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A note on one-way quantum deficit and quantum discord

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Abstract One-way quantum deficit and quantum discord are two important measures of quantum correlations. We revisit the relationship between them in two-qubit systems. We investigate the conditions that both one-way quantum deficit and quantum discord have the same optimal measurement ensembles, and demonstrate that one-way quantum deficit can be derived from the quantum discord for a class of X states. Moreover, we give an explicit relation between one-way quantum deficit and entanglement of formation. We show that under phase damping channel both one-way quantum deficit and quantum discord evolve exactly in the same way for four parameter X states. Some examples are presented in details.

Keywords One-way quantum deficit · Quantum discord · Entanglement of formation

1 Introduction

Quantum entanglement plays important roles in quantum information and quantum computation [1]. However, some quantum states without quantum entanglement can also perform quantum tasks [2,3] such as quantum state discrimination [4,5], remote state preparation [6], quantum state merging [7,8], which have led to new definitions of quantum correlations such as quantum discord [9,10], one-way quantum deficit [11–14], and various "discord-like" measures [15].

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One-way quantum deficit was first proposed by Oppenheim et al. [11] for studying thermodynamical systems. They considered the amount of work which could be extracted from a heat bath by local operations. It quantifies the minimum distillable entanglement generated between the whole system and the measurement apparatus in measuring one subsystem of the whole system [16]. The analytical formulae of one-way quantum deficit are not known even for two-qubit states. With limited analytical results [17,18], many discussions on quantum deficit only rely on numerical results, since it involves minimization of sum of local and conditional entropies.

Another famous measure of quantum correlations, the quantum discord [9,10], is defined to be the difference of two classically equivalent expressions for the mutual information. There have been a lot of results on quantum discord for bipartite as well as multipartite mixed quantum states [15]. Nevertheless, due to the optimization problem involved, it has been recently shown that calculating quantum discord is an NP complexity problem [19].

It is meaningful to link directly one-way quantum deficit to quantum discord. The relationship between quantum discord and one-way quantum deficit was first discussed in Ref. [14]. Horodecki et al. show that the one-way quantum deficit is upper bounded by the quantum discord for any bipartite quantum states. In Ref. [20], a tradeoff relationship between one-way unlocalizable quantum discord and one-way unlocalizable quantum deficit has been presented. The tradeoff relationship between quantum discord and one-way quantum deficit is obtained [21].

Anyway, decisive results between quantum discord and one-way quantum deficit are not fully explored even for the two-qubit X states yet. Here, we revisit the relationship between one-way quantum deficit and quantum discord. We find that for special two-qubit X states the one-way quantum deficit can be derived from quantum discord exactly in some optimal measurement bases. Furthermore, we connect one-way quantum deficit to entanglement of formation directly.

To capture the non-classical correlations in bipartite systems, let us recall the following two popular measures of quantum correlations.

One-way quantum deficit Suppose Alice and Bob are allowed to perform only local operations. Consider a one-way classical communication, say, from Alice to Bob. The amount of information extractable from quantum system ϱ^{AB} is given by $\mathcal{I}_e = \log_2 \mathcal{D} - S(\varrho^{AB})$, where $\mathcal{D}$ is the dimension of the Hilbert space and $S(\varrho) = -\mathrm{Tr}[\varrho \log_2 \varrho]$ is the von Neumann entropy of a quantum state ϱ.

The classical operations to extract the amount of information from the quantum state is $\mathcal{I}_o = \log_2 \mathcal{D} - \min S((\varrho^{AB})')$, where $(\varrho^{AB})' = \sum_k M_k^A \varrho^{AB} M_k^A$ is the quantum state after measurement M_k^A has been performed on A. The one-way quantum deficit [11–14] is given by the difference of $\mathcal{I}_e$ and $\mathcal{I}_o$ [16],

$$\begin{aligned}\overrightarrow{\Delta} &= \mathcal{I}_e - \mathcal{I}_o \\ &= \min S\left(\sum_k M_k^A \varrho^{AB} M_k^A\right) - S\left(\varrho^{AB}\right). \end{aligned} \tag{1}$$

The minimum is taken over all local measurements M_k^A. This quantity is equal to the thermal discord [22].

Quantum discord The quantum discord is defined as the minimal difference between quantum mutual information and classical correlation. The quantum mutual information is denoted by $\mathcal{I}(\varrho^{AB}) = S(\varrho^A) + S(\varrho^B) - S(\varrho^{AB})$, which is also identified as the total correlation of the bipartite quantum system ϱ^{AB}. The $\varrho^{A(B)}$ are the reduced density matrices $\mathrm{Tr}_{B(A)}\varrho^{AB}$, respectively. Let $\{M_k^A\}$ be a measurement on subsystem A. Classical correlation is given as $\mathcal{J}(\varrho^{AB}) = S(\varrho^B) - \min\sum_k p_k S(\varrho^B_{M_k^A})$, where $p_k = \mathrm{Tr}(M_k^A \otimes I_2\varrho^{AB})$ is the probability of kth measurement outcome and $\varrho^B_{M_k^A} = \mathrm{Tr}_A[M_k^A \otimes I_2\varrho^{AB}]/p_k$ is the post-measurement state.

The quantum discord [9,10] is defined by

$$\begin{aligned} \overrightarrow{\delta} &= \mathcal{I}\left(\varrho^{AB}\right) - \mathcal{J}\left(\varrho^{AB}\right) \\ &= S\left(\varrho^{A}\right) + \min\sum_k p_k S\left(\varrho^B_{M_k^A}\right) - S\left(\varrho^{AB}\right). \end{aligned} \tag{2}$$

The superscript "$\rightharpoonup$" stands for that the measurement performed on subsystem A. The minimum is taken over all possible measurements $\{M_k^A\}$ on the subsystem A.

2 Linking one-way quantum deficit to quantum discord

Let us consider bipartite systems in Hilbert space $\mathbb{C}^2 \otimes \mathbb{C}^2$. Generally, the quantum correlations are invariant under local unitary operations [15]. Hence, one can write the X states [23] in the form

$$\varrho^{AB} = \frac{1}{4}\left(I_2 \otimes I_2 + a\sigma_z \otimes I_2 + bI_2 \otimes \sigma_z + \sum_{i\in\{x,y,z\}} c_i\sigma_i \otimes \sigma_i\right), \tag{3}$$

where σ_i $(i \in \{x, y, z\})$ are Pauli matrices, I_2 is the identity matrix, and the parameters $\{a, b, c_x, c_y, c_z\} \in [-1, 1]$ are real numbers.

The optimal measurement with measurement operators satisfying $M_k^A \geqslant 0$, $\sum_k M_k^A = I$, is generally positive operator-valued measurement (POVM). For rank-two two-qubit systems, the optimal measurement is just projective ones [24]. It is also sufficient to consider projective measurement for rank three and four [25].

Let $M_k^A = |k'\rangle\langle k'|$, $k \in \{0, 1\}$, where

$$|0'\rangle = \cos(\theta/2)|0\rangle - e^{-i\phi}\sin(\theta/2)|1\rangle, \tag{4}$$
$$|1'\rangle = e^{i\phi}\sin(\theta/2)|0\rangle + \cos(\theta/2)|1\rangle. \tag{5}$$

For the given system (3), we obtain $\overrightarrow{\delta} = S(\varrho^A) + \min\sum_k p_k S(\varrho^B_{M_k^A}) - S(\varrho^{AB})$, in which

$$p_{k\in\{0,1\}} = \frac{1}{2}(1 \pm a\cos\theta), \tag{6}$$

$S(\varrho^A) = h(\frac{1+a}{2})$ with $h(x) = -x\log_2 x - (1-x)\log_2(1-x)$,

$$\begin{aligned}\sum_k p_k S\left(\varrho^B_{M_k^A}\right) &= p_0 S\left(\varrho^B_{M_0^A}\right) + p_1 S\left(\varrho^B_{M_1^A}\right)\\ &= -\sum_{k,j\in\{0,1\}} p_k w_{kj}\log w_{kj},\end{aligned} \tag{7}$$

with w_{00}, w_{01} and w_{10}, w_{11} the eigenvalues of $\varrho^B_{M_0^A}$ and $\varrho^B_{M_1^A}$, respectively,

$$\begin{aligned}w_{kj\in\{0,1\}} = &\Big\{1 + (-1)^k a\cos\theta\\ &+ (-1)^j\sqrt{\left[c_x^2\cos^2\phi + c_y^2\sin^2\phi\right]\sin^2\theta + \left[b + (-1)^k c_z\cos\theta\right]^2}\Big\}\Big/ (4p_k).\end{aligned} \tag{8}$$

The corresponding quantity $S(\sum_k M_k^A\varrho^{AB}M_k^A)$ in the definition of one-way quantum deficit is given by

$$\begin{aligned}S\left(\sum_k M_k^A\varrho^{AB}M_k^A\right) &= S\left(M_0^A\otimes p_0\varrho^B_{M_0^A} + M_1^A\otimes p_1\varrho^B_{M_1^A}\right)\\ &= S\left(p_0\varrho^B_{M_0^A}\right) + S\left(p_1\varrho^B_{M_1^A}\right)\\ &= -\sum_{k,j\in\{0,1\}} p_k w_{kj}\log p_k w_{kj}\\ &= h(p_0) - \sum_{k,j\in\{0,1\}} p_k w_{kj}\log w_{kj}.\end{aligned} \tag{9}$$

Substituting Eq. (7) into above equation, we have

$$S\left(\sum_k M_k^A\varrho^{AB}M_k^A\right) = h(p_0) + \sum_k p_k S\left(\varrho^B_{M_k^A}\right), \tag{10}$$

which is joint entropy theorem [26].

Let us set

$$\mathcal{F} = S(\varrho^A) + \sum_k p_k S\left(\varrho^B_{M_k^A}\right) - S\left(\varrho^{AB}\right), \tag{11}$$

$$\mathcal{G} = S\left(\sum_k M_k^A\varrho^{AB}M_k^A\right) - S\left(\varrho^{AB}\right). \tag{12}$$

Inserting Eqs. (7), (9) into Eqs. (11), (12), respectively, we have

$$\mathcal{F} = S(\varrho^A) - \sum_{k,j\in\{0,1\}} p_k w_{kj} \log w_{kj} - S(\varrho^{AB}), \tag{13}$$

$$\mathcal{G} = h(p_0) - \sum_{k,j\in\{0,1\}} p_k w_{kj} \log w_{kj} - S(\varrho^{AB}). \tag{14}$$

To search for the minimization involved in computing quantum discord and one-way quantum deficit is equivalent to seek for the minimal value of the function $\mathcal{F}$ and $\mathcal{G}$ with respect to the two parameters θ and ϕ in the measurement operators. According to the similar technique used in calculating the quantum discord which only need to minimize $\mathcal{F}(\theta, 0)$ [27], here to minimize $\mathcal{G}(\theta, \phi)$, we only need to minimize $\mathcal{G}(\theta, 0)$ in calculating the one-way quantum deficit. We denote

$$G(\theta,\phi) = S\left(\sum_k M_k^A \varrho^{AB} M_k^A\right) = -\sum_{k,j\in\{0,1\}} p_k w_{kj} \log p_k w_{kj} = -\sum_{l=1}^{4} \lambda_l \log_2 \lambda_l,$$

where,

$$\lambda_{1,2} = \frac{1}{4}\left(p_0 \pm \sqrt{R+T_0}\right), \quad \lambda_{3,4} = \frac{1}{4}\left(p_1 \pm \sqrt{R+T_1}\right),$$

and $p_0 = 1 + a\cos\theta$, $p_1 = 1 - a\cos\theta$, $R = [c_x^2 \cos^2\phi + c_y^2 \sin^2\phi]\sin^2\theta$, $T_0 = (b + c_z\cos\theta)^2$, $T_1 = (b - c_z\cos\theta)^2$. Since $\lambda_l \geqslant 0$, one has $p_k \geqslant \sqrt{R+T_k} \geqslant 0$.

Noting that $G(\theta,\phi) = G(\pi-\theta,\phi) = G(\theta, 2\pi-\phi)$ and $G(\theta,\phi)$ is symmetric with respect to $\theta = \pi/2$ and $\phi = \pi$, we only need to consider the case of $\theta \in [0, \pi/2]$ and $\phi \in [0, \pi)$. The extreme points of $G(\theta,\phi)$ are determined by the first partial derivatives of G with respect to θ and ϕ,

$$\frac{\partial G}{\partial \theta} = -\frac{\sin\theta}{4} H_\theta, \tag{15}$$

with

$$H_\theta = \frac{R\csc\theta\cot\theta - c_z\sqrt{T_0}}{\sqrt{R+T_0}} \log_2 \frac{p_0 + \sqrt{R+T_0}}{p_0 - \sqrt{R+T_0}} + a\log_2 \frac{p_1^2 - (R+T_1)}{p_0^2 - (R+T_0)} + \frac{R\csc\theta\cot\theta + c_z\sqrt{T_1}}{\sqrt{R+T_1}} \log_2 \frac{p_1 + \sqrt{R+T_1}}{p_1 - \sqrt{R+T_1}}, \tag{16}$$

and

$$\frac{\partial G}{\partial \phi} = 2\,ef \sin^2\theta \, \sin 2\phi \, H_\phi, \tag{17}$$

with

$$H_\phi = \frac{1}{\sqrt{R+T_0}} \log_2 \frac{p_0 + \sqrt{R+T_0}}{p_0 - \sqrt{R+T_0}} + \frac{1}{\sqrt{R+T_1}} \log_2 \frac{p_1 + \sqrt{R+T_1}}{p_1 - \sqrt{R+T_1}}, \quad (18)$$

$e = \frac{1}{4}|c_x + c_y|$ and $f = \frac{1}{4}|c_x - c_y|$ where the absolute values have been taken since the phase for X states can be always removed by local unitary operation [15].

As H_ϕ is always positive, $\frac{\partial G}{\partial \phi} = 0$ implies that either $\phi = 0, \pi/2$ for any θ, or $\theta = 0$ for any ϕ which implies that Eq. (15) is zero and the minimization is independent on ϕ. If $\theta \neq 0$, one gets the second derivative of G,

$$\left.\frac{\partial^2 G}{\partial \phi^2}\right|_{(\theta,0)} = 4ef \sin^2(\theta) H_{\phi=0} > 0,$$

and

$$\left.\frac{\partial^2 G}{\partial \phi^2}\right|_{(\theta,\pi/2)} = -4ef \sin^2(\theta) H_{\phi=\pi/2} < 0.$$

Since for any θ the second derivative $\partial^2 G/\partial\phi^2$ is always negative for $\phi = \pi/2$, we only need to deal with the minimization problem for the case of $\phi = 0$. To minimize $G(\theta, \phi)$ becomes to minimize $G(\theta, 0)$. Thus, we need only to find the minimal value of $\mathcal{F}$ and $\mathcal{G}$ by varying θ only.

Denote $\mathcal{F}(\theta) = \mathcal{F}|_{\phi=0}$, $\mathcal{G}(\theta) = \mathcal{G}|_{\phi=0}$ and $\mathcal{H}(\theta) = \mathcal{G}(\theta) - \mathcal{F}(\theta)$. The first derivative of $\mathcal{H}(\theta)$ with respect to θ is given by

$$\mathcal{H}(\theta)' = \frac{a}{2} \sin\theta \log_2 \frac{1 + a\cos\theta}{1 - a\cos\theta}. \quad (19)$$

From $\mathcal{H}(\theta)' = 0$, we have either $a = 0$ or $\theta = 0, \pi/2$. Since these stationary points make $\mathcal{F}(\theta)' = \mathcal{G}(\theta)'$, they are the sufficient conditions that both $\mathcal{G}(\theta)$ and $\mathcal{F}(\theta)$ reach the minimum with the same optimal measurement ensemble. Here a is a parameter of the X states and θ is a parameter related to measurement. Substituting $a = 0$ or $\theta = 0, \pi/2$ into $\mathcal{F}(\theta)$ and $\mathcal{G}(\theta)$, we have the following results:

Theorem *For two-qubit X states, if the measurement is performed on the subsystem A (resp. B), then $\overset{\rightharpoonup}{\Delta} = \overset{\rightharpoonup}{\delta}$ for $a = 0$ (resp. $b = 0$). Depending on the parameters of the state, the optimum is either at $\theta = 0$ or at $\theta = \pi/2$. If the optimum is at $\theta = 0$, then $\overset{\rightharpoonup}{\Delta} = \overset{\rightharpoonup}{\delta}$. If the optimum is at $\theta = \pi/2$, then $\overset{\rightharpoonup}{\Delta} = \overset{\rightharpoonup}{\delta} - S(\varrho^A) + 1$.*

Recently, we notice that in Ref. [28], the authors assumed that the quantum discord and one-way quantum deficit get their minimal values in the same measurement ensemble simultaneously. Thus similar to the quantum discord, the frozen quantum phenomenon under bit flip channels of one-way quantum deficit happens. Here, our *Theorem* gives the explicit conditions that both quantum discord and one-way quantum deficit have the same optimal measurement bases.

Corollary 1 *The one-way quantum deficit is bounded by the quantum discord for two-qubit X states,*

$$\overrightarrow{\delta} \leqslant \overrightarrow{\Delta} \leqslant S(\rho^A). \tag{20}$$

Proof Since $0 \leqslant \mathcal{H} \leqslant 1$, we have $\overrightarrow{\delta} \leqslant \overrightarrow{\Delta} \leqslant \overrightarrow{\delta} + 1$. By using the tight bound about one-way quantum deficit $\Delta \leqslant S(\rho^A)$ in Ref. [26], we obtain (20). □

Corollary 2 *One-way quantum deficit and the entanglement of formation satisfy the following relations for two-qubit X states,*

$$\overrightarrow{\Delta} = E_f(\varrho^{BC}) - S(\varrho^{AB}) + \begin{cases} 1, & a = 0 \text{ or } \theta = \pi/2; \\ h\left(\frac{1-a}{2}\right), & \theta = 0, \end{cases} \tag{21}$$

where C is the assisted system to purify the state ϱ^{AB}, and $E_f(\varrho^{BC})$ is the entanglement of formation of ϱ^{BC}, while ϱ^{BC} is the reduced state from a pure state $|\psi\rangle_{ABC}$.

Proof From the Koashi–Winter equality [29]

$$S(\varrho^B) = \mathcal{J}(\varrho^{AB}) + E_f(\varrho^{BC}), \tag{22}$$

and $\mathcal{J}(\varrho^{AB}) = S(\varrho^B) - \min \sum_k p_k S(\varrho^B_{M^A_k})$, one has $E_f(\varrho^{BC}) = \min \sum_k p_k S(\varrho^B_{M^A_k})$. Consequently, quantum discord is rewritten as

$$\overrightarrow{\delta} = S(\varrho^A) + E_f(\varrho^{BC}) - S(\varrho^{AB}). \tag{23}$$

Thus, we have

$$\overrightarrow{\Delta} = \overrightarrow{\delta} = h\left(\frac{1-a}{2}\right) + E_f(\varrho^{BC}) - S(\varrho^{AB}), \tag{24}$$

where both of the optimal measurement bases are taken at $\theta = 0$. Hence for $a = 0$, we have $\overrightarrow{\Delta} = E_f(\varrho^{BC}) - S(\varrho^{AB}) + 1$ indeed. For $\theta = \pi/2$, by using the relations in *Theorem* and Eq. (23) we also get $\overrightarrow{\Delta} = E_f(\varrho^{BC}) - S(\varrho^{AB}) + 1$. □

Remark Recently, in Ref. [21] by using measure of relative entropy of coherence,

$$C_{RE}(\varrho^A) = \min_{\sigma \in \mathcal{I}} S(\varrho^A || \sigma), \tag{25}$$

where $\mathcal{I}$ stands for the set of decoherence states $\sigma = \sum_i \mu_i |i\rangle\langle i|$ with $\mu_i \in [0, 1]$ and $\sum_i \mu_i = 1$, and the authors provided a tradeoff relationship between $\overrightarrow{\delta}$ and $\overrightarrow{\Delta}$, i. e., $\overrightarrow{\delta} + C_{RE}(\varrho^A) = \overrightarrow{\Delta}$.

In fact, one-way quantum deficit can be derived from quantum discord directly. We consider the exact relationship between quantum discord and one-way quantum deficit in the following examples.

Example 1 The Bell-diagonal state $\varrho_{Bell}^{AB} = \frac{1}{4}(I_2 \otimes I_2 + \sum_{i\in\{x,y,z\}} c_i\sigma_i \otimes \sigma_i)$. In this case $a = 0$ and

$$\overrightarrow{\Delta} = \overrightarrow{\delta} = h\left(\frac{1-c}{2}\right) + \sum_{s\in\{jkl\}} A_s \log_2 A_s, \tag{26}$$

where s is the set $\{jkl\} = \{111, 100, 010, 001\}$, $A_{jkl} = \frac{1}{4}(1 + (-1)^j c_x + (-1)^k c_y + (-1)^l c_z)$, and $c \equiv \max\{|c_x|, |c_y|, |c_z|\}$. Therefore, from *Theorem* we get the analytical expression of one-way quantum deficit from quantum discord given in [30].

Example 2 Consider a class of X-state,

$$\varrho_q^{AB} = q|\psi^-\rangle\langle\psi^-| + (1-q)|00\rangle\langle 00|, \tag{27}$$

where $|\psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$.

For this state, quantum discord is derived at $\theta = \pi/2$ for $q \in [0, 1]$. The optimal basis of one-way quantum deficit for $q \in [0.67, 1]$ is also at $\theta = \pi/2$. The value 0.67 is the solution of $H'_\theta|_{\theta=\pi/2,\phi=0} = 0$ in Eq. (16) for the state ϱ_q^{AB}. According to the *Theorem*, we have

$$\overrightarrow{\Delta} = E_f(\varrho^{BC}) - S(\varrho^{AB}) + 1, \tag{28}$$

where the entanglement of formation

$$E_f(\varrho^{BC}) = h\left(\frac{1+\sqrt{1-\mathcal{C}^2}}{2}\right) \tag{29}$$

with concurrence $\mathcal{C} = \sqrt{2q(1-q)}$. So analytical one-way quantum deficit of state ϱ_q^{AB} is

$$\overrightarrow{\Delta} = h\left(\frac{1+\sqrt{1-\mathcal{C}^2}}{2}\right) - h(q) + 1 \tag{30}$$

for $q \in [0.67, 1]$, see Fig. 1.

3 Quantum correlations under phase damping channel

A quantum system would be subject to interaction with environments. We consider now the evolution of one-way quantum deficit and quantum discord under noisy channels. Consider a class of initial two-qubit states,

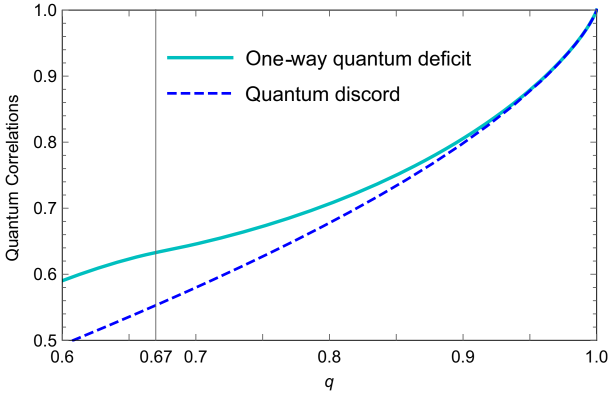

Fig. 1 One-way quantum deficit (*turquoise solid line*) and quantum discord (*blue dashed line*) versus q. The interval $q \in [0.67, 1]$ one-way quantum deficit and quantum discord both get their optimum at $\theta = \pi/2$ (Color figure online)

$$\Omega = \frac{1}{4}\left(I_2 \otimes I_2 + bI_2 \otimes \sigma_z + \sum_{i \in \{x,y,z\}} c_i \sigma_i \otimes \sigma_i \right). \tag{31}$$

If both two qubits independently go through a channel given by the Kraus operators $\{K_i\}$, $\sum_i K_i^\dagger K_i = I$. The state Ω evolves into

$$\tilde{\Omega} = \sum_{i,j \in \{1,2\}} K_i^A \otimes K_j^B \cdot \Omega \cdot \left[K_i^A \otimes K_j^B \right]^\dagger. \tag{32}$$

For phase damping channels [31], the Kraus operators are given by $K_1^{A(B)} = |0\rangle\langle 0| + \sqrt{1-\gamma}|1\rangle\langle 1|$, and $K_2^{A(B)} = \sqrt{\gamma}|1\rangle\langle 1|$ with the decoherence rate $\gamma \in [0, 1]$. Thus we have

$$\tilde{\Omega} = \frac{1}{4}\left[I_2 \otimes I_2 + bI_2 \otimes \sigma_z + c_z \sigma_z \otimes \sigma_z + \sum_{i \in \{x,y\}} (1-\gamma) c_i \sigma_i \otimes \sigma_i \right], \tag{33}$$

which is a two-qubit X state with $a = 0$. From the *Theorem*, we obtain one-way quantum deficit and quantum discord performed on the subsystem A evolve coincidentally with each other all the time.

For example, we draw the quantum discord and one-way quantum deficit vs parameter γ in Fig. 2 for $b = 0.26$, $c_x = 0.13$, $c_z = 0.08$, and $c_y = 0.15, 0.25, 0.35, 0.45, 0.55$, respectively.

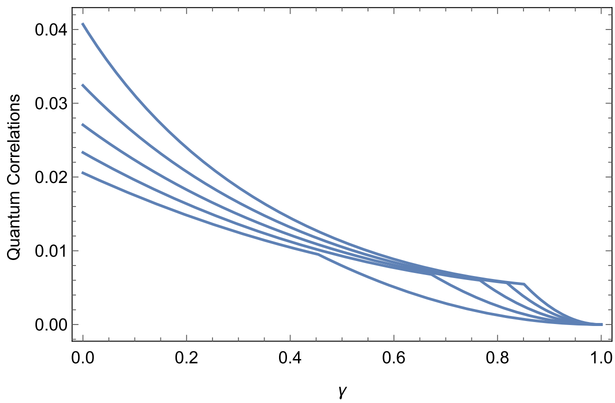

Fig. 2 One-way quantum deficit and quantum discord evolve exactly in the same way under phase damping channel. The *solid lines* from *bottom* to *top* correspond $c_y = 0.15, 0.25, 0.35, 0.45, 0.55$, respectively, for fixed parameters $b = 0.26$, $c_x = 0.13$, and $c_z = 0.08$

4 Conclusions

We have investigated the connections between one-way quantum deficit and quantum discord for two-qubit X states. Sufficient conditions are given that the one-way quantum deficit can be derived from quantum discord directly. The explicit relation between one-way quantum deficit and entanglement of formation is also presented. Moreover, we have shown that the one-way quantum deficit and quantum discord of a class of four parameters X states evolve coincidentally under phase damping channel. Our results may enlighten the understanding on the relations between one-way quantum deficit and quantum discord. It is also interesting to study the relationship between one-way quantum deficit and quantum discord for higher-dimensional and multipartite systems.

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