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# Two efficient schemes for probabilistic remote state preparation and the combination of both schemes

**Jiahua Wei** · **Hong-Yi Dai** · **Ming Zhang**

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**Abstract** We propose two novel schemes for probabilistic remote preparation of an arbitrary quantum state with the aid of the introduction of auxiliary particles and appropriate local unitary operations. The first new proposal could be used to improve the total successful probability of the remote preparation of a general quantum state, and the successful probability is twice as much as the one of the preceding schemes. Meanwhile, one can make use of the second proposal to realize the remote state preparation when the information of the partially entangled state is only available for the sender. This is in contrast to the fact that the receiver must know the non-maximally entangled state in previous typical schemes. Hence, our second proposal could enlarge the applied range of probabilistic remote state preparation. Additionally, we will illustrate how to combine these novel proposals in detail, and our results show that the union has the advantages of both schemes. Of course, our protocols are implemented at the cost of the increased complexity of the practical realizations.

**Keywords** Quantum information · Remote state preparation · Successful probability · Partially entangled state

J. Wei · M. Zhang (✉)
Department of Automatic Control, College of Mechatronics and Automation, National University of Defense Technology, Changsha 410073, Hunan, Peoples Republic of China
e-mail: zhangming_2@yeah.net; zhangming@nudt.edu.cn

J. Wei
e-mail: weijiahua@126.com

H.-Y. Dai
Department of Physics, College of Science, National University of Defense Technology, Changsha 410073, Hunan, Peoples Republic of China
e-mail: daihongyi1@163.com

## 1 Introduction

Quantum teleportation, originally proposed by Bennett et al. [1], is the process that transmits unknown quantum states from a sender to a remote receiver via a quantum channel with the help of some classical information. Due to its potential applications in the realm of quantum communication, the teleportation has acquired lots of attention over the past decades [2–12]. Recently, Lo [13] has presented an interesting method to transmit a pure known quantum state using a prior shared entanglement and some classical communication when the sender knows fully the transmitted state. This communication protocol is called remote state preparation (RSP). Its main difference from the usual teleportation [1] is that the sender is assumed to know completely the transmitted state to be prepared by receiver, while in the teleportation neither the sender nor receiver has a knowledge of the transmitted state.

For the sake of that the RSP could be applied to quantum computation and quantum information, a growing number of works [13–28] have appeared in the RSP recently. Pati [14] has found that the RSP is more economical than quantum teleportation and requires only one classical bit from the sender to the receiver for some special ensembles, but for general states, the RSP requires as much classical communication cost as quantum teleportation does. Bennett et al. [15] have studied the trade-off between the required entanglement and classical communication in the RSP. Dai et al. [16] have presented a protocol for probabilistic remote preparation of the four-particle GHZ class state from a sender to either one of the two receivers via two non-maximally entangled GHZ states. Meantime, the schemes of the RSP have been implemented experimentally using linear optical elements [29,30]. Nevertheless, there are also many important and open subjects to be taken into account for the RSP.

The purpose of this paper was to give two novel schemes to probabilistically prepare an arbitrary quantum state with the help of introducing auxiliary particles and appropriate local unitary operations. In most schemes for probabilistic RSP using a partially entangled state as a quantum channel, the total successful probability of the RSP is considered as one of the most important parameters. In Ref. [26], the authors calculated the successful total probabilities in general and particular cases, respectively. It should be underlined that the successful probability of the RSP for a maximally entangled channel would come to be one for the particular case, in which the sender chooses to prepare a qubit from the equatorial line on the Bloch sphere or from the polar line. However, the successful probability in general case is only equal to the square of the norm of the minimum amplitude coefficient of the partially entangled channel. So that, it is very natural to ask the following question: How to improve the successful probability in the general case? In view of that, we presented the first novel scheme to implement the RSP of a general quantum state with the definite successful probability, twice as much as the probability of the preceding schemes for the RSP. On the other hand, we would like to point out that the receiver not only must introduce an auxiliary particle and make a corresponding unitary transformation to construct the original quantum state, but also needs to know fully the information of the non-maximally entangled state in most schemes for probabilistic RSP. Evidently, the previous schemes [13–16,26] are not valid on condition that only the sender has total knowledge of the partially entangled state. In order to overcome this drawback, we proposed the second novel protocol to probabilistically prepare quantum states to the receiver when the information of the partially entangled state is only available for the sender, i.e., the receiver does not know the information of non-maximally entangled state completely. Thus, the second novel proposal could enlarge the applied range of probabilistic RSP. Furthermore, we will illustrate how to combine the first novel proposal with the second one in detail. It should be presented that the combination has the benefits of both two new schemes. Actually, our proposals are obtained at the cost of the increased complexity of the practical realizations.

The rest of this paper is organized as follows: In Sect. 2, a previous typical scheme of probabilistic RSP is stated. In Sect. 3, we propose the first novel scheme to prepare a general quantum state with the fixed successful probability, twice as much as the probability of the preceding schemes. Moreover, the concrete implementation processes of this scheme are elaborated. The second novel scheme for probabilistically preparing a quantum state would be presented in Sect. 4, and this proposal could be valid when the information of the partially entangled state is only available for the sender. In Sect. 5, we will illustrate how to combine these two new schemes in detail. The paper concludes with Sect. 6.

## 2 A typical scheme of probabilistic RSP in the general case

Let us begin with a brief statement of probabilistic RSP using a partially entangled state. Suppose that the sender Alice wants to help the receiver Bob remotely prepare the following quantum state

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \alpha|0\rangle + |\beta|e^{i\phi}|1\rangle \tag{1}$$

where $0 \leq \phi < 2\pi$, $\alpha$ is real and $\beta$ is a complex number, and $|\alpha|^2 + |\beta|^2 = 1$. Without loss of generality, the entanglement channel is composed of a partially entangled two-particle state below

$$|\Psi\rangle_{12} = (a|00\rangle + b|11\rangle)_{12} \tag{2}$$

where the real coefficient $a$ and the complex one $b$ satisfy $|a|^2 + |b|^2 = 1$, and $|a| \geq |b| > 0$. Particle 1 belongs to the sender Alice, while particle 2 belongs to the receiver Bob. In order to realize the RSP, particle 1 needs to be performed a projective measurement with respect to the orthogonal basis $\{|\psi\rangle, |\psi_\perp\rangle\}$, and $|\psi\rangle$ is given by Eq. (1), while $|\psi_\perp\rangle$ could be shown as

$$|\psi_\perp\rangle = \beta^*|0\rangle - \alpha|1\rangle \tag{3}$$

Thus, the partially entangled state described in Eq. (2) can be rewritten as

$$|\Psi\rangle_{12} = |\psi\rangle_1 \otimes (\alpha a|0\rangle + \beta^* b|1\rangle)_2 + |\psi_\perp\rangle_1 \otimes (\beta a|0\rangle - \alpha b|1\rangle)_2 \tag{4}$$

Then, Alice measures particle 1 and informs Bob of her measurement results via a classical channel. When Alice's measurement outcome on particle 1 is $|\psi\rangle_1$, the state of particle 2, as shown by Eq. (4), will become $(\alpha a|0\rangle + \beta^* b|1\rangle)_2 = (\alpha a|0\rangle + |\beta|e^{-i\phi} b|1\rangle)_2$. Since Bob has no knowledge of this state, he cannot unitarily convert the state of particle 2 into the original state $|\psi\rangle$; thus, the RSP fails. Similarly, if Alice's measurement result is $|\psi_\perp\rangle_1$, the state of particle 2 will collapse into

$$(\beta a|0\rangle - \alpha b|1\rangle)_2 \tag{5}$$

Actually, the original state shown in Eq. (1) can be reconstructed with the successful probability of $|b|^2$ from the state, as given by Eq. (5), via introducing an auxiliary particle and an unitary operation, which is relative to the parameters $a$ and $b$ of Eq. (2). Therefore, the total successful probability of the RSP for a general quantum state is equal to $|b|^2$ in this typical proposal [13]. Meanwhile, If $|a| = |b| = \frac{1}{\sqrt{2}}$, i.e., the quantum channel is composed of a maximally entangled state, the total probability equals 50 %.

## 3 Adding the successful probability of the RSP

As a matter of fact, we would like to point out that a general quantum state is only prepared with the total successful probability of $|b|^2$ in the typical proposals. In this section, a novel scheme would be presented to perform the remote preparation of a general quantum state, and it should be underlined that this proposal could improve the total successful probability of the RSP to $2|b|^2$ via the aid of auxiliary particles, introduced by the sender. When the quantum channel is composed of a maximally entangled state, the total successful probability equals one. Moreover, the detailed processes of our proposal are elaborated as follows

***Step 1***: Alice introduces an auxiliary particle $m$ with an initial state $|0\rangle_m$, and then, the partially entangled state could be written as

$$|\Psi^0\rangle_{1m2} = (a|000\rangle + b|101\rangle)_{1m2} \tag{6}$$

***Step 2***: On particles 1 and $m$, Alice performs the C-NOT operation $U_{CNOT}$, where particle 1 works as controlling qubit and auxiliary particle $m$ as target qubit. After that, the above state $|\Psi^0\rangle_{1m2}$ will become

$$\begin{aligned} |\Psi\rangle_{1m2} &= [U_{CNOT}|\Psi^0\rangle_{1m}] \otimes |\Psi^0\rangle_2 = (a|000\rangle + b|111\rangle)_{1m2} \\ &= |\psi\rangle_1 \otimes (\alpha a|00\rangle + \beta^* b|11\rangle)_{m2} + |\psi_\perp\rangle_1 \otimes (\beta a|00\rangle - \alpha b|11\rangle)_{m2} \end{aligned} \tag{7}$$

***Step 3***: Alice measures the state of this particle 1. If her measurement result is $|\psi_\perp\rangle_1$, the state of particles $m$ and 2, as shown by Eq. (7), will collapse into

$$(\beta a|00\rangle - \alpha b|11\rangle)_{m2} \tag{8}$$

On this situation, Alice needs to perform the Hadamard operation $H$ on this auxiliary particle $m$

$$\beta a(H|0\rangle_m) \otimes |0\rangle_2 - \alpha b(H|1\rangle_m) \otimes |1\rangle_2 \\ = \frac{1}{\sqrt{2}}|0\rangle_m \otimes (\beta a|0\rangle - \alpha b|1\rangle)_2 + \frac{1}{\sqrt{2}}|1\rangle_m \otimes (\beta a|0\rangle + \alpha b|1\rangle)_2 \quad (9)$$

Thus, Alice measures the state of this auxiliary particle $m$. If Alice obtains the result of $|0\rangle_m$, and then, the state of particle 2 will become

$$\frac{1}{\sqrt{2}}(\beta a|0\rangle - \alpha b|1\rangle)_2 \quad (10)$$

Otherwise, if the measurement result is $|1\rangle_m$, the state of particle 2 will be projected to

$$\frac{1}{\sqrt{2}}(\beta a|0\rangle + \alpha b|1\rangle)_2 \quad (11)$$

Similarly, from Eq. (7), if Alice's measurement result on particle 1 is $|\psi\rangle_1$, the state of particles $m$ and 2 will become

$$(\alpha a|00\rangle + \beta^* b|11\rangle)_{m2} = (\alpha a|00\rangle + \beta b e^{-2i\phi}|11\rangle)_{m2} \quad (12)$$

For particle $m$, Alice performs an unitary transformation $U_1$, which takes the form of the following $2 \times 2$ matrix

$$U_1 = \begin{pmatrix} e^{-i\phi} & 0 \\ 0 & e^{i\phi} \end{pmatrix} \quad (13)$$

After that, the state described in Eq. (12) will become

$$e^{-i\phi}(\alpha a|00\rangle + \beta b|11\rangle)_{m2}$$

Since the global phase $e^{-i\phi}$ cannot be distinguished by any physical meaning, the above state could be shown as

$$(\alpha a|00\rangle + \beta b|11\rangle)_{m2} \quad (14)$$

Then, Alice performs the Hadamard operation $H$ on the auxiliary particle $m$, and measures it. If Alice obtains the result of $|0\rangle_m$, the state of particle 2 should be written as follows

$$\frac{1}{\sqrt{2}}(\alpha a|0\rangle + \beta b|1\rangle)_2 \quad (15)$$

Otherwise, the state of particle 2 will be projected to

$$\frac{1}{\sqrt{2}}(\alpha a|0\rangle - \beta b|1\rangle)_2 \quad (16)$$

**Table 1** The measurement results and the unitary transformation

| Measurement results | | $U_F$ | Results after the transformation $U_F$ | | |
|---|---|---|---|---|---|
| Particle 1 | Particle $m$ | | Particle $n$ | Particle 2 | Probabilities |
| $\vert\psi_\perp\rangle_1$ | $\vert 0\rangle_m$ | $U_F^0$ | $\vert 0\rangle_n$ | $(\alpha\vert 0\rangle+\beta\vert 1\rangle)_2$ | $\frac{\vert b\vert^2}{2}$ |
| | | | $\vert 1\rangle_n$ | $\vert 0\rangle_2$ | $\frac{\vert\alpha\vert^2}{2}(1-2\vert b\vert^2)$ |
| | $\vert 1\rangle_m$ | $U_F^1$ | $\vert 0\rangle_n$ | $(\alpha\vert 0\rangle+\beta\vert 1\rangle)_2$ | $\frac{\vert b\vert^2}{2}$ |
| | | | $\vert 1\rangle_n$ | $\vert 0\rangle_2$ | $\frac{\vert\alpha\vert^2}{2}(1-2\vert b\vert^2)$ |
| $\vert\psi\rangle_1$ | $\vert 0\rangle_m$ | $U_F^2$ | $\vert 0\rangle_n$ | $(\alpha\vert 0\rangle+\beta\vert 1\rangle)_2$ | $\frac{\vert b\vert^2}{2}$ |
| | | | $\vert 1\rangle_n$ | $\vert 1\rangle_2$ | $\frac{\vert\beta\vert^2}{2}(1-2\vert b\vert^2)$ |
| | $\vert 1\rangle_m$ | $U_F^3$ | $\vert 0\rangle_n$ | $(\alpha\vert 0\rangle+\beta\vert 1\rangle)_2$ | $\frac{\vert b\vert^2}{2}$ |
| | | | $\vert 1\rangle_n$ | $\vert 1\rangle_2$ | $\frac{\vert\beta\vert^2}{2}(1-2\vert b\vert^2)$ |

After the measurements of particles 1 and $m$, Alice informs Bob of her measurement results via a classical channel. In order to construct the original state shown in Eq. (1), Bob needs to introduce another auxiliary particle $n$ with an initial state $|0\rangle_n$ and performs a conditional unitary transformation $U_F$ on particles 2 and $n$, which depends on the measurement results of particles 1 and $m$. Table 1 shows the measurement results of Alice on particles 1 and $m$, and the unitary transformation on particles 2 and $n$. The unitary transformations $U_F^i (i = 0, 1, 2, 3)$ in Table 1 are described as

$$\begin{aligned} U_F^0 &= \begin{pmatrix} \mathbf{0} & \sigma_z \\ -A(a,b) & \mathbf{0} \end{pmatrix} \quad U_F^1 = \begin{pmatrix} \mathbf{0} & \sigma_z \\ A(a,b) & \mathbf{0} \end{pmatrix} \\ U_F^2 &= \begin{pmatrix} A(a,b) & \mathbf{0} \\ \mathbf{0} & \sigma_z \end{pmatrix} \quad U_F^3 = \begin{pmatrix} A(a,b) & \mathbf{0} \\ \mathbf{0} & -\sigma_z \end{pmatrix} \end{aligned} \tag{17}$$

where $\mathbf{0}$ is the $2 \times 2$ zero matrix, $\sigma_z$ is known as one of pauli matrixes, and $A(a, b)$ is the $2 \times 2$ matrix relative to the parameters $a$ and $b$ of Eq. (2). $\sigma_z$ and $A(a, b)$ could be expressed as

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad A(a,b) = \begin{pmatrix} \frac{b}{a} & \sqrt{1-\frac{|b|^2}{|a|^2}} \\ \sqrt{1-\frac{|b|^2}{|a|^2}} & -\frac{b}{a} \end{pmatrix}$$

In reality, projective measurements together with unitary operations are sufficient to obtain the original state from the state shown in Eq. (10) with the aid of the introduction of auxiliary particles, and the successful probability is equal to $\frac{|b|^2}{2}$. Moreover, it has been demonstrated that the original state would be successfully prepared with the equal probability from these four kinds of the states shown in Eqs. (10), (11), (15) and (16); hence, the whole successful probability is $2|b|^2$ in our proposal for the RSP. This is in contrast to the fact that the whole successful probability of the RSP for a general quantum state is equal to $|b|^2$ in previous typical schemes [16–26]. It should be presented that our proposal can be directly generalized to improve the successful probability for the three-parties controlled remote state preparation from a sender to either of two receivers.

## 4 The RSP when only the sender knows the entangled state

Based on the analyses in Sect. 2, it should be underlined that the receiver Bob must know the parameters $a$ and $b$ of the partially entangled state shown in Eq. (2) to construct the original state in the typical scheme. It is revealed that one cannot take advantages of this scheme to realize the RSP in the situation that only the sender Alice has full knowledge of the non-maximally entangled state in the RSP processes. In order to overcome this drawback, we propose a novel scheme to prepare a quantum state, whether the receiver knows the partially entangled state or not. The concrete implementation procedures of this new scheme are presented as follows

***Step 1***: Alice introduces an auxiliary particle $l$ with an initial state $|0\rangle_l$, and then, the partially entangled state described by Eq. (2) would become

$$|\Psi^0\rangle_{1l2} = (a|000\rangle + b|101\rangle)_{1l2} \tag{18}$$

***Step 2***: On particles 1 and $l$, Alice performs an unitary transformation $U_F^3$, which is given by Eq. (17). Then, the state $|\Psi^0\rangle_{1l2}$ could be transported into

$$\begin{aligned}|\Psi\rangle_{1l2} &= [U_F^3|\Psi^0\rangle_{1l}] \otimes |\Psi^0\rangle_2 = (b|000\rangle + b|101\rangle + a\sqrt{1-\frac{|b|^2}{|a|^2}}|010\rangle)_{1l2} \\ &= b|\psi\rangle_1 \otimes |0\rangle_l \otimes (\alpha|0\rangle + \beta^*|1\rangle)_2 + b|\psi_\perp\rangle_1 \otimes |0\rangle_l \otimes (\beta|0\rangle + \alpha|1\rangle)_2 \\ &\quad + a\sqrt{1-\frac{|b|^2}{|a|^2}}|0\rangle_1 \otimes |1\rangle_l \otimes |0\rangle_2\end{aligned} \tag{19}$$

where the measurements $|\psi\rangle$ and $|\psi_\perp\rangle$ are given by Eqs. (1) and (3), respectively.

***Step 3***: Alice measures the states of particles 1 and $l$. Subsequently, Alice informs Bob of her measurement results using a classical channel. It should be emphasized that the RSP could be realized successfully when the state of particle 1 is $|\psi_\perp\rangle_1$ and particle $l$ is $|0\rangle_l$ with the probability of $|b|^2$; otherwise, it fails.

***Step 4***: If the state of particles 1 and $l$ is $|\psi_\perp\rangle_1 \otimes |0\rangle_l$, Bob only needs to perform the unitary transformation $\sigma_x$ on particle 2 to obtain the original state.

It should be presented that one can use our scheme to carry out probabilistic RSP when only the sender Alice fully knows the partially entangled state. This is different from the former typical one that the receiver Bob must have complete information of the entangled state. The cost of an entangled state is necessary for both the previous typical scheme and our new scheme in this section. In addition, classical communication is also essential to realize the novel RSP.

## 5 The combination of our novel proposals

According to the aforementioned analyses in Sects. 3 and 4, we would like to point out that each one of our two proposals owns itself advantages. Fortunately, these novel proposals could be united to realize the RSP with the total successful probability of $2|b|^2$, whether the receiver Bob knows the partially entangled state or not. In this section, we will show how to combine our two proposals in detail. Furthermore, the implementation processes could be reduced to these steps

***Step 1***: From the steps 1 and 2 in Sect. 3, one can find that Alice can change the state shown in Eq. (2) into the partially entangled state $|\Psi\rangle_{1m2}$ given by Eq. (7) with the aid of the introduction of an auxiliary particle $m$.

***Step 2***: On the basis of the steps 1 and 2 in Sect. 4, Alice introduces an auxiliary particle $l$ with the initial state $|0\rangle_l$ and performs the unitary transformation $U_F^3$ given by Eq. (17) on particles 1 and $l$. Then, the state $|\Psi\rangle_{1m2}$ could be transported into

$$\begin{aligned}|\Psi\rangle_{1lm2} = b|\psi\rangle_1 \otimes |0\rangle_l \otimes (\alpha|00\rangle + \beta^*|11\rangle)_{m2}\\ + b|\psi_\perp\rangle_1 \otimes |0\rangle_l \otimes (\beta|00\rangle - \alpha|11\rangle)_{m2}\\ + a\sqrt{1-\frac{|b|^2}{|a|^2}}|0\rangle_1 \otimes |1\rangle_l \otimes |00\rangle_{m2}\end{aligned} \tag{20}$$

***Step 3***: Alice measures the state of particle $l$. When the state of particle $l$ is $|0\rangle_l$ with the probability of $2|b|^2$, the RSP could be realized successfully; otherwise, it fails.

***Step 4***: Alice measures the state of this particle 1. If her measurement result is $|\psi_\perp\rangle_1$, the state of particles $m$ and 2, as shown by Eq. (20), will collapse into

$$b(\beta|00\rangle - \alpha|11\rangle)_{m2} \tag{21}$$

Then, Bob can perform the Hadamard operation $H$ on particle $m$

$$\begin{aligned}&b\beta(H|0\rangle_m) \otimes |0\rangle_2 - b\alpha(H|1\rangle_m) \otimes |1\rangle_2\\ &= \frac{b}{\sqrt{2}}|0\rangle_m \otimes (\beta|0\rangle - \alpha|1\rangle)_2 + \frac{b}{\sqrt{2}}|1\rangle_m \otimes (\beta|0\rangle + \alpha|1\rangle)_2\end{aligned} \tag{22}$$

Similarly, from Eq. (20), if Alice's measurement result on particle 1 is $|\psi\rangle_1$, the state of particles $m$ and 2 will become

$$b(\alpha|00\rangle + \beta^*|11\rangle)_{m2} = b(\alpha|00\rangle + \beta e^{-2i\phi}|11\rangle)_{m2} \tag{23}$$

On particle $m$, Alice performs the unitary transformations $U_1$ and $H$, respectively; hence, the above state will become

**Table 2** The measurement results and the unitary transformation $U_2$

| Measurement results of Alice | | | States of particle 2 | Probabilities | $U_2$ |
|---|---|---|---|---|---|
| Particle *l* | Particle 1 | Particle *m* | | | |
| $\|0\rangle_l$ | $\|\psi_\perp\rangle_1$ | $\|0\rangle_m$ | $(\beta\|0\rangle - \alpha\|1\rangle)_2$ | $\frac{\|b\|^2}{2}$ | $-i\sigma_y$ |
| | | $\|1\rangle_m$ | $(\beta\|0\rangle + \alpha\|1\rangle)_2$ | $\frac{\|b\|^2}{2}$ | $\sigma_x$ |
| | $\|\psi\rangle_1$ | $\|0\rangle_m$ | $(\alpha\|0\rangle + \beta\|1\rangle)_2$ | $\frac{\|b\|^2}{2}$ | $I$ |
| | | $\|1\rangle_m$ | $(\alpha\|0\rangle - \beta\|1\rangle)_2$ | $\frac{\|b\|^2}{2}$ | $\sigma_z$ |
| $\|1\rangle_l$ | $\|0\rangle_1$ | – | – | $1 - 2\|b\|^2$ | – |

$$e^{-i\phi}[\frac{b}{\sqrt{2}}|0\rangle_m \otimes \frac{b}{\sqrt{2}}(\alpha|0\rangle + \beta|1\rangle)_2 + \frac{b}{\sqrt{2}}|1\rangle_m \otimes (\frac{b}{\sqrt{2}}(\alpha|0\rangle - \beta|1\rangle)_2] \tag{24}$$

where the global phase $e^{-i\phi}$ could be ignored.

After the measurements of particles 1, *l* and *m*, Alice informs Bob of her measurement results via a classical channel. In order to construct the original state shown in Eq. (1), Bob only needs to perform a conditional unitary transformation $U_2$ on particle 2, which depends on Alice's measurement results. Table 2 shows the measurement results on particles *l*, 1 and *m*, and the unitary transformation on particle 2.

From the above analyses, one can find that these two proposals in Sects. 3 and 4 could be united to perform the RSP, and the combination has the advantages of both two schemes. Hence, one can make use of the combination of these two schemes to realize the RSP with the total successful probability of $2|b|^2$, whether the receiver knows the partially entangled state or not.

## 6 Discussions and conclusions

In the following, we will discuss the physical realizations of our proposals. Nowadays, some linear optical protocols [27,28] for the RSP have been presented theoretically, specially some ones have been experimentally realized with the use of linear optics [29,30]. According to the aforementioned analyses, one can find that the unitary transformation $U_1$, Hadamard operation $H$ and C-NOT gate $U_{CNOT}$ need to be completed for our proposals. Fortunately, arbitrary single-qubit operations can be performed to the optical qubit via phase shifters and beam splitters together [31], namely the single-qubit operations $U_1$ and $H$ could be implemented using linear optical elements. Meanwhile, it has been demonstrated experimentally that the C-NOT gate $U_{CNOT}$ can be achieved in linear optical [32]. Hence, all of these operations introduced for these new proposals could be realizable in optical experiments. Moreover, it is worthwhile to point out that the linear optical elements required for these special operations have been widely used; for instance, the action of half-wave plate(HWP) is equal to the Hadamard operation $H$. Consequently, our protocols may be implemented with the use of linear optics in near future. Additionally, no more than two auxiliary particles are necessary to be introduced for the novel schemes with the exception of these three unitary operations. Practically speaking, our protocols are finished at the cost of the increased complexity of the practical realizations.

In summary, we put forward two novel schemes to probabilistically prepare an arbitrary quantum state with the help of the introduction of auxiliary particles and appropriate local unitary operations. In contrast to the previous schemes [16–26], our present ones have the following advantages. First, one can use the former of our proposals to improve the successful probability for the RSP. This scheme could be realized with the total successful probability of $2|b|^2$, which is twice as much as the one of the previous schemes [13–21] to prepare a general quantum state. Therefore, our proposal is valid for adding the total successful probability of the RSP. Second, the latter novel proposal could be utilized to perform the RSP when the information of the partially entangled state is only available for the sender, whereas the previous typical schemes [16–26] are not feasible unless the receiver completely knows the partially entangled state. Thus, the second proposal could enlarge the applied range of probabilistic RSP. Third, our protocols can been combined together. Meanwhile, the combination has the advantages of both schemes in essential, i.e., this union would be useful not only in improving the successful probability, but also in expanding the applied range of the RSP. Thus, from the point of the potential applications of the RSP, we believe that our schemes will play an important role in expanding the field of quantum information processing.

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