*|11\rangle \tag{17}$$
$$|\lambda_2\rangle = c_1|00\rangle + c_2|01\rangle + c_3|10\rangle + c_4|11\rangle \tag{18}$$
$$|\lambda_3\rangle = \eta_2^0 c_2^*|00\rangle - \eta_2^0 c_1|01\rangle - (\eta_2^0)^{-1} c_4^*|10\rangle + (\eta_2^0)^{-1} c_3^*|11\rangle \tag{19}$$
$$|\lambda_4\rangle = \eta_2^0 c_1|00\rangle + \eta_2^0 c_2|01\rangle - (\eta_2^0)^{-1} c_3|10\rangle + (\eta_2^0)^{-1} c_4 11\rangle \tag{20}$$
$$|\Phi_1\rangle = c_2|00\rangle - c_1|01\rangle + c_4|10\rangle - c_3|11\rangle \tag{21}$$
$$|\Phi_2\rangle = c_1|00\rangle + c_2^*|01\rangle + c_3^*|10\rangle + c_4^*|11\rangle \tag{22}$$
$$|\Phi_3\rangle = \eta_2^0 c_2|00\rangle - \eta_2^0 c_1|01\rangle - (\eta_2^0)^{-1} c_4|10\rangle + (\eta_2^0)^{-1} c_3|11\rangle \tag{23}$$
$$|\Phi_4\rangle = \eta_2^0 c_1|00\rangle + \eta_2^0 c_2^*|01\rangle - (\eta_2^0)^{-1} c_3^*|10\rangle - (\eta_2^0)^{-1} c_4^*|11\rangle \tag{24}$$
$$\eta_2^0 = \sqrt{(|c_3|^2 + |c_4|^2)/(|c_1|^2 + |c_2|^2)} \tag{25}$$

After the two-qubit projective measurements $\{|\lambda_i\rangle \mid i = 1, 2, 3, 4\}$ on the qubit pair $(1,\ 3)$, the sender Alice informs the receiver Bob of her measurement results via classical channel. As we see, if the measurement result is $|\lambda_1\rangle$, the particles of Bob would be $|\Phi_1\rangle$ in Eq. (21). Hence, by performing $\sigma_x \cdot \sigma_z$ on particles 4, Bob can reconstruct the original state on particles $(2,\ 4)$. Note that the successful probability of RSP is $1/4$.

Furthermore, if $c_j$ $(j = 2, 3, 4)$ are real numbers, and Alice's measurement outcome is $|\lambda_2\rangle$, particles $(2, 4)$ would be on the original state, shown as Eq. (1). The RSP can be realized successfully for the measurement results $\{|\lambda_1\rangle, |\lambda_2\rangle\}$, and then the whole probability would be improved to $1/2$. By the way, when $|c_j| = 1/2$ $(j = 1, 2, \ldots, n)$, the parameter $\eta_2^0$ is equal to 1, and $|\Phi_3\rangle_{24}$ shown as Eq. (23) would be rewritten as

$$|\Phi_3\rangle_{24} = (c_2|00\rangle - c_1|01\rangle - c_4|10\rangle + c_3|11\rangle)_{24}$$

Hence, Bob can reconstruct the original state on particles $(2,\ 4)$ by performing $\sigma_z$ on particle 2 and $\sigma_x \cdot \sigma_z$ on particle 4, respectively. Note that the successful probability of RSP in this case is $1/2$.

### 4.2 Three-qubit states

Suppose that the receiver Bob wants to prepare the follow three-qubit state

$$|\psi\rangle = [c_1,\ c_2,\ c_3,\ c_4,\ c_5,\ c_6,\ c_7,\ c_8]\,[|\Gamma\rangle]^{\mathrm{T}}$$

where $[|\Gamma\rangle] = [|000\rangle,\ |001\rangle,\ |010\rangle,\ |011\rangle,\ |100\rangle,\ |101\rangle,\ |110\rangle,\ |111\rangle]$. Note that $U[\Theta_3^3]$ can be described by

$$U[\Theta_3^3] = \begin{pmatrix} c_2^* & -c_1 & c_4^* & -c_3^* & c_6^* & -c_5^* & c_8^* & -c_7^* \\ c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8 \\ \eta_2^0 c_2^* & -\eta_2^0 c_1 & -(\eta_2^0)^{-1} c_4^* & (\eta_2^0)^{-1} c_3^* & \eta_2^2 c_6^* & -\eta_2^2 c_5^* & -(\eta_2^2)^{-1} c_8^* & (\eta_2^2)^{-1} c_7^* \\ \eta_2^0 c_1 & \eta_2^0 c_2 & -(\eta_2^0)^{-1} c_3 & -(\eta_2^0)^{-1} c_4 & \eta_2^2 c_5 & \eta_2^2 c_6 & -(\eta_2^2)^{-1} c_7 & -(\eta_2^2)^{-1} c_8 \\ \eta_3^0 c_2^* & -\eta_3^0 c_1 & \eta_3^0 c_4^* & -\eta_3^0 c_3^* & -(\eta_3^0)^{-1} c_6^* & (\eta_3^0)^{-1} c_5^* & -(\eta_3^0)^{-1} c_8^* & (\eta_3^0)^{-1} c_7^* \\ \eta_3^0 c_1 & \eta_3^0 c_2 & \eta_3^0 c_3 & \eta_3^0 c_4 & -(\eta_3^0)^{-1} c_5 & -(\eta_3^0)^{-1} c_6 & -(\eta_3^0)^{-1} c_7 & -(\eta_3^0)^{-1} c_8 \\ \eta_3^0\eta_2^0 c_2^* & -\eta_3^0\eta_2^0 c_1 & -\eta_3^0(\eta_2^0)^{-1} c_4^* & \eta_3^0(\eta_2^0)^{-1} c_3^* & -(\eta_3^0)^{-1}\eta_2^2 c_6^* & (\eta_3^0)^{-1}\eta_2^2 c_5^* & (\eta_3^0\eta_2^2)^{-1} c_8^* & -(\eta_3^0\eta_2^2)^{-1} c_7^* \\ \eta_3^0\eta_2^0 c_1 & \eta_3^0\eta_2^0 c_2 & -\eta_3^0(\eta_2^0)^{-1} c_3 & -\eta_3^0(\eta_2^0)^{-1} c_4 & -(\eta_3^0)^{-1}\eta_2^2 c_5 & -(\eta_3^0)^{-1}\eta_2^2 c_6 & (\eta_3^0\eta_2^2)^{-1} c_7 & (\eta_3^0\eta_2^2)^{-1} c_8 \end{pmatrix} \tag{26}$$

Quantum channel is composed of three maximally entangled states below

$$|\Psi\rangle_{jk} = \frac{\sqrt{2}}{2}(|00\rangle + |11\rangle)_{jk} \quad j = 1, 3, 5; \quad k = j + 1. \tag{27}$$

Thus, the whole particles could be rewritten as

$$|\Psi\rangle_{12} \otimes |\Psi\rangle_{34} \otimes |\Psi\rangle_{56} = \frac{1}{2\sqrt{2}} \sum_{i=1}^{8} (|\lambda_i\rangle_{135}|\Phi_i\rangle_{246}) \tag{28}$$

where

$$|\lambda_1\rangle = \left[c_2^*,\ -c_1,\ c_4^*,\ -c_3^*,\ c_6^*,\ -c_5^*,\ c_8^*,\ -c_7^*\right] \cdot [|\Gamma\rangle]^{\mathrm{T}} \tag{29}$$

$$|\lambda_2\rangle = [c_1,\ c_2,\ c_3,\ c_4,\ c_5,\ c_6,\ c_7,\ c_8] \cdot [|\Gamma\rangle]^{\mathrm{T}} \tag{30}$$

$$\begin{aligned}|\lambda_3\rangle = \Big[&\eta_2^0 c_2^*,\ -\eta_2^0 c_1,\ -(\eta_2^2)^{-1} c_4^*,\ (\eta_2^2)^{-1} c_3^*,\\ &-\eta_2^2 c_6^*,\ \eta_2^2 c_5^*,\ -(\eta_2^2)^{-1} c_8^*,\ (\eta_2^2)^{-1} c_7^*\Big] \cdot [|\Gamma\rangle]^{\mathrm{T}}\end{aligned} \tag{31}$$

$$\begin{aligned}|\lambda_4\rangle = \Big[&\eta_2^0 c_1,\ -\eta_2^0 c_2,\ -(\eta_2^2)^{-1} c_3,\ (\eta_2^2)^{-1} c_4,\\ &-\eta_2^2 c_5,\ \eta_2^2 c_6,\ -(\eta_2^2)^{-1} c_7,\ (\eta_2^2)^{-1} c_8\Big] \cdot [|\Gamma\rangle]^{\mathrm{T}}\end{aligned} \tag{32}$$

$$\begin{aligned}|\lambda_5\rangle = \Big[&\eta_3^0 c_2^*,\ -\eta_3^0 c_1,\ \eta_3^0 c_4^*,\ -\eta_3^0 c_3^*,\\ &-(\eta_3^0)^{-1} c_6^*,\ (\eta_3^0)^{-1} c_5^*,\ -(\eta_3^0)^{-1} c_8^*,\ (\eta_3^0)^{-1} c_7^*\Big] \cdot [|\Gamma\rangle]^{\mathrm{T}}\end{aligned} \tag{33}$$

$$\begin{aligned}|\lambda_6\rangle = \Big[&\eta_3^0 c_1,\ -\eta_3^0 c_2,\ \eta_3^0 c_3,\ -\eta_3^0 c_4,\\ &-(\eta_3^0)^{-1} c_5,\ -(\eta_3^0)^{-1} c_6,\ -(\eta_3^0)^{-1} c_7,\ -(\eta_3^0)^{-1} c_8\Big] \cdot [|\Gamma\rangle]^{\mathrm{T}}\end{aligned} \tag{34}$$

$$\begin{aligned}|\lambda_7\rangle = \Big[&\eta_3^0\eta_2^0 c_2^*,\ -\eta_3^0\eta_2^0 c_1,\ -\eta_3^0(\eta_2^2)^{-1} c_4^*,\ \eta_3^0(\eta_2^2)^{-1} c_3^*,\\ &-(\eta_3^0)^{-1}\eta_2^2 c_6^*,\ (\eta_3^0)^{-1}\eta_2^2 c_5^*,\ (\eta_3^0\eta_2^2)^{-1} c_8^*,\ -(\eta_3^0\eta_2^2)^{-1} c_7^*\Big] \cdot [|\Gamma\rangle]^{\mathrm{T}}\end{aligned} \tag{35}$$

$$\begin{aligned}|\lambda_8\rangle = \Big[&\eta_3^0\eta_2^0 c_1,\ -\eta_3^0\eta_2^0 c_2,\ -\eta_3^0(\eta_2^2)^{-1} c_3,\ \eta_3^0(\eta_2^2)^{-1} c_4,\\ &(\eta_3^0)^{-1}\eta_2^2 c_5,\ -(\eta_3^0)^{-1}\eta_2^2 c_6,\ (\eta_3^0\eta_2^2)^{-1} c_7,\ (\eta_3^0\eta_2^2)^{-1} c_8\Big] \cdot [|\Gamma\rangle]^{\mathrm{T}}\end{aligned} \tag{36}$$

$$|\Phi_1\rangle = [c_2,\ -c_1,\ c_4,\ -c_3,\ c_6,\ -c_5,\ c_8,\ -c_7] \cdot [|\Gamma\rangle]^{\mathrm{T}} \tag{37}$$

$$|\Phi_2\rangle = \left[c_1,\ c_2^*,\ c_3^*,\ c_4^*,\ c_5^*,\ c_6^*,\ c_7^*,\ c_8^*\right] \cdot [|\Gamma\rangle]^{\mathrm{T}} \tag{38}$$

$$\begin{aligned}|\Phi_3\rangle = \Big[&\eta_2^0 c_2,\ -\eta_2^0 c_1,\ -(\eta_2^2)^{-1} c_4,\ (\eta_2^2)^{-1} c_3,\\ &-\eta_2^2 c_6,\ \eta_2^2 c_5,\ -(\eta_2^2)^{-1} c_8,\ (\eta_2^2)^{-1} c_7\Big] \cdot [|\Gamma\rangle]^{\mathrm{T}}\end{aligned} \tag{39}$$

$$|\Phi_4\rangle = \Big[\eta_2^0 c_1,\ -\eta_2^0 c_2^*,\ -(\eta_2^2)^{-1} c_3^*,\ (\eta_2^2)^{-1} c_4^*,\\ -\eta_2^2 c_5^*,\ \eta_2^2 c_6^*,\ -(\eta_2^2)^{-1} c_7^*,\ (\eta_2^2)^{-1} c_8^*\Big] \cdot [|\Gamma\rangle]^{\mathrm{T}} \tag{40}$$

$$|\Phi_5\rangle = \Big[\eta_3^0 c_2,\ -\eta_3^0 c_1,\ \eta_3^0 c_4,\ -\eta_3^0 c_3,\\ -(\eta_3^0)^{-1} c_6,\ (\eta_3^0)^{-1} c_5,\ -(\eta_3^0)^{-1} c_8,\ (\eta_3^0)^{-1} c_7\Big] \cdot [|\Gamma\rangle]^{\mathrm{T}} \tag{41}$$

$$|\Phi_6\rangle = \Big[\eta_3^0 c_1,\ -\eta_3^0 c_2^*,\ \eta_3^0 c_3^*,\ -\eta_3^0 c_4^*,\\ -(\eta_3^0)^{-1} c_5^*,\ -(\eta_3^0)^{-1} c_6^*,\ -(\eta_3^0)^{-1} c_7^*,\ -(\eta_3^0)^{-1} c_8^*\Big] \cdot [|\Gamma\rangle]^{\mathrm{T}} \tag{42}$$

$$|\Phi_7\rangle = \Big[\eta_3^0\eta_2^0 c_2,\ -\eta_3^0\eta_2^0 c_1,\ -\eta_3^0(\eta_2^2)^{-1} c_4,\ \eta_3^0(\eta_2^2)^{-1} c_3,\\ -(\eta_3^0)^{-1}\eta_2^2 c_6,\ (\eta_3^0)^{-1}\eta_2^2 c_5,\ (\eta_3^0\eta_2^2)^{-1} c_8,\ -(\eta_3^0\eta_2^2)^{-1} c_7\Big] \cdot [|\Gamma\rangle]^{\mathrm{T}} \tag{43}$$

$$|\Phi_8\rangle = \Big[\eta_3^0\eta_2^0 c_1,\ -\eta_3^0\eta_2^0 c_2^*,\ -\eta_3^0(\eta_2^2)^{-1} c_3^*,\ \eta_3^0(\eta_2^2)^{-1} c_4^*,\\ (\eta_3^0)^{-1}\eta_2^2 c_5^*,\ -(\eta_3^0)^{-1}\eta_2^2 c_6^*,\ (\eta_3^0\eta_2^2)^{-1} c_7^*,\ (\eta_3^0\eta_2^2)^{-1} c_8^*\Big] \cdot [|\Gamma\rangle]^{\mathrm{T}} \tag{44}$$

$$\eta_2^0 = \sqrt{(|c_3|^2 + |c_4|^2)/(|c_1|^2 + |c_2|^2)} \tag{45}$$

$$\eta_2^2 = \sqrt{(|c_7|^2 + |c_8|^2)/(|c_5|^2 + |c_6|^2)} \tag{46}$$

$$\eta_3^0 = \sqrt{(\Sigma_{j=5}^{8}|c_j|^2)/(\Sigma_{i=1}^{4}|c_i|^2)} \tag{47}$$

For arbitrary three-qubit states, when the measurement result is $|\lambda_1\rangle_{135}$, Bob can construct the original state from $|\Phi_1\rangle_{246}$ by performing $\sigma_x \cdot \sigma_z$ on particle 6. The successful probability of RSP in the general case is $1/8$.

Furthermore, we would like to point out that one can convert $|\Phi_2\rangle_{246}$ to the original state when $c_j$ ($j = 2, 3, \ldots, 8$) are real numbers. The RSP can be realized successfully for the measurement results $\{|\lambda_1\rangle, |\lambda_2\rangle\}$, and then the successful probability would be improved to $1/4$. Others, if $|c_j| = \frac{1}{2\sqrt{2}}$ ($j = 1, 2, \ldots, 8$), i.e., $\eta_2^0 = \eta_2^2 = \eta_3^0 = 1$, Bob can reconstruct $|\Phi_k\rangle$ ($k = 1, 3, 5, 7$) to the original state on particles (2, 4, 6). The total successful probability of RSP is $1/2$. For example, if the measurement result is $|\lambda_7\rangle_{135}$,

$$|\Phi_7\rangle = [c_2,\ -c_1,\ -c_4,\ c_3,\ -c_6,\ c_5,\ c_8,\ -c_7] \cdot [|\Gamma\rangle]^{\mathrm{T}}$$

the original state can be reconstructed from $|\Phi_7\rangle$ on particles (2, 4, 6) by performing $\sigma_z$ on particle (2, 4) and $\sigma_x \cdot \sigma_z$ on particle 6, respectively.

Actually, the real-parameter states ($c_i = c_i^*,\ i = 2, \ldots, 2^n$) and the equatorial states ($|c_i| = \sqrt{2^{-n}}$) have some special properties that make them interesting for quantum information processing [37–40]. Since they contain less information compared to general states, it should be easier to prepare special states than arbitrary states. Jiang and Dong [37] presented a two-phase general protocol for deterministic remote preparation of the real-parameter states. Moreover, Li and Ghose [38] presented a scheme for optimal joint RSP of $n$-qubit equatorial states with the 100% success probability. It is noting that one can use the similar procedures of these two proposals to perform the RSP of special states in a deterministic manner, even though these proposals are not available for preparing general states, which could be realized by using of our scheme. Hence, the method in this paper and the former two proposals complement each other.

## 5 Discussions and conclusions

In summary, we put forward an efficient scheme to prepare an arbitrary $n$-qubit state with the aid of $n$ maximally entangled two-qubit states and appropriate local $2^n \times 2^n$ unitary operation, which is described by the form of iterative process. The total successful probabilities of the RSP in the general and particular cases are calculated, respectively. The feasibility of this proposal is proved by theoretical studies and concrete examples. In contrast to the preceding schemes [10–25], our method has the following advantages. First, this scheme can in principle be applied to prepare any arbitrary multi-qubit states including the real-parameter states and the equatorial states in a reliable way. Second, quantum channel for our proposal is composed of $n$ maximally two-qubit entangled states. The creation for entangled two-qubit states has been widely discussed and realized; it is relatively easier for us to obtain this resource. Third, this method can be generalized to the controlled RSP scheme by substituting the GHZ state for one of maximally two-qubit entangled states. In this case, only one qubit would be added to decide whether the RSP is successful or not. Forth, our method can be simply changed to the RSP protocol with $n$ remote receivers when particles $(2, 4, \ldots, 2n)$ belong to each receiver, respectively. Thus, the original state could be prepared in the particles of separate receivers. This makes our iterative protocol promising for wide applications. From the point of the potential applications of the RSP, we believe that our scheme will play an important role in expanding the field of quantum information processing.

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