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One-way deficit of two-qubit X states

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Abstract Quantum deficit originates in questions regarding work extraction from quantum systems coupled to a heat bath (Oppenheim et al. in Phys Rev Lett 89:180402, 2002). It links quantum correlations with quantum thermodynamics and provides a new standpoint for understanding quantum non-locality. In this paper, we propose a new method to evaluate the one-way deficit for a class of two-qubit states. The dynamic behavior of the one-way deficit under decoherence channel is investigated, and it is shown that the one-way deficit of the X states with five parameters is more robust against decoherence than entanglement.

Keywords One-way deficit · Concurrence · Phase flip channel

1 Introduction

Quantum entanglement is a resource in quantum information processing such as teleportation [1], super-dense coding [2], quantum cryptography [3], remote-state

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preparation [4,5], and so on. However, there are quantum correlations other than entanglement which are also useful and have attracted much attention recently [6–11]. One remarkable and widely accepted quantum correlation is quantum discord. Quantum discord is a measure of the difference between the mutual information and maximum classical mutual information, which is generally difficult to calculate even for two-qubit quantum system [12–16].

Other nonclassical correlations besides entanglement and quantum discord have arisen recently; for example, quantum deficit [17,18], measurement-induced disturbance [19], geometric discord [20,21], and continuous-variable discord [22,23], see a review [11]. Quantum deficit originates on asking how to use nonlocal operation to extract work from a correlated system coupled to a heat bath [17]. It is also closely related to other forms of quantum correlations. Oppenheim et al. [17] defined the work deficit

$$\Delta \equiv W_{\mathrm{t}} - W_{\mathrm{l}}, \tag{1}$$

where W_{t} is the information of the whole system and W_{l} is the localizable information [24,25]. As with quantum discord, quantum deficit is also equal to the difference in the mutual information and classical deficit [26]. Recently, Streltsov et al. [27,28] give the definition of the one-way information deficit (one-way deficit) in terms of relative entropy, which reveals an important role of quantum deficit as a resource for the distribution of entanglement. One-way deficit by von Neumann measurement on one side is given by [29]

$$\Delta^{\rightarrow}(\rho^{ab}) = \min_{\{\Pi_k\}} S\left(\sum_k \Pi_k \rho^{ab} \Pi_k\right) - S(\rho^{ab}). \tag{2}$$

From the definition, we can see that the one-way deficit and quantum discord are different kinds of quantum correlations. The one-way deficit is related to the work that can be extracted from the total system, and the work that can be extracted from the subsystems after suitable LOCC operations. Quantum discord quantifies the difference between the mutual information and maximal classical mutual information. Moreover, the minimizations involved in computing one-way deficit and quantum discord are also different. One may wonder whether the analytical formula or the calculation method for a class of two-qubit states like quantum discord can be obtained. In this paper, we will endeavor to calculate the one-way deficit for two-qubit X states with five parameters.

2 One-way deficit for X states with five parameters

We first introduce the form of two-qubit X states. By using appropriate local unitary transformations, we can write ρ^{ab} as

$$\rho^{ab} = \frac{1}{4}\left(I \otimes I + \mathbf{r} \cdot \sigma \otimes I + I \otimes \mathbf{s} \cdot \sigma + \sum_{i=1}^{3} c_i \sigma_i \otimes \sigma_i\right), \tag{3}$$

where $\mathbf{r}$ and $\mathbf{s}$ are Bloch vectors and $\{\sigma_i\}_{i=1}^{3}$ are standard Pauli matrices. When $\mathbf{r} = \mathbf{s} = \mathbf{0}$, ρ reduces to two-qubit Bell-diagonal states. When we assume that Bloch vectors are in the z direction, that is, $\mathbf{r} = (0, 0, r)$, $\mathbf{s} = (0, 0, s)$, the state in Eq. (3) has the following form

$$\rho^{ab} = \frac{1}{4}\left(I \otimes I + r\sigma_3 \otimes I + I \otimes s\sigma_3 + \sum_{i=1}^{3} c_i \sigma_i \otimes \sigma_i\right). \tag{4}$$

In the computational basis $|00\rangle, |01\rangle, |10\rangle, |11\rangle$, the density matrix of ρ^{ab} is

$$\rho = \frac{1}{4}\begin{pmatrix} 1 + r + s + c_3 & 0 & 0 & c_1 - c_2 \\ 0 & 1 + r - s - c_3 & c_1 + c_2 & 0 \\ 0 & c_1 + c_2 & 1 - r + s - c_3 & 0 \\ c_1 - c_2 & 0 & 0 & 1 - r - s + c_3 \end{pmatrix}. \tag{5}$$

From Eq. (4) in [14], after some algebraic calculations, we can obtain that parameters x, y, s, u, t in [14] can be substituted for r, s, c_1, c_2, c_3 of the X states in Eq. (5) successively and

$$r, s, c_1, c_2, c_3 \in [-1, 1]. \tag{6}$$

One can also change them to be x or y direction via an appropriate local unitary transformation without losing its diagonal property of the correlation terms [30].

The eigenvalues of the X states in Eq. (5) are given by

$$u_{\pm} = \frac{1}{4}\left[1 - c_3 \pm \sqrt{(r - s)^2 + (c_1 + c_2)^2}\right],$$
$$v_{\pm} = \frac{1}{4}\left[1 + c_3 \pm \sqrt{(r + s)^2 + (c_1 - c_2)^2}\right].$$

The entropy is given by

$$\begin{aligned} S(\rho) = 2 - \Bigg[&\frac{1}{4}\left(1 - c_3 + \sqrt{(r - s)^2 + (c_1 + c_2)^2}\right) \log\left(1 - c_3 + \sqrt{(r - s)^2 + (c_1 + c_2)^2}\right) \\ &+ \frac{1}{4}\left(1 - c_3 - \sqrt{(r - s)^2 + (c_1 + c_2)^2}\right) \log\left(1 - c_3 - \sqrt{(r - s)^2 + (c_1 + c_2)^2}\right) \\ &+ \frac{1}{4}\left(1 + c_3 + \sqrt{(r + s)^2 + (c_1 - c_2)^2}\right) \log\left(1 + c_3 + \sqrt{(r + s)^2 + (c_1 + c_2)^2}\right) \\ &+ \frac{1}{4}\left(1 + c_3 - \sqrt{(r + s)^2 + (c_1 - c_2)^2}\right) \log\left(1 + c_3 - \sqrt{(r + s)^2 + (c_1 - c_2)^2}\right)\Bigg]. \end{aligned} \tag{7}$$

Next, we evaluate the one-way deficit of the X states in Eq. (5). Let $\{\Pi_k = |k\rangle\langle k|, k = 0, 1\}$ be the local measurement for the party b along the computational base $|k\rangle$; then any von Neumann measurement for the party b can be written as

$$\{B_k = V\Pi_k V^\dagger : \quad k = 0, 1\}$$

for some unitary $V \in U(2)$. For any unitary V,

$$V = tI + i\vec{y}\cdot\vec{\sigma}$$

with $t \in \mathbb{R}$, $\vec{y} = (y_1, y_2, y_3) \in \mathbb{R}^3$, and $t^2 + y_1^2 + y_2^2 + y_3^2 = 1$, after the measurement B_k, the state ρ^{ab} will be changed into the ensemble $\{\rho_k, p_k\}$ with

$$\rho_k = \frac{1}{p_k}(I\otimes B_k)\rho(I\otimes B_k), \quad p_k = tr(I\otimes B_k)\rho(I\otimes B_k).$$

To evaluate ρ_k and p_k, we write

$$\begin{aligned} p_k\rho_k &= (I\otimes B_k)\rho(I\otimes B_k)\\ &= \frac{1}{4}(I\otimes V)(I\otimes\Pi_k)\Big[I + r\sigma_3\otimes I + sI\otimes V^\dagger\sigma_3 V^\dagger\\ &\quad + \sum_{j=1}^{3} c_j\sigma_j\otimes(V^\dagger\sigma_j V)\Big](I\otimes\Pi_k)(I\otimes V^\dagger). \end{aligned}$$

By the relations [19]

$$\begin{aligned} V^\dagger\sigma_1 V &= \left(t^2 + y_1^2 - y_2^2 - y_3^2\right)\sigma_1 + 2(ty_3 + y_1y_2)\sigma_2 + 2(-ty_2 + y_1y_3)\sigma_3,\\ V^\dagger\sigma_2 V &= 2(-ty_3 + y_1y_2)\sigma_1 + \left(t^2 + y_2^2 - y_1^2 - y_3^2\right)\sigma_2 + 2(ty_1 + y_2y_3)\sigma_3,\\ V^\dagger\sigma_3 V &= 2(ty_2 + y_1y_3)\sigma_1 + 2(-ty_1 + y_2y_3)\sigma_2 + \left(t^2 + y_3^2 - y_1^2 - y_2^2\right)\sigma_3, \end{aligned}$$

and

$$\Pi_0\sigma_3\Pi_0 = \Pi_0, \Pi_1\sigma_3\Pi_1 = -\Pi_1, \Pi_j\sigma_k\Pi_j = 0, \quad \text{for } j = 0, 1, \quad k = 1, 2,$$

we obtain

$$\begin{aligned} p_0\rho_0 &= \frac{1}{4}\left[I + sz_3I + c_1z_1\sigma_1 + c_2z_2\sigma_2 + (r + c_3z_3)\sigma_3\right]\otimes(V\Pi_0V^\dagger),\\ p_1\rho_1 &= \frac{1}{4}\left[I - sz_3I - c_1z_1\sigma_1 - c_2z_2\sigma_2 + (r - c_3z_3)\sigma_3\right]\otimes(V\Pi_1V^\dagger), \end{aligned}$$

where

$$z_1 = 2(-ty_2 + y_1y_3), \quad z_2 = 2(ty_1 + y_2y_3), \quad z_3 = t^2 + y_3^2 - y_1^2 - y_2^2.$$

Then, we will evaluate the eigenvalues of $\sum_k \Pi_k \rho^{ab} \Pi_k$ by

$$\sum_k \Pi_k \rho^{ab} \Pi_k = p_0\rho_0 + p_1\rho_1, \tag{8}$$

and

$$\begin{aligned} p_0\rho_0 + p_1\rho_1 &= \frac{1}{4}\left[(I + r\sigma_3) + (sz_3 I + c_1z_1\sigma_1 + c_2z_2\sigma_2 + c_3z_3\sigma_3)\right] \otimes (V\Pi_0V^\dagger) \\ &\quad + \frac{1}{4}\left[(I + r\sigma_3) - (sz_3 I + c_1z_1\sigma_1 + c_2z_2\sigma_2 + c_3z_3\sigma_3)\right] \otimes (V\Pi_1V^\dagger) \\ &= \frac{1}{4}(I + r\sigma_3) \otimes (V\Pi_0V^\dagger + V\Pi_1V^\dagger) \\ &\quad + \frac{1}{4}(sz_3 I + c_1z_1\sigma_1 + c_2z_2\sigma_2 + c_3z_3\sigma_3) \otimes (V\Pi_0V^\dagger - V\Pi_1V^\dagger) \\ &= \frac{1}{4}(I + r\sigma_3) \otimes I + \frac{1}{4}(sz_3 I + c_1z_1\sigma_1 + c_2z_2\sigma_2 + c_3z_3\sigma_3) \otimes V\sigma_3V^\dagger. \end{aligned}$$

The eigenvalues of $p_0\rho_0 + p_1\rho_1$ are the same with the eigenvalues of the states $(I \otimes V^\dagger)(p_0\rho_0 + p_1\rho_1)(I \otimes V)$, and

$$\begin{aligned} (I \otimes V^\dagger)(p_0\rho_0 + p_1\rho_1)(I \otimes V) &= \frac{1}{4}(I + r\sigma_3) \otimes I \\ &\quad + \frac{1}{4}(sz_3 I + c_1z_1\sigma_1 + c_2z_2\sigma_2 + c_3z_3\sigma_3) \otimes \sigma_3. \end{aligned} \tag{9}$$

The eigenvalues of the states in the Eq. (9) are

$$\begin{aligned} \lambda_{1,2} &= \frac{1}{4}\left(1 - sz_3 \pm \sqrt{r^2 - 2rc_3z_3 + c_1^2z_1^2 + c_2^2z_2^2 + c_3^2z_3^2}\right), \\ \lambda_{3,4} &= \frac{1}{4}\left(1 + sz_3 \pm \sqrt{r^2 + 2rc_3z_3 + c_1^2z_1^2 + c_2^2z_2^2 + c_3^2z_3^2}\right). \end{aligned} \tag{10}$$

It can be directly verified that $z_1^2 + z_2^2 + z_3^2 = 1$. Set $\phi = z_3$, and

$$\phi \in [-1, 1]. \tag{11}$$

Let us put $\theta = c_1^2z_1^2 + c_2^2z_2^2 + c_3^2z_3^2$, $c = \min\{|c_1|, |c_2|, |c_3|\}$, $C = \max\{|c_1|, |c_2|, |c_3|\}$, then $c^2 = \min\{c_1^2, c_2^2, c_3^2\}$, $C^2 = \max\{c_1^2, c_2^2, c_3^2\}$, $c^2 \le \theta \le C^2$, and the equality can be readily obtained by appropriate choice of t, y_j [19]. Therefore, we see that the range of values allowed for θ is $[c^2, C^2]$. The entropy of $\sum_k \Pi_k \rho^{ab} \Pi_k$ is

$$S\left(\sum_k \Pi_k \rho^{ab} \Pi_k\right) = f(\phi, \theta) = -\sum_{i=1}^{4} \lambda_i \log \lambda_i$$

$$
\begin{aligned}
&= 2 - \frac{1}{4}\Big[(1 - s\phi + \sqrt{r^2 - 2rc_3\phi + \theta}) \log(1 - s\phi + \sqrt{r^2 - 2rc_3\phi + \theta}) \\
&\quad + (1 - s\phi - \sqrt{r^2 - 2rc_3\phi + \theta}) \log(1 - s\phi - \sqrt{r^2 - 2rc_3\phi + \theta}) \\
&\quad + (1 + s\phi + \sqrt{r^2 + 2rc_3\phi + \theta}) \log(1 + s\phi + \sqrt{r^2 + 2rc_3\phi + \theta}) \\
&\quad + (1 + s\phi - \sqrt{r^2 + 2rc_3\phi + \theta}) \log(1 + s\phi - \sqrt{r^2 + 2rc_3\phi + \theta})\Big].
\end{aligned} \tag{12}
$$

From Eqs. (6) and (11), we can obtain $1 \mp s\phi \geq 0$ and

$$
\begin{aligned}
\frac{\partial f}{\partial \theta} &= \frac{1}{\ln 256}\Bigg(\frac{\ln(1 - s\phi - \sqrt{r^2 - 2rc_3\phi + \theta}) - \ln(1 - s\phi + \sqrt{r^2 - 2rc_3\phi + \theta})}{\sqrt{r^2 - 2rc_3\phi + \theta}} \\
&\quad + \frac{\ln(1 + s\phi - \sqrt{r^2 + 2rc_3\phi + \theta}) - \ln(1 + s\phi + \sqrt{r^2 + 2rc_3\phi + \theta})}{\sqrt{r^2 + 2rc_3\phi + \theta}}\Bigg) \\
&= \frac{1}{\ln 256}\left(\frac{\ln \frac{1 - s\phi - \sqrt{r^2 - 2rc_3\phi + \theta}}{1 - s\phi + \sqrt{r^2 - 2rc_3\phi + \theta}}}{\sqrt{r^2 - 2rc_3\phi + \theta}} + \frac{\ln \frac{1 + s\phi - \sqrt{r^2 + 2rc_3\phi + \theta}}{1 + s\phi + \sqrt{r^2 + 2rc_3\phi + \theta}}}{\sqrt{r^2 + 2rc_3\phi + \theta}}\right) < 0.
\end{aligned} \tag{13}
$$

It converts the problem about $\min_{\{\Pi_k\}} S(\sum_k \Pi_k \rho^{ab} \Pi_k)$ to the problem about the function of one variable ϕ for minimum. That is

$$
\begin{aligned}
&\min_{\{\Pi_k\}} S\left(\sum_k \Pi_k \rho^{ab} \Pi_k\right) \\
&\quad = \min_{\phi} f(\phi, C) \\
&\quad = \min_{\phi}\Big\{2 - \frac{1}{4}\Big[(1 - s\phi + \sqrt{r^2 - 2rc_3\phi + C^2}) \log(1 - s\phi + \sqrt{r^2 - 2rc_3\phi + C^2}) \\
&\qquad + (1 - s\phi - \sqrt{r^2 - 2rc_3\phi + C^2}) \log(1 - s\phi - \sqrt{r^2 - 2rc_3\phi + C^2}) \\
&\qquad + (1 + s\phi + \sqrt{r^2 + 2rc_3\phi + C^2}) \log(1 + s\phi + \sqrt{r^2 + 2rc_3\phi + C^2}) \\
&\qquad + (1 - s\phi - \sqrt{r^2 + 2rc_3\phi + C^2}) \log(1 + s\phi - \sqrt{r^2 + 2rc_3\phi + C^2})\Big]\Big\}.
\end{aligned} \tag{14}
$$

By Eqs. (2), (7), (14), the one-way deficit of the X states in Eq. (5) is given by

$$
\begin{aligned}
\Delta^{\rightarrow}(\rho^{ab}) &= \min_{\{\Pi_k\}} S\left(\sum_k \Pi_k \rho^{ab} \Pi_k\right) - S(\rho^{ab}) \\
&= \frac{1}{4}\Big[(1 - c_3 + \sqrt{(r - s)^2 + (c_1 + c_2)^2}) \log(1 - c_3 + \sqrt{(r - s)^2 + (c_1 + c_2)^2}) \\
&\quad + (1 - c_3 - \sqrt{(r - s)^2 + (c_1 + c_2)^2}) \log(1 - c_3 - \sqrt{(r - s)^2 + (c_1 + c_2)^2})
\end{aligned}
$$

$$
\begin{aligned}
&+(1+c_3+\sqrt{(r+s)^2+(c_1-c_2)^2})\log(1+c_3+\sqrt{(r+s)^2+(c_1-c_2)^2})\\
&+(1+c_3-\sqrt{(r+s)^2+(c_1-c_2)^2})\log(1+c_3-\sqrt{(r+s)^2+(c_1-c_2)^2})\Big]\\
&-\max_{\phi}\frac{1}{4}\Big[(1-s\phi+\sqrt{r^2-2rc_3\phi+C^2})\log(1-s\phi+\sqrt{r^2-2rc_3\phi+C^2})\\
&+(1-s\phi-\sqrt{r^2-2rc_3\phi+C^2})\log(1-s\phi-\sqrt{r^2-2rc_3\phi+C^2})\\
&+(1+s\phi+\sqrt{r^2+2rc_3\phi+C^2})\log(1+s\phi+\sqrt{r^2+2rc_3\phi+C^2})\\
&+(1+s\phi-\sqrt{r^2+2rc_3\phi+C^2})\log(1+s\phi-\sqrt{r^2+2rc_3\phi+C^2})\Big],
\end{aligned}
\tag{15}
$$

where $C=\max\{|c_1|,|c_2|,|c_3|\}$, $\phi\in[-1,1]$.

For example, we set $r=0.2$, $s=0.3$, $c_1=0.3$, $c_2=-0.4$, $c_3=0.56$, and use the minimum command

$$
\mathrm{MinValue}\Big[\{\Delta^{\rightarrow}(\rho^{ab}),-1\le\phi\le 1\},\phi\Big] \tag{16}
$$

in “Wolfram Mathematics8.0” software, and obtain that the value of the one-way deficit is 0.130614.

When $r=s=0$, ρ reduces to two-qubit Bell-diagonal states. One-way deficit of Bell-diagonal states is

$$
\begin{aligned}
\Delta^{\rightarrow}(\rho^{ab}) &= \min_{\{\Pi_k\}} S\left(\sum_k \Pi_k\rho^{ab}\Pi_k\right)-S(\rho^{ab})\\
&=\frac{1}{4}\big[(1-c_1-c_2-c_3)\log(1-c_1-c_2-c_3)\\
&\quad+(1-c_1+c_2+c_3)\log(1-c_1+c_2+c_3)\\
&\quad+(1+c_1-c_2+c_3)\log(1+c_1-c_2+c_3)\\
&\quad+(1+c_1+c_2-c_3)\log(1+c_1+c_2-c_3)\big]\\
&\quad-\frac{1-C}{2}\log(1-C)-\frac{1+C}{2}\log(1+C),
\end{aligned}
\tag{17}
$$

which is in consistent with the result using the simultaneous diagonalization theorem obtained in [31].

It is worth mentioning that we have obtained a formula for solving one-way deficit. It is simpler than the method using the joint entropy theorem [32].

3 Dynamics of one-way deficit under local nondissipative channels

The concurrence of the X states in Eq. (5) can be calculated in terms of the eigenvalues of $\rho\widetilde{\rho}$, where $\widetilde{\rho}=\sigma_y\otimes\sigma_y\rho^*\sigma_y\otimes\sigma_y$. The eigenvalues of $\rho\widetilde{\rho}$ are

$$\lambda_5 = \frac{1}{16}\left(c_1 - c_2 - \sqrt{(1+c_3)^2-(r+s)^2}\right)^2 = \frac{1}{16}\left(c_1 - c_2 - \sqrt{(1+r+s+c_3)(1-r-s+c_3)}\right)^2,$$
$$\lambda_6 = \frac{1}{16}\left(c_1 - c_2 + \sqrt{(1+c_3)^2-(r+s)^2}\right)^2 = \frac{1}{16}\left(c_1 - c_2 + \sqrt{(1+r+s+c_3)(1-r-s+c_3)}\right)^2,$$
$$\lambda_7 = \frac{1}{16}\left(c_1 + c_2 - \sqrt{(1-c_3)^2-(r-s)^2}\right)^2 = \frac{1}{16}\left(c_1 + c_2 - \sqrt{(1+r-s-c_3)(1-r+s-c_3)}\right)^2,$$
$$\lambda_8 = \frac{1}{16}\left(c_1 + c_2 + \sqrt{(1-c_3)^2-(r-s)^2}\right)^2 = \frac{1}{16}\left(c_1 + c_2 + \sqrt{(1+r-s-c_3)(1-r+s-c_3)}\right)^2.$$

The concurrence of the X states in Eq. (5) is given by

$$C(\rho^{ab}) = \max\left\{2\max\{\sqrt{\lambda_5}, \sqrt{\lambda_6}, \sqrt{\lambda_7}, \sqrt{\lambda_8}\right\} - \sqrt{\lambda_5} - \sqrt{\lambda_6} - \sqrt{\lambda_7} - \sqrt{\lambda_8}, 0\}. \tag{18}$$

In the following, we consider that the X states in Eq. (5) undergo the phase flip channel [33], with the Kraus operators $\Gamma_0^{(A)} = \mathrm{diag}(\sqrt{1-p/2}, \sqrt{1-p/2}) \otimes I$, $\Gamma_1^{(A)} = \mathrm{diag}(\sqrt{p/2}, -\sqrt{p/2}) \otimes I$, $\Gamma_0^{(B)} = I \otimes \mathrm{diag}(\sqrt{1-p/2}, \sqrt{1-p/2})$, $\Gamma_1^{(B)} = I \otimes \mathrm{diag}(\sqrt{p/2}, -\sqrt{p/2})$, where $p = 1-\exp(-\gamma t)$, γ is the phase damping rate [33,34]. Let ε represent the operator of decoherence. Then, under the phase flip channel, we have

$$\varepsilon(\rho) = \frac{1}{4}\big(I\otimes I + r\sigma_3\otimes I + I\otimes s\sigma_3 + (1-p)^2c_1\sigma_1\otimes\sigma_1 + (1-p)^2c_2\sigma_2\otimes\sigma_2 + c_3\sigma_3\otimes\sigma_3\big). \tag{19}$$

We will only consider the following further simplified family of the X states in Eq. (5), where

$$|c_1| < |c_2| < |c_3|. \tag{20}$$

As $\varepsilon(\rho)$ satisfies conditions in Eqs. (5), (20) and the one-way deficit of the ρ^{ab} under the phase flip channel is given by

$$\Delta^{\rightarrow}(\varepsilon(\rho^{ab})) = \frac{1}{4}\Bigg[\left(1-c_3+\sqrt{(r-s)^2+(1-p)^4(c_1+c_2)^2}\right) \times \log\left(1-c_3+\sqrt{(r-s)^2+(1-p)^4(c_1+c_2)^2}\right)$$

$$
\begin{aligned}
&+\left(1-c_3-\sqrt{(r-s)^2+(1-p)^4(c_1+c_2)^2}\right)\\
&\times\log\left(1-c_3-\sqrt{(r-s)^2+(1-p)^4(c_1+c_2)^2}\right)\\
&+\left(1+c_3+\sqrt{(r+s)^2+(1-p)^4(c_1-c_2)^2}\right)\\
&\times\log\left(1+c_3+\sqrt{(r+s)^2+(1-p)^4(c_1-c_2)^2}\right)\\
&+\left(1+c_3-\sqrt{(r+s)^2+(1-p)^4(c_1-c_2)^2}\right)\\
&\times\log\left(1+c_3-\sqrt{(r+s)^2+(1-p)^4(c_1-c_2)^2}\right)\Bigg]\\
&-\max_{\phi}\frac{1}{4}\Bigg[\left(1-s\phi+\sqrt{r^2-2rc_3\phi+c_3^2}\right)\log\left(1-s\phi+\sqrt{r^2-2rc_3\phi+c_3^2}\right)\\
&+\left(1-s\phi-\sqrt{r^2-2rc_3\phi+c_3^2}\right)\log\left(1-s\phi-\sqrt{r^2-2rc_3\phi+c_3^2}\right)\\
&+\left(1+s\phi+\sqrt{r^2+2rc_3\phi+c_3^2}\right)\log\left(1+s\phi+\sqrt{r^2+2rc_3\phi+c_3^2}\right)\\
&+\left(1+s\phi-\sqrt{r^2+2rc_3\phi+c_3^2}\right)\log\left(1+s\phi-\sqrt{r^2+2rc_3\phi+c_3^2}\right)\Bigg].
\end{aligned}
\tag{21}
$$

As an example, for $r=0.2$, $s=0.3$, $c_1=0.3$, $c_2=-0.4$, $c_3=0.56$, the dynamic behavior of correlations of the state under the phase flip channel is depicted in Fig. 1. Here one sees that the concurrence becomes zero after the transition. We find that sudden death of entanglement appears at $p=0.217617$. Therefore, for these states, concurrence is weaker against decoherence than one-way deficit.

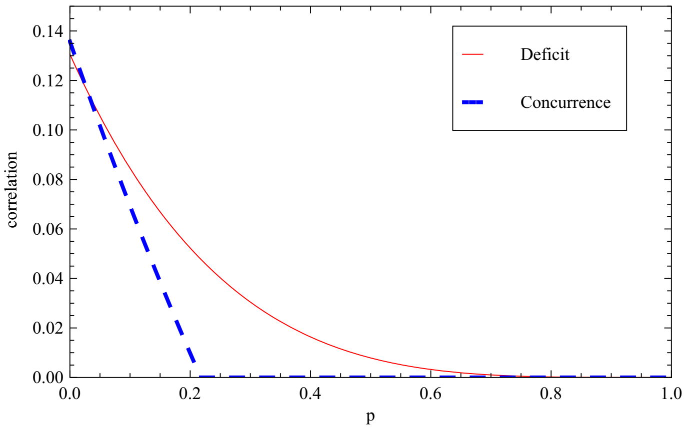

Fig. 1 Concurrence (*blue dashed line*) and one-way deficit (*red solid line*) under phase flip channel for $r=0.2$, $s=0.3$, $c_1=0.3$, $c_2=-0.4$ and $c_3=0.56$ (Color figure online)

4 summary

We have given a new method to evaluate the one-way deficit for X states with five parameters. By this way, we can evaluate one-way deficit of the wide range states than the method using the simultaneous diagonalization theorem. Meanwhile, this way is more simpler than the method using the joint entropy theorem. The dynamic behavior of the one-way deficit under decoherence channel is investigated. It is shown that one-way deficit of the X states is more robust against the decoherence than concurrence.

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