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# Geometric measure of quantum discord with weak measurements

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**Abstract** Super quantum discord based on weak measurements was introduced by Singh and Pati (Ann Phys 343:141–152, 2014). We propose a geometric way of quantifying quantum discord with weak measurements. It is shown that this geometric measure of quantum discord with weak measurements (GQDW) is linearly dependent on geometric measure of quantum discord (Dakic et al. in Phys Rev Lett 105:190502, 2010) and only captures partial quantumness of the states. It is found that the quantum correlation can be extracted by a sequence of infinitesimal weak measurements. Finally, the level surfaces of GQDW for Bell-diagonal states are depicted and the results are demonstrated by explicit example.

## 1 Introduction

Quantum discord (QD) introduced by Ollivier and Zurek [1] is fundamentally different from entanglement [2] which plays a vital role in quantum information processing. The QD, which can be nonzero even for some separable states, represents the locally inaccessible information, which is the difference between total and classical corre-

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lation. From this, we can see that QD may capture the quantum correlation which cannot be detected by entanglement. Meanwhile, the deterministic quantum computation with one quantum bit (QDC1) [3–6] demonstrates the quantum advantage of QD in mixed states which are absent of entanglement. However, from a computational point of view, the evaluation of QD is a tough task due to the complicated optimization procedure [7]. Consequently, a lot of alternative measures of quantum correlation have been proposed , such as geometric quantum discord (GQD) [8], relative entropy of discord [9], continuous-variable discord [10], quantum deficit [11].

On the other hand, QD is quantum measurement dependent. The original definition of QD is obtained by the optimal general positive operator-valued measurements (POVMs) and subsequently improved to restrict the measurements within the range of the projective measurements which have strong influence on the initial states. In 1988, Aharonov et al. [12] introduced the concept of weak measurements. It is universal in the sense that any generalized measurement can be realized as a sequence of weak measurements which make small changes to the quantum state for all outcomes [13]. Recently, Singh and Pati [14] introduced the super quantum discord (SQD) considering such weak measurements in identifying the quantum correlations. It has been shown that SQD is always larger than QD captured by strong measurements and that SQD is a monotone function of measurement strength. The SQD has attracted intensive attention and interest in the past few years [15–17].

In this paper, motivated by GQD [8], we present the GQDW which employs the norm in the Hilbert-Schmidt space as the distance between pre- and post-measurement states under weak measurement. It should be noted that in Ref. [18], Xiang and Jing proposed the geometric analog of quantum discord under weak measurement using the same distance measure. They showed that for correlated state shared by two relatively accelerated observers the GQDW was always smaller than GQD, so they named it inferior "Geometric discord." However, in this work, we will present the more rigorous relationship between GQDW and GQD for any state in an analytical way. What is more, we find that the residual quantumness which the weak measurements fail to extract can be extracted by strong measurements on the post-state after weak measurements. In particular, we obtain that the quantum correlation can be resurrected by a sequence of infinitesimal weak measurements. This suggests a conservation law for quantum correlation in any composite system. Finally, we depict the level surfaces of GQDW for Bell-diagonal states and demonstrate our results by explicit example.

## 2 Some concepts and definition of GQDW

In this section, we will firstly present a brief review of weak measurement and GQD and then give the definition of GQDW.

The weak measurement operators are given by [14]

$$P(+) = t_1\Pi_1 + t_2\Pi_2,\ P(-) = t_2\Pi_1 + t_1\Pi_2, \tag{1}$$

where

$$t_1 = \sqrt{\frac{1-\tanh x}{2}}, \quad t_2 = \sqrt{\frac{1+\tanh x}{2}}, \tag{2}$$

with $x$ denoting the strength in the weak measurement process, and $\Pi_1$, $\Pi_2$ are two orthogonal projectors with $\Pi_1 + \Pi_2 = I$ . One can easily find that weak measurement will reduce to orthogonal projectors when $x \to \infty$, and $P^\dagger(+)P(+)+P^\dagger(-)P(-) = I$.

For any bipartite quantum state $\rho_{AB}$, the GQD [8] was firstly defined as

$$Q_G(\rho_{AB}) = \min_{\chi\in\Omega_0} \|\rho_{AB} - \chi\|, \tag{3}$$

where $\Omega_0$ denotes the set of zero-discord states and $\|\cdot\|$ is the norm in the Hilbert-Schmidt space. Then Luo and Fu [19] proposed an equivalent form of GQD as

$$Q_G(\rho_{AB}) = \min_{\Pi} \|\rho_{AB} - \Pi(\rho_{AB})\|, \tag{4}$$

where $\Pi$ is over all von Neumann measurements on the subsystem $A$. Based on the above ideas, we define GQDW which is related to weak measurements as follows.

**Definition** The GQDW for a bipartite quantum state $\rho_{AB}$ is defined as

$$Q_{GW}(\rho_{AB}) = \min_{\hat{\Pi}} \|\rho_{AB} - \hat{\Pi}(\rho_{AB})\|, \tag{5}$$

where

$$\hat{\Pi}(\rho_{AB}) = P(+)\rho_{AB}P(+) + P(-)\rho_{AB}P(-)$$

with $P(+)$, $P(-)$ being weak measurements on subsystem $A$ defined by Eq. (1).

When the strength of the weak measurement $x \to \infty$ , for any bipartite quantum state, the GQDW defined in Eq. (5) will be equal to GQD defined in Eq. (3) and Eq. (4), that is

$$Q_G(\rho_{AB}) = \lim_{x\to\infty} Q_{GW}(\rho_{AB}). \tag{6}$$

Another point one should note is that for the case $x = 0$, there is no measurement performed on the system. In such case, the value of SQD is equal to quantum mutual information while the value of GQDW becomes zero. This observation could confirm the viewpoint [18] that quantum discord and geometric discord are not too concordant with each other. Besides, we find a more rigorous relation between GQDW and GQD.

**Theorem 1** *For any bipartite quantum state* $\rho_{AB}$

$$Q_{GW}(\rho_{AB}) = (1 - 2t_1t_2)Q_G(\rho_{AB}) \tag{7}$$

*with* $t_1, t_2$ *defined by Eq.* (2).

*Proof* By straightforward calculation, we have

$$\begin{aligned}\hat{\Pi}(\rho_{AB}) &= P(+)\rho_{AB}P(+) + P(-)\rho_{AB}P(-)\\ &= 2t_1t_2\rho_{AB} + (1 - 2t_1t_2)(\Pi_1\rho_{AB}\Pi_1 + \Pi_2\rho_{AB}\Pi_2).\end{aligned}$$

Thus,

$$
\begin{aligned}
&Q_{GW}(\rho_{AB})\\
&\quad= \min_{\hat{\Pi}} \|\rho_{AB} - \hat{\Pi}(\rho_{AB})\|\\
&\quad= \min_{\hat{\Pi}} \sqrt{tr\rho_{AB}^2 - 2tr\rho_{AB}\hat{\Pi}(\rho_{AB}) + tr\hat{\Pi}(\rho_{AB})\hat{\Pi}(\rho_{AB})}\\
&\quad= \min_{\Pi} \sqrt{(1+4t_1^2t_2^2 - 4t_1t_2)(tr\rho_{AB}^2 - tr\rho_{AB}\Pi_1\rho_{AB}\Pi_1 - tr\rho_{AB}\Pi_2\rho_{AB}\Pi_2)}\\
&\quad= (1-2t_1t_2)\min_{\Pi} \|\rho_{AB} - \Pi(\rho_{AB})\|\\
&\quad= (1-2t_1t_2)Q_G(\rho_{AB}),
\end{aligned}
$$

which completes the proof. □

From Eq. (7), it can be easily seen that $Q_{GW}(\rho_{AB}) \leq Q_G(\rho_{AB})$ which is in sharp contrast to the result in Ref. [14]. Singh and Pati [14] have proved that for any state the SQD under weak measurement was always greater than QD revealed by projective measurement. On the other hand, this result is consistent with the statement in Ref. [20] where they proved that weak one-way deficit is less than one-way deficit [11]. This may be interpreted as that weak measurements are less invasive than projective measurements, so the Hilbert-Schmidt distance or von Neumann entropy from pre-measurement state to post-weak-measured state is less than that to post-projective-measured state. One can also find that since $(1-2t_1t_2)'_x \geq 0$, the GQDW is a monotonically increasing function of the measurement strength $x$. On the contrary, it has been showed [14] that SQD is a monotonically decreasing function of the measurement strength $x$. Accordingly, SQD and GQDW seem to be measuring different things.

One may raise the problem about non-contractivity of Hilbert-Schmidt norm [21, 22]. Fortunately, we can remedy this drawback by two skillful approaches and at the same time efficiently employ the low computational demands of Hilbert-Schmidt norm. We can replace $\rho_{AB}$ by $\sqrt{\rho_{AB}}$ [21] or $\frac{\rho_{AB}}{\|\rho_{AB}\|}$ [23]. The similar results will be obtained if one follows the analogous proof in Theorem 1.

Another significant meaning of Theorem 1 lies in the fact that we can give the explicit form of GQDW for any two-qubit state by drawing on the known results in [8]. Considering any two-qubit state $\rho_{AB}$ in Bloch representation:

$$
\rho_{AB} = \frac{1}{4}\left(\mathbf{I}\otimes\mathbf{I} + \vec{x}\vec{\sigma}\otimes\mathbf{I} + \mathbf{I}\otimes\vec{y}\vec{\sigma} + \sum_{ij=1}^{3} t_{ij}\sigma_i\otimes\sigma_j\right),
$$

where $\sigma_1, \sigma_2, \sigma_3$ are Pauli operators, we have

$$
Q_{GW}(\rho_{AB}) = \frac{1}{2}(1-2t_1t_2)\sqrt{\|\vec{x}\|^2 + \|T\|^2 - \lambda_{max}}, \tag{8}
$$

where $\lambda_{max}$ is the largest eigenvalue of matrix $\vec{x}\vec{x}^{\mathrm{T}} + TT^{\mathrm{T}}$.

## 3 Extraction processing

To begin with, we would like to present the concept of residual quantumness. From Theorem 1, we can see that GQDW merely captures the partial quantumness of quantum state compared with GQD. Thus, the residual quantumness can be written as

$$\Delta Q(\rho_{AB}) = Q_G(\rho_{AB}) - Q_{GW}(\rho_{AB}) = 2t_1t_2Q_G(\rho_{AB}). \tag{9}$$

Next, we will show that the residual quantumness can be further extracted by performing a projective measurement on the post-weak-measurement state.

**Theorem 2** *Let $\tilde{\rho}_{AB}$ denote the optimal post-weak-measurement state such that $Q_{GW}(\rho_{AB}) = \|\rho_{AB} - \tilde{\rho}_{AB}\|$, then we have*

$$Q_G(\tilde{\rho}_{AB}) = \Delta Q(\rho_{AB}).$$

*Proof* First noting that

$$\tilde{\rho}_{AB} = P(+)\rho_{AB}P(+) + P(-)\rho_{AB}P(-),$$

where $P(+) = t_1\tilde{\Pi}_1 + t_2\tilde{\Pi}_2$, $P(-) = t_2\tilde{\Pi}_1 + t_1\tilde{\Pi}_2$, then we have

$$Q_G(\tilde{\rho}_{AB}) = \min_{\Pi} \|\tilde{\rho}_{AB} - \sum_{i=1}^{2} \Pi_i\tilde{\rho}_{AB}\Pi_i\|, \tag{10}$$

where $\{\Pi_i\}$ is the projective measurement on $\tilde{\rho}_{AB}$. Since for $x \to \infty$, the weak measurement will reduce to a projective measurement, one can obtain

$$\lim_{x\to\infty} Q_G(\tilde{\rho}_{AB}) = \min_{\Pi} \|\sum_{j=1}^{2} \tilde{\Pi}_j\rho_{AB}\tilde{\Pi}_j - \sum_{i,j=1}^{2} \Pi_i\tilde{\Pi}_j\rho_{AB}\tilde{\Pi}_j\Pi_i\| = 0, \tag{11}$$

where $\{\tilde{\Pi}_j\}$ is the projective measurement on $\rho_{AB}$. Equality (11) holds if and only if $\{\Pi_i\} = \{\tilde{\Pi}_j\}$. Therefore the optimization process in Eq. (10) can be omitted, and it can be rewritten as

$$Q_G(\tilde{\rho}_{AB}) = \|\tilde{\rho}_{AB} - \sum_{i=1}^{2} \tilde{\Pi}_i\tilde{\rho}_{AB}\tilde{\Pi}_i\|. \tag{12}$$

Meanwhile, noticing that

$$P(+) = t_1\tilde{\Pi}_1 + t_2\tilde{\Pi}_2,\ P(-) = t_2\tilde{\Pi}_1 + t_1\tilde{\Pi}_2. \tag{13}$$

Substituting Eq. (13) into Eq. (12), we will arrive at

$$Q_G(\tilde{\rho}_{AB}) = 2t_1t_2\|\tilde{\Pi}_1\rho_{AB}\tilde{\Pi}_2 + \tilde{\Pi}_2\rho_{AB}\tilde{\Pi}_1\|. \tag{14}$$

On the other hand, the weak measurement in $\tilde{\rho}_{AB}$ is optimal if and only if $Q_{GW}(\rho_{AB}) = \|\rho_{AB} - \tilde{\rho}_{AB}\|$. Thus, the projectors in the weak measurement are also optimal for the corresponding $Q_G(\rho_{AB})$ for $x \to \infty$; otherwise, it will lead to two different $Q_G(\rho_{AB})s$ for the same $\rho_{AB}$. So we have

$$Q_G(\rho_{AB}) = \|\rho_{AB} - \sum_{i=1}^{2} \tilde{\Pi}_i \rho_{AB} \tilde{\Pi}_i\| \\ = \|\tilde{\Pi}_1 \rho_{AB} \tilde{\Pi}_2 + \tilde{\Pi}_2 \rho_{AB} \tilde{\Pi}_1\|. \quad (15)$$

Combining Eq. (14) and Eq. (15), we complete the proof. □

Since any generalized measurement can be realized as a sequence of weak measurements, a natural question is whether the quantum correlation can be extracted little by little by these weak measurements. We will give an affirmative answer to this question in the following theorem.

**Theorem 3** *Let $\rho_n$ be the final state after n steps optimal weak measurements with the same infinitesimal strength on the subsystem A such that $Q_{GW}(\rho_n) = \|\rho_n - \rho_{n+1}\|$ with $\rho_0 = \rho_{AB}$, then we will have*

$$Q_G(\rho_{AB}) = \lim_{x\to 0} \sum_{n=0}^{\infty} Q_{GW}(\rho_n).$$

*Proof* According to Theorem 2, we have

$$Q_G(\rho_n) = 2t_1 t_2 Q_G(\rho_{n-1})$$

which implies

$$Q_G(\rho_n) = (2t_1 t_2)^n Q_G(\rho_{AB}). \quad (16)$$

Eq. (7) shows that

$$Q_{GW}(\rho_n) = (1 - 2t_1 t_2) Q_G(\rho_n),$$

together with Eq. (16) mean

$$Q_{GW}(\rho_n) = (1 - 2t_1 t_2)(2t_1 t_2)^n Q_G(\rho_{AB}). \quad (17)$$

If the measurement strength is infinitesimal, that is $x \to 0$, summing both sides of Eq. (17) over $n$, we have

$$\lim_{x\to 0} \sum_{n=0}^{\infty} Q_{GW}(\rho_n) = \lim_{x\to 0} (1 - 2t_1 t_2) \sum_{n=0}^{\infty} (2t_1 t_2)^n Q_G(\rho_{AB}) = Q_G(\rho_{AB}).$$

The proof is completed. □

Theorem 3 shows that the quantum correlation can be extracted little by little by the infinitesimal weak measurements which suggests a conservation law for quantum correlation in any composite system. However, if we firstly carry out the projective measurements, the quantum state will become classical-quantum (zero-discord) state. Thus according to Theorem 2, the GQDW and GQD are simultaneously zero.

## 4 Level surfaces of GQDW and explicit example

In this section, we will depict the level surfaces of GQDW for two-qubit Bell-diagonal states and demonstrate the validity of our results by explicit example. We investigate the two-qubit Bell-diagonal states [26], which have the form

$$\rho_c = \frac{1}{4}\left(\mathbf{I}\otimes\mathbf{I} + \sum_{i=1}^{3} c_i\sigma_i\otimes\sigma_i\right),$$

where $\sigma_1, \sigma_2, \sigma_3$ are Pauli operators. The state is valid if $\tilde{c} = (c_1, c_2, c_3)$ belongs to the tetrahedron defined by the set of vertices $(-1, -1, -1)$, $(-1, 1, 1)$, $(1, -1, 1)$ and $(1, 1, -1)$ [26]. According to Eq. (8), we can obtain the explicit GQDW of $\rho_c$ as

$$Q_{GW}(\rho_c) = \frac{1}{2}(1-2t_1t_2)\sqrt{c_1^2+c_2^2+c_3^2-max\{c_1^2, c_2^2, c_3^2\}}. \tag{18}$$

For a certain strength of weak measurement, the zero-GQDW states are located on Cartesian axes while Eq. (18) reaches its maximal value at the vertices of the tetrahedron. On the other hand, by simple calculation, we achieve $(1-2t_1t_2)'_x \geq 0$ which implies GQDW will increase with the growth of strength of weak measurement for a certain Bell-diagonal state. Due to the symmetric properties of Eq. (18), the level surface [24,25] of GQDW can be easily depicted (see Fig. 1). Similar to GQD [8], the level surface of GQDW is composed of three identical intersecting cylinders which are running along the three Cartesian axes and their ends are cut off by the valid state tetrahedron. If the value of GQDW increases, the cylinders are cut into four identical pieces reaching out toward the vertices of tetrahedron, which stand for the four Bell states.

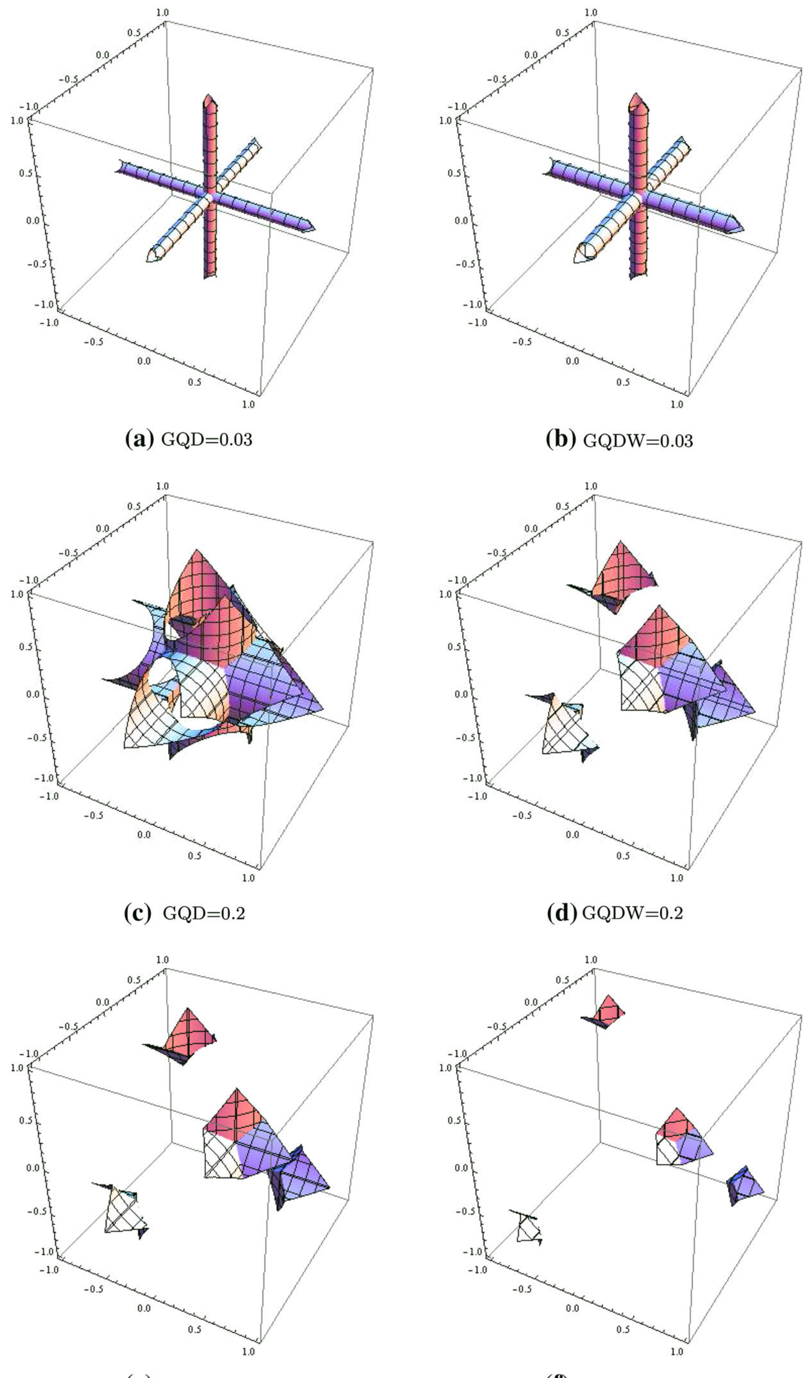
If we choose $|c_3| > |c_2| > |c_1|$, then from Eq. (18), we have

$$Q_G(\rho_c) = \frac{1}{2}\sqrt{c_1^2+c_2^2}, \tag{19}$$

and

$$Q_{GW}(\rho_c) = \frac{1}{2}(1-2t_1t_2)\sqrt{c_1^2+c_2^2}. \tag{20}$$

In addition, the optimal projectors for $Q_G(\rho_c)$ and $Q_{GW}(\rho_c)$ are $\tilde{\Pi}_1 = |0\rangle\langle 0|$ and $\tilde{\Pi}_2 = |1\rangle\langle 1|$. Then the corresponding weak measurements can be written as $P(+) = t_1\tilde{\Pi}_1 + t_2\tilde{\Pi}_2$ and $P(-) = t_2\tilde{\Pi}_1 + t_1\tilde{\Pi}_2$, and the post-weak-measurement state as [27]




**(a)** GQD=0.03 **(b)** GQDW=0.03

**(c)** GQD=0.2 **(d)** GQDW=0.2

**(e)** GQD=0.35 **(f)** GQDW=0.35

**Fig. 1** Level surfaces of constant GQD (*left*) and GQDW (*right*). All of level surfaces are cut off by the valid state tetrahedron

$$\begin{aligned}\tilde{\rho}_c &= P(+)\rho_c P(+) + P(-)\rho_c P(-) \\ &= \frac{1}{4}(\mathbf{I}\otimes\mathbf{I} + \sum_{i=1}^{3}\tilde{c}_i\sigma_i\otimes\sigma_i),\end{aligned}$$

where $\tilde{c}_1 = 2t_1t_2c_1$, $\tilde{c}_2 = 2t_1t_2c_2$ and $\tilde{c}_3 = c_3$. One will find that

$$Q_G(\tilde{\rho}_c) = t_1t_2\sqrt{c_1^2 + c_2^2}. \tag{21}$$

Furthermore, if we continue performing a weak measurement on $\tilde{\rho}_c$, then the twice post-weak-measurement state denoted by $\tilde{\rho}_2$ is still Bell-diagonal state with $\tilde{c}_1, \tilde{c}_2$ multiplying $2t_1t_2$ and $c_3$ leaving invariant. So we can obtain

$$Q_G(\tilde{\rho}_2) = (2t_1t_2)^2(\frac{1}{2}\sqrt{c_1^2 + c_2^2}), \tag{22}$$

and

$$Q_{GW}(\tilde{\rho}_2) = ((1 - 2t_1t_2))(2t_1t_2)^2(\frac{1}{2}\sqrt{c_1^2 + c_2^2}). \tag{23}$$

In sum, Eqs. (19, 20) demonstrate the validity of Theorem 2, Eqs. (19–21) show the validity of Theorem 3, and Eqs. (19–23) show the validity of Theorem 3.

## 5 Conclusion

In this paper, we have presented the definition of GQDW. It was shown that, for any bipartite quantum state, GQDW is linearly dependent on and less than GQD [8]. To quantify their differences, we proposed the term "residual quantumness." It is interesting that the residual quantumness can be further extracted by performing projective measurement on the post-weak-measurement state. Besides, we found that the quantum correlation can be extracted little by little by a sequence of weak measurements. However, if we firstly carry out the projective measurements on the quantum state, both of GQD and GQDW will be zero. This is natural since the projective measurement can influence the state strongly, and then we can capture more quantumness from the perspective of distance. Finally, as a demonstration, we showed the geometric properties of GQDW for the Bell-diagonal states and the validity of our results.

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