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C-NOT three-gates performance by coherent cavity field and its optimized quantum applications

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Abstract A new realization model of controlled-not (C-NOT) three gates operations, between two atoms and coherent light. The proposed interaction model allows to earn more interaction time compared with the C-NOT two gates model of the reference work. As investigation of the obtained results, we enhance and optimize a recent teleportation work via coherent cavity field, by using less cavities number during the teleportation process. A higher probability have seen in the coherent state teleportation compared with the reference teleportation protocol (single photon teleportation). Furthermore, a generation and teleportation scheme of multipartite GHZ-type entangled coherent states is established. We note that more efficiency is given to our scheme because of simplicity of homodyne detection for coherent light.

Keywords C-NOT three gates operations · Coherent state teleportation · Coherent cavity field · Teleportation and generation of multipartite GHZ-type entangled coherent states

1 Introduction

Quantum information processing (QIP) using cavity quantum electrodynamics (QED) [1,2] system is one of the rich diffusion information tools. Also in this context quantum entanglement has been widely used in quantum information field, such as quantum teleportation [3,4], quantum dense encoding [5,6], quantum cryptography [7,8] and so on.

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In addition to linear optics [9,10] as a method of generation of entanglement, the cavity QED [11,12] also guarantees the preparation of entangled states. Nowadays, technics for generating entangled photon pairs and quantum teleportation of unknown photonic states [13] can be experimentally realized. However for the entangled atomic (or ionic) states much still to be done, as a very small number of schemes that can really be realized experimentally have been proposed so far.

Up to now, for the above reasons, the quantum controlled-not (C-NOT) gate is the scheme that is mostly proposed to achieve the preparation of entangled photonic states before realizing the teleportation process through Bell state measurement. So, many schemes try to realize these two aims by avoiding the thermal fields and cavity decay which affect schemes badly. In [14], the authors proposed a scheme for the quantum gate operations by using dispersive atom-field interaction, where the controlled collision of two Rydberg atoms described in an interaction process assisted induced by a non-resonant cavity. But the disadvantage of this theoretical scheme, is that it doesn't allow to realize C-NOT operations from atoms to cavity, cavity to cavity, cavity to atom.

After that, [15] suggests an efficient scheme to realize the C-NOT two gates between one atom and field of coherent light using only the non-resonant interaction between one atom and the cavity fields realizing the C-NOT gate from atom to field of coherent light, C-NOT gate from coherent light to atom, C-NOT gate from one field to another, C-NOT gate from one atom to another. The problem in the [15] model, is in the process of sending two atoms to interact with the coherent field; the first atom in a time $t_1 = \frac{\pi}{2\lambda}$ and the second one in the interaction time $t_1 = \frac{\pi}{2\lambda}$, by this process[15] didn't find C-NOT gates, but only bell atomic and coherent states. So, we spend more time without realizing the C-NOT gates.

For solving the problem of the previous work. As continuation, we establish a new model of C-NOT three gates based on coherent light (two Rydberg atoms and coherent field). To achieve this goal, we use the interaction between two atoms sent simultaneously through a cavity QED submitted to coherent field. We note that [15] realized C-NOT of two gates such as the use of two successive interaction times is done without obtaining this realization gates. But, the present model perform C-NOT three gates in one minimized interaction time thanks to the modification we do it for the Hamiltonian [15] to hold in the interaction between two Rydberg atoms simultaneously and coherent cavity field with the same initial conditions [15].

After that, we exploit the results of our model, to generate entangled atomic states that we will be used as a quantum channel during the teleportation process in the next step.

It is reported that we used an alternative protocol to teleport a single atomic state, an entangled atomic state and an unknown coherent state. The present protocol is an optimization and evolution of [16] work, which exploited the input-output evolution photon states based on faraday rotation, by eastablishing a teleportation process via cavity QED. The current work makes a similar teleportation process based on C-NOT three gates instead of faraday rotation. That's gives many optimization advantages compared to [16].

These advantages appear on the requirement of less coherent cavities QED than [16] through the teleportation process. Indeed, we need just one coherent cavity field instead of two cavities [16] in the single atomic teleportation process. Also, only the need of two coherent cavities field instead of for [16] in the entangled atomic teleportation process. As another major advantage of the present work, we get a higher probability in the coherent state teleportation than the probability of photonic state teleportation [16].

Finally, we eastablish an efficient scheme which investigate the C-NOT three gates to generate n-cavities GHZ-type entangled coherent states which will be helpful as a quantum channel in the teleportation scheme, to teleport multipartite coherent qubit via multipartite GHZ-type Entangled Coherent cavity field. It is useful to recall that [17] presented Quantum teleportation of arbitrary two-qubit state via entangled cavity fields. The main imperfection of the present work comes from homodyne detection of coherent light.

The paper is organized as follows,

In Sect. 2, we present a C-NOT three gates model, also a scheme presentation allowing to generate entangled atomic states. These are then used in Sect. 3 to present protocols allowing to teleport single atomic states, as entangled atomic pairs. We also propose a protocol for teleporting a superposition of unkonown coherent states. In Sect. 4, we present an efficient scheme for generating and teleporting n-cavities GHZ-type entangled coherent states. A conclusion is then presented in Sect. 5.

2 Proposed scheme for generating entangled atomic pairs

2.1 The model of C-NOT three gates

Consider the interaction between two three-level atoms of Λ-type and a coherent optical field. The two lower levels of the atoms are degenerate, and the frequency of the coherent optical field ω_c is largely detuned from the atomic transition frequency ω_0 between the degenerated lower levels and the upper level. In this large detuning limit the upper level can be adiabatically eliminated during the interaction, if we consider the following initial conditions [15] ,

$$g = g_1 = g_2 \quad ; \quad \lambda = \beta_1 = \beta_2 = \frac{g^2}{\Delta}$$

with $\Delta = \omega_0 - \omega_c$ being the detuning between atomic transition frequency and the frequency of the coherent light, g_1 ,g_2 being the coupling constant between the cavity mode and the transitions $| i\rangle \rightarrow | e\rangle$, $| i\rangle \rightarrow | g\rangle$, respectively (before the adiabatic elimination of the upper level to have in the end interaction of two level of two atoms with the coherent cavity).

The effective Hamiltonian [15,18] of the system then can be modified to be written as follow

$$\begin{aligned} H_{\text{eff}} = -\lambda a^{+} a\{ & | e\rangle_{11}\langle g | + | g\rangle_{11}\langle e | + | e\rangle_{11}\langle e | + | g\rangle_{11}\langle g | \\ & + | e\rangle_{22}\langle g | + | g\rangle_{22}\langle e | + | e\rangle_{22}\langle e | + | g\rangle_{22}\langle g |\} \end{aligned}$$

Remind that this Hamiltonian is also expressed in the basis $\{| e\rangle, | g\rangle\}$, but the aim by modifying the expression of the Hamiltonian [15] is to extract the evolution states of a system contains two atoms 1 and 2 sent simultaneously to interact with a coherent cavity field.

In our model to verify the possibility of establishing a three CNOT gates in the basis $\{\mid +\rangle, \mid -\rangle\}$ using the Hamiltonian expression, following the work [15] that establishes only two CNOT gates, let us define the following two states for each atom:

$$\mid +\rangle_i = \frac{1}{\sqrt{2}} (\mid e\rangle_i + \mid g\rangle_i) \tag{1}$$

$$\mid -\rangle_i = \frac{1}{\sqrt{2}} (\mid e\rangle_i - \mid g\rangle_i) \quad i = 1, 2 \tag{2}$$

For the overall state of the two atoms one has the following possible combinations

$$\mid -\rangle_1 \mid -\rangle_2 = \frac{1}{2} \{\mid e\rangle_1 \mid e\rangle_2 - \mid e\rangle_1 \mid g\rangle_2 - \mid g\rangle_1 \mid e\rangle_2 + \mid g\rangle_1 \mid g\rangle_2\} \tag{3}$$

$$\mid -\rangle_1 \mid +\rangle_2 = \frac{1}{2} \{\mid e\rangle_1 \mid e\rangle_2 + \mid e\rangle_1 \mid g\rangle_2 - \mid g\rangle_1 \mid e\rangle_2 - \mid g\rangle_1 \mid g\rangle_2\} \tag{4}$$

$$\mid +\rangle_1 \mid -\rangle_2 = \frac{1}{2} \{\mid e\rangle_1 \mid e\rangle_2 - \mid e\rangle_1 \mid g\rangle_2 + \mid g\rangle_1 \mid e\rangle_2 - \mid g\rangle_1 \mid g\rangle_2\} \tag{5}$$

$$\mid +\rangle_1 \mid +\rangle_2 = \frac{1}{2} \{\mid e\rangle_1 \mid e\rangle_2 + \mid e\rangle_1 \mid g\rangle_2 + \mid g\rangle_1 \mid e\rangle_2 + \mid g\rangle_1 \mid g\rangle_2\} \tag{6}$$

If we suppose that both atoms 1 and 2 are initially prepared in $\mid -\rangle_1 \mid -\rangle_2$ states and sent simultaneously into a cavity that contains an optical field described by the coherent states $\mid \alpha\rangle$, with the help of schrodinger equation and (3) in order to convert the initial state to be suitable with the basis hamiltonian expression, then the interaction between the atoms and the optical field yields the following state

$$\mid -\rangle_1 \mid -\rangle_2 \mid \alpha\rangle \rightarrow \mid -\rangle_1 \mid -\rangle_2 \mid e^{4i\lambda t}\alpha\rangle. \tag{7}$$

By adjusting the interaction time $t = \frac{\pi}{2\lambda}$, the previous equation, then, becomes:

$$\mid -\rangle_1 \mid -\rangle_2 \mid \alpha\rangle \rightarrow \mid -\rangle_1 \mid -\rangle_2 \mid \alpha\rangle \tag{8}$$

Similarly, the other possible initial states of the system, keeping the same interaction time, will lead to the following final states:

$$\mid +\rangle_1 \mid +\rangle_2 \mid \alpha\rangle \rightarrow \mid +\rangle_1 \mid +\rangle_2 \mid \alpha\rangle \tag{9}$$

$$\mid +\rangle_1 \mid -\rangle_2 \mid \alpha\rangle \rightarrow \mid +\rangle_1 \mid -\rangle_2 \mid -\alpha\rangle \tag{10}$$

$$\mid -\rangle_1 \mid +\rangle_2 \mid \alpha\rangle \rightarrow \mid -\rangle_1 \mid +\rangle_2 \mid -\alpha\rangle \tag{11}$$

$$\mid -\rangle_1 \mid -\rangle_2 \mid \alpha\rangle \rightarrow \mid -\rangle_1 \mid -\rangle_2 \mid \alpha\rangle \tag{12}$$

$$\mid +\rangle_1 \mid +\rangle_2 \mid -\alpha\rangle \rightarrow \mid +\rangle_1 \mid +\rangle_2 \mid -\alpha\rangle \tag{13}$$

$$\mid +\rangle_1 \mid -\rangle_2 \mid -\alpha\rangle \rightarrow \mid +\rangle_1 \mid -\rangle_2 \mid \alpha\rangle \tag{14}$$

$$\mid -\rangle_1 \mid +\rangle_2 \mid -\alpha\rangle \rightarrow \mid -\rangle_1 \mid +\rangle_2 \mid \alpha\rangle \tag{15}$$

$$\mid -\rangle_1 \mid -\rangle_2 \mid -\alpha\rangle \rightarrow \mid -\rangle_1 \mid -\rangle_2 \mid -\alpha\rangle \tag{16}$$

These results show that the states of the atoms remain unchanged after the interaction and it is the coherent field that changes if the initial atomic states are different. These are the performed C-NOT three-qubit gates.

It is useful to recall that [15] presented a model which is limited in the C-NOT two-qubit gates creation, using one atomic state and the optical field. Even though the [15] model uses the interaction of two atoms with the coherent field, it does in two successive interaction times without reaching the C-NOT realization. Therefore, it consumes much time, and don't reach the C-NOT three-qubit gates in the case of C-NOT realization.

2.2 Generation of entangled atomic state

If the initial atomic states are in terms of the states $\mid e\rangle$ and/or $\mid g\rangle$ then keeping the same interaction time as before and using Eqs (3–16), one can realize the following transformations:

$$\mid e\rangle_1 \mid e\rangle_2 \mid \alpha\rangle \rightarrow \frac{1}{\sqrt{2}}[\mid \Psi_{12}^{+}\rangle \mid \alpha\rangle + \mid \Psi_{12}^{-}\rangle \mid -\alpha\rangle] \tag{17}$$

$$\mid g\rangle_1 \mid g\rangle_2 \mid \alpha\rangle \rightarrow \frac{1}{\sqrt{2}}[\mid \Psi_{12}^{+}\rangle \mid \alpha\rangle - \mid \Psi_{12}^{-}\rangle \mid -\alpha\rangle] \tag{18}$$

$$\mid e\rangle_1 \mid g\rangle_2 \mid \alpha\rangle \rightarrow \frac{1}{\sqrt{2}}[\mid \Phi_{12}^{+}\rangle \mid \alpha\rangle + \mid \Phi_{12}^{-}\rangle \mid -\alpha\rangle] \tag{19}$$

$$\mid g\rangle_1 \mid e\rangle_2 \mid \alpha\rangle \rightarrow \frac{1}{\sqrt{2}}[- \mid \Phi_{12}^{+}\rangle \mid \alpha\rangle + \mid \Psi_{12}^{-}\rangle \mid -\alpha\rangle] \tag{20}$$

$$\mid e\rangle_1 \mid e\rangle_2 \mid -\alpha\rangle \rightarrow \frac{1}{\sqrt{2}}[\mid \Psi_{12}^{+}\rangle \mid -\alpha\rangle + \mid \Psi_{12}^{-}\rangle \mid \alpha\rangle] \tag{21}$$

$$\mid g\rangle_1 \mid g\rangle_2 \mid -\alpha\rangle \rightarrow \frac{1}{\sqrt{2}}[\mid \Psi_{12}^{+}\rangle \mid -\alpha\rangle - \mid \Psi_{12}^{-}\rangle \mid \alpha\rangle] \tag{22}$$

$$\mid e\rangle_1 \mid g\rangle_2 \mid -\alpha\rangle \rightarrow \frac{1}{\sqrt{2}}[\mid \Phi_{12}^{+}\rangle \mid -\alpha\rangle + \mid \Phi_{12}^{-}\rangle \mid \alpha\rangle] \tag{23}$$

$$\mid g\rangle_1 \mid e\rangle_2 \mid -\alpha\rangle \rightarrow \frac{1}{\sqrt{2}}[- \mid \Phi_{12}^{+}\rangle \mid -\alpha\rangle + \mid \Psi_{12}^{-}\rangle \mid \alpha\rangle] \tag{24}$$

where we have used the Bell states notations:

$$\mid \Phi_{12}^{+}\rangle = \frac{1}{\sqrt{2}}[\mid e\rangle_1 \mid g\rangle_2 + \mid g\rangle_1 \mid e\rangle_2]$$

$$\mid \Phi_{12}^{-}\rangle = \frac{1}{\sqrt{2}}[\mid e\rangle_1 \mid g\rangle_2 - \mid g\rangle_1 \mid e\rangle_2]$$

$$\mid \psi_{12}^{+}\rangle = \frac{1}{\sqrt{2}}[\mid e\rangle_1 \mid e\rangle_2 + \mid g\rangle_1 \mid g\rangle_2]$$

$$\mid \psi_{12}^{-}\rangle = \frac{1}{\sqrt{2}}[\mid e\rangle_1 \mid e\rangle_2 - \mid g\rangle_1 \mid g\rangle_2]$$

The atomic Bell states can thus be obtained by controling the optical field in the cavity. For instance, for the first case (17), if the optical field detection are in the state $\mid \alpha\rangle$ then the atomic state will be in the entangled Bell state $\mid \psi^{+}_{12}\rangle$.

Note that the detection of coherent field has been realized by B. Yurke et al [19], and they can distinguish $\mid \alpha\rangle$ and $\mid -\alpha\rangle$ by homodyne detection.

It is useful to recall that in [15] the extraction of entanglement need more than one interaction time but by sending two atoms simultaneously through the coherent cavity field in the present work we need just one interaction time.

3 Cavity QED teleportation scheme!!!

Many protocols, to achieve a secret sharing of quantum information, have been proposed in terms of cavity QED via entanglement swapping or teleportation [16,20,21]. Here, based on C-NOT three-qubit gates approach, we propose to develop and optimize the teleportation protocol used in [16] which is based on faraday rotation approach. By the same scheme as [16] which uses more number of cavities then our currrent work through the teleportation process, we reach a good improvement. That's thanks to the used Hamiltonian of interaction between two atoms and coherent field. Then, the present work gives a lower cost then the others teleportation protocol.

Also, the main evolution of the current protocol compared to the other appears in the performance for unknown coherent state teleportation with higher probability.

3.1 Teleportation of single atomic state

We assume that Alice wants to teleport the following unknown atomic state to Bob:

$$\mu \mid e\rangle_1 + \beta \mid g\rangle_1. \tag{25}$$

The communication will be carried over a quantum channel formed by atoms 2 and 3 that are maximally entangled: $\frac{1}{\sqrt{2}}\left(\mid ee\rangle_{23}+ \mid gg\rangle_{23}\right)$, and we consider that the cavity field is a superposition of coherent states: $\frac{1}{\sqrt{2}}\left(\mid \alpha\rangle_4+ \mid -\alpha\rangle_4\right)$.

The overall state is then

$$\begin{aligned}\mid \Psi\rangle = &\frac{1}{2}[\mu(\mid eee\rangle_{123}+ \mid egg\rangle_{123}) + \beta(\mid gee\rangle_{123}+ \mid ggg\rangle_{123})] \mid \alpha\rangle_4 \\ &+\frac{1}{2}[\mu(\mid eee\rangle_{123}+ \mid egg\rangle_{123}) + \beta(\mid gee\rangle_{123}+ \mid ggg\rangle_{123})] \mid -\alpha\rangle_4\end{aligned} \tag{26}$$

Alice sends atoms 1 and 2 simultaneously into the cavity. Using the Eqs. (17)–(24) the global state after interaction is written as:

$$\mid \Psi\rangle_{\text{int}} = \frac{1}{2}(\mid \alpha\rangle_4+ \mid -\alpha\rangle_4)[\mu \mid eee\rangle_{123} + \beta \mid ggg\rangle_{123}+\mu \mid egg\rangle_{123}-\beta \mid gee\rangle_{123}] \tag{27}$$

Table 1 Alice measurement outcomes on atomic states 1 and 2, the corresponding quantum states of atom 3 and the unitary operations to be applied by Bob in order to recover the initial state

Alice measurement on atoms 1 and 2	Bob recovered state on atom 3	Transformation
$\mid ee\rangle_{12}$	$\mu \mid e\rangle_3 + \beta \mid g\rangle_3$	I
$\mid gg\rangle_{12}$	$\mu \mid e\rangle_3 - \beta \mid g\rangle_3$	σ_Z
$\mid eg\rangle_{12}$	$-\beta \mid e\rangle_3 + \mu \mid g\rangle_3$	σ_Y
$\mid ge\rangle_{12}$	$\beta \mid e\rangle_3 + \mu \mid g\rangle_3$	σ_X

Alice then applies the Hadamard operator on atoms 1 and 2:

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix} \tag{28}$$

which will allow to achieve the following transitions :

$$\begin{aligned} H \mid ee\rangle &= \frac{\mid ee\rangle + \mid gg\rangle}{\sqrt{2}} \\ H \mid eg\rangle &= \frac{\mid eg\rangle + \mid ge\rangle}{\sqrt{2}} \\ H \mid ge\rangle &= \frac{\mid eg\rangle - \mid ge\rangle}{\sqrt{2}} \\ H \mid gg\rangle &= \frac{\mid ee\rangle - \mid gg\rangle}{\sqrt{2}} \end{aligned}$$

So Alice rotates his atoms 1 and 2 simultaneously with the help of the defined Hadamard operator transitions, and the global state after interaction becomes:

$$\begin{aligned} \mid \Psi\rangle_{\text{int}} = \frac{1}{2\sqrt{2}}\left[\mid \alpha\rangle_4 + \mid -\alpha\rangle_4\right]\Big[&\mid ee\rangle_{12}(\mu \mid e\rangle_3 + \beta \mid g\rangle_3) + \mid gg\rangle_{12}(\mu \mid e\rangle_3 \\ &-\beta \mid g\rangle_3) + \mid eg\rangle_{12}(\mu \mid g\rangle_3 - \beta \mid e\rangle_3) + \mid ge\rangle_{12}(\mu \mid g\rangle_3 + \beta \mid e\rangle_3)\Big] \end{aligned}$$

Alice then makes her measurements on atoms 1 and 2. By informing Bob, over a classical channel, about her measurement outcomes, Alice allows him to reconstruct the initial atomic state by performing one of the unitary transformations summarized in Table 1 below:

Where

$$\sigma_Z = \mid e\rangle\langle e \mid - \mid g\rangle\langle g \mid ; \sigma_X = \mid g\rangle\langle e \mid + \mid e\rangle\langle g \mid ; \sigma_Y = - \mid g\rangle\langle e \mid + \mid g\rangle\langle g \mid \tag{29}$$

So that, for example, if Alice measures the state $\mid eg\rangle_{12}$ then Bob should apply the operator σ_Y on atom 3 in order to recover the initial state (25), while if she measures

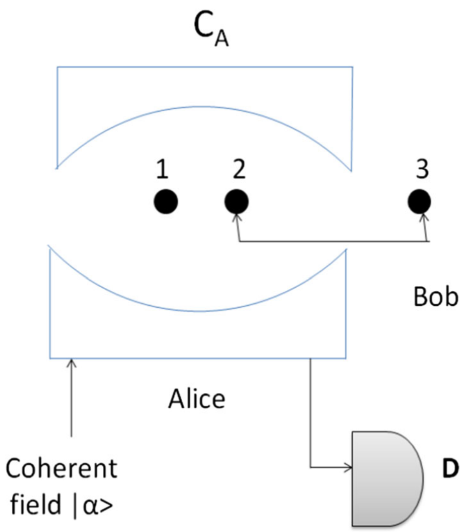

Fig. 1 The schematic of single atomic state teleportation

the state $| ee\rangle_{12}$ then no further action is required from Bob as he recovers the initial state (25) on atom 3 directly.

It's clear in the Fig. 1, that the present proposed protocol based on C-NOT three gates, reduce the used coherent cavities number compared with the single atomic teleportation in [16] which is based on faraday rotation.

3.2 Teleportation of an entangled atomic state

We consider in this protocol, that Bob possesses two atoms 1 and 2 trapped in a low-Q cavity c_B and Alice possesses two atoms 3 and 4 trapped in a separate low-Q cavity c_A.

Bob has his two atoms in the initial states

$$\frac{1}{\sqrt{2}}[| +\rangle_1 + | -\rangle_1]$$
$$\frac{1}{\sqrt{2}}[| +\rangle_2 + | -\rangle_2]$$

then sends a coherent field $| \alpha\rangle$ into the cavity c_B. After interaction, using the the performed C-NOT three-qubit gates (9)–(16) the overall state at Bob's becomes

$$\frac{1}{\sqrt{2}}[| \Psi^+_{12}\rangle \ | \alpha\rangle + | \Phi^+_{12}\rangle \ | -\alpha\rangle] \tag{30}$$

where

$$| \Psi^+_{12}\rangle = \frac{1}{\sqrt{2}}[| ++\rangle_{12} + | --\rangle_{12}] \tag{31}$$

$$| \Phi_{12}^{+}\rangle = \frac{1}{\sqrt{2}}[| +-\rangle_{12}+ | -+\rangle_{12}] \tag{32}$$

On the other hand Alice possesses the unknown entangled atomic state

$$\mu \mid ++\rangle_{34} + \beta \mid --\rangle_{34} \tag{33}$$

In order to transfer this unknown state to Bob, Alice first applies a two dimensional Hadamard operator on atom 3 by a classical field. Then, she guides the coherent light from c_B to c_A and let it interact with atoms 3 and 4, by using the performed C-NOT three-qubit gates (17)–(24). This leads the whole system of the four atoms and the coherent light to the following state :

$$\begin{aligned} | \Psi\rangle_{\text{int}} = &\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}(\mu \mid ++\rangle_{34} - \beta \mid --\rangle_{34})(| \Psi\rangle_{12}^{+}\rangle \mid \alpha\rangle + \mid \Phi\rangle_{12}^{+}\rangle \mid -\alpha\rangle) \\ &+\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}(\mu \mid -+\rangle_{34} + \beta \mid +-\rangle_{34})(| \Phi\rangle_{12}^{+}\rangle \mid \alpha\rangle + \mid \Psi\rangle_{12}^{+}\rangle \mid -\alpha\rangle) \end{aligned}$$

After that, Alice applies again a Hadamard operation on atom 4 to get the total state of the four atoms in the following form:

$$\begin{aligned} |\Psi_{\text{int}}\rangle = \frac{1}{2\sqrt{2}}\Big[&| \alpha\rangle \mid ++\rangle_{34}(\mu \mid \Psi_{12}^{+}\rangle + \beta \mid \Phi_{12}^{+}\rangle) + | -\alpha\rangle \mid ++\rangle_{34}(\mu \mid \Phi_{12}^{+}\rangle + \beta \mid \Psi_{12}^{+}\rangle) \\ &+| \alpha\rangle \mid +-\rangle_{34}(\mu \mid \Psi\rangle_{12}^{+}\rangle - \beta \mid \Phi_{12}^{+}\rangle) + \mid -\alpha\rangle \mid +-\rangle_{34}(\mu \mid \Phi\rangle_{12}^{+}\rangle - \beta \mid \Psi_{12}^{+}\rangle) \\ &+| \alpha\rangle \mid -+\rangle_{34}(\mu \mid \Phi_{12}^{+}\rangle - \beta \mid \Psi_{12}^{+}\rangle) + \mid -\alpha\rangle \mid -+\rangle_{34}(\mu \mid \Psi_{12}^{+}\rangle - \beta \mid \Phi_{12}^{+}\rangle) \\ &+| \alpha\rangle \mid --\rangle_{34}(\mu \mid \Phi_{12}^{+}\rangle + \beta \mid \Psi_{12}^{+}\rangle) + \mid -\alpha\rangle \mid --\rangle_{34}(\mu \mid \Psi_{12}^{+}\rangle + \beta \mid \Phi_{12}^{+}\rangle)\Big] \end{aligned}$$

At the end Alice measures the output coherent field and also atoms 3 and 4, then sends the measurement outcomes to Bob via a classical channel.

On the other hand, Bob applies the following unitary transformations on atoms 1 and 2 [22,23]:

$$\begin{aligned} | \Psi\rangle_{12}^{+} &\rightarrow | +\rangle_1 \mid +\rangle_2 \\ | \Phi\rangle_{12}^{+} &\rightarrow | -\rangle_1 \mid -\rangle_2 \end{aligned}$$

Then based on Alice's results he decides which operation should be applied on his atoms in order to get the initial state (33) as is summarized in Table 2 below.

Where

$$\begin{aligned} \sigma_Z &= | ++\rangle\langle ++ | - | --\rangle\langle -- | \\ \sigma_X &= | --\rangle\langle ++ | + | ++\rangle\langle -- | \\ \sigma_Y &= - | --\rangle\langle ++ | + | ++\rangle\langle -- | \end{aligned} \tag{34}$$

Table 2 Alice measurement outcomes on the coherent field and atoms 3 and 4, and the corresponding quantum state of atom 3 and unitary operations of Bob for atoms 1 and 2

Coherent state	Atom 3	Atom 4	Operation of Bob
$\mid\alpha\rangle$	$\mid+\rangle$	$\mid+\rangle$	I
$\mid\alpha\rangle$	$\mid+\rangle$	$\mid-\rangle$	σ_Z
$\mid\alpha\rangle$	$\mid-\rangle$	$\mid+\rangle$	σ_Y
$\mid\alpha\rangle$	$\mid-\rangle$	$\mid-\rangle$	σ_X
$\mid-\alpha\rangle$	$\mid+\rangle$	$\mid+\rangle$	σ_X
$\mid-\alpha\rangle$	$\mid+\rangle$	$\mid-\rangle$	σ_Y
$\mid-\alpha\rangle$	$\mid-\rangle$	$\mid+\rangle$	σ_Z
$\mid-\alpha\rangle$	$\mid-\rangle$	$\mid-\rangle$	I

It appears obviously in the Fig. 2, that the current proposed protocol based on C-NOT three gates, reduce the used coherent cavities number compared with the entangled atomic teleportation in [16] which is based on faraday rotation.

3.3 Teleportation of a coherent state superposition

In order to prepare the entangled quantum channel, two atoms a and b in the entangled state

$$\frac{1}{\sqrt{2}}[\mid ee\rangle_{ab}+\mid gg\rangle_{ab}] \tag{35}$$

are put in a low Q-cavity driven by a superposition coherent field 1:

$$\frac{1}{\sqrt{2}}[\mid \alpha\rangle_1+\mid -\alpha\rangle_1] \tag{36}$$

After interaction,by using the Eqs. (17)–(24), the two atoms-coherent field state can be expressed as

$$\frac{1}{2}[(\mid ee\rangle_{ab}+\mid gg\rangle_{ab})\mid \alpha\rangle_1+(\mid ee\rangle_{ab}+\mid gg\rangle_{ab})\mid -\alpha\rangle_1]$$

In order to establish the quantum channel, Alice sends the coherent field 1 (after interaction with atoms a and b) to Bob.

To teleport the unknown coherent state 2

$$\mu\mid \alpha\rangle_2+\beta\mid -\alpha\rangle_2 \tag{37}$$

Alice guides it into the cavity and lets it interact with atoms a and b. So, using (17)–(24), the global quantum state is:

$$\mid\Psi\rangle_{ab12}=\frac{1}{2}[(\mid ee\rangle_{ab}+\mid gg\rangle_{ab})\mu\mid \alpha\rangle_1\mid \alpha\rangle_2+(\mid ee\rangle_{ab}+\mid gg\rangle_{ab})\beta\mid -\alpha\rangle_1\mid -\alpha\rangle_2]$$

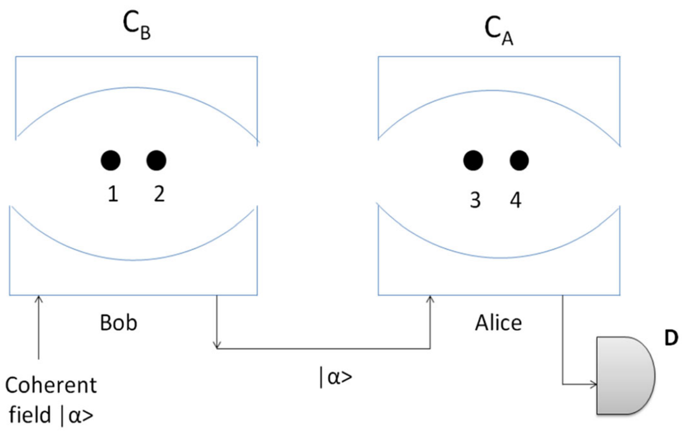

Fig. 2 The schematic of entangled atomic state teleportation

Table 3 Alice measurement outcomes on the coherent field 2, with the corresponding quantum states of coherent state 1 and the unitary operations required for Bob

Measurement on coherent field 2	Coherent state 1	Transformation
$\mid \alpha\rangle_2$	$\mu \mid \alpha\rangle_1 + \beta \mid -\alpha\rangle_1$	I
$\mid -\alpha\rangle_2$	$\mu \mid \alpha\rangle_1 - \beta \mid -\alpha\rangle_1$	σ_Z

Where $\sigma_Z = \mid \alpha\rangle\langle\alpha \mid - \mid -\alpha\rangle\langle-\alpha \mid$

Now Alice applies a 2-dimensional Hadamard operation on the coherent field 2. The final state of the whole system becomes

$$\begin{aligned} \mid \Psi\rangle_{ab12} = &\frac{1}{2\sqrt{2}}(\mid ee\rangle_{ab} + \mid gg\rangle_{ab})(\mu \mid \alpha\rangle_1 + \beta \mid -\alpha\rangle_1) \mid \alpha\rangle_2 \\ &+ \frac{1}{2\sqrt{2}}(\mid ee\rangle_{ab} + \mid gg\rangle_{ab})(\mu \mid \alpha\rangle_1 - \beta \mid -\alpha\rangle_1) \mid -\alpha\rangle_2 \end{aligned} \quad (38)$$

So Alice makes her measurement on the coherent field 2 and sends the outcomes to Bob who makes the corresponding unitary transformations on the coherent field 1 according to the results in Table 3. This will allow him to recover the initial state (37) that Alice wants to teleport.

From this table, we have seen that the probability for recovering the initial unknown coherent state is higher than the the probability for recovering the initial unknown photonic state presented in [16].

4 Generation and teleportation of n-qubit GHZ-type entangled coherent states

Several sechmes have been proposed to prepare entangled coherent states, as a previous Greenberger-Horne-Zeilinger (GHZ) states [24–26] schemes.

In this section, we propose to eastablish a generation scheme of n-qubit GHZ-type Entangled Coherent States by investigating the realized C-NOT three-qubit gates. So, that's help to present a new teleportation scheme of unknown multipartite coherent qubit via multipartite GHZ-type entangled coherent cavity field. We note that an earlier work [17] achieves the quantum teleportation of arbitrary two-qubit states via entangled cavity fields.

4.1 Generation of n-qubit GHZ-type entangled coherent states

Let us use two atoms intially prepared in the state: $\mid ++\rangle_{1,2}$, n-cavities governed by the coherent fields $\mid \alpha, \alpha, \ldots, \alpha\rangle_{1,2,\ldots,n}$, and apply a classical field just on the second atom to have the following transitions

$$\mid +\rangle_2 \rightarrow (\mid +\rangle_2 + \mid -\rangle_2) \tag{39}$$

$$\mid -\rangle_2 \rightarrow (\mid +\rangle_2 - \mid -\rangle_2) \tag{40}$$

We propose to send the two atoms through the n-cavities successively from C_1 to C_n to interact with the previous coherent fields, so the initial state of system will be in this form

$$\mid ++\rangle_{1,2} \mid \alpha, \alpha, \ldots, \alpha\rangle_{1,2,\ldots,n} + \mid +-\rangle_{1,2} \mid \alpha, \alpha, \ldots, \alpha\rangle_{1,2,\ldots,n} \tag{41}$$

After interaction, using the realized C-NOT three-qubit gates (9)–(16), the state of the system is

$$\mid ++\rangle_{1,2} \mid \alpha, \alpha, \ldots, \alpha\rangle_{1,2,\ldots,n} + \mid +-\rangle_{1,2} \mid -\alpha, -\alpha, \ldots, -\alpha\rangle_{1,2,\ldots,n} \tag{42}$$

Applying again the same transitions by classical field on the second atom (40), then

$$N_e\Big[\mid ++\rangle_{1,2}\Big(\mid \alpha, \alpha, \ldots, \alpha\rangle_{1,2,\ldots,n} + \mid -\alpha, -\alpha, \ldots, -\alpha\rangle_{1,2,\ldots,n}\Big) + \mid +-\rangle_{1,2}\Big(\mid \alpha, \alpha, \ldots, \alpha\rangle_{1,2,\ldots,n} - \mid -\alpha, -\alpha, \ldots, -\alpha\rangle_{1,2,\ldots,n}\Big)\Big] \tag{43}$$

With N_e is the normalization factor, in this case $N_e = \frac{1}{2}$. If the two atoms are selected by detectors after mesurement in the state: $\mid ++\rangle_{1,2}$, then the N-cavities field will be $\{\mid \alpha, \alpha, \ldots, \alpha\rangle_{1,2,\ldots,n} + \mid -\alpha, -\alpha, \ldots, -\alpha\rangle_{1,2,\ldots,n}\}$, else the state $\{\mid \alpha, \alpha, \ldots, \alpha\rangle_{1,2,\ldots,n} - \mid -\alpha, -\alpha, \ldots, -\alpha\rangle_{1,2,\ldots,n}\}$ will be realized, which are just the so called n-cavities GHZ-type ECSs.

4.2 Teleportation of multipartite coherent qubit via multipartite GHZ-type Entangled coherent cavity field

Let us consider in this scheme, two atoms a and b in the initial entangled state:

$$\frac{1}{\sqrt{2}}[| +-\rangle_{ab}+ | -+\rangle_{ab}] \tag{44}$$

The two atoms sent through a cavity undergoes the following n-GHZ -type Entangled Coherent field:

$$N_e\Big[| \alpha, \alpha, \ldots, \alpha\rangle_{1,2,\ldots,n} + | -\alpha, -\alpha, \ldots, -\alpha\rangle_{1,2,\ldots,n}\Big] \tag{45}$$

With N_e is the normalization factor. After interaction, following the performed C-NOT three-qubit gates (9)–(16). The whole system becomes

$$\frac{1}{\sqrt{2}}N_e\Big[\Big(| +-\rangle_{ab} + | -+\rangle_{ab}\Big) | -\alpha, -\alpha, \ldots, -\alpha\rangle_{1,2,\ldots,n} + \Big(| +-\rangle_{ab} + | -+\rangle_{ab}\Big) | \alpha, \alpha, \ldots, \alpha\rangle_{1,2,\ldots,n}\Big] \tag{46}$$

To teleport the unknown n-qubit GHZ-type entangled coherent states

$$\mu | \alpha, \alpha, \ldots, \alpha\rangle_{n+1,n+2,\ldots,2n} + \beta | -\alpha, -\alpha, \ldots, -\alpha\rangle_{n+1,n+2,\ldots,2n} \tag{47}$$

Alice guides it into the cavity and lets it interact with atoms a and b. So, using (9)–(16), the global quantum state is:

$$\frac{1}{\sqrt{2}}N_e\Big[\mu\Big(| +-\rangle_{ab} + | -+\rangle_{ab}\Big) | -\alpha, -\alpha, \ldots, -\alpha\rangle_{1,2,\ldots,n} | -\alpha, -\alpha, \ldots, -\alpha\rangle_{n+1,n+2,\ldots,2n} + \beta\Big(| +-\rangle_{ab} + | -+\rangle_{ab}\Big) | \alpha, \alpha, \ldots, \alpha\rangle_{1,2,\ldots,n} | \alpha, \alpha, \ldots, \alpha\rangle_{n+1,n+2,\ldots,2n}\Big] \tag{48}$$

Then, Alice applies a 2-dimensional Hadamard operation on the n coherent field from $n+1$ to $2n$. So, after sending one of the $2 \times n$ possible outcome measurements to Bob, he can recover easly the state (47).

5 Conclusion

To enhance a previous work which proposed the realization of quantum two gates controlled-NOT between (one atom—coherent field) [15], we eastablish an other model which allows to realize Controlled-NOT three gates between (two rydberg atoms—coherent field), based on the interaction between two three-level atoms simultaneously and coherent cavity field. We reach this realization by using suitable Hamiltonian of the appropriate interaction, after adiabatic elimination of the upper level of atoms because of the large detuned between the coherent field frequency and the atomic transition frequency. Compared to [15], we get the Controlled-NOT three gates with minimum of interaction time without spending two successive interaction times as it was doing in the reference work. We note that the change of interaction time value in our proposed model, can gives an other logic quantum gates.

As a matter of fact, we investigated the obtained results to generate entangled atomic states in the ground and excited atomic basis which was then used as a quantum channel for a proposed teleportation protocol of atomic states, entangled atomic states, and coherent state. The proposed teleportation protocol improve an earlier work [16] which do the teleportation process based on faraday rotation. But, with a near process based on Controlled-NOT three gates, we optimize the use coherent cavities number compared to [16]. Also, we reach a high probability through the coherent state teleportation process, and these are another major advantages of the present work.

The performed controlled-NOT three gates are investigated also, to eastablish the generation of n-cavities GHZ-type entangled coherent states which will be helpful as a quantum channel in the proposed teleportation process, to teleport multipartite coherent qubit via multipartite GHZ-type entangled coherent cavity field. It is useful to recall that [17] presented Quantum teleportation of arbitrary two-qubit state via entangled cavity fields.

The present protocol is based on coherent light to avoid the low efficiency of single photon detectors, to increase the detection probability and to reduce the measurement difficulties, also the simplicity of scheme makes it possible that it will be implemented in the near future as a consequence of the main imperfection comes from homodyne detection of coherent light. Another important advantage in the presented scheme here its lower cost compared with other protocols such as in as the number of cavities used in these protocols is reduced compared with the other protocols and less interaction time which is sufficient time to achieve our scheme aiming to do the same task.

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