

Practical single-photon-assisted remote state preparation with non-maximally entanglement

Dong Wang^{1,2} · Ai-Jun Huang¹ ·
Wen-Yang Sun¹ · Jia-Dong Shi¹ · Liu Ye¹

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Abstract Remote state preparation (RSP) and joint remote state preparation (JRSP) protocols for single-photon states are investigated via linear optical elements with partially entangled states. In our scheme, by choosing two-mode instances from a polarizing beam splitter, only the sender in the communication protocol needs to prepare an ancillary single-photon and operate the entanglement preparation process in order to retrieve an arbitrary single-photon state from a photon pair in partially entangled state. In the case of JRSP, i.e., a canonical model of RSP with multi-party, we consider that the information of the desired state is split into many subsets and in prior maintained by spatially separate parties. Specifically, with the assistance of a single-photon state and a three-photon entangled state, it turns out that an arbitrary single-photon state can be jointly and remotely prepared with certain probability, which is characterized by the coefficients of both the employed entangled state and the target state. Remarkably, our protocol is readily to extend to the case for RSP and JRSP of mixed states with the all optical means. Therefore, our protocol is promising for communicating among optics-based multi-node quantum networks.

Keywords Remote state preparation · Joint remote state preparation · Partially entangled states

✉ Dong Wang
dwang@ahu.edu.cn
Liu Ye
yeliu@ahu.edu.cn

¹ School of Physics and Material Science, Anhui University, Hefei 230601, China

² National Laboratory for Infrared Physics, Shanghai Institute of Technical Physics, Chinese Academy of Sciences, Shanghai 200083, China

1 Introduction

Quantum entanglement is at the heart of quantum information theory. It combines three basic structural elements of quantum theory: superposition principle, quantum non-separability property, and exponential scaling of the state space with the number of partitions [1]. This unique resource, termed as particular non-classical correlations among separated quantum systems, can be used for efficiently achieving many tasks, including quantum secret sharing [2, 3], quantum dense coding [4], and quantum entanglement concentration [5–7], quantum computation [8–11], which are impossible in classical physics.

Quantum communication is concerned with the transmission, manipulation, and detection of quantum information, of which, quantum teleportation is used for transmitting an unknown quantum state from a sender (Alice) to a spatially separate receiver (Bob) with the aid of local operations and classical communication (LOCC). Notably, there also exists another situation that for an arbitrary target state, each sender knows partially of the classical information of the target state. In this case, it naturally arises an interesting question that whether it is possible to reconstruct the wanted state in a spatially separate site. Surprisingly, this can be achieved by the so-called remote state preparation (RSP) [12–14] and joint remote state preparation (JRSP).

The basic protocol of RSP involves two parties during the quantum communication process, say, Alice and Bob, which relies on correlations between two entangled qubits to similarly prepare Bob's qubit in a particular state determined by Alice, conditioned on the result of a measurement on her qubit. This means RSP transmits a known state within LOCC, requiring that they succeed not exactly but only with high fidelity. However, unlike teleportation, RSP does not require the sender to perform full Bell-state analysis, which is currently experimental challenging for optical implementations. Meanwhile, it has been shown that classical communication cost for transmitting a known state via RSP is less than that of teleportation [14]. On the other hand, JRSP is designed for manipulation of quantum information in quantum communication, namely retrieval of the target state in a quantum networks between multi-sender and one receiver. In this case, the transmitted information is generally split into many subsets, and each sender owns one of the pieces. To recover the target state in receiver's place, this protocol requires all senders to collaborate with each other. In this sense, the information security is essentially guaranteed to a large extent. Therefore, it is of practical significance to concentrate on RSP and JRSP on the demand for long-distance quantum communication and information processing.

Up to now, many theoretical investigations have been put forward for RSP [15–29] and JRSP [30–39]. For examples, optimal RSP [15], generalized RSP [16], faithful RSP [17], oblivious RSP [18, 19], RSP without oblivious conditions [20], and RSP for qubit states [21–29] have been explored. Remarkably, several RSP demonstrations have been reported by controlling over various degrees of the remotely prepared qubits [40–43]. To be explicit, Peng et al. investigated a RSP scheme by using NMR [40], Xiang et al. [41] and Peters et al. [42] proposed two RSP schemes with spontaneous parametric down-conversion. Julio et al. [43] reported the remote preparation of two-qubit hybrid entangled states, including a family of vector-polarization beams where single-photon states are encoded in the photon spin and orbital angular momentum,

and then the desired state is reconstructed by means of spin-orbit state tomography and transverse polarization tomography. Recently, Liu et al. [44] demonstrated a novel implementation for arbitrary single-qubit states using Bell states as quantum resource with linear optical system. Nevertheless, after the transmission of entanglement through a practical channel with noise and storage into entangled systems, the employed Bell state, used as the entangled resource, will become a mixed entangled state or a partially entangled pure state. Furthermore, in comparison with the maximally entangled state, a partially entangled state is more general and relatively practical to be realized in the laboratory. Therefore, we explore single-photon-assistance RSP and JRSP protocols of an arbitrary single-photon state with partially entangled states as quantum channels. In particular, we also derive the success probability and classical communication cost (CCC) for the two communication protocols.

2 The RSP scheme

In this section, we will demonstrate how to remotely prepare an arbitrary single-photon pure state $|P\rangle = \alpha|H\rangle + \beta e^{i\theta}|V\rangle$ from a partially entangled biphotonic state with an ancillary single photon and linear optical elements, where H (V) denotes the horizontal (vertical) photon polarization, the real-valued α and β satisfy the normalized condition, and $\theta \in [0, 2\pi)$. Assuming that the system of two photons A and B is in the following polarization entangled state

$$|\phi_{in}\rangle_{AB} = a|H_1\rangle_A|V_2\rangle_B + b|V_1\rangle_A|H_2\rangle_B, \tag{1}$$

where a and b are complex numbers and satisfy the normalized condition $|a|^2 + |b|^2 = 1$, the subscripts 1 and 2 denote spatial modes of photon transmission, and A and B represent a pair of photons shared by two remote parties in quantum communication, say Alice and Bob. To be explicit, Alice is the sender of the desired state, and Bob is the receiver. Provided that photon A is at the Bob's laboratory, while photon B at Alice's location.

In order to remotely prepare each single-photon system in the desired state, Alice prepares a single photon C , the initial state of which is

$$|\phi\rangle_C = a|H_3\rangle_C + b|V_3\rangle_C. \tag{2}$$

Thus, the state of the composite system ABC is expressed as

$$|\Phi\rangle_{ABC} = (a|H_1\rangle_A|V_2\rangle_B + b|V_1\rangle_A|H_2\rangle_B) \otimes (a|H_3\rangle_C + b|V_3\rangle_C). \tag{3}$$

Alice can pick up the items with the same parameter ab with linear optical elements such as a polarizing beam splitter (PBS) and a post-selection. It is found that the a photon pair B and C have identical polarization states when the two items have the same parameter ab ; otherwise, they are in two different polarization states. The experimental setup of generating entanglement for employing one pure partially entangled state as quantum channel is shown in Fig. 1. Alice lets his two photons B and C enter into

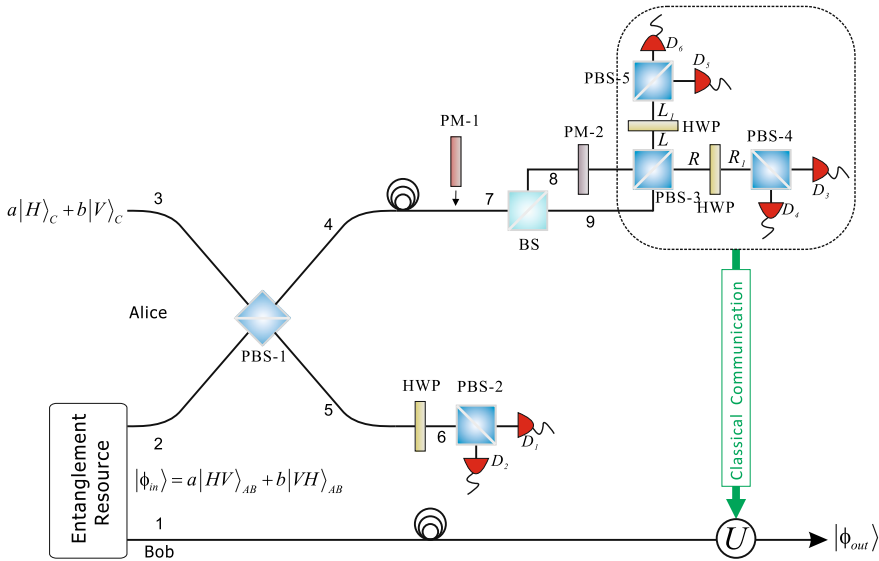


Fig. 1 The setup for implementing RSP protocol of a single-photon state. PBS denotes polarizing beam splitter which transmits the horizontal component and reflect the vertical component, BS denotes beam splitter which is with the ratio of the transmission and reflection α/β , HWP denotes half-wave plate with included angle between its major axis and the horizontal direction $\eta = 22.5^\circ$, PM denotes phase modulator, and D_i denotes the i -th photon detector

PBS-1. When the two photons have the identical polarization states $|H\rangle_B|H\rangle_C$ or $|V\rangle_B|V\rangle_C$, they emit from the two outputs of the PBS. That is, each of the spatial modes of the PBS has one and only one photon in this time. Usually, one can call it as the two-mode instance. The polarization state $|H\rangle_B|V\rangle_C$ will lead to the fact that the two photons emit from the same spatial mode, i.e., the upper mode, while the state $|V\rangle_B|H\rangle_C$ will lead the two photons to emit from the down-spatial mode. That is, when the two photons emit from different spatial modes, Alice can pick up the state

$$|\phi^1\rangle_{ABC} = ab(|H_1\rangle_A|V_4\rangle_B|V_5\rangle_C + |V_1\rangle_A|H_4\rangle_B|H_5\rangle_C) \tag{4}$$

with occurrence probability of $2|ab|^2$.

The element HWP in Fig. 1 represents a half-wave plate which operation is used to accomplish the transformation as follows

$$\begin{aligned} |H\rangle &\mapsto \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle), \\ |V\rangle &\mapsto \frac{1}{\sqrt{2}}(|H\rangle - |V\rangle). \end{aligned} \tag{5}$$

After passing through the first HWP, the state of the composite system becomes

$$|\phi^2\rangle_{ABC} = \frac{1}{2} [|H_1\rangle_A|V_4\rangle_B(|H_6\rangle_C - |V_6\rangle_C) + |V_1\rangle_A|H_4\rangle_B(|H_6\rangle_C + |V_6\rangle_C)]$$

$$\begin{aligned}
 &= \frac{1}{2} [|H_6\rangle_C (|H_1\rangle_A |V_4\rangle_B + |V_1\rangle_A |H_4\rangle_B) \\
 &\quad + |V_6\rangle_C (|V_1\rangle_A |H_4\rangle_B - |H_1\rangle_A |V_4\rangle_B)]. \tag{6}
 \end{aligned}$$

If the photon detector D_1 is triggered, that is, the state $|H\rangle_C$ is probed by Alice, the subsystem AB will collapse into the standard Bell state $|\phi^+\rangle_{AB} = \frac{1}{\sqrt{2}}(|H_1\rangle_A |V_4\rangle_B + |V_1\rangle_A |H_4\rangle_B)$; Otherwise, the photon is detected in D_2 , this implies $|V\rangle_C$ is attained, the remaining photons will collapse into the standard Bell state $\frac{1}{\sqrt{2}}(|H_1\rangle_A |V_4\rangle_B - |V_1\rangle_A |H_4\rangle_B)$, which can be converted into $|\phi^+\rangle_{AB}$ by inserting a phase modulator PM-1 plate with transformations $|H_4\rangle \rightarrow -|H_7\rangle$ and $|V_4\rangle \rightarrow |V_7\rangle$.

To achieve the state with prescribed coefficients α and β , a polarization-independent beam splitter (BS-1) is used, the ratio of which transmission and reflection is equal to α/β . After passing the BS, the state of $|\phi^+\rangle_{AB}$ will evolve into

$$|\phi^3\rangle_{AB} = \frac{1}{\sqrt{2}} [|H_1\rangle_A (\alpha |V_9\rangle_B + \beta |V_8\rangle_B) + |V_1\rangle_A (\alpha |H_9\rangle_B + \beta |H_8\rangle_B)]. \tag{7}$$

Incidentally, PM-2 is a phase modulator which is used to bring on a relative phase shift $e^{i\theta}$ to the state, namely the state of

$$|\phi^4\rangle_{AB} = \frac{1}{\sqrt{2}} [|H_1\rangle_A (\alpha |V_9\rangle_B + \beta e^{i\theta} |V_8\rangle_B) + |V_1\rangle_A (\alpha |H_9\rangle_B + \beta e^{i\theta} |H_8\rangle_B)] \tag{8}$$

will be obtained as PM-2 is applied to the path 8.

Next, let the two photons pass the third PBS, and thus the systemic state is given by

$$|\phi^5\rangle_{AB} = \frac{1}{\sqrt{2}} [|H_1\rangle_A (\alpha |V_R\rangle_B + \beta e^{i\theta} |V_L\rangle_B) + |V_1\rangle_A (\alpha |H_L\rangle_B + \beta e^{i\theta} |H_R\rangle_B)]. \tag{9}$$

Subsequently, photons A and B go through a HWP, respectively. This leads to the state of

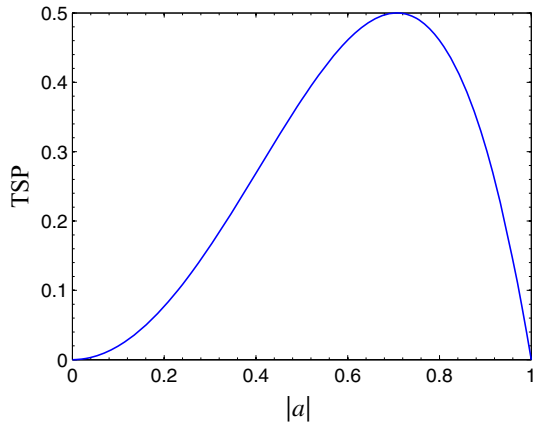
$$\begin{aligned}
 |\phi^6\rangle_{AB} &= \frac{1}{2} \left\{ |H_1\rangle_A \left[\alpha (|H_{R_1}\rangle_B - |V_{R_1}\rangle_B) + \beta e^{i\theta} (|H_{L_1}\rangle_B - |V_{L_1}\rangle_B) \right] \right. \\
 &\quad \left. + |V_1\rangle_A \left[\alpha (|H_{L_1}\rangle_B + |V_{L_1}\rangle_B) + \beta e^{i\theta} (|H_{R_1}\rangle_B + |V_{R_1}\rangle_B) \right] \right\} \\
 &= \frac{1}{2} \left[|H_{R_1}\rangle_B (\alpha |H_1\rangle_A + \beta e^{i\theta} |V_1\rangle_A) + |V_{R_1}\rangle_B (-\alpha |H_1\rangle_A + \beta e^{i\theta} |V_1\rangle_A) \right. \\
 &\quad \left. + |H_{L_1}\rangle_B (\alpha |V_1\rangle_A + \beta e^{i\theta} |H_1\rangle_A) + |V_{L_1}\rangle_B (\alpha |V_1\rangle_A - \beta e^{i\theta} |H_1\rangle_A) \right]. \tag{10}
 \end{aligned}$$

From Eq. (10), one can see that if the detector D_3 is triggered, Alice realizes his photon is the state of $|H\rangle_B$ in the path R_1 , and then she informs Bob of this outcome via classical bits “00,” and Bob realizes that the output state $|\phi_{out}\rangle$ of his photon has been the target state $|P\rangle$. If any one of other detectors is triggered, Alice informs Bob of this

Table 1 OAD represents outcome of Alice detector, CCC represents classical communication cost, BCS represents Bob’s photon collapsed state and U represents appropriate unitary operations performed by Alice on her photon A

OAD	CCC	BCS	U
$ H_{R_1}\rangle_B$	00	$\alpha H\rangle_A + \beta e^{i\theta} V\rangle_A$	$ H\rangle\langle H + V\rangle\langle V $
$ V_{R_1}\rangle_B$	01	$-\alpha H\rangle_A + \beta e^{i\theta} V\rangle_A$	$ V\rangle\langle V - H\rangle\langle H $
$ H_{L_1}\rangle_B$	10	$\alpha V\rangle_A + \beta e^{i\theta} H\rangle_A$	$ V\rangle\langle H + H\rangle\langle V $
$ V_{L_1}\rangle_B$	11	$\alpha V\rangle_A - \beta e^{i\theta} H\rangle_A$	$ H\rangle\langle V - V\rangle\langle H $

Fig. 2 The total success probability (TSP) is plotted as a function of the coefficient $|a|$ from the employed channel



outcome via classical channels, Bob realizes that his photon has been the target state as long as operating an appropriate unitary transformation U , which has been explicitly shown in Table 1. The total success probability (TSP) of the current scheme is $2|ab|^2$ associated with the employed entanglement as shown in Fig. 2, and CCC is 2 cbits.

3 The JRSP scheme

At this stage, we turn to present a feasible JRSP scheme for remotely preparing a single-photon pure state $|P\rangle = \alpha|H\rangle + \beta e^{i\theta}|V\rangle$ from a partially entangled GHZ state among multipartite agents. Assume that $\alpha = \alpha_1 \cdot \alpha_2$, $\beta = \beta_1 \cdot \beta_2$ and $\theta = \theta_1 + \theta_2$, and Alice has knowledge of α_1 , β_1 and θ_1 , while Bob knows the remaining information of the state. In this case, there is a system of three-photon ABC with a general polarization entangled state

$$|\varphi_{in}\rangle_{ABC} = a|H_1\rangle_A|H_2\rangle_B|H_3\rangle_C + b|V_1\rangle_A|V_2\rangle_B|V_3\rangle_C, \tag{11}$$

where the complex numbers a and b satisfy the normalized condition $|a|^2 + |b|^2 = 1$, and the subscripts 1, 2 and 3 represent spatial transmission modes of photons, and A , B and C represent the three photons shared by three remote parties in quantum communication, say Alice, Bob and Charlie. To be explicit, Alice and Bob are the

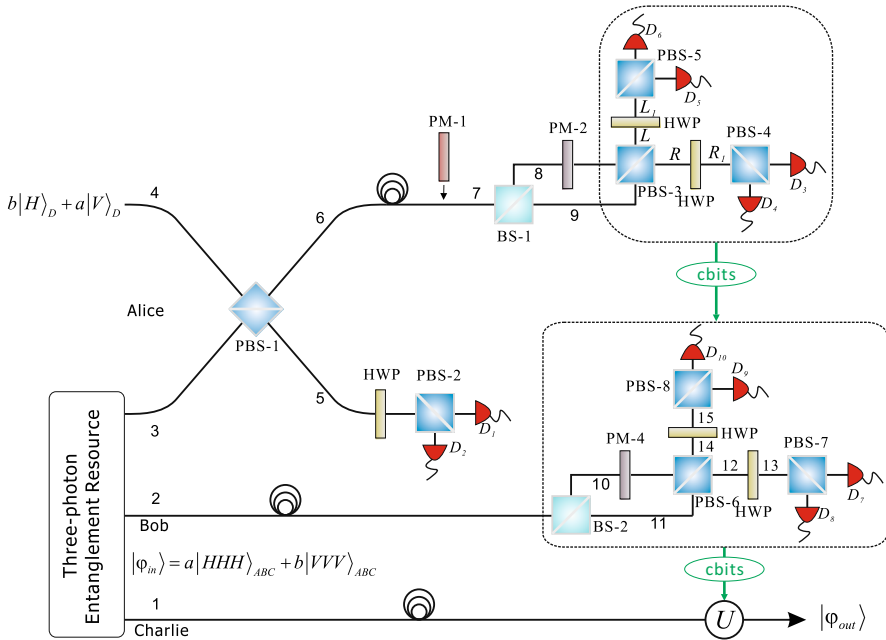


Fig. 3 The setup for implementing JRSP protocol of a single-photon polarization state. PBS denotes polarizing beam splitter which transmits the horizontal component and reflects the vertical component, BS-1 denotes a beam splitter which is with the ratio of the transmission and reflection α_1/β_1 , BS-2 denotes a beam splitter which is with the ratio of the transmission and reflection α_2/β_2 , HWP denotes half-wave plate with $\eta = 22.5^\circ$, PM denotes phase modulator, D_i denotes the i -th photon detector, and U denotes an unitary operation on Bob's photon A

senders of the wanted state, Charlie is the receiver, and photon A , B , and C belong to Charlie, Bob and Alice, respectively.

In order to remotely prepare each single-photon system in the desired state, Alice needs to prepare a single photon D in a state of

$$|\varphi\rangle_D = b|H_4\rangle_D + a|V_4\rangle_D. \tag{12}$$

Thus, the state of the composite system of $ABCD$ will be

$$|\Psi\rangle_{ABCD} = (a|H_1\rangle_A|H_2\rangle_B|H_3\rangle_C + b|V_1\rangle_A|V_2\rangle_A|V_3\rangle_C) \otimes (b|H_4\rangle_D + a|V_4\rangle_D). \tag{13}$$

Alice can pick up the items with the same parameter ab with linear optical elements, e.g., a polarizing beam splitter (PBS) and a post-selection. It is not difficult to find that the two photons C and D have identical polarization states when the two items have the same parameter ab ; otherwise, they are in two different polarization states. The experimental setup of jointly generating entanglement for arbitrary pure single-photon states is shown in Fig. 3. Alice lets his two photons C and D enter into PBS-1. Alice chooses the two outputs of the PBS via two-mode instance. That is, when C and D emit from different spatial modes, Alice can pick up

$$|\varphi^1\rangle_{ABCD} = ab(|H_1\rangle_A|H_2\rangle_B|H_6\rangle_C|H_5\rangle_D + |V_1\rangle_A|V_2\rangle_B|V_6\rangle_C|V_5\rangle_D). \quad (14)$$

with probability of $2|ab|^2$.

Through a HWP, the state of the composite system evolves to

$$\begin{aligned} |\varphi^2\rangle_{ABCD} &= \frac{1}{\sqrt{2}} [(|H_1\rangle_A|H_2\rangle_B|H_6\rangle_C(|H_5\rangle_D + |V_5\rangle_D) \\ &\quad + |V_1\rangle_A|V_2\rangle_B|V_6\rangle_C(|H_5\rangle_D - |V_5\rangle_D)] \\ &= \frac{1}{\sqrt{2}} [(|H_5\rangle_D(|H_1\rangle_A|H_2\rangle_B|H_6\rangle_C + |V_1\rangle_A|V_2\rangle_B|V_6\rangle_C) \\ &\quad + |V_5\rangle_D(|H_1\rangle_A|H_2\rangle_B|H_6\rangle_C - |V_1\rangle_A|V_2\rangle_B|V_6\rangle_C)]. \end{aligned} \quad (15)$$

When the photon detector D_1 is triggered, i.e., Alice obtains the state $|H\rangle_D$, the system ABC will collapse into a three-photon Greenberger–Horne–Zeilinger state $|\varphi^+\rangle_{ABC} = \frac{1}{\sqrt{2}}(|H_1\rangle_A|H_2\rangle_B|H_6\rangle_C + |V_1\rangle_A|V_2\rangle_B|V_6\rangle_C)$. Otherwise, the photon is detected in D_2 , this implies $|V\rangle_D$ is attained, the subsystem ABC will collapse into the state $\frac{1}{\sqrt{2}}(|H_1\rangle_A|H_2\rangle_B|H_6\rangle_C - |V_1\rangle_A|V_2\rangle_B|V_6\rangle_C)$, which can be converted into $|\varphi^+\rangle_{ABC}$ by inserting a phase modulator (PM-1) plate with transformations $|H_6\rangle \rightarrow |H_7\rangle$ and $|V_6\rangle \rightarrow -|V_7\rangle$.

To get α_1 and β_1 , Alice incorporates a polarization-independent beam splitter (BS-1) with the ratio of transmission and reflection coefficients equals to α_1/β_1 . Meanwhile, PM-2 is added in path 9, which is a phase modulator with changing a relative phase $e^{i\theta_1}$ to the state. After going through the BS-1 and PM-2, the state of $|\varphi^+\rangle_{ABC}$ will evolve into

$$\begin{aligned} |\varphi^3\rangle_{ABC} &= \frac{1}{\sqrt{2(\alpha_1^2 + \beta_1^2)}} \left[|H_1\rangle_A|H_2\rangle_B(\alpha_1|H_9\rangle_C \right. \\ &\quad \left. + \beta_1 e^{i\theta_1}|H_8\rangle_C) + |V_1\rangle_A|V_2\rangle_B(\alpha_1|V_9\rangle_C + \beta_1 e^{i\theta_1}|V_8\rangle_C) \right]. \end{aligned} \quad (16)$$

To achieve JRSP, let the photon C pass PBS-3, the whole state is now reduced to

$$\begin{aligned} |\varphi^4\rangle_{ABC} &= \frac{1}{\sqrt{2(\alpha_1^2 + \beta_1^2)}} \left[|H_1\rangle_A|H_2\rangle_B(\alpha_1|H_L\rangle_C \right. \\ &\quad \left. + \beta_1 e^{i\theta_1}|H_R\rangle_C) + |V_1\rangle_A|V_2\rangle_B(\alpha_1|V_R\rangle_C + \beta_1 e^{i\theta_1}|V_L\rangle_C) \right]. \end{aligned} \quad (17)$$

Then, photons B and C will pass a HWP, which leads the above reduced state to

$$\begin{aligned} |\varphi^5\rangle_{ABC} &= \frac{1}{2\sqrt{\alpha_1^2 + \beta_1^2}} \left\{ |H_1\rangle_A|H_2\rangle_B \left[\alpha_1(|H_{L_1}\rangle_C + |V_{L_1}\rangle_C) \right. \right. \\ &\quad \left. \left. + \beta_1 e^{i\theta_1}(|H_{R_1}\rangle_C + |V_{R_1}\rangle_C) \right] + |V_1\rangle_A|V_2\rangle_B \left[\alpha_1(|H_{R_1}\rangle_C - |V_{R_1}\rangle_C) \right. \right. \\ &\quad \left. \left. + \beta_1 e^{i\theta_1}(|H_{L_1}\rangle_C - |V_{L_1}\rangle_C) \right] \right\} \end{aligned}$$

$$\begin{aligned}
 & + \beta_1 e^{i\theta_1} (|H_{L_1}\rangle_C - |V_{L_1}\rangle_C) \Big] \Big\} \\
 = & \frac{1}{2\sqrt{\alpha_1^2 + \beta_1^2}} \Big[|H_{L_1}\rangle_C (\alpha_1 |H_1\rangle_A |H_2\rangle_B \\
 & + \beta_1 e^{i\theta_1} |V_1\rangle_A |V_2\rangle_B) + |V_{L_1}\rangle_C (\alpha_1 |H_1\rangle_A |H_2\rangle_B \\
 & - \beta_1 e^{i\theta_1} |V_1\rangle_A |V_2\rangle_B) + |H_{R_1}\rangle_C (\beta_1 e^{i\theta_1} |H_1\rangle_A |H_2\rangle_B \\
 & + \alpha_1 |V_1\rangle_A |V_2\rangle_B) + |V_{R_1}\rangle_C (\beta_1 e^{i\theta_1} |H_1\rangle_A |H_2\rangle_B - \alpha_1 |V_1\rangle_A |V_2\rangle_B) \Big]. \tag{18}
 \end{aligned}$$

Without losing the generality, suppose that the detector D_3 is triggered, Alice realizes his photon is the state of $|H_{R_1}\rangle_C$, and then she informs Bob of this outcome via classical bits “10,” and Bob realizes that the output state of his photon has been the state of $\frac{1}{\sqrt{\alpha_1^2 + \beta_1^2}} (\alpha_1 |V_1\rangle_A |V_2\rangle_B + \beta_1 e^{i\theta_1} |H_1\rangle_A |H_2\rangle_B)$. After that, Bob lets his photon pass a beam splitter (BS-2) with the ratio of transmission and reflection to be α_2/β_2 and a phase modulator (PM-4) with altering a phase of θ_2 , the state of AB will become

$$\begin{aligned}
 |\varphi^6\rangle_{AB} = & \frac{1}{\sqrt{(\alpha_1^2 + \beta_1^2)(\alpha_2^2 + \beta_2^2)}} \Big[\alpha_1 |V_1\rangle_A (\alpha_2 |V_{11}\rangle_B \\
 & + \beta_2 e^{i\theta_2} |V_{10}\rangle_B) + \beta_1 e^{i\theta_1} |H_1\rangle_A (\alpha_2 |H_{11}\rangle_B + \beta_2 e^{i\theta_2} |H_{10}\rangle_B) \Big]. \tag{19}
 \end{aligned}$$

Then, it will lead to

$$\begin{aligned}
 |\varphi^7\rangle_{AB} = & \frac{1}{\sqrt{(\alpha_1^2 + \beta_1^2)(\alpha_2^2 + \beta_2^2)}} \Big[\alpha_1 |V_1\rangle_A (\alpha_2 |V_{12}\rangle_B \\
 & + \beta_2 e^{i\theta_2} |V_{14}\rangle_B) + \beta_1 e^{i\theta_1} |H_1\rangle_A (\alpha_2 |H_{14}\rangle_B + \beta_2 e^{i\theta_2} |H_{12}\rangle_B) \Big] \tag{20}
 \end{aligned}$$

on the basis of PBS-6. Subsequently, two HWPs in paths 12 and 14 will transfer $|\varphi^7\rangle_{AB}$ into

$$\begin{aligned}
 & \frac{1}{\sqrt{2(\alpha_1^2 + \beta_1^2)(\alpha_2^2 + \beta_2^2)}} \Big\{ \Big[\alpha_1 |V_1\rangle_A [\alpha_2 (|H_{13}\rangle_B - |V_{13}\rangle_B) \\
 & + \beta_2 e^{i\theta_2} (|H_{15}\rangle_B - |V_{15}\rangle_B)] + \beta_1 e^{i\theta_1} |H_1\rangle_A [\alpha_2 (|H_{15}\rangle_B \\
 & + |V_{15}\rangle_B) + \beta_2 e^{i\theta_2} (|H_{13}\rangle_B + |V_{13}\rangle_B) \Big] \Big\} \\
 = & \frac{1}{\sqrt{2(\alpha_1^2 + \beta_1^2)(\alpha_2^2 + \beta_2^2)}} \Big[|H_{13}\rangle_B (\alpha_1 \alpha_2 |V_1\rangle_A \\
 & + \beta_1 \beta_2 e^{i(\theta_1 + \theta_2)} |H_1\rangle_A) + |V_{13}\rangle_B (-\alpha_1 \alpha_2 |V_1\rangle_A \\
 & + \beta_1 \beta_2 e^{i(\theta_1 + \theta_2)} |H_1\rangle_A) + |H_{15}\rangle_B (\alpha_1 \beta_2 e^{i\theta_2} |V_1\rangle_A \\
 & + \beta_1 \alpha_2 e^{i\theta_1} |H_1\rangle_A) + |V_{15}\rangle_B (-\alpha_1 \beta_2 e^{i\theta_2} |V_1\rangle_A + \beta_1 \alpha_2 e^{i\theta_1} |H_1\rangle_A) \Big]. \tag{21}
 \end{aligned}$$

Table 2 The symbol “√” represents the triggered detector, the symbol “×” represents the untriggered detector and U represents appropriate unitary recovering operations Charlie needs to make on his photon A

D_3	D_4	D_5	D_6	D_7	D_8	D_9	D_{10}	U
√	×	×	×	×	×	×	√	$ H\rangle\langle H + V\rangle\langle V $
√	×	×	×	×	×	√	×	$ H\rangle\langle H - V\rangle\langle V $
×	√	×	×	×	×	×	√	$ H\rangle\langle H - V\rangle\langle V $
×	√	×	×	×	×	√	×	$ H\rangle\langle H + V\rangle\langle V $
×	×	√	×	√	×	×	×	$ H\rangle\langle V + V\rangle\langle H $
×	×	√	×	×	√	×	×	$ V\rangle\langle H - H\rangle\langle V $
×	×	×	√	√	×	×	×	$ V\rangle\langle H - H\rangle\langle V $
×	×	×	√	×	√	×	×	$ H\rangle\langle V + V\rangle\langle H $

According to Eq. (21), one can see that the target state has been recovered by Charlie with the operation of σ_x or $\sigma_x\sigma_z$ on photon A , when Bob’s photon detector D_7 or D_8 is triggered. In this case, the success probability should be $2|ab|^2 \times \frac{1}{4} \times 2 \times \left(\frac{1}{\sqrt{2(\alpha_1^2 + \beta_1^2)(\alpha_2^2 + \beta_2^2)}}\right)^2 = \frac{|ab|^2}{2(\alpha_1^2 + \beta_1^2)(\alpha_2^2 + \beta_2^2)}$.

In addition, Alice’s outcome may be one of other three outcomes $|H_{L_1}\rangle_C$, $|V_{L_1}\rangle_C$ and $|V_{R_1}\rangle_C$. If so, the target state can be recovered in Charlie’s photon A by making the corresponding transformation on his photon. For clearness, possible positive outcomes of the senders’ detectors and Charlie’s required operations have been shown in Table 2, respectively. Therefore, the total success probability is $\frac{2|ab|^2}{(\alpha_1^2 + \beta_1^2)(\alpha_2^2 + \beta_2^2)}$ as shown in Figs. 4 and 5. From the figures, one can conclude that: (1) TSP increases with the increase of α_1/β_1 when $\alpha = 0$, i.e., the target state is reduced into $|V\rangle$. (2) TSP will reach an extremum when $\alpha_1/\beta_1 = 1$ and $\alpha \neq 0$. (3) TSP is less than 0.5 all the time. (4) TSP is 25%, which is independent of the prepared state, if $\alpha_1/\beta_1 = 1$ and $|a| = 1/\sqrt{2}$. Meanwhile, according to the required communication among Alice, Bob and Charlie, one can get that CCC is up to 4 cbits.

4 Extensions

4.1 RSP for a mixed state

In this subsection, we briefly derive RSP protocol for a mixed state with the current setup as displayed in Fig. 1. To prepare a mixed state, PBS-4, D_3 and D_4 are replaced by a polarization analyzer (called as PA-1) and PBS-5, D_5 and D_6 are replaced by another polarization analyzer (PA-2), and thus photon B is measured in a polarization-insensitive way. By this, one can prepare photon A in the mixed state. In terms of detection of photon B on path L or R , the mixed state is

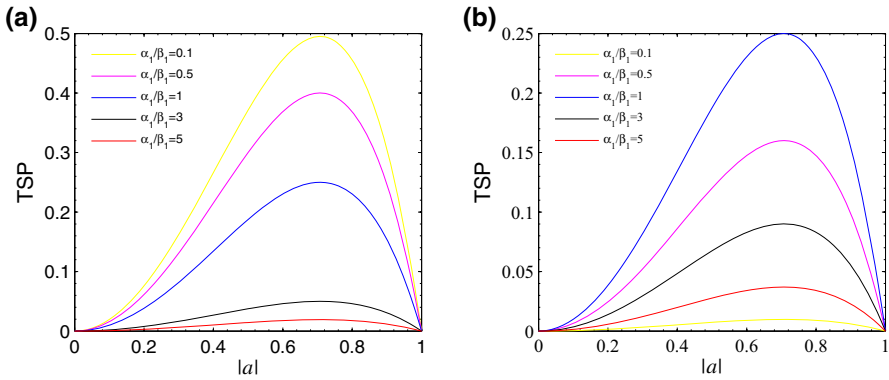


Fig. 4 The TSP is plotted as a function of coefficient $|a|$ with different ensembles of target states. Graph **a**: $\alpha = 0$ and Graph **b**: $\alpha = 1/\sqrt{2}$

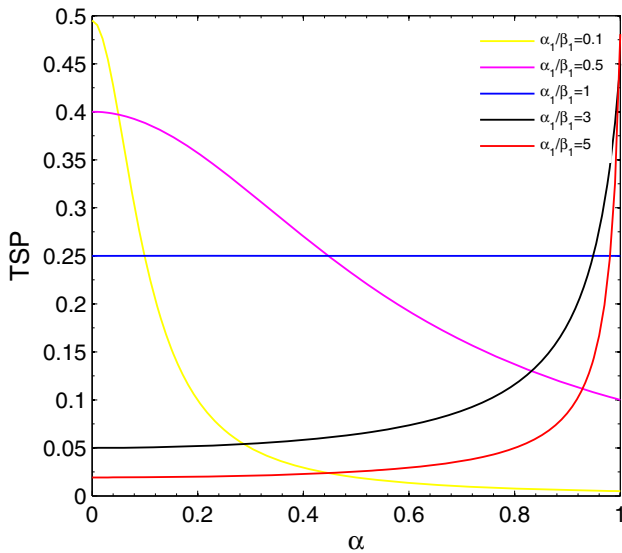


Fig. 5 The TSP is plotted as a function of coefficient α with different values of α_1/β_1 when $|a| = 1/\sqrt{2}$

$$\rho_A^L = \text{Tr}_B \left[(|H_L\rangle_B \langle H_L| + |V_L\rangle_B \langle V_L|) \rho_{AB} \right] = \alpha^2 |V_1\rangle_A \langle V_1| + \beta^2 |H_1\rangle_A \langle H_1| \quad (22)$$

or

$$\rho_A^R = \text{Tr}_B \left[(|H_R\rangle_B \langle H_R| + |V_R\rangle_B \langle V_R|) \rho_{AB} \right] = \alpha^2 |H_1\rangle_A \langle H_1| + \beta^2 |V_1\rangle_A \langle V_1| \quad (23)$$

where $\rho_{AB} = |\phi^5\rangle_{AB} \langle \phi^5|$.

4.2 JRSP for a mixed state

For a mixed state, our JRSP setup shown as Fig. 2 is available as well. Here, we simply illustrate the JRSP protocol for a mixed state with the current setup. To prepare mixed states, PBS-7, D_7 and D_8 are replaced by a polarization analyzer and PBS-8, D_9 and D_{10} are replaced by another polarization analyzer. In this way, photon B is measured in a polarization-insensitive way. Relying on such actions, one can prepare photon A in mixed states. According to detection of photon B on path 12 or 14, one can compute that the mixed state is given by

$$\rho_A^{12} = \text{Tr}_B [(|H_{12}\rangle_B \langle H_{12}| + |V_{12}\rangle_B \langle V_{12}|) \rho'_{AB}] = \alpha^2 |V_1\rangle_A \langle V_1| + \beta^2 |H_1\rangle_A \langle H_1| \quad (24)$$

or

$$\rho_A^{14} = \text{Tr}_B [(|H_{14}\rangle_B \langle H_{14}| + |V_{14}\rangle_B \langle V_{14}|) \rho'_{AB}] = \alpha^2 |H_1\rangle_A \langle H_1| + \beta^2 |V_1\rangle_A \langle V_1| \quad (25)$$

where $\rho'_{AB} = |\varphi^7\rangle_{AB} \langle \varphi^7|$.

4.3 Single-photon-assisted JRSP with N -party

Note that our JRSP scheme can be extended to a N -sender ($N > 2$) case. Suppose there are N sender, say, Alice₁, Alice₂, ..., Alice _{N} , who would like to recover a single-photon pure state $|P\rangle = \alpha|H\rangle + \beta e^{i\theta}|V\rangle$ in Bob's site. Assume that $\alpha = \alpha_1 \alpha_2 \cdots \alpha_N$, $\beta = \beta_1 \beta_2 \cdots \beta_N$ and $\theta = \theta_1 + \theta_2 + \cdots + \theta_N$, and Alice _{i} has knowledge of α_i , β_i and θ_i . Now there is a system with a N -qubit GHZ-type state

$$|\psi_{\text{in}}\rangle_{A_1 A_2 \cdots A_N B} = a |H_1\rangle_{A_1} \cdots |H_N\rangle_{A_N} |H_{N+1}\rangle_B + b |V_1\rangle_{A_1} \cdots |V_N\rangle_{A_N} |V_{N+1}\rangle_B, \quad (26)$$

shared by multipartite participants. In Eq. (26), the complex numbers a and b satisfy the normalized condition, and the subscript i ($i = 1, 2, \dots, N$) represents spatial transmission modes of photons, and A_i labels the photons shared by those parties in quantum communication. To be explicit, photon A_i belongs to Alice _{i} , and B to Bob. In order to remotely prepare each single-photon system in the desired state, Alice₁ needs to prepare a single photon C with state of

$$|\varphi\rangle_C = b |H_{N+1}\rangle_C + a |V_{N+1}\rangle_C. \quad (27)$$

By applying two-mode instances and the similar analysis methods as the case of two senders, Alice _{i} ($i = 3, 4, \dots, N$) can construct the same optical elements as those by Alice₂, except that PM- $(i + 2)$ added by Alice _{i} is with a relative phase $e^{i\theta_i}$ in her path after the BS- i . For simplicity, we do not depict it any more. Upon classical communication for their measuring outcomes, Bob can realize which state his photon

is. If his collapsed state can be converted into the desired state via appropriate unitary transformation, this shows that our JRSP can be successful; otherwise, Bob needs to restart the preparation procedure. Through some recursive calculations, one can obtain that TSP of the protocol is $\frac{|ab|^2}{2^{(N-3)}(\alpha_1^2+\beta_1^2)(\alpha_2^2+\beta_2^2)\dots(\alpha_N^2+\beta_N^2)}$ and CCC is up to 2^N cbits.

5 Conclusion

To conclude, we have designed two optics-based implementations for RSP and JRSP of an arbitrary single-photon pure state by taking advantage of linear optics elements and partially entangled states as quantum resources, respectively. With the aid of suitable LOCC, our RSP protocol can be realized with certain success probability related to the employed entanglements, and the JRSP protocol can be achieved with a success probability characterized by the coefficients of both the employed entanglements and the ensembles of the target states, and we compute CCC for the two communication protocols as well, respectively. Interestingly, we have illustrated that the current proposals are available for implementing RSP and JRSP for a mixed state by means of the required replacement of photon detectors with polarization analyzers. In addition, the JRSP protocol can also be generalized to the case of N -sender with taking multiphoton partial entanglements as quantum channels. Furthermore, we believe that the present proposals might be importance of optics-based quantum communication in prospective multi-node quantum networks.

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