

Deterministic remote two-qubit state preparation in dissipative environments

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Abstract We propose a new scheme for efficient remote preparation of an arbitrary two-qubit state, introducing two auxiliary qubits and using two Einstein–Podolsky–Rosen (EPR) states as the quantum channel in a non-recursive way. At variance with all existing schemes, our scheme accomplishes deterministic remote state preparation (RSP) with only one sender and the simplest entangled resource (say, EPR pairs). We construct the corresponding quantum logic circuit using a unitary matrix decomposition procedure and analytically obtain the average fidelity of the deterministic RSP process for dissipative environments. Our studies show that, while the average fidelity gradually decreases to a stable value without any revival in the Markovian regime, it decreases to the same stable value with a dampened revival amplitude in the non-Markovian regime. We also find that the average fidelity’s approximate maximal value can be preserved for a long time if the non-Markovian and the detuning conditions are satisfied simultaneously.

Keywords Remote state preparation · Dissipative environment · Average fidelity · Detuning

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1 Introduction

As one of the most important discoveries in quantum information science, quantum teleportation (QT), first proposed by Bennett et al. [1] in 1993, is a process that a sender intends to teleport an unknown quantum state to a remote receiver with the assistance of an entangled channel and some classical communication. Subsequently, similar to QT, Lo [2], Pati [3], Bennett et al. [4] presented another quantum state transmission protocol called remote state preparation (RSP), in which the sender has complete knowledge of the original state to be transferred. Thus, in RSP there may exist a trade-off between the required entanglement and the classical information cost. Since then, various RSP protocols have been extensively investigated [5–18] in theory, and experimental implementation of RSP has been demonstrated in optical systems [19–24] and using liquid-state nuclear magnetic resonance techniques [25].

Generally, a realistic quantum system is not closed and will undergo unavoidable interaction with its external environment, which makes the system lose its coherence. Due to the decoherence effect of the system resulting from the ambient noises, usually the initial state at the side of the sender cannot be exactly transferred to the final state at the side of the remote receiver during the quantum state transmission process [26–29]. What happens to the RSP protocols in realistic environments? We notice that Chen et al. [30] studied how to remotely prepare a qubit via a W state subject to Pauli noises by using the trace distance to characterize how close the initial state is to the final state. Zhang et al. [31] investigated the scheme for remotely preparing a general two-qubit pure state with real parameters through two Bell-like states in non-Markovian environment. Furthermore, Liang et al. [32] examined the remote preparation of a single qubit and of a bipartite entangled state through a GHZ-class channel in the presence of dephasing and bit-flip noises. Recently, Sharma et al. [33] suggested a protocol for probabilistic controlled bidirectional RSP under the influence of the amplitude-damping and the phase-damping noises. Chen et al. [34] discussed the influence of Pauli noises on the deterministic joint RSP of an arbitrary state of two qubits via two GHZ states. On the experimental realization, Xiang et al. [35] reported the remote preparation of pure and mixed states via dephasing noisy entanglement by using spontaneous parametric down conversion and linear optical elements.

To our knowledge, the authors of Refs. [36–41] presented some schemes for deterministic remote preparation of an arbitrary pure state of two qubits via various types of entangled channels. However, these schemes need two or multi-senders who jointly share the complete information of the two-qubit state to be prepared, and they require at least one tripartite or multi-particle entangled state as shared quantum resource. Meanwhile, An et al. [42] utilized four EPR pairs as entangled resource to joint remotely prepare an arbitrary two-qubit state with unity probability and six classical bits. Moreover, An et al. [43] designed a protocol for RSP of a single-qubit state with unit success probability and two classical bits by using an EPR state and one ancillary qubit. In this paper, we put forward a new protocol to realize deterministic RSP of an arbitrary two-qubit state through two EPR pairs by employing two auxiliary qubits, and then its corresponding quantum logic circuit is designed by means of the decomposition procedure of unitary matrix. Our protocol can economize two EPR pairs and two bits of classical communication cost, compared with An et al.'s [42].

Further, the protocol that we suggest takes into account the presence of dissipative environments, and the average fidelity of the deterministic RSP process is derived analytically. Through numerical analysis, our study reveals that the average fidelity of the RSP subject to the Markovian environment and that subject to the non-Markovian environment exhibit quite different evolution behaviors. To be specific, the average fidelity decreases to a stable value asymptotically without any revival in the Markovian regime, while it diminishes to the same stable value with a damping of its revival amplitude in the non-Markovian regime. Besides, we find that the average fidelity can preserve the approximate maximum being one for a long time if the non-Markovian and the detuning conditions are satisfied at the same time.

This paper is organized as follows. In Sect. 2, we first propose a new deterministic protocol for remotely preparing an arbitrary pure state of two qubits through two EPR pairs, and then construct the corresponding quantum circuit of the RSP process by virtue of unitary matrix decomposition method. In Sect. 3, we examine the average fidelity of our RSP protocol under the influence of dissipative environments, and we end in Sect. 4 a brief conclusion.

2 Deterministic RSP via two EPR pairs

In this section, we put forward a new deterministic RSP protocol for remotely preparing an arbitrary two-qubit pure state in a non-recursive manner through two EPR states by incorporating two auxiliary qubits. First of all, suppose that the sender Alice wants to help the receiver Bob remotely prepare an arbitrary two-qubit state taking the following form

$$|\psi\rangle = \varepsilon_0 |00\rangle + \varepsilon_1 e^{i\theta_1} |01\rangle + \varepsilon_2 e^{i\theta_2} |10\rangle + \varepsilon_3 e^{i\theta_3} |11\rangle, \tag{1}$$

where the real parameters $\theta_s \in [0, 2\pi)(s = 1, 2, 3)$ and $\varepsilon_l \geq 0 (l = 0, 1, 2, 3)$ with $\sum_{l=0}^3 (\varepsilon_l)^2 = 1$. The parameters θ_s and ε_l are assumed to be known completely to Alice but unknown to Bob. To realize the RSP task, we also suppose that Alice and Bob share the following two EPR pairs as the quantum channel

$$\begin{aligned} |\phi\rangle_{12} &= \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)_{12}, \\ |\phi\rangle_{34} &= \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)_{34}, \end{aligned} \tag{2}$$

where qubits 1 and 3 belong to Alice, as well as qubits 2 and 4 to Bob. For the sake of efficiently analyzing the RSP process in dissipative environments, Bob first performs Pauli operation σ_x on each of his two qubits. In this way, the total state of the quantum channel shared by Alice and Bob can be written as

$$[(\sigma_x)_2 |\phi\rangle_{12}] \otimes [(\sigma_x)_4 |\phi\rangle_{34}] = \frac{1}{2}(|0000\rangle + |0011\rangle + |1100\rangle + |1111\rangle)_{1234}. \tag{3}$$

To help Bob prepare the original state $|\psi\rangle$, Alice introduces two auxiliary qubits a and b with their respective initial states $|0\rangle_a$ and $|0\rangle_b$ and performs two CNOT operations

with particles 1 and 3 as the controlled qubits and auxiliary particles a and b as the target qubits, respectively. Then the total state consisting of the six qubits 1, 2, 3, 4, a , and b is given by

$$|\Phi\rangle_T = \frac{1}{2}(|00\rangle|00\rangle|00\rangle + |00\rangle|11\rangle|01\rangle + |11\rangle|00\rangle|10\rangle + |11\rangle|11\rangle|11\rangle)_{1234ab}. \tag{4}$$

Since Alice has complete knowledge of the original state to be prepared, she performs a two-qubit projective measurement on her auxiliary two qubits, which depends on the amplitude information of $|\psi\rangle$ and is described by

$$\begin{aligned} |\eta_0\rangle_{ab} &= (\varepsilon_0|00\rangle + \varepsilon_1|01\rangle + \varepsilon_2|10\rangle + \varepsilon_3|11\rangle)_{ab}, \\ |\eta_1\rangle_{ab} &= (\varepsilon_1|00\rangle - \varepsilon_0|01\rangle + \varepsilon_3|10\rangle - \varepsilon_2|11\rangle)_{ab}, \\ |\eta_2\rangle_{ab} &= (\varepsilon_2|00\rangle - \varepsilon_3|01\rangle - \varepsilon_0|10\rangle + \varepsilon_1|11\rangle)_{ab}, \\ |\eta_3\rangle_{ab} &= (\varepsilon_3|00\rangle + \varepsilon_2|01\rangle - \varepsilon_1|10\rangle - \varepsilon_0|11\rangle)_{ab}, \end{aligned} \tag{5}$$

where the four states $\{|\eta_m\rangle_{ab}\} (m = 0, 1, 2, 3)$ constitute a set of complete orthonormal basis vectors in a four-dimensional Hilbert space. Using Eqs. (4) and (5), we can rewrite the total state $|\Phi\rangle_T$ as follows

$$\begin{aligned} |\Phi\rangle_T &= \frac{1}{2} [|\eta_0\rangle_{ab} (\varepsilon_0|00\rangle_{13}|00\rangle_{24} + \varepsilon_1|01\rangle_{13}|01\rangle_{24} \\ &\quad + \varepsilon_2|10\rangle_{13}|10\rangle_{24} + \varepsilon_3|11\rangle_{13}|11\rangle_{24}) \\ &\quad + |\eta_1\rangle_{ab} (\varepsilon_1|00\rangle_{13}|00\rangle_{24} - \varepsilon_0|01\rangle_{13}|01\rangle_{24} \\ &\quad + \varepsilon_3|10\rangle_{13}|10\rangle_{24} - \varepsilon_2|11\rangle_{13}|11\rangle_{24}) \\ &\quad + |\eta_2\rangle_{ab} (\varepsilon_2|00\rangle_{13}|00\rangle_{24} - \varepsilon_3|01\rangle_{13}|01\rangle_{24} \\ &\quad - \varepsilon_0|10\rangle_{13}|10\rangle_{24} + \varepsilon_1|11\rangle_{13}|11\rangle_{24}) \\ &\quad + |\eta_3\rangle_{ab} (\varepsilon_3|00\rangle_{13}|00\rangle_{24} + \varepsilon_2|01\rangle_{13}|01\rangle_{24} \\ &\quad - \varepsilon_1|10\rangle_{13}|10\rangle_{24} - \varepsilon_0|11\rangle_{13}|11\rangle_{24})]. \end{aligned} \tag{6}$$

After making the projective measurement on the two auxiliary qubits a and b , Alice transmits her measurement outcomes to Bob through two classical bits. According to Eq. (6), if Alice obtains four possible measurement results $\{|\eta_m\rangle_{ab}\}$, the particles 1, 2, 3, and 4 are correspondingly collapsed to four different states ${}_{ab}\langle\eta_m|\Phi\rangle_T$, which can be converted into the identical state $|\Phi\rangle_{1324} = \varepsilon_0|00\rangle_{13}|00\rangle_{24} + \varepsilon_1|01\rangle_{13}|01\rangle_{24} + \varepsilon_2|10\rangle_{13}|10\rangle_{24} + \varepsilon_3|11\rangle_{13}|11\rangle_{24}$ via suitable unitary operations on particles a and b . To be specific, when the outcome of Alice’s measurement on the two auxiliary qubits is $|\eta_0\rangle, |\eta_1\rangle, |\eta_2\rangle$ or $|\eta_3\rangle$, the corresponding collapsed state of particles 1, 2, 3, and 4 can be transformed into the state $|\Phi\rangle_{1324}$ depending on the amplitude information of the original state $|\psi\rangle$ by Alice performing unitary operation $(I)_1 \otimes (I)_3 \otimes (I)_2 \otimes (I)_4, (I)_1 \otimes (\sigma_x)_3 \otimes (I)_2 \otimes (i\sigma_y)_4, (\sigma_x)_1 \otimes (I)_3 \otimes (i\sigma_y)_2 \otimes (\sigma_z)_4$ or $(\sigma_x)_1 \otimes (\sigma_x)_3 \otimes (i\sigma_y)_2 \otimes (\sigma_x)_4$, respectively. Here I is an identity operation, σ_x, σ_y and σ_z are Pauli operations.

At this stage, the RSP task is to retrieve the original state $|\psi\rangle$ from the collapsed one $|\Phi\rangle_{1324}$. To do so, Alice proceeds the two-qubit projective measurement on her own particles 1 and 3 under the following orthonormal basis

$$\begin{aligned}
 |\chi_0\rangle_{13} &= \frac{1}{2} \left(|00\rangle + e^{-i\theta_1} |01\rangle + e^{-i\theta_2} |10\rangle + e^{-i\theta_3} |11\rangle \right)_{13}, \\
 |\chi_1\rangle_{13} &= \frac{1}{2} \left(|00\rangle - e^{-i\theta_1} |01\rangle + e^{-i\theta_2} |10\rangle - e^{-i\theta_3} |11\rangle \right)_{13}, \\
 |\chi_2\rangle_{13} &= \frac{1}{2} \left(|00\rangle + e^{-i\theta_1} |01\rangle - e^{-i\theta_2} |10\rangle - e^{-i\theta_3} |11\rangle \right)_{13}, \\
 |\chi_3\rangle_{13} &= \frac{1}{2} \left(|00\rangle - e^{-i\theta_1} |01\rangle - e^{-i\theta_2} |10\rangle + e^{-i\theta_3} |11\rangle \right)_{13}. \tag{7}
 \end{aligned}$$

Here the four states $\{|\chi_n\rangle_{13}\} (n = 0, 1, 2, 3)$ are related to the phase information of the original state to be remotely prepared. According to the above equation, we can expand the state $|\Phi\rangle_{1324}$ of particles 1, 2, 3, and 4 as

$$\begin{aligned}
 |\Phi\rangle_{1324} &= \frac{1}{2} \left[|\chi_0\rangle_{13} \left(\varepsilon_0 |00\rangle_{24} + \varepsilon_1 e^{i\theta_1} |01\rangle_{24} + \varepsilon_2 e^{i\theta_2} |10\rangle_{24} + \varepsilon_3 e^{i\theta_3} |11\rangle_{24} \right) \right. \\
 &\quad + |\chi_1\rangle_{13} \left(\varepsilon_0 |00\rangle_{24} - \varepsilon_1 e^{i\theta_1} |01\rangle_{24} + \varepsilon_2 e^{i\theta_2} |10\rangle_{24} - \varepsilon_3 e^{i\theta_3} |11\rangle_{24} \right) \\
 &\quad + |\chi_2\rangle_{13} \left(\varepsilon_0 |00\rangle_{24} + \varepsilon_1 e^{i\theta_1} |01\rangle_{24} - \varepsilon_2 e^{i\theta_2} |10\rangle_{24} - \varepsilon_3 e^{i\theta_3} |11\rangle_{24} \right) \\
 &\quad \left. + |\chi_3\rangle_{13} \left(\varepsilon_0 |00\rangle_{24} - \varepsilon_1 e^{i\theta_1} |01\rangle_{24} - \varepsilon_2 e^{i\theta_2} |10\rangle_{24} + \varepsilon_3 e^{i\theta_3} |11\rangle_{24} \right) \right]. \tag{8}
 \end{aligned}$$

After Alice performs the projective measurement on her two qubits under the basis $\{|\chi_n\rangle_{13}\} (n = 0, 1, 2, 3)$, she informs Bob of her measurement results with two bits of classical communication. For instance, when Alice obtains the measurement result $|\chi_0\rangle_{13}$, the particles 2 and 4 are then collapsed to the state $\varepsilon_0 |00\rangle_{24} + \varepsilon_1 e^{i\theta_1} |01\rangle_{24} + \varepsilon_2 e^{i\theta_2} |10\rangle_{24} + \varepsilon_3 e^{i\theta_3} |11\rangle_{24}$, which is exactly the desired state $|\psi\rangle$ that Alice wishes to prepare remotely. Moreover, if the outcome of Alice’s measurement on her two qubits 1 and 3 is $|\chi_1\rangle, |\chi_2\rangle$ or $|\chi_3\rangle$, the particles 2 and 4 held by Bob will correspondingly be collapsed to the state $[(I)_2 \otimes (\sigma_z)_4] |\psi\rangle, [(\sigma_z)_2 \otimes (I)_4] |\psi\rangle$, or $[(\sigma_z)_2 \otimes (\sigma_z)_4] |\psi\rangle$ with the probability of 1/4, respectively. Summing up all the four possible measurement outcomes of Alice, Bob is always able to restore the original state $|\psi\rangle$ at his side with unit probability by means of proper local unitary operations on his two particles. Thus our protocol for remotely preparing an arbitrary two-qubit pure state can be achieved deterministically with four bits of classical communication from Alice to Bob [44–47].

Based on the decomposition methods of unitary matrices [48,49], following the aforementioned procedure, we design the quantum circuit for implementing the deterministic RSP protocol in Fig. 1, where ‘ H ’ is the Hadamard operation, the dot ‘ \bullet ’ denotes the control qubit, ‘ \oplus ’, ‘ $i\sigma_y$ ’, ‘ σ_z ’ represent the target qubits, ‘ σ_x ’, ‘ σ_y ’, ‘ σ_z ’ are three Pauli operations, ‘ M_1 ’, ‘ M_3 ’, ‘ M_a ’, ‘ M_b ’ represent the single-qubit projective measurement, as well as the double line denotes the classical communication.

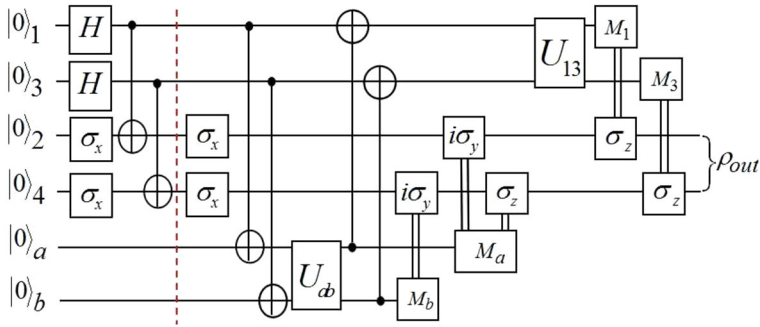


Fig. 1 (Color online) Quantum circuit for deterministic remote preparation of an arbitrary two-qubit state via two EPR states, where the *top two lines* belong to Alice, the *third and fourth lines* belong to Bob, and the *two bottom lines* represent the two auxiliary qubits

Furthermore, the unitary amplitude operator U_{ab} and the unitary phase operator U_{13} take the following respective forms

$$U_{ab} = \begin{pmatrix} \varepsilon_0 & \varepsilon_1 & \varepsilon_2 & \varepsilon_3 \\ \varepsilon_1 & -\varepsilon_0 & \varepsilon_3 & -\varepsilon_2 \\ \varepsilon_2 & -\varepsilon_3 & -\varepsilon_0 & \varepsilon_1 \\ \varepsilon_3 & \varepsilon_2 & -\varepsilon_1 & -\varepsilon_0 \end{pmatrix}, \tag{9}$$

$$U_{13} = \frac{1}{2} \begin{pmatrix} 1 & e^{i\theta_1} & e^{i\theta_2} & e^{i\theta_3} \\ 1 & -e^{i\theta_1} & e^{i\theta_2} & -e^{i\theta_3} \\ 1 & e^{i\theta_1} & -e^{i\theta_2} & -e^{i\theta_3} \\ 1 & -e^{i\theta_1} & -e^{i\theta_2} & e^{i\theta_3} \end{pmatrix}. \tag{10}$$

3 Deterministic RSP in dissipative environments

An open quantum system will interact with its surrounding environment unavoidably, which causes the system to lose its coherence gradually. Thus it is necessary to consider and explore the realistic quantum state transmission process in the presence of environmental noises. In this section, we will investigate the influence of dissipative environments on the aforementioned deterministic RSP protocol. Here we assume the four qubits 1, 2, 3, and 4 constituting the quantum channel given in Sect. 2 interact with their respective independent vacuum reservoirs at zero temperature. The single-qubit reservoir Hamiltonian (in units of $\hbar = 1$) is described by the well-known damped Jaynes–Cummings model as follows [50–52]

$$H = \omega_0 \sigma_+ \sigma_- + \sum_{k=1}^N \omega_k \hat{b}_k^+ \hat{b}_k + \sum_{k=1}^N \left(g_k \sigma_+ \hat{b}_k + g_k^* \sigma_- \hat{b}_k^+ \right), \tag{11}$$

where ω_0 is the transition frequency of two-level system, σ_+ (σ_-) is the raising (lowering) operator of the qubit, \hat{b}_k^+ (\hat{b}_k) is the creation (annihilation) operator of mode k of

the reservoir with frequency ω_k , g_k is the strength of coupling between the qubit and the reservoir mode k . Now the initial state of the EPR pair and the vacuum reservoir can be written as

$$|\Omega(0)\rangle = |\phi\rangle_{q_1q_2} |\bar{0}\rangle_{e_1} |\bar{0}\rangle_{e_2} = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)_{q_1q_2} |\bar{00}\rangle_{e_1e_2}, \tag{12}$$

where $|\bar{0}\rangle_e = \prod_{k=1}^N |0_k\rangle_e$ denotes the vacuum environment with N modes. Then the time evolution of $|\Omega(0)\rangle$ can be given by

$$|\Omega(t)\rangle = \frac{1}{\sqrt{2}} \left(|0\rangle_{q_1} |\bar{0}\rangle_{e_1} |\omega(t)\rangle_{q_2e_2} + |\omega(t)\rangle_{q_1e_1} |0\rangle_{q_2} |\bar{0}\rangle_{e_2} \right), \tag{13}$$

where $|\omega(t)\rangle_{q_je_j}$ ($j = 1, 2$) takes the form

$$\begin{aligned} |\omega(t)\rangle_{q_je_j} &= \xi_0(t) |1\rangle_{q_j} |\bar{0}\rangle_{e_j} + \sum_{k=1}^N \xi_k(t) |0\rangle_{q_j} |1_k\rangle_{e_j} \\ &= \xi_0(t) |1\rangle_{q_j} |\bar{0}\rangle_{e_j} + \xi(t) |0\rangle_{q_j} |\bar{1}\rangle_{e_j}. \end{aligned} \tag{14}$$

Here $|\bar{1}\rangle_{e_j} = \frac{1}{\xi(t)} \sum_{k=1}^N \xi_k(t) |1_k\rangle_{e_j}$, $|1_k\rangle_{e_j}$ represents the state of the N -mode environment with only a single excitation in the k th mode, $\xi_0(t)$ and $\xi_k(t)$ are the time-dependent probability amplitudes, and $|\xi_0(t)|^2 + |\xi(t)|^2 = 1$. Generally, the dynamics of state $|\omega(t)\rangle_{q_je_j}$ is governed by the schrödinger equation: $i\hbar \frac{\partial}{\partial t} |\omega(t)\rangle_{q_je_j} = H |\omega(t)\rangle_{q_je_j}$. So the evolution of $\xi_0(t)$ and $\xi_k(t)$ obeys the following equations

$$\begin{aligned} \dot{\xi}_0(t) &= -i \sum_k^N g_k \exp [i (\omega_0 - \omega_k) t] \xi_k(t), \\ \dot{\xi}_k(t) &= -i g_k^* \exp [-i (\omega_0 - \omega_k) t] \xi_0(t), \end{aligned} \tag{15}$$

with $\xi_0(t) = 1$. By solving the second equation and substituting the solution into the first one, we can obtain an integro-differential formulas for $\xi_0(t)$ as follows

$$\dot{\xi}_0(t) = - \int_0^t dt_1 f(t - t_1) \xi_0(t_1), \tag{16}$$

where the correlation function $f(t-t_1)$ can be described by the Fourier transform of the environmental spectral density $J(\omega)$ with $f(t-t_1) = \int d\omega J(\omega) \exp [i(\omega_0 - \omega)(t-t_1)]$. Clearly, the explicit form of $\xi_0(t)$ depends on the particular choice of the environmental spectral density. Next we take into account the spectral distribution of an electromagnetic field inside a lossy cavity supporting the fundamental mode ω_c . The effective spectral density of the intracavity field exhibits a Lorentzian broadening with [53,54]

$$J(\omega) = \frac{1}{2\pi} \frac{\Gamma\lambda^2}{(\omega_0 - \omega - \Delta)^2 + \lambda^2}, \tag{17}$$

in which Γ is related to the decay of the excited state for the qubit in the Markovian limit of a flat spectrum, λ is the spectral width of the coupling, and Δ is the detuning between the qubit transition frequency ω_0 and the cavity center frequency ω_c . By judging the relation of Γ and λ , we can distinguish the weak and strong coupling regimes. Specifically, $\lambda/\Gamma > 2$ corresponds to the weak regime, called Markovian environment. On the contrary, $\lambda/\Gamma < 2$ means the strong regime, called non-Markovian environment. Substituting Eq. (17) into Eq. (16) and using Laplace transform, the time-dependent function $\xi_0(t)$ can be derived as

$$\xi_0(t) = e^{-\frac{1}{2}(\lambda-i\Delta)t} \left[\cosh\left(\frac{\Theta t}{2}\right) + \frac{\lambda-i\Delta}{\Theta} \sinh\left(\frac{\Theta t}{2}\right) \right], \tag{18}$$

where $\Theta = \sqrt{(\lambda - i\Delta)^2 - 2\lambda\Gamma}$.

Following the above procedure to deal with system dynamics and after tracing out the reservoir modes, we obtain the time evolution of the density matrix elements of each EPR pair as follows

$$\rho_{12}(t) = \rho_{34}(t) = \frac{1}{2} \begin{pmatrix} 2(1 - |\xi_0(t)|^2) & 0 & 0 & 0 \\ 0 & |\xi_0(t)|^2 & |\xi_0(t)|^2 & 0 \\ 0 & |\xi_0(t)|^2 & |\xi_0(t)|^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{19}$$

under the basis $\{|11\rangle, |10\rangle, |01\rangle, |00\rangle\}$. Then the whole dynamics of the two EPR pairs serving as quantum channel is given by $\rho_{12}(t) \otimes \rho_{34}(t)$. Based on Fig. 1, the output state of our RSP protocol via the noisy channel can be expressed as

$$\rho_{\text{out}} = \rho_{24} = Tr_{13ab} \left\{ U_{\text{RSP}} (\rho_{12}(t) \otimes \rho_{34}(t) \otimes |0\rangle_{aa} \langle 0| \otimes |0\rangle_{bb} \langle 0|) U_{\text{RSP}}^\dagger \right\}, \tag{20}$$

where Tr_{13ab} is the partial trace over qubits 1, 3, a, b , and the unitary operator

$$U_{\text{RSP}} = (\sigma_z)_{3 \rightarrow 4} (\sigma_z)_{1 \rightarrow 2} (U_{13}) (\sigma_z)_{a \rightarrow 4} (i\sigma_y)_{b \rightarrow 4} (i\sigma_y)_{a \rightarrow 2} (\sigma_x)_{b \rightarrow 3} (\sigma_x)_{a \rightarrow 1} (U_{ab}) (\sigma_x)_{3 \rightarrow b} (\sigma_x)_{1 \rightarrow a}. \tag{21}$$

Here $(\sigma_q)_{e \rightarrow f}$ denotes a two-qubit unitary controlled— σ_q ($q = x, y, z$) operation with e ($e = 1, 3, a, b$) as control qubit and f ($f = 1, 2, 3, 4, a, b$) as target qubit. Through analytical derivation, the output state ρ_{out} at Bob’s side takes the following form

$$\begin{aligned} \rho_{\text{out}} = \rho_{24} = & \left[\varepsilon_0^2 |\xi_0(t)|^4 + (\varepsilon_1^2 + \varepsilon_2^2) |\xi_0(t)|^2 (1 - |\xi_0(t)|^2) + \varepsilon_3^2 (1 - |\xi_0(t)|^2)^2 \right] |00\rangle \langle 00| \\ & + \left[\varepsilon_0 \varepsilon_1 e^{-i\theta_1} |\xi_0(t)|^4 + \varepsilon_2 \varepsilon_3 e^{i(\theta_2 - \theta_3)} |\xi_0(t)|^2 (1 - |\xi_0(t)|^2) \right] |00\rangle \langle 01| \\ & + \left[\varepsilon_0 \varepsilon_2 e^{-i\theta_2} |\xi_0(t)|^4 + \varepsilon_1 \varepsilon_3 e^{i(\theta_1 - \theta_3)} |\xi_0(t)|^2 (1 - |\xi_0(t)|^2) \right] |00\rangle \langle 10| \end{aligned}$$

$$\begin{aligned}
 & + \varepsilon_0 \varepsilon_3 e^{-i\theta_3} |\xi_0(t)|^4 |00\rangle \langle 11| \\
 & + \left[\varepsilon_0 \varepsilon_1 e^{i\theta_1} |\xi_0(t)|^4 + \varepsilon_2 \varepsilon_3 e^{i(\theta_3 - \theta_2)} |\xi_0(t)|^2 \left(1 - |\xi_0(t)|^2 \right) \right] |01\rangle \langle 00| \\
 & + \left[\varepsilon_1^2 |\xi_0(t)|^4 + \left(\varepsilon_0^2 + \varepsilon_3^2 \right) |\xi_0(t)|^2 \left(1 - |\xi_0(t)|^2 \right) + \varepsilon_2^2 \left(1 - |\xi_0(t)|^2 \right)^2 \right] |01\rangle \langle 01| \\
 & + \varepsilon_1 \varepsilon_2 e^{i(\theta_1 - \theta_2)} |\xi_0(t)|^4 |01\rangle \langle 10| \\
 & + \left[\varepsilon_1 \varepsilon_3 e^{i(\theta_1 - \theta_3)} |\xi_0(t)|^4 + \varepsilon_0 \varepsilon_2 e^{-i\theta_2} |\xi_0(t)|^2 \left(1 - |\xi_0(t)|^2 \right) \right] |01\rangle \langle 11| \\
 & + \left[\varepsilon_0 \varepsilon_2 e^{i\theta_2} |\xi_0(t)|^4 + \varepsilon_1 \varepsilon_3 e^{i(\theta_3 - \theta_1)} |\xi_0(t)|^2 \left(1 - |\xi_0(t)|^2 \right) \right] |10\rangle \langle 00| \\
 & + \varepsilon_1 \varepsilon_2 e^{i(\theta_2 - \theta_1)} |\xi_0(t)|^4 |10\rangle \langle 01| \\
 & + \left[\varepsilon_2^2 |\xi_0(t)|^4 + \left(\varepsilon_0^2 + \varepsilon_3^2 \right) |\xi_0(t)|^2 \left(1 - |\xi_0(t)|^2 \right) + \varepsilon_1^2 \left(1 - |\xi_0(t)|^2 \right)^2 \right] |10\rangle \langle 10| \\
 & + \left[\varepsilon_2 \varepsilon_3 e^{i(\theta_2 - \theta_3)} |\xi_0(t)|^4 + \varepsilon_0 \varepsilon_1 e^{-i\theta_1} |\xi_0(t)|^2 \left(1 - |\xi_0(t)|^2 \right) \right] |10\rangle \langle 11| \\
 & + \varepsilon_0 \varepsilon_3 e^{i\theta_3} |\xi_0(t)|^4 |11\rangle \langle 00| \\
 & + \left[\varepsilon_1 \varepsilon_3 e^{i(\theta_3 - \theta_1)} |\xi_0(t)|^4 + \varepsilon_0 \varepsilon_2 e^{i\theta_2} |\xi_0(t)|^2 \left(1 - |\xi_0(t)|^2 \right) \right] |11\rangle \langle 01| \\
 & + \left[\varepsilon_2 \varepsilon_3 e^{i(\theta_3 - \theta_2)} |\xi_0(t)|^4 + \varepsilon_0 \varepsilon_1 e^{i\theta_1} |\xi_0(t)|^2 \left(1 - |\xi_0(t)|^2 \right) \right] |11\rangle \langle 10| \\
 & + \left[\varepsilon_3^2 |\xi_0(t)|^4 + \left(\varepsilon_1^2 + \varepsilon_2^2 \right) |\xi_0(t)|^2 \left(1 - |\xi_0(t)|^2 \right) + \varepsilon_0^2 \left(1 - |\xi_0(t)|^2 \right)^2 \right] |11\rangle \langle 11|.
 \end{aligned}
 \tag{22}$$

To depict how much quantum information is transmitted from the initial state to the output state through the noisy channel, it is quite useful to calculate the fidelity defined by

$$F = \langle \psi | \rho_{\text{out}} | \psi \rangle.
 \tag{23}$$

By straightforward calculation, the fidelity of our RSP protocol subject to the noisy environment can be derived as

$$\begin{aligned}
 F & = \varepsilon_0^4 |\xi_0(t)|^4 + \varepsilon_1^4 |\xi_0(t)|^4 + \varepsilon_2^4 |\xi_0(t)|^4 + \varepsilon_3^4 |\xi_0(t)|^4 + 4\varepsilon_0^2 \varepsilon_3^2 |\xi_0(t)|^4 + 4\varepsilon_1^2 \varepsilon_2^2 |\xi_0(t)|^4 \\
 & - 8\varepsilon_0 \varepsilon_1 \varepsilon_2 \varepsilon_3 \cos(\theta_1 + \theta_2 - \theta_3) |\xi_0(t)|^4 + 2\varepsilon_0^2 \varepsilon_1^2 |\xi_0(t)|^2 + 2\varepsilon_0^2 \varepsilon_2^2 |\xi_0(t)|^2 \\
 & - 4\varepsilon_0^2 \varepsilon_3^2 |\xi_0(t)|^2 - 4\varepsilon_1^2 \varepsilon_2^2 |\xi_0(t)|^2 + 2\varepsilon_1^2 \varepsilon_3^2 |\xi_0(t)|^2 + 2\varepsilon_2^2 \varepsilon_3^2 |\xi_0(t)|^2 \\
 & + 8\varepsilon_0 \varepsilon_1 \varepsilon_2 \varepsilon_3 \cos(\theta_1 + \theta_2 - \theta_3) |\xi_0(t)|^2 + 2\varepsilon_0^2 \varepsilon_3^2 + 2\varepsilon_1^2 \varepsilon_2^2.
 \end{aligned}
 \tag{24}$$

Generally, the initial state to be remotely prepared is an arbitrary two-qubit state. Thus it is of great significance for us to examine the average fidelity over all possible initial states, which can be formulated by [34]

$$F_{\text{av}} = \left(\int_0^{2\pi} \int_0^{2\pi} \int_0^{2\pi} d\theta_1 d\theta_2 d\theta_3 \int_0^\pi \int_0^\pi \int_0^\pi F \sin \alpha \sin \beta \sin \delta d\alpha d\beta d\delta \right) / 64\pi^3,
 \tag{25}$$

with $\varepsilon_0 = \cos \frac{\alpha}{2} \cos \frac{\beta}{2}$, $\varepsilon_1 = \cos \frac{\alpha}{2} \sin \frac{\beta}{2}$, $\varepsilon_2 = \sin \frac{\alpha}{2} \cos \frac{\delta}{2}$, $\varepsilon_3 = \sin \frac{\alpha}{2} \sin \frac{\delta}{2}$, and $\alpha, \beta, \delta \in [0, \pi]$. Substituting Eq. (24) into Eq. (25), we can analytically calculate the average fidelity to be

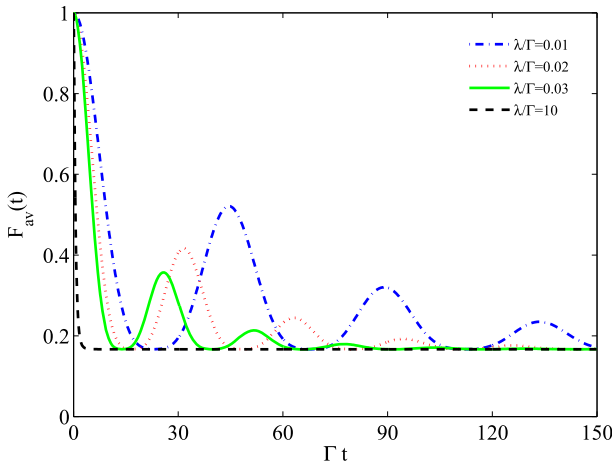


Fig. 2 (Color online) Average fidelity $F_{av}(t)$ is plotted versus the dimensionless quantity Γt for different amount of non-Markovianity λ/Γ with $\Delta = 0$

$$F_{av}(t) = \frac{7}{9} |\xi_0(t)|^4 + \frac{1}{18} |\xi_0(t)|^2 + \frac{1}{6}. \tag{26}$$

Clearly, the average fidelity $F_{av}(0)$ is equal to 1 when the two EPR pairs serving as the entangled channel are in the absence of noisy environments (say, $\xi(0) = 1$). Next we numerically discuss the average fidelity of our RSP protocol under the influence of Markovian and non-Markovian regimes.

In Fig. 2, we plot the average fidelity as a function of Γt for different values of λ/Γ with $\Delta = 0$. It can be seen from the figure that the evolution behavior of $F_{av}(t)$ in non-Markovian regime is essentially different from that in Markovian regime. More specifically, the average fidelity in non-Markovian regime ($\lambda/\Gamma < 2$) decreases gradually to the stable value of $1/6$ and then revives for a period of finite time with a damping amplitude, whereas the average fidelity in Markovian regime ($\lambda/\Gamma > 2$) decays asymptotically to the same value of $1/6$ without any revival. The revival phenomenon is due to the memory effects of non-Markovian environment which allow the backflow of information from the environment to the system [55–57]. Moreover, it can also be seen that the revival amplitude and the non-Markovian decay rate are related to the amount of non-Markovianity λ/Γ . With the decreasing λ/Γ , the revival amplitude increases and the non-Markovian decay rate diminishes, this is mainly owed to more information fed from the narrower environmental spectrum.

In Fig. 3, we plot $F_{av}(t)$ as a function of Γt for different detuning. As shown in Fig. 3a, b, the average fidelity decreases exponentially to the stable value of $1/6$ in Markovian regime without the oscillation phenomenon no matter whether the detuning is zero or not. Furthermore, the detuning is helpful to prolong the decay time of the average fidelity, which implies that the larger the detuning is, the longer the average fidelity takes to decay. From Fig. 3c, d, we can see that with the amount of the non-Markovianity increasing, the average fidelity takes longer to survive and its revival amplitude becomes larger. Besides, it is interesting to notice from Fig. 3c, d that the

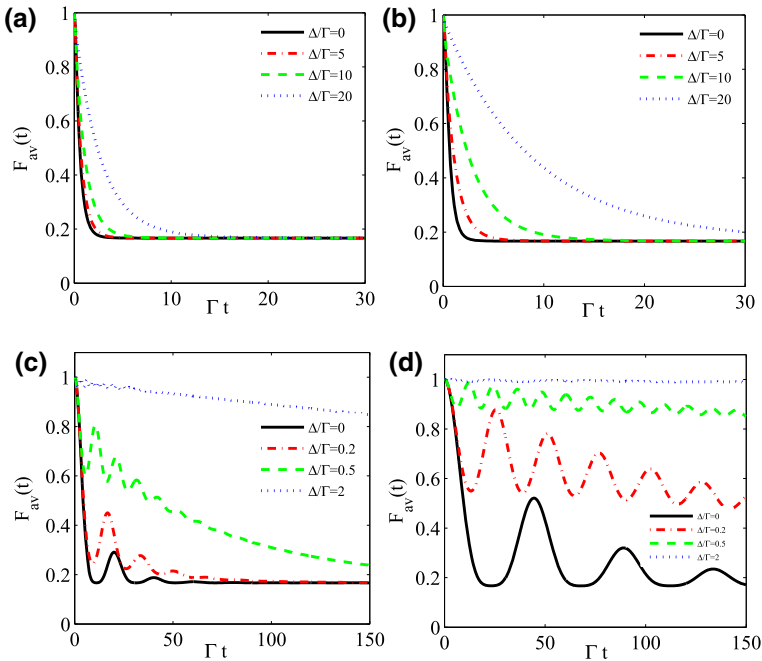


Fig. 3 (Color online) Average fidelity $F_{av}(t)$ is plotted as a function of Γt for different detuning. Here **a** the Markovian regime $\lambda/\Gamma = 10$, **b** the Markovian regime $\lambda/\Gamma = 5$, **c** the non-Markovian regime $\lambda/\Gamma = 0.05$, **d** the non-Markovian regime $\lambda/\Gamma = 0.01$

average fidelity in non-Markovian regime exhibits the phenomenon of slow decay for large detuning. That is to say, the average fidelity oscillates rapidly at the very beginning for large value of the detuning, but as Γt increases it can preserve a nearly maximal stationary value of 1 together with the great amount of non-Markovianity. This robust preserving of the average fidelity can efficiently overcome the environmental impact, which is of great significance in quantum information processing. Therefore, if the non-Markovian condition and the detuning condition are satisfied simultaneously in appropriate physical systems [58,59], the average fidelity of the RSP can maintain the approximate ideal value of one for a long time.

4 Conclusions

In summary, we have proposed a new protocol for deterministic remote preparation of an arbitrary two-qubit state via two EPR states serving as the entanglement resource by incorporating two ancillary qubits. Our protocol mainly includes two stages: the first stage is amplitude manipulation by means of Alice adjusting the initial quantum channel into the target channel which has the same amplitude parameters with the prepared state, and the second stage is phase manipulation by Alice performing suitable unitary phase operation on her own qubits. In this way, our remote preparation of the two-qubit pure state with complex parameters can be achieved faithfully in a

nonrecursive way, which is relatively simpler than previous RSP protocols [60–62], where the recursive channel modulation needs to be implemented until it succeeds. In contrast to An et al.'s protocol [42], ours can economize two EPR pairs and two bits of classical information. Moreover, the quantum circuit of the deterministic RSP protocol we have proposed is designed by means of unitary matrix decomposition technique, and the corresponding average fidelity is derived analytically for the case that the quantum channel is subject to dissipative environments. Through numerical analysis, our findings have revealed that the average fidelity decreases to a stable value exponentially in the Markovian regime, but it attenuates to the identical stable value oscillatorily with a damping of revival amplitude in the non-Markovian regime. The revival phenomenon is due to the non-Markovian dynamics of single qubit resulting from the feedback effect of environment with memory [52,53]. Furthermore, the detuning between the qubit frequency and the cavity center frequency is useful to prolong the decay of the average fidelity no matter whether the quantum channel is affected by Markovian environment or non-Markovian environment. We have also found that the average fidelity can achieve preserving the nearly ideal value of one for a long time if the detuning and the non-Markovian conditions are satisfied simultaneously in suitable physical systems. Our results might strengthen the understandings of the RSP process and could stimulate the further studies on quantum communication protocols in actual noisy environments.

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