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Optimal joint remote state preparation of equatorial states

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Abstract We present a scheme for optimal joint remote state preparation of two-qubit equatorial states. Our protocol improves on a previous scheme (Choudhury and Dhara in Quantum Inf Process 14:373–379, 2015) that had a success probability of 25 %, which increased to 50 % when extra classical information is sent to the receiver. We show that using our modified scheme, the desired state can be prepared deterministically with the same quantum channel. Moreover, we generalize the scheme to prepare N-qubit equatorial states in which the receiver can reconstruct the original state with 100 % success probability.

Keywords Joint remote state preparation · Equatorial N-qubit state · Succeed deterministically

1 Introduction

Remote state preparation (RSP) [1], like quantum teleportation [2], is a novel way to transmit a quantum state between distant parties without physically sending the

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state itself. Although it is only applicable to known states, RSP requires less classical communication than quantum teleportation [3]. Moreover, the two resources for quantum communication—quantum entanglement and classical communication—can be traded off against each other in RSP schemes. The interesting properties of RSP have been widely studied theoretically [4–8]. Several schemes have been proposed for RSP of different input states, RSP using various entangled channels, and RSP with different numbers of senders and receivers [9–16]. Remote state preparation has also been studied experimentally in recent years [17–24].

To satisfy the requirements of different communication scenarios, RSP has several variants, one of which is called joint remote state preparation (JRSP) [25,26]. In JRSP, the knowledge of the state to be prepared is shared by several senders, each of them having partial information. The receiver has no information about the state. Only when all the senders collaborate can the receiver reconstruct the desired state via some operations on his/her own particles. Many novel JRSP schemes have been designed for different types of quantum states using a variety of quantum channels [25–35]. Most recently, a JRSP scheme for preparing two-qubit equatorial states was proposed by Choudhury and Dhara [36]. In this scheme, two senders each have partial information about the state to be prepared and the quantum channel is composed of two maximally entangled three-qubit Greenberger–Horne–Zeilinger (GHZ) states. After the two senders apply projective measurements on their qubits and transmit their measurement outcomes, the receiver can reconstruct the original state with a success probability of 25 %. The authors also showed that the success probability can be increased to 50 % if one sender transmits extra classical information to the receiver. We henceforth refer to this scheme as the CD protocol.

In this paper, we revisit the scenario for JRSP of two-qubit equatorial states explored in Ref. [36] and show that the CD protocol is not optimal. Our analysis demonstrates that by modifying the measurement basis of the two senders, the receiver can deterministically obtain the desired state with proper unitary operations. Moreover, we extend the scheme to JRSP of N-qubit equatorial states and explicitly describe the general form of the senders' measurement bases. In our scheme, the receiver can always perform a unitary operation corresponding to every possible measurement outcome of the senders and reconstruct the state with 100 % success probability.

2 JRSP of an arbitrary equatorial two-qubit state

There are three spatially separated parties in this JRSP scheme, the two senders Alice and Bob and the receiver Charlie. Alice and Bob wish to help Charlie prepare an arbitrary equatorial two-qubit state written as

$$|\Phi\rangle = \frac{1}{2}(|00\rangle + \mathrm{e}^{i\delta_1}|01\rangle + \mathrm{e}^{i\delta_2}|10\rangle + \mathrm{e}^{i\delta_3}|11\rangle). \tag{1}$$

Here $\delta_j (j = 1, 2, 3)$ is a real phase parameter shared by the two senders. Alice and Bob have partial information about this state. They each know the parameters a_j and

b_j, respectively, such that

$$\delta_j = a_j + b_j, \quad (j = 1, 2, 3). \tag{2}$$

The receiver has no knowledge about the desired state at all. Only when the two senders collaborate will the receiver be able to reconstruct the two-qubit equatorial state in his location.

The quantum channel is composed of two maximally entangled three-qubit GHZ states.

$$|\Psi\rangle = |\mathrm{GHZ}_3\rangle_{A_1B_1C_1} \otimes |\mathrm{GHZ}_3\rangle_{A_2B_2C_2}, \tag{3}$$

$$|\mathrm{GHZ}_3\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle). \tag{4}$$

Particles A_j belong to Alice, and B_j, C_j belong to Bob and Charlie, respectively.

To help Charlie prepare the desired state, Alice and Bob perform projective measurements on their own qubits based on the partial information they have. In the scheme proposed by Choudhury and Dhara [36], the measurement basis for Alice and Bob was selected to be

$$|\varphi_0\rangle = \frac{1}{4}(|00\rangle + \mathrm{e}^{-ix_1}|01\rangle + \mathrm{e}^{-ix_2}|10\rangle + \mathrm{e}^{-ix_3}|11\rangle), \tag{5}$$

$$|\varphi_1\rangle = \frac{1}{4}(\mathrm{e}^{ix_1}|00\rangle - |01\rangle + \mathrm{e}^{ix_3}|10\rangle - \mathrm{e}^{ix_2}|11\rangle), \tag{6}$$

$$|\varphi_2\rangle = \frac{1}{4}(|00\rangle + \mathrm{e}^{-ix_1}|01\rangle - \mathrm{e}^{-ix_2}|10\rangle - \mathrm{e}^{-ix_3}|11\rangle), \tag{7}$$

$$|\varphi_3\rangle = \frac{1}{4}(\mathrm{e}^{ix_1}|00\rangle - |01\rangle - \mathrm{e}^{ix_3}|10\rangle + \mathrm{e}^{ix_2}|11\rangle). \tag{8}$$

Here $x = a(b)$ for Alice (Bob). Alice and Bob then send their measurement outcomes to Charlie, who performs unitary operations on his qubits to prepare the state $|\Phi\rangle$. However, for the measurement basis states described in Choudhury and Dhara's proposal [36], it is not always possible for Charlie to find a unitary operation that will recover the state $|\Phi\rangle$. For 4 of the 16 possible measurement outcomes, Charlie can perform operations to recover the input state $|\Phi\rangle$. Hence, the probability of success is 25 %. If Alice assists Charlie by sending him the values a_1, a_2 and a_3, then Charlie can successfully prepare the state $|\Phi\rangle$ for 8 of the 16 measurement outcomes, and thus the probability of success increases to 50 %.

We now show that the probability of success can be increased to 100 % by selecting a different measurement basis. The measurement basis for Alice and Bob is selected to be

$$|\varphi_0\rangle = \frac{1}{2}(|00\rangle + \mathrm{e}^{-ix_1}|01\rangle + \mathrm{e}^{-ix_2}|10\rangle + \mathrm{e}^{-ix_3}|11\rangle), \tag{9}$$

$$|\varphi_1\rangle = \frac{1}{2}(|00\rangle + i\mathrm{e}^{-ix_1}|01\rangle - \mathrm{e}^{-ix_2}|10\rangle - i\mathrm{e}^{-ix_3}|11\rangle), \tag{10}$$

$$|\varphi_2\rangle = \frac{1}{2}(|00\rangle - \mathrm{e}^{-ix_1}|01\rangle + \mathrm{e}^{-ix_2}|10\rangle - \mathrm{e}^{-ix_3}|11\rangle), \tag{11}$$

$$|\varphi_3\rangle = \frac{1}{2}(|00\rangle - i\mathrm{e}^{-ix_1}|01\rangle - \mathrm{e}^{-ix_2}|10\rangle + i\mathrm{e}^{-ix_3}|11\rangle). \tag{12}$$

Here $x = a(b)$ for Alice (Bob). The four states are mutually orthogonal, and Alice (Bob) can obtain each one of them with equal probability. The quantum channel can be rewritten in terms of Alice's and Bob's measurement basis as

$$\begin{aligned}|\Psi\rangle = \frac{1}{8}\ &[|\varphi_0\rangle_{A_1A_2}|\varphi_0\rangle_{B_1B_2}(|00\rangle + \mathrm{e}^{i\delta_1}|01\rangle + \mathrm{e}^{i\delta_2}|10\rangle + \mathrm{e}^{i\delta_3}|11\rangle)_{C_1C_2}\\
&+|\varphi_0\rangle_{A_1A_2}|\varphi_1\rangle_{B_1B_2}(|00\rangle - i\mathrm{e}^{i\delta_1}|01\rangle - \mathrm{e}^{i\delta_2}|10\rangle + i\mathrm{e}^{i\delta_3}|11\rangle)_{C_1C_2}\\
&+|\varphi_0\rangle_{A_1A_2}|\varphi_2\rangle_{B_1B_2}(|00\rangle - \mathrm{e}^{i\delta_1}|01\rangle + \mathrm{e}^{i\delta_2}|10\rangle - \mathrm{e}^{i\delta_3}|11\rangle)_{C_1C_2}\\
&+|\varphi_0\rangle_{A_1A_2}|\varphi_3\rangle_{B_1B_2}(|00\rangle + i\mathrm{e}^{i\delta_1}|01\rangle - \mathrm{e}^{i\delta_2}|10\rangle - i\mathrm{e}^{i\delta_3}|11\rangle)_{C_1C_2}\\
&+|\varphi_1\rangle_{A_1A_2}|\varphi_0\rangle_{B_1B_2}(|00\rangle - i\mathrm{e}^{i\delta_1}|01\rangle - \mathrm{e}^{i\delta_2}|10\rangle + i\mathrm{e}^{i\delta_3}|11\rangle)_{C_1C_2}\\
&+|\varphi_1\rangle_{A_1A_2}|\varphi_1\rangle_{B_1B_2}(|00\rangle - \mathrm{e}^{i\delta_1}|01\rangle + \mathrm{e}^{i\delta_2}|10\rangle - \mathrm{e}^{i\delta_3}|11\rangle)_{C_1C_2}\\
&+|\varphi_1\rangle_{A_1A_2}|\varphi_2\rangle_{B_1B_2}(|00\rangle + i\mathrm{e}^{i\delta_1}|01\rangle - \mathrm{e}^{i\delta_2}|10\rangle - i\mathrm{e}^{i\delta_3}|11\rangle)_{C_1C_2}\\
&+|\varphi_1\rangle_{A_1A_2}|\varphi_3\rangle_{B_1B_2}(|00\rangle + \mathrm{e}^{i\delta_1}|01\rangle + \mathrm{e}^{i\delta_2}|10\rangle + \mathrm{e}^{i\delta_3}|11\rangle)_{C_1C_2}\\
&+|\varphi_2\rangle_{A_1A_2}|\varphi_0\rangle_{B_1B_2}(|00\rangle - \mathrm{e}^{i\delta_1}|01\rangle + \mathrm{e}^{i\delta_2}|10\rangle - \mathrm{e}^{i\delta_3}|11\rangle)_{C_1C_2}\\
&+|\varphi_2\rangle_{A_1A_2}|\varphi_1\rangle_{B_1B_2}(|00\rangle + i\mathrm{e}^{i\delta_1}|01\rangle - \mathrm{e}^{i\delta_2}|10\rangle - i\mathrm{e}^{i\delta_3}|11\rangle)_{C_1C_2}\\
&+|\varphi_2\rangle_{A_1A_2}|\varphi_2\rangle_{B_1B_2}(|00\rangle + \mathrm{e}^{i\delta_1}|01\rangle + \mathrm{e}^{i\delta_2}|10\rangle + \mathrm{e}^{i\delta_3}|11\rangle)_{C_1C_2}\\
&+|\varphi_2\rangle_{A_1A_2}|\varphi_3\rangle_{B_1B_2}(|00\rangle - i\mathrm{e}^{i\delta_1}|01\rangle - \mathrm{e}^{i\delta_2}|10\rangle + i\mathrm{e}^{i\delta_3}|11\rangle)_{C_1C_2}\\
&+|\varphi_3\rangle_{A_1A_2}|\varphi_0\rangle_{B_1B_2}(|00\rangle + i\mathrm{e}^{i\delta_1}|01\rangle - \mathrm{e}^{i\delta_2}|10\rangle - i\mathrm{e}^{i\delta_3}|11\rangle)_{C_1C_2}\\
&+|\varphi_3\rangle_{A_1A_2}|\varphi_1\rangle_{B_1B_2}(|00\rangle + \mathrm{e}^{i\delta_1}|01\rangle + \mathrm{e}^{i\delta_2}|10\rangle + \mathrm{e}^{i\delta_3}|11\rangle)_{C_1C_2}\\
&+|\varphi_3\rangle_{A_1A_2}|\varphi_2\rangle_{B_1B_2}(|00\rangle - i\mathrm{e}^{i\delta_1}|01\rangle - \mathrm{e}^{i\delta_2}|10\rangle + i\mathrm{e}^{i\delta_3}|11\rangle)_{C_1C_2}\\
&+|\varphi_3\rangle_{A_1A_2}|\varphi_3\rangle_{B_1B_2}(|00\rangle - \mathrm{e}^{i\delta_1}|01\rangle + \mathrm{e}^{i\delta_2}|10\rangle - \mathrm{e}^{i\delta_3}|11\rangle)_{C_1C_2}.\end{aligned} \tag{13}$$

From this expression, it is clear that no matter what measurement results Alice and Bob get, the state of C_1C_2 can always be transformed into Eq.(1) via a unitary operation. For example, if Alice's and Bob's measurement results are $|\varphi_3\rangle_{A_1A_2}$ and $|\varphi_3\rangle_{B_1B_2}$, the unitary operation for C_1C_2 is $U = |00\rangle\langle 00| - |01\rangle\langle 01| + |10\rangle\langle 10| - |11\rangle\langle 11| = (I)_{C_1} \otimes (\sigma_z)_{C_2}$. Only when the measurement outcomes of both Alice and Bob are sent to Charlie can he prepare the desired state. The success probability of this JRSP scheme is 100 % in principle.

3 JRSP of an arbitrary equatorial N-qubit state

We now generalize our scheme to describe the deterministic JRSP of an arbitrary equatorial N-qubit state, which can be written as

$$|\Phi\rangle = \frac{1}{(\sqrt{2})^N}\sum_{l_1,\ldots,l_N=0}^{1}\exp(i\delta_j)|l_N, l_{N-1}, \ldots, l_1\rangle. \tag{14}$$

Here $j = 0, 1, 2, \ldots, 2^N - 1$, which is the decimal form of the binary string $(l_N, l_{N-1}, \ldots, l_1)$.

$$j = \sum_{n=1}^{N} 2^{n-1} l_n. \tag{15}$$

The two senders share partial information about the desired state with $a_j + b_j = \delta_j$ $(j = 0, 1, 2, \ldots 2^N - 1)$. The three parties share N three-qubit GHZ states in advance.

$$|\mathrm{GHZ}_3\rangle_{A_n B_n C_n} = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{A_n B_n C_n}, (n = 1, 2, \ldots, N). \tag{16}$$

Alice and Bob perform projective measurements on her/his particles $A_1, A_2, \ldots, A_N$ and $B_1, B_2, \ldots, B_N$, respectively. Their measurement basis can be written as

$$|\varphi_k\rangle = \frac{1}{(\sqrt{2})^N} \sum_{l_1,\ldots,l_N=0}^{1} \exp\left(\frac{2\pi ijk - ix_j}{4}\right) |l_N, l_{N-1}, \ldots, l_1\rangle, \tag{17}$$

where $k = 0, 1, 2, \ldots, 2^N - 1$ and $x = a(b)$ for Alice (Bob). Then the original N GHZ states can be rewritten in terms of Alice's and Bob's measurement bases as

$$\begin{aligned}
|\Psi\rangle &= |\mathrm{GHZ}_3\rangle_{A_1B_1C_1} \otimes |\mathrm{GHZ}_3\rangle_{A_2B_2C_2} \otimes \cdots \otimes |\mathrm{GHZ}_3\rangle_{A_NB_NC_N} \\
&= \frac{1}{2^N} \sum_{k_A=0}^{2^N-1} \sum_{k_B=0}^{2^N-1} |\varphi_{k_A}\rangle_{A_1A_2\ldots A_N} |\varphi_{k_B}\rangle_{B_1B_2\ldots B_N} \otimes \\
&\quad \left[\frac{1}{(\sqrt{2})^N} \sum_{l_1,\ldots,l_N=0}^{1} \exp\left(\frac{-2\pi ij(k_A + k_B)}{4}\right) \exp(i\delta_j)) |l_N, l_{N-1}, \ldots, l_1\rangle_{C_1C_2\ldots C_N} \right].
\end{aligned} \tag{18}$$

We find that no matter what measurement results Alice and Bob obtain, the state of Charlie's particles can always be transformed into the desired form via unitary operations:

$$\begin{aligned}
U_{k_Ak_B} &= \sum_{l_1,\ldots,l_N=0}^{1} \exp\left(\frac{2\pi ij(k_A + k_B)}{4}\right) |l_N, l_{N-1}, \ldots, l_1\rangle\langle l_N, l_{N-1}, \ldots, l_1| \\
&= \prod^{\otimes} \sum_{l_n=0}^{1} \exp\left(\frac{2\pi i \times 2^{n-1} l_n (k_A + k_B)}{4}\right) |l_n\rangle\langle l_n|.
\end{aligned} \tag{19}$$

Charlie only has to perform N single-qubit unitary operations based on Alice's and Bob's measurement results k_A and k_B in order to obtain the state.

4 Discussion

We have revisited the scheme described in Ref. [36] for JRSP of two-qubit equatorial states. We have shown that the success probability of the scheme can be improved from 25 to 100 % by choosing optimal measurement bases for the two senders. We have also successfully generalized our scheme to remotely prepare N-qubit equatorial states in a deterministic manner.

In our scheme, there are only two senders. Actually, the scheme can be simply changed to a JRSP protocol with M senders. Suppose each sender holds partial information where $a_j + b_j + \cdots + m_j = \delta_j$ $(j = 0, 1, \ldots 2^N - 1)$. In this case, the quantum channel should be N $(M + 1)$-qubit GHZ states. Each of the senders measures her/his own N particles in an appropriate basis $|\varphi\rangle$ based on her/his information. Then the receiver can obtain the desired state via unitary operations conditioned on the M senders' measurement results. Moreover, the scheme can also be generalized to a controlled RSP (CRSP) scheme [37–41] by increasing the number of qubits in the GHZ states. Generally, the controller has no information about the quantum state to be prepared. Therefore, he/she only needs to perform N single-particle measurements in the diagonal basis $|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$.

The equatorial states have some special properties that make them interesting for quantum information processing [42–44]. Since they contain less information compared to arbitrary quantum states, it should be easier to prepare equatorial states than arbitrary states. It was previously shown that a single-qubit equatorial state can be remotely prepared with one classical bit via the maximally entangled channel [4]. From our results, we can conjecture that the equatorial state can always be deterministically prepared via a proper maximally entangled channel.

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