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Deterministic joint remote preparation of arbitrary two- and three-qubit entangled states

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Abstract We present several schemes for joint remote preparation of arbitrary two- and three-qubit entangled states with complex coefficients via two and three GHZ states as the quantum channel, respectively. In these schemes, two senders (or N senders) share the original state which they wish to help the receiver to remotely prepare. To complete the JRSP schemes, some novel sets of mutually orthogonal basis vectors are introduced. It is shown that, only if two senders (or N senders) collaborate with each other, and perform projective measurements under suitable measuring basis on their own qubits, respectively, the receiver can reconstruct the original state by means of some appropriate unitary operations. The advantage of the present schemes is that the success probability in all the considered JRSP can reach 1.

Keywords Joint remote state preparation · General two-qubit entangled state · Arbitrary three-qubit entangled state · Two- and three-qubit projective measurement

1 Introduction

Quantum communication plays a significant role in the ongoing field of information theory. It is well known that novel phenomena including quantum teleportation [1], quantum key distribution [2] and quantum dense coding [3] are striking application of quantum entanglement in quantum information processing. In the last decade, Lo [4], Pati [5] and Bennett et al. [6] presented a new quantum communication scheme that uses classical communication and a previously shared entangled resource to remotely prepare a quantum state. This communication scheme is called

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remote state preparation (RSP). Compared with teleportation, RSP requires less classical communication cost than teleportation for some special ensembles [5] Since then, various theoretical protocols for generalization of RSP have been proposed and experimental implementations of RSP scheme have been presented [7–26]. One can note easily that the above schemes assume the case that only one sender knows the original state.

Recently, a novel aspect of PSP, called as the joint remote state preparation(JRSP), has been proposed [27–37]. In these schemes of the JRSP [27–37], two senders (or N senders) know partly of original state they want to remotely preparation, respectively. If and only if all the senders agree to collaborate, the receiver can reconstruct the original quantum state. Up to now, however, no schemes have been reported for JRSP which can be realized with unit success probability. In this paper, we propose several schemes for joint remote preparation of arbitrary two- and three-qubit entangled states with complex coefficients. In our schemes, two senders (or N senders) share the original state, but each sender only partly knows the state. To complete the JRSP schemes, inspired by the method in Ref. [21], several novel sets of two-qubit and three-qubit mutually orthogonal basis vectors have been introduced. It is shown that, if and only if all the sender agree to collaboration, and perform projective measurements under appropriate measuring basis on their own qubit(s), respectively, the receiver can recover the original state with unit successful probability.

This paper is organized as follows. In Sect. 2, the joint remote preparation of a general two-qubit entangled state with two senders and N senders is studied, respectively, by two schemes using two three-qubit GHZ states (Sect. 2.1) and two (N + 1)-qubit GHZ states (Sect. 2.2) as the quantum channel. Furthermore, Sect. 3 presents two schemes for the JRSP of an arbitrary three-qubit entangled state with two senders and N senders, respectively, where the quantum channel is composed of three three-qubit (Sect. 3.1) and three (N + 1)-qubit (Sect. 3.2) GHZ entangled states, respectively. Conclusions are given in Sect. 4.

2 JRSP of an arbitrary two-qubit entangled state with complex coefficients

We firstly present the joint remote preparation of an arbitrary two-qubit entangled state with complex coefficients. In the first scheme the original state is shared by two senders, while the prepared state is shared by N senders in the second scheme.

2.1 JRSP with two senders

Suppose that the state Alice and Bob wish to help the receiver Charlie remotely prepare is an arbitrary two-qubit entangled state

$$|p\rangle = x|00\rangle + ye^{i\alpha}|01\rangle + ze^{i\beta}|10\rangle + we^{i\gamma}|11\rangle, \tag{1}$$

where $x, y, z, w, \alpha, \beta, \gamma$ are real, and $x^2 + y^2 + z^2 + w^2 = 1$. We find out that, similar to Ref. [21], if the coefficients in Eq. (1) are split into two subsets, modulus coefficients (x, y, z, w) and phase coefficients (α, β, γ), the unit success probability can be achieved. Accordingly, we suppose that Alice and Bob share the original state Eq. (1) and they know the state partly, that is Alice know x, y, z, w, Bob knows α, β, γ, but Charlie does not know them at all. Thus, neither Alice nor Bob can complete the RSP alone by means of usual RSP schemes only if they agree to collaborate. We also suppose that the quantum channel shared by Alice, Bob, and Charlie are two GHZ states

$$|\phi_1\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{A_1B_1C_1},$$
$$|\phi_2\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{A_2B_2C_2}, \tag{2}$$

where the qubits A_1 and A_2 belong to Alice, qubits B_1 and B_2 to Bob, and qubits C_1 and c_2 to Charlie, respectively. In order to help Charlie remotely prepare the original state $|p\rangle$, what Alice and Bob need to do is to perform two-qubit projective measurement on their own qubits A_1, A_2 and B_1, B_2, respectively. The first measuring basis chosen by Alice is a set of mutually orthogonal basis vectors (MOBVs) $\{|\mu_k\rangle\}(k = 0, 1, 2, 3)$, which is given by

$$\begin{pmatrix} |\mu_0\rangle \\ |\mu_1\rangle \\ |\mu_2\rangle \\ |\mu_3\rangle \end{pmatrix} = F \begin{pmatrix} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{pmatrix}, \tag{3}$$

where

$$F = \begin{pmatrix} x & y & z & w \\ y & -x & w & -z \\ z & -w & -x & y \\ w & z & -y & -x \end{pmatrix}. \tag{4}$$

The second measuring bases chosen by Bob are four sets of MOBVs $\{|\lambda_j^{(k)}\rangle\}(k, j = 0, 1, 2, 3)$, which are given by

$$\begin{pmatrix} |\lambda_0^{(k)}\rangle \\ |\lambda_1^{(k)}\rangle \\ |\lambda_2^{(k)}\rangle \\ |\lambda_3^{(k)}\rangle \end{pmatrix} = G^{(k)} \begin{pmatrix} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{pmatrix}, \tag{5}$$

where

$$G^{(0)} = \begin{pmatrix} 1 & g_1 & g_2 & g_3 \\ 1 & -g_1 & g_2 & -g_3 \\ 1 & -g_1 & -g_2 & g_3 \\ 1 & g_1 & -g_2 & -g_3 \end{pmatrix}, \quad G^{(1)} = \begin{pmatrix} g_1 & 1 & g_3 & g_2 \\ g_1 & -1 & g_3 & -g_2 \\ g_1 & -1 & -g_3 & g_2 \\ g_1 & 1 & -g_3 & -g_2 \end{pmatrix},$$

Table 1 Corresponding relation between the measurement results (MR) of Alice and Bob and the local unitary operations U_C performed by Charlie

MR	U_C	MR	U_C
$\xi_0\zeta_0^{(0)}$	$(I)_{C_1}\otimes(I)_{C_2}$	$\xi_2\zeta_0^{(2)}$	$(-i\sigma_y)_{C_1}\otimes(\sigma_z)_{C_2}$
$\xi_0\zeta_1^{(0)}$	$(I)_{C_1}\otimes(\sigma_z)_{C_2}$	$\xi_2\zeta_1^{(2)}$	$(-i\sigma_y)_{C_1}\otimes(I)_{C_2}$
$\xi_0\zeta_2^{(0)}$	$(\sigma_z)_{C_1}\otimes(\sigma_z)_{C_2}$	$\xi_2\zeta_2^{(2)}$	$(\sigma_x)_{C_1}\otimes(I)_{C_2}$
$\xi_0\zeta_3^{(0)}$	$(\sigma_z)_{C_1}\otimes(I)_{C_2}$	$\xi_2\zeta_3^{(2)}$	$(\sigma_x)_{C_1}\otimes(\sigma_z)_{C_2}$
$\xi_1\zeta_0^{(1)}$	$(I)_{C_1}\otimes(-i\sigma_y)_{C_2}$	$\xi_3\zeta_0^{(3)}$	$(-i\sigma_y)_{C_1}\otimes(\sigma_x)_{C_2}$
$\xi_1\zeta_1^{(1)}$	$(I)_{C_1}\otimes(\sigma_x)_{C_2}$	$\xi_3\zeta_1^{(3)}$	$(-i\sigma_y)_{C_1}\otimes(-i\sigma_y)_{C_2}$
$\xi_1\zeta_2^{(1)}$	$(\sigma_z)_{C_1}\otimes(\sigma_x)_{C_2}$	$\xi_3\zeta_2^{(3)}$	$(\sigma_x)_{C_1}\otimes(-i\sigma_y)_{C_2}$
$\xi_1\zeta_3^{(1)}$	$(\sigma_z)_{C_1}\otimes(-i\sigma_y)_{C_2}$	$\xi_3\zeta_3^{(3)}$	$(\sigma_x)_{C_1}\otimes(\sigma_x)_{C_2}$

$(\xi_k \to |\mu_k\rangle_{A_1A_2}), (\zeta_j^{(k)} \to |\lambda_j^{(k)}\rangle_{B_1B_2}, k, j = 0, 1, 2, 3)$

$$G^{(2)} = \begin{pmatrix} g_2 & g_3 & 1 & g_1 \\ g_2 & -g_3 & 1 & -g_1 \\ g_2 & -g_3 & -1 & g_1 \\ g_2 & g_3 & -1 & -g_1 \end{pmatrix}, \quad G^{(3)} = \begin{pmatrix} g_3 & g_2 & g_1 & 1 \\ g_3 & -g_2 & g_1 & -1 \\ g_3 & -g_2 & -g_1 & 1 \\ g_3 & g_2 & -g_1 & -1 \end{pmatrix}, \tag{6}$$

where $g_1 = e^{-i\alpha}$, $g_2 = e^{-i\beta}$, and $g_3 = e^{-i\gamma}$.

In order to realize the JRSP, Alice first performs the two-qubit projective measurement on the qubits A_1 andA_2 under the basis $\{|\mu_k\rangle\}(k = 0, 1, 2, 3)$ and publicly announces her measurement outcome. Next, according to Alice's measurement result, Bob should choose one of the measuring bases $\{|\lambda_j^{(k)}\rangle\}(k, j = 0, 1, 2, 3)$ to measure his qubits B_1 and B_2. After the measurement, Bob informs Charlie of his result of measurement by the classical channel. In accord with Alice's and Bob's results, Charlie can recover the original state $|p\rangle$ by suitable unitary operation. For instance, without loss of generality, assume Alice's measurement outcome is $|\mu_0\rangle_{A_1A_2}$, Bob should choose measuring basis $\{|\lambda_j^{(0)}\rangle\}(j = 0, 1, 2, 3)$ to measure the qubits B_1 and B_2, and then inform Charlie of his measurement outcome by classical channel. If Bob's measurement result is $|\lambda_2^{(0)}\rangle_{B_1B_2}$, the qubits C_1 and C_2 will collapse into the state $\frac{1}{2}(x|00\rangle - ye^{i\alpha}|01\rangle + ze^{i\beta}|10\rangle - we^{i\gamma}|11\rangle)_{C_1C_2}$. According to Alice's and Bob's public announcement, Charlie can perform the local unitary operation $U_C = (I)_{C_1}\otimes(\sigma_z)_{C_2}$ on his qubits C_1 and C2, then the original state $|p\rangle$ can be reconstructed at his side. If Alice's measurement results are the other 3 cases, i.e. $|\mu_1\rangle_{A_1A_2}$, $|\mu_2\rangle_{A_1A_2}$ and $|\mu_3\rangle_{A_1A_2}$, Bob should choose appropriate measuring bases $\{|\lambda_j^{(1)}\rangle\}$, $\{|\lambda_j^{(2)}\rangle\}$, and $\{|\lambda_j^{(3)}\rangle\}(j = 0, 1, 2, 3)$ to measure his qubits B_1 and B_2, respectively. Then Charlie performs appropriate unitary transformation on qubits C_1 and C_2, and the original state can be recovered. The relation between the results obtained by Alice and Bob and appropriate unitary operation performed by Charlie is shown in Table 1. It is easily found that, for all the 16 measurement outcomes of Alice and Bob, the receiver Charlie can reconstruct the original state $|p\rangle$ by performing suitable unitary operations U_C.

Compared with the previous scheme in Ref. [31], in our scheme, the consumed amount of entanglement and the required classical communication cost (four bits) are the same as that in [31] and the advantage of our scheme is that the total success probability process being 1. In this sense, our scheme is an optimal one.

2.2 JRSP with N senders

Now we will generalize the above scheme to the case of N senders. Suppose that N senders Alice and $\text{Bob}_1, \text{Bob}_2, \ldots, \text{Bob}_{N-1}$ share an arbitrary two-qubit entangled state

$$|P\rangle = x|00\rangle + ye^{i\alpha}|01\rangle + ze^{i\beta}|10\rangle + we^{i\gamma}|11\rangle, \tag{7}$$

where $x, y, z, w, \alpha, \beta, \gamma$ are real, and $x^2 + y^2 + z^2 + w^2 = 1$, Assume the N senders wish to help the receiver Charlie remotely prepare the original state $|P\rangle$, and they know the state $|P\rangle$ partly, i.e. Alice knows x, y, z, w, Bob_1 knows $\alpha_1, \beta_1, \gamma_1$, Bob_2 knows $\alpha_2, \beta_2, \gamma_2, \ldots, \text{Bob}_{N-1}$ knows $\alpha_{N-1}, \beta_{N-1}, \gamma_{N-1}$, where $\alpha = \alpha_1 + \alpha_2 + \cdots + \alpha_{N-1}$, $\beta = \beta_1 + \beta_2 + \cdots + \beta_{N-1}$, $\gamma = \gamma_1 + \gamma_2 + \cdots + \gamma_{N-1}$, but Charlie does not know them at all. We also suppose that two (N + 1)-qubit GHZ states are shared by N senders and Charlie as the quantum channel, which are given by

$$\begin{aligned}
|\Phi_1\rangle &= \frac{1}{\sqrt{2}}(|0\rangle^{\otimes(N+1)} + |1\rangle^{\otimes(N+1)})_{A^{(1)}B_1^{(1)}B_2^{(1)}\cdots B_{N-1}^{(1)}C^{(1)}},\\
|\Phi_2\rangle &= \frac{1}{\sqrt{2}}(|0\rangle^{\otimes(N+1)} + |1\rangle^{\otimes(N+1)})_{A^{(2)}B_1^{(2)}B_2^{(2)}\cdots B_{N-1}^{(2)}C^{(2)}},
\end{aligned} \tag{8}$$

where qubits $A^{(1)}$ and $A^{(2)}$ belong to Alice, qubits $B(1)_1$ and $B_1^{(2)}$ to Bob_1, ..., qubits $B_{N-1}^{(1)}$ and $B_{N-1}^{(2)}$ to Bob_{N-1} and qubits $C^{(1)}$ and $C^{(2)}$ to Charlie, respectively. Similar to above scheme, the N senders must construct their own measuring basis. The first measuring basis chosen by Alice is still in Eqs. (3) and (4), and the measuring bases chosen by $\text{Bob}_1, \ldots, \text{Bob}_{N-1}$ are $4(N-1)$ sets of MOBVs $\{|\lambda_{jl}^{(k)}\rangle\}(k, j = 0, 1, 2, 3; l = 1, 2, \ldots, N-1)$, which are given by

$$\begin{pmatrix} |\lambda_{0l}^{(k)}\rangle \\ |\lambda_{1l}^{(k)}\rangle \\ |\lambda_{2l}^{(k)}\rangle \\ |\lambda_{3l}^{(k)}\rangle \end{pmatrix} = G_l^{(k)} \begin{pmatrix} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{pmatrix}, \tag{9}$$

where

$$G_l^{(0)} = \begin{pmatrix} 1 & g_{1l} & g_{2l} & g_{3l} \\ 1 & -g_{1l} & g_{2l} & -g_{3l} \\ 1 & -g_{1l} & -g_{2l} & g_{3l} \\ 1 & g_{1l} & -g_{2l} & -g_{3l} \end{pmatrix}, \quad G_l^{(1)} = \begin{pmatrix} g_{1l} & 1 & g_{3l} & g_{2l} \\ g_{1l} & -1 & g_{3l} & -g_{2l} \\ g_{1l} & -1 & -g_{3l} & g_{2l} \\ g_{1l} & 1 & -g_{3l} & -g_{2l} \end{pmatrix},$$

$$G_l^{(2)} = \begin{pmatrix} g_{2l} & g_{3l} & 1 & g_{1l} \\ g_{2l} & -g_{3l} & 1 & -g_{1l} \\ g_{2l} & -g_{3l} & -1 & g_{1l} \\ g_{2l} & g_{3l} & -1 & -g_{1l} \end{pmatrix}, \quad G_l^{(3)} = \begin{pmatrix} g_{3l} & g_{2l} & g_{1l} & 1 \\ g_{3l} & -g_{2l} & g_{1l} & -1 \\ g_{3l} & -g_{2l} & -g_{1l} & 1 \\ g_{3l} & g_{2l} & -g_{1l} & -1 \end{pmatrix}, \quad (10)$$

where $g_{1l} = e^{-i\alpha_l}$, $g_{2l} = e^{-i\beta_l}$, and $g_{3l} = e^{-i\gamma_l} (l = 1, 2, \ldots, N-1)$.

Now let Alice first perform the two-qubit projective measurement on her qubits $A^{(1)}$ and $A^{(2)}$ under the basis $\{|\mu_k\rangle\}$ [see Eqs. (3) and (4)] and publicly announces her outcome of measurement. In accord with the Alice's result, $Bob_1, Bob_2, \ldots, Bob_{N-1}$ should choose suitable measuring basis in the MOBVs $\{|\lambda_{jl}^{(k)}\rangle\}$ to measure their own qubits $B_l^{(1)}$ and $B_l^{(2)} (l = 1, 2, \ldots, N-1)$, and then inform Charlie of their measurement outcomes, respectively. According to the results of N senders, the receiver Charlie can reconstruct the original state $|P\rangle$. For instance, without loss of generality, suppose that Alice's measurement outcome is $|\mu_0\rangle_{A_1A_2}$, then $Bob_1, Bob_2, \ldots, Bob_{N-1}$ should choose suitable measuring bases $\{|\lambda_{j1}^{(0)}\rangle\}, \{|\lambda_{j2}^{(0)}\rangle\}, \ldots, \{|\lambda_{j(N-1)}^{(0)}\rangle\} (j = 0, 1, 2, 3)$ to measure their own qubits $(B1^{(1)}, B_1^{(2)})$, $(B2^{(1)}, B_2^{(2)})$,..., and $(BN-1^{(1)}, B_{N-1}^{(2)})$, respectively. Assume that the Bob_1's measurement outcome is only $|\lambda_{11}^{(0)}\rangle_{B_1^{(1)}B_1^{(2)}}$ while all other sender's results are $|\lambda_{0m}^{(0)}\rangle_{B_m^{(1)}B_m^{(2)}} (m = 2, 3, \ldots, N-1)$, respectively, the qubits $C^{(1)}$ and$C^{(2)}$ will be collapsed into the state $\frac{1}{2}(x|00\rangle - ye^{i\alpha}|01\rangle + ze^{i\beta}|10\rangle - we^{i\gamma}|11\rangle)_{C^{(1)}C^{(2)}}$. In accord with the outcomes of N senders, Charlie should perform the unitary operation $(I)_{C^{(1)}} \otimes (\sigma_z)_{C^{(2)}}$ on the qubits $C^{(1)}$ and$C^{(2)}$, then the original state $|P\rangle$ can be reconstructed. If N senders obtain other measurement results, similar to above method, the receiver Charlie can reconstruct the original state $|P\rangle$ by appropriate unitary operations, and the total success probability of the JRSP is still 1. Here we no longer depict them one by one. The required classical communication cost is 2N bits.

3 JRSP of an arbitrary three-qubit state with complex coefficients

Now let us further propose the joint remote preparation of an arbitrary three-qubit entangled state with complex coefficients. In the first case the original state is shared by two senders, while N senders are considered in the second case.

3.1 JRSP with two senders

Suppose that two senders Alice and Bob wish to help the receiver Charlie remotely prepare the state

$$\begin{aligned} |q\rangle = & x_0|000\rangle + x_1e^{i\delta_1}|001\rangle + x_2e^{i\delta_2}|010\rangle + x_3e^{i\delta_3}|011\rangle \\ & + x_4e^{i\delta_4}|100\rangle + x_5e^{i\delta_5}|101\rangle + x_6e^{i\delta_6}|110\rangle + x_7e^{i\delta_7}|111\rangle, \end{aligned} \quad (11)$$

where x_j and $\delta_j (j = 0, 1, \ldots, 7)$ are real, $\delta_0 = 0$ and $\sum_{j=0}^{7} x_j^2 = 1$. Assume that Alice and Bob share the state $|q\rangle$ and they know the state partly, that is Alice knows

$x_j (j = 0, 1, \ldots, 7)$, and Bob knows $\delta_j (j = 0, 1, \ldots, 7)$, but Charlie does not know them at all. We also suppose that the state shared by Alice, Bob, and Charlie as quantum channel are three GHZ states

$$|\psi_{1(2,3)}\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{A_1B_1C_1(A_2B_2C_2,A_3B_3C_3)}, \tag{12}$$

where the qubits A_1, A_2, A_3 belong to Alice, qubits B_1, B_2, B_3 to Bob, and qubits C_1, C_2, C_3 to Charlie, respectively.

In order to complete the JRSP, Alice and Bob should construct their measuring bases. The first measuring basis chosen by Alice is a set of MOBVs $\{|\eta_k\rangle\}(k = 0, 1, \ldots, 7)$, which is given by

$$\begin{pmatrix} |\eta_0\rangle \\ |\eta_1\rangle \\ |\eta_2\rangle \\ |\eta_3\rangle \\ |\eta_4\rangle \\ |\eta_5\rangle \\ |\eta_6\rangle \\ |\eta_7\rangle \end{pmatrix} = H \begin{pmatrix} |000\rangle \\ |001\rangle \\ |010\rangle \\ |011\rangle \\ |100\rangle \\ |101\rangle \\ |110\rangle \\ |111\rangle \end{pmatrix}, \tag{13}$$

where

$$H = \begin{pmatrix} x_0 & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ x_1 & -x_0 & x_3 & -x_2 & x_5 & -x_4 & x_7 & -x_6 \\ x_2 & -x_3 & -x_0 & x_1 & -x_6 & x_7 & x_4 & -x_5 \\ x_3 & x_2 & -x_1 & -x_0 & x_7 & x_6 & -x_5 & -x_4 \\ x_4 & -x_5 & x_6 & -x_7 & -x_0 & x_1 & -x_2 & x_3 \\ x_5 & x_4 & -x_7 & -x_6 & -x_1 & -x_0 & x_3 & x_2 \\ x_6 & -x_7 & -x_4 & x_5 & x_2 & -x_3 & -x_0 & x_1 \\ x_7 & x_6 & x_5 & x_4 & -x_3 & -x_2 & -x_1 & -x_0 \end{pmatrix}. \tag{14}$$

The second measuring bases chosen by Bob are eight sets of MOBVs $\{|\tau_j^{(k)}\rangle\}$, which are given by

$$\begin{pmatrix} |\tau_0^{(k)}\rangle \\ |\tau_1^{(k)}\rangle \\ |\tau_2^{(k)}\rangle \\ |\tau_3^{(k)}\rangle \\ |\tau_4^{(k)}\rangle \\ |\tau_5^{(k)}\rangle \\ |\tau_6^{(k)}\rangle \\ |\tau_7^{(k)}\rangle \end{pmatrix} = M^{(k)} \begin{pmatrix} |000\rangle \\ |001\rangle \\ |010\rangle \\ |011\rangle \\ |100\rangle \\ |101\rangle \\ |110\rangle \\ |111\rangle \end{pmatrix}, \tag{15}$$

where $k = 0, 1, \ldots, 7$, and $M^{(k)}$is a 8×8 matrix,

$$\begin{aligned}
M^{(0)} &= M(1, r_1, r_2, r_3, r_4, r_5, r_6, r_7), \\
M^{(1)} &= M(r_1, 1, r_3, r_2, r_5, r_4, r_7, r_6), \\
M^{(2)} &= M(r_2, r_3, 1, r_1, r_6, r_7, r_4, r_5), \\
M^{(3)} &= M(r_3, r_2, r_1, 1, r_7, r_6, r_5, r_4), \\
M^{(4)} &= M(r_4, r_5, r_6, r_7, 1, r_1, r_2, r_3), \\
M^{(5)} &= M(r_5, r_4, r_7, r_6, r_1, 1, r_3, r_2), \\
M^{(6)} &= M(r_6, r_7, r_4, r_5, r_2, r_3, 1, r_1), \\
M^{(7)} &= M(r_7, r_6, r_5, r_4, r_3, r_2, r_1, 1),
\end{aligned} \tag{16}$$

where $r_j = e^{-i\delta_j}$ $(j = 0, 1, \ldots, 7)$, $\delta_0 = 0$, and $M_{(a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8)}$ in Eq. (16) is given by

$$M_{(a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8)} = \begin{pmatrix}
a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \\
a_1 & -a_2 & a_3 & -a_4 & a_5 & -a_6 & a_7 & -a_8 \\
a_1 & -a_2 & -a_3 & a_4 & -a_5 & a_6 & a_7 & -a_8 \\
a_1 & a_2 & -a_3 & -a_4 & a_5 & a_6 & -a_7 & -a_8 \\
a_1 & -a_2 & a_3 & -a_4 & -a_5 & a_6 & -a_7 & a_8 \\
a_1 & a_2 & -a_3 & -a_4 & -a_5 & -a_6 & a_7 & a_8 \\
a_1 & -a_2 & -a_3 & a_4 & a_5 & -a_6 & -a_7 & a_8 \\
a_1 & a_2 & a_3 & a_4 & -a_5 & -a_6 & -a_7 & -a_8
\end{pmatrix}. \tag{17}$$

Now let Alice first perform three-qubit projective measurement on the qubits A_1, A_2, A_3 by using the basis $\{|\eta_k\rangle\}(k = 0, 1, \ldots, 7)$ and publicly announces her measurement outcome. Next, in accord with Alice's result of measurement, Bob should choose one of the measuring bases $\{|\tau_j^{(k)}\rangle\}(k, j = 0, 1, \ldots, 7)$ to measure his qubits B_1, B_2 and B_3. After the measurement, Bob informs Charlie of his result of measurement by the classical channel. According to Alice's and Bob's results, Charlie can reconstruct the original state $|q\rangle$ by suitable unitary operation. For example, without loss of generality, assume Alice's measurement outcome is $|\eta_1\rangle_{A_1A_2A_3}$, Bob should choose measuring basis $\{|\tau_j^{(1)}\rangle\}$, which is given by

$$\begin{pmatrix}
|\tau_0^{(1)}\rangle \\ |\tau_1^{(1)}\rangle \\ |\tau_2^{(1)}\rangle \\ |\tau_3^{(1)}\rangle \\ |\tau_4^{(1)}\rangle \\ |\tau_5^{(1)}\rangle \\ |\tau_6^{(1)}\rangle \\ |\tau_7^{(1)}\rangle
\end{pmatrix} = \begin{pmatrix}
r_1 & 1 & r_3 & r_2 & r_5 & r_4 & r_7 & r_6 \\
r_1 & -1 & r_3 & -r_2 & r_5 & -r_4 & r_7 & -r_6 \\
r_1 & -1 & -r_3 & r_2 & -r_5 & r_4 & r_7 & -r_6 \\
r_1 & 1 & -r_3 & -r_2 & r_5 & r_4 & -r_7 & -r_6 \\
r_1 & -1 & r_3 & -r_2 & -r_5 & r_4 & -r_7 & r_6 \\
r_1 & 1 & -r_3 & -r_2 & -r_5 & -r_4 & r_7 & r_6 \\
r_1 & -1 & -r_3 & r_2 & r_5 & -r_4 & -r_7 & r_6 \\
r_1 & 1 & r_3 & r_2 & -r_5 & -r_4 & -r_7 & -r_6
\end{pmatrix} \begin{pmatrix}
|000\rangle \\ |001\rangle \\ |010\rangle \\ |011\rangle \\ |100\rangle \\ |101\rangle \\ |110\rangle \\ |111\rangle
\end{pmatrix}, \tag{18}$$

to measure the qubits B_1, B_2, B_3, and then inform Charlie of his measurement result by classical channel. If Bob's measurement result is $|\tau_3^{(1)}\rangle_{B_1B_2B_3}$, the qubits C_1, C_2 and C_3 will collapse into the state $\frac{1}{2\sqrt{2}}(x_1e^{i\delta_1}|000\rangle + x_0|001\rangle - x_3e^{i\delta_3}|010\rangle - x_2e^{i\delta_2}|011\rangle - x_5e^{i\delta_5}|100\rangle - x_4e^{i\delta_4}|101\rangle + x_7e^{i\delta_7}|110\rangle + x_6e^{i\delta_6}|111\rangle)_{C_1C_2C_3}$. According to Alice's and Bob's public announcements, Charlie can perform the local unitary operation $(\sigma_z)_{C_1} \otimes (\sigma_z)_{C_2} \otimes (\sigma_x)_{C_3}$ on his qubits C_1, C_2 and C_3, thus the original state can be recovered. If Alice's measurement outcomes are the other 7 cases in the basis $\{|\eta_k\rangle\}(k = 0, 1, \ldots, 7)$, Bob should choose appropriate measuring bases $\{|\tau_j^{(k)}\rangle\}(k, j = 0, 1, \ldots, 7)$ to measure his qubits B_1, B_2 and B_3. The corresponding relation of Alice's measurement result $|\eta_k\rangle_{A_1A_2A_3}$ and the measuring basis $\{|\tau_j^{(k)}\rangle\}$ performed by Bob can be described as $|\eta_k\rangle_{A_1A_2A_3} \rightarrow \{|\tau_j^{(k)}\rangle\}(k, j = 0, 1, \ldots, 7)$. Explicitly,

$$\begin{aligned}
|\eta_0\rangle_{A_1A_2A_3} &\longrightarrow \{|\tau_j^{(0)}\rangle\},\\
|\eta_1\rangle_{A_1A_2A_3} &\longrightarrow \{|\tau_j^{(1)}\rangle\},\\
|\eta_2\rangle_{A_1A_2A_3} &\longrightarrow \{|\tau_j^{(2)}\rangle\},\\
|\eta_3\rangle_{A_1A_2A_3} &\longrightarrow \{|\tau_j^{(3)}\rangle\},\\
|\eta_4\rangle_{A_1A_2A_3} &\longrightarrow \{|\tau_j^{(4)}\rangle\},\\
|\eta_5\rangle_{A_1A_2A_3} &\longrightarrow \{|\tau_j^{(5)}\rangle\},\\
|\eta_6\rangle_{A_1A_2A_3} &\longrightarrow \{|\tau_j^{(6)}\rangle\},\\
|\eta_7\rangle_{A_1A_2A_3} &\longrightarrow \{|\tau_j^{(7)}\rangle\},
\end{aligned} \tag{19}$$

where $j = 0, 1, \ldots, 7$. Similar to above approach, after Alice's and Bob's measurements, Charlie can reconstruct the original state $|q\rangle$ by appropriate unitary operation at his side. Compared with the previous scheme in Ref. [34], in our scheme, the consumed amount of entanglement is the same as that in [34], and the required classical communication cost is six bits. Especially, it is easily found that the total success probability of our scheme can reach 1.

3.2 JRSP with N senders

The scheme in Sect. 3.1 can be generalized to the case of N senders. Suppose that Alice and $\text{Bob}_1, \text{Bob}_2, \ldots, \text{Bob}_{N-1}$ wish to help the receiver Charlie remotely prepare an arbitrary three-qubit entangled state

$$\begin{aligned}
|Q\rangle = x_0|000\rangle + x_1e^{i\varphi_1}|001\rangle + x_2e^{i\varphi_2}|010\rangle + x_3e^{i\varphi_3}|011\rangle\\
+x_4e^{i\varphi_4}|100\rangle + x_5e^{i\varphi_5}|101\rangle + x_6e^{i\varphi_6}|110\rangle + x_7e^{i\varphi_7}|111\rangle,
\end{aligned} \tag{20}$$

where x_j and $\varphi_j(j = 0, 1, \ldots, 7)$ are real, $\varphi_0 = 0$ and $\sum_{j=0}^{7} x_j^2 = 1$. Assume that the N senders know the state $|Q\rangle$ partly, i.e. Alice knows $x_j(j = 0, 1, \ldots, 7)$, Bob_1

knows $\varphi_j^{(1)}$, Bob$_2$ knows $\varphi_j^{(2)}, \ldots,$ Bob$_{N-1}$ knows $\varphi_j^{(N-1)}$, where $\varphi_j = \varphi_j^{(1)} + \varphi_j^{(2)} + \cdots + \varphi_j^{(N-1)}$ $(j = 0, 1, \ldots, 7)$, but Charlie does not know them at all. We also suppose that the N sender and receiver Charlie share three (N + 1)-qubit GHZ states as the quantum channel, which are given by

$$
\begin{aligned}
|\Psi_1\rangle &= \frac{1}{\sqrt{2}}(|0\rangle^{\otimes(N+1)} + |1\rangle^{\otimes(N+1)})_{A^{(1)}B_1^{(1)}B_2^{(1)}\cdots B_{N-1}^{(1)}C^{(1)}},\\
|\Psi_2\rangle &= \frac{1}{\sqrt{2}}(|0\rangle^{\otimes(N+1)} + |1\rangle^{\otimes(N+1)})_{A^{(2)}B_1^{(2)}B_2^{(2)}\cdots B_{N-1}^{(2)}C^{(2)}},\\
|\Psi_3\rangle &= \frac{1}{\sqrt{2}}(|0\rangle^{\otimes(N+1)} + |1\rangle^{\otimes(N+1)})_{A^{(3)}B_1^{(3)}B_2^{(3)}\cdots B_{N-1}^{(3)}C^{(3)}},
\end{aligned}
\tag{21}
$$

where qubits $A^{(1)}$, $A^{(2)}$ and $A^{(3)}$ belong to Alice, qubits $B_1^{(1)}$, $B_1^{(2)}$ and $B_1^{(3)}$ to Bob$_1$, …, qubits $B_{N-1}^{(1)}$, $B_{N-1}^{(2)}$ and $B_{N-1}^{(3)}$ to Bob$_{N-1}$, and qubits $C^{(1)}$, $C^{(2)}$ and $C^{(3)}$ to Charlie, respectively. As in the above scheme, the N senders must construct their own measurement basis. The first measuring basis chosen by Alice is still in Eqs. (13) and (14), and the measuring bases chosen by Bob$_1$, Bob$_2, \ldots,$ Bob$_{N-1}$ are $8(N-1)$ sets of MOBVs $\{|\tau_{jl}^{(k)}\rangle\}(k, j = 0, 1, \ldots, 7, l = 1, 2, \ldots, N-1)$, which are given by

$$
\begin{pmatrix}
|\tau_{0l}^{(k)}\rangle\\
|\tau_{1l}^{(k)}\rangle\\
|\tau_{2l}^{(k)}\rangle\\
|\tau_{3l}^{(k)}\rangle\\
|\tau_{4l}^{(k)}\rangle\\
|\tau_{5l}^{(k)}\rangle\\
|\tau_{6l}^{(k)}\rangle\\
|\tau_{7l}^{(k)}\rangle
\end{pmatrix}
= R_l^{(k)}
\begin{pmatrix}
|000\rangle\\
|001\rangle\\
|010\rangle\\
|011\rangle\\
|100\rangle\\
|101\rangle\\
|110\rangle\\
|111\rangle
\end{pmatrix},
\tag{22}
$$

where $k = 0, 1, \ldots, 7$; $l = 1, 2, \ldots, N-1$, and

$$
\begin{aligned}
R_l^{(0)} &= R(1, r_{1l}, r_{2l}, r_{3l}, r_{4l}, r_{5l}, r_{6l}, r_{7l}),\\
R_l^{(1)} &= R(r_{1l}, 1, r_{3l}, r_{2l}, r_{5l}, r_{4l}, r_{7l}, r_{6l}),\\
R_l^{(2)} &= R(r_{2l}, r_{3l}, 1, r_{1l}, r_{6l}, r_{7l}, r_{4l}, r_{5l}),\\
R_l^{(3)} &= R(r_{3l}, r_{2l}, r_{1l}, 1, r_{7l}, r_{6l}, r_{5l}, r_{4l}),\\
R_l^{(4)} &= R(r_{4l}, r_{5l}, r_{6l}, r_{7l}, 1, r_{1l}, r_{2l}, r_{3l}),\\
R_l^{(5)} &= R(r_{5l}, r_{4l}, r_{7l}, r_{6l}, r_{1l}, 1, r_{3l}, r_{2l}),\\
R_l^{(6)} &= R(r_{6l}, r_{7l}, r_{4l}, r_{5l}, r_{2l}, r_{3l}, 1, r_{1l}),\\
R_l^{(7)} &= R(r_{7l}, r_{6l}, r_{5l}, r_{4l}, r_{3l}, r_{2l}, r_{1l}, 1),
\end{aligned}
\tag{23}
$$

where $r_{jl} = e^{-i\varphi_j^{(l)}}$ $(j = 0, 1, \ldots, 7, l = 1, 2, \ldots, N-1)$, and $R(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ is also a 8×8 matrix which similar to Eq. (17).

Alice first performs the three-qubit projective measurement on her qubits $A^{(1)}$, $A^{(2)}$ and $A^{(3)}$ under the basis $\{|\eta_k\rangle\}$ [see Eqs. (13) and (14)] and publicly announces her result of measurement. According to Alice's outcome, $\mathrm{Bob}_1, \mathrm{Bob}_2, \ldots, \mathrm{Bob}_{N-1}$ should choose suitable measuring basis in the MOBVs $\{|\tau_{jl}^{(k)}\rangle\}(k, j = 0, 1, \ldots, 7, l = 1, 2, \ldots, N-1)$ to measure their own qubits $(B_1^{(1)}, B_1^{(2)}, B_1^{(3)}), (B_2^{(1)}, B_2^{(2)}, B_2^{(3)}), \ldots, (B_{N-1}^{(1)}, B_{N-1}^{(2)}, B_{N-1}^{(3)})$, and then inform Charlie of their measurement results, respectively. In accord with the announcement of N senders, the receiver Charlie can reconstruct the original state $|Q\rangle$ by using appropriate unitary operation. For example, without loss of generality, suppose that Alice's measurement result is $|\eta_0\rangle_{A_1A_2A_3}$, then $\mathrm{Bob}_1, \mathrm{Bob}_2, \ldots, \mathrm{Bob}_{N-1}$ should choose suitable measuring bases $\{|\tau_{j1}^{(0)}\rangle\}, \{|\tau_{j2}^{(0)}\rangle\}, \ldots, \{|\tau_{j(N-1)}^{(0)}\rangle\}$ [see Eqs. (22) and (23)] to measure their own qubits, respectively. Assume that the Bob_1's measurement result is only $|\tau_{11}^{(0)}\rangle_{B_1^{(1)}B_1^{(2)}B_1^{(3)}}$ while all other senders' results are $|\tau_{0m}^{(0)}\rangle_{B_m^{(1)}B_m^{(2)}B_m^{(3)}}(m = 2, 3, \ldots, N-1)$, respectively, the qubits $C^{(1)}$, $C^{(2)}$ and $C^{(3)}$ will be collapsed into the state $\frac{1}{\sqrt{2}}(x_0|000\rangle - x_1e^{i\varphi_1}|001\rangle + x_2e^{i\varphi_2}|010\rangle - x_3e^{i\varphi_3}|011\rangle + x_4e^{i\varphi_4}|100\rangle - x_5e^{i\varphi_5}|101\rangle + x_6e^{i\varphi_6}|110\rangle - x_7e^{i\varphi_7}|111\rangle)_{C^{(1)}C^{(2)}C^{(3)}}$. According to the results of N senders, Charlie can perform the unitary operation $(I)_{C^{(1)}} \otimes (I)_{C^{(2)}} \otimes (\sigma_z)_{C^{(3)}}$ on the qubits $C^{(1)}$, $C^{(2)}$ and $C^{(3)}$, then the original state $|Q\rangle$ can be reconstructed. If Alice's measurement results are the other 7 cases in the basis $\{|\eta_k\rangle\}(k = 0, 1, \ldots, 7)$, $\mathrm{Bob}_1, \mathrm{Bob}_2, \ldots, \mathrm{Bob}_{N-1}$ should choose suitable measuring bases $\{|\tau_{jl}^{(k)}\rangle\}(k, j = 0, 1, \ldots, 7, l = 1, 2, \ldots, N-1)$ to measure their own qubits, respectively, then Charlie can recover the original state $|Q\rangle$ by appropriate unitary operations. Here we no longer depict them one by one. The corresponding relation of Alice's measurement outcome $|\eta_k\rangle_{A_1A_2A_3}$ and the measuring basis $\{|\tau_{jl}^{(k)}\rangle\}$ performed by $\mathrm{Bob}_1, \mathrm{Bob}_2, \ldots, \mathrm{Bob}_{N-1}$ can be described as

$$\begin{aligned}
|\eta_0\rangle_{A_1A_2A_3} &\longrightarrow \{\tau_{jl}^{(0)}\rangle\}, \\
|\eta_1\rangle_{A_1A_2A_3} &\longrightarrow \{\tau_{jl}^{(1)}\rangle\}, \\
|\eta_2\rangle_{A_1A_2A_3} &\longrightarrow \{\tau_{jl}^{(2)}\rangle\}, \\
|\eta_3\rangle_{A_1A_2A_3} &\longrightarrow \{\tau_{jl}^{(3)}\rangle\}, \\
|\eta_4\rangle_{A_1A_2A_3} &\longrightarrow \{\tau_{jl}^{(4)}\rangle\}, \\
|\eta_5\rangle_{A_1A_2A_3} &\longrightarrow \{\tau_{jl}^{(5)}\rangle\}, \\
|\eta_6\rangle_{A_1A_2A_3} &\longrightarrow \{\tau_{jl}^{(6)}\rangle\}, \\
|\eta_7\rangle_{A_1A_2A_3} &\longrightarrow \{\tau_{jl}^{(7)}\rangle\},
\end{aligned} \tag{24}$$

where $j = 0, 1, \ldots, 7, l = 1, 2, \ldots, N-1$. In this scheme, the total successful probability of the JRSP is still 1, and the required classical communication cost is $3N$ bits.

4 Conclusion

In conclusion, we have presented several new schemes for joint remote preparation of arbitrary two- and three-qubit entangled states. In these schemes, the coefficients of the original states to be co-prepared are all complex. In the first scheme, two sender share an arbitrary two-qubit state, but each sender only partly knows the state, and two three-qubit GHZ states are exploited as the quantum channel. In order to help the receiver remotely prepare the original state, in accord with the knowledge of the original state which she/he known, each sender must construct her/his own two-qubit measuring basis. Firstly, a sender performs a two-qubit projective measurement on her qubits, then another sender should choose, according to the measurement result of the first sender, an appropriate two-qubit measuring basis to measure his qubits. After these projective measurements, the receiver can reconstruct the original state by means of appropriate unitary operation. Then we generalize the scheme to N senders case. In the generalized scheme, the original state is shared by the N senders and the quantum channel shared by the N senders and the receiver are two $(\mathrm{N}+1)$-qubit GHZ states. It is shown that, only if when N senders collaborate with each other, the receiver can remotely reconstruct the original state. Next, we have proposed two schemes for JRSP of arbitrary three-qubit entangled state with two senders and N senders via three three-qubit GHZ states and three $(\mathrm{N}+1)$-qubit GHZ states as the quantum channel, respectively. To complete the JRSP schemes, some novel sets of two-qubit mutually orthogonal basis vectors have been introduced. After the projective measurements by two senders (or N senders) under these bases, respectively, the original state can be recovered by the receiver. Compared with the previous schemes of JSRP in Refs. [27–37], the advantage of all the present schemes is that the total success probability reaches 1. In this sense, our schemes are optimal. Thus, we hope that our schemes will be helpful in the deeper understanding of the process of RSP, and may be useful for the further studies on quantum information science, such as quantum secret sharing and quantum network communication.

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