

Quantum teleportation in a dissipative environment

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Received: 11 November 2011 / Accepted: 16 December 2011 / Published online: 23 December 2011
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Abstract We investigate quantum teleportation of a single-qubit state for the situation in which both qubits of the entangled channel are subjected to local structured reservoirs. We consider the effect of entanglement sudden death (ESD) of the channel on the average fidelity of the teleportation. It is shown the appearance of ESD leads to an abrupt variation of the fidelity of quantum teleportation. In addition, we show the fidelity exhibits oscillations in the non-Markovian reservoir due to the memory effect of the reservoir.

Keywords Quantum teleportation · Dissipative reservoir · Entanglement sudden death · Non-Markovian effect

1 Introduction

By using prior shared entanglement as a channel, quantum teleportation [1] can transmit an unknown quantum state from the sender Alice to the receiver Bob without transferring the physical carrier of the state [2–5]. The same as other entanglement dependent quantum protocols, a perfect implementation of quantum teleportation relies on the quality of the shared entanglement [6–9]. Since a real quantum system unavoidably interacts with its surroundings undergoing consequent decoherence and entanglement degradation, the study of quantum teleportation in the presence of various noisy environments proves to be significant and has attracted more and more attentions [9–20].

Recently, an intriguing dynamical feature of entanglement has been experimentally confirmed for the case of two qubits [21–23] that entanglement may disappear in a

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finite time, an interesting phenomenon termed as entanglement sudden death (ESD) [24–27]. The phenomenon of ESD has been extensively studied, partially due to a concern about its detrimental effect on the entanglement dependent quantum protocol. That is, an entangled resource undergoing ESD might put a limitation on its work time in practice. However, it is not yet clear what the explicit effect of ESD on quantum task besides the limited results [28–31]. In the area of quantum error correction, an explicit study of the three-qubit phase flip code concludes that there is no fundamental relationship between ESD and the failure of the code, i.e., this specific code is indifferent to ESD [28]. In Ref. [29], the authors have studied the entanglement evolution of a four-qubit cluster state in a dephasing environment concentrating on the effect of ESD on the utilization of this cluster state as a means of implementing a single-qubit rotation in the measurement-based cluster state model of quantum computation. Through comparing the evolution of the entanglement to the fidelity, the authors find that ESD does not cause a change in behavior or discontinuity in the fidelity [29]. However, the question of whether ESD affects quantum information processing requires a further study and may be related to the role of entanglement in that processing [29]. To the best of our knowledge, the specific effect of ESD on the implementation of quantum teleportation is not clear so far. It is worth to know if ESD results in some abrupt variation for the fidelity of quantum teleportation. In this work, focusing on a situation in which both two qubits of the entangled channel are subjected to local structured reservoirs, we show that when ESD occurs for the channel the average fidelity of quantum teleportation abruptly drops to $2/3$, i.e., the best possible value obtained only by the classical communication [6]. By contrast, in the absence of ESD, the fidelity only asymptotically decays to $2/3$. The result demonstrates an obvious influence of ESD on quantum teleportation.

Due to the advance of experimental technique, the non-Markovian features have been observed in some physical systems. It has been shown that the entanglement of two noninteracting qubits embedded in separated non-Markovian environments can revive after a period of sudden death [32, 33]. The similar result has been obtained for two qubits interacting with a common non-Markovian environment [34]. In this connection, it is interesting to know if the fidelity of quantum teleportation can revive after it has dropped to $2/3$ in the non-Markovian regime. We shall show in the following that the answer is positive, namely, the quantum advantage of teleportation can recover after it has been lost thanks to the memory effect of the non-Markovian reservoir.

2 The model and result

Consider each of the two qubits A and B of the entangled channel locally interacts with its own multimode vacuum reservoir. The whole system can be described via the sum of two independent qubit-reservoir Hamiltonian of the form ($\hbar = 1$)

$$\hat{H} = \omega_0 \hat{\sigma}_+ \hat{\sigma}_- + \sum_j \omega_j \hat{a}_j^+ \hat{a}_j + \sum_j (g_j \hat{\sigma}_+ \hat{a}_j + g_j^* \hat{\sigma}_- \hat{a}_j^+), \quad (1)$$

where $\hat{\sigma}_+ = |1\rangle\langle 0|$ and $\hat{\sigma}_- = |0\rangle\langle 1|$ are the raising and lowering operators of the qubit $S = A, B$ with the transition frequency ω_0 ; \hat{a}_j^+ (\hat{a}_j) is the creation (annihilation) operator of mode j of the reservoir with the frequency ω_j ; g_j measures the strength of coupling between the qubit and the reservoir mode j . The overall dynamics can simply be obtained from the evolution of the individual qubit-reservoir subsystem. The dynamics of the single qubit S is known to be described by the reduced density matrix [35,36]

$$\rho^S(t) = \begin{pmatrix} \rho_{11}^S(0)|h(t)|^2 & \rho_{10}^S(0)h(t) \\ \rho_{01}^S(0)h^*(t) & 1 - \rho_{11}^S(0)|h(t)|^2 \end{pmatrix}, \tag{2}$$

in the qubit basis $\{|1\rangle, |0\rangle\}$. The function $h(t)$ is defined as the solution of the integro-differential equation

$$\frac{d}{dt}h(t) = - \int_0^t dt_1 f(t - t_1)h(t), \tag{3}$$

where $f(t - t_1)$ denotes the two-point reservoir correlation function which can be written as the Fourier transform of the spectral density $J(\omega)$

$$f(t - t_1) = \int d\omega J(\omega) \exp[i(\omega_0 - \omega)(t - t_1)]. \tag{4}$$

The exact form of $h(t)$ thus depends on the particular choice of the spectral density of the reservoir. In the following we consider the structured reservoir as the electromagnetic field inside a lossy cavity. In this case, the fundamental mode ω_c supported by the cavity displays a Lorentzian broadening due to the non-perfect reflectivity of the cavity mirrors. The effective spectral density of the intracavity field can be modeled as

$$J(\omega) = \frac{1}{2\pi} \frac{\gamma_0 \lambda^2}{(\omega_0 - \omega)^2 + \lambda^2}, \tag{5}$$

in which γ_0 is related to the decay of the excited state of the qubit in the Markovian limit of a flat spectrum and λ is the half width at half-maximum of the intracavity field spectrum profile. We may distinguish the Markovian and non-Markovian regimes using γ_0 and λ : $\gamma_0 < \lambda/2$ means the Markovian regime and $\gamma_0 > \lambda/2$ correspond to the non-Markovian regime. The function $h(t)$ can be obtained as

$$h(t) = e^{-\lambda t/2} \left[\cosh\left(\frac{dt}{2}\right) + \frac{\lambda}{d} \sinh\left(\frac{dt}{2}\right) \right], \tag{6}$$

where $d = \sqrt{\lambda^2 - 2\lambda\gamma_0}$. By means of the reduced density matrix elements of a single qubit given in Eq. (2), we can construct the reduced matrix for the two-qubit system.

For the quantum channel, we consider a general Bell-like state in the form

$$|\Psi(0)\rangle_{AB} = \alpha |00\rangle + \sqrt{1 - \alpha^2} |11\rangle, \quad (7)$$

in which the parameter α is assumed the real number for simplicity. Following the procedure presented in Ref. [32], we can obtain in the standard product basis $\{|1\rangle \equiv |11\rangle, |2\rangle \equiv |10\rangle, |3\rangle \equiv |01\rangle, |4\rangle \equiv |00\rangle\}$, the reduced matrix elements of qubits A and B at any time $t > 0$ as

$$\begin{aligned} \rho_{11}(t) &= (1 - \alpha^2)|h(t)|^4, \\ \rho_{22}(t) &= \rho_{33}(t) = (1 - \alpha^2)|h(t)|^2(1 - |h(t)|^2), \\ \rho_{44}(t) &= \alpha^2 + (1 - \alpha^2)(1 - |h(t)|^2)^2, \\ \rho_{14}(t) &= \rho_{41}^*(t) = \alpha\sqrt{1 - \alpha^2}h^2(t). \end{aligned} \quad (8)$$

To examine quantitatively the environmental effects on fidelity of quantum teleportation, it is convenient to write the unknown state to be teleported of a qubit a as $|\psi_a\rangle = c_1 |0\rangle + c_2 |1\rangle$ with $c_1 = \cos(\theta/2)e^{i\phi/2}$ and $c_2 = \sin(\theta/2)e^{-i\phi/2}$ ($0 \leq \theta \leq \pi, 0 \leq \phi \leq 2\pi$). Suppose the teleportation is implemented perfectly, then after a series of teleportation procedures, Bob gets the teleported state $|\varphi_B\rangle$ which can be expressed in terms of density operator as

$$\rho_{\text{out}} = \text{Tr}_{a,A} \{ \mathcal{U}_{\text{tel}} \rho_{\text{in}} \otimes \mathcal{E}(\rho_{\text{en}}) \mathcal{U}_{\text{tel}}^\dagger \}, \quad (9)$$

where $\rho_{\text{in}} = |\psi_a\rangle \langle \psi_a|$, $\rho_{\text{en}} = |\Psi(0)\rangle_{AB} \langle \Psi(0)|$, and $\text{Tr}_{a,A}$ is a partial trace over qubits a and A in Alice's hand. \mathcal{E} represents the actions of the reservoirs on the qubits AB and $\mathcal{E}(\rho_{\text{en}})$ thus means the evolved density operator of the quantum channel. $\mathcal{U}_{\text{tel}} = C_{aB}^Z C_{AB}^X \mathcal{H}_a C_{aA}^X$ is a unitary operator with \mathcal{H}_a stands for the Hadamard operation on qubit a , and C_{mn}^P ($P = X, Z$) denotes the controlled- P operation with m as the control qubit and n the target qubit. The quality of a quantum teleportation process is usually quantified by the teleportation fidelity, defined as the overlap between the unknown input state and the teleported state

$$F(\theta, \phi) = \langle \psi_a | \rho_{\text{out}} | \psi_a \rangle. \quad (10)$$

Since in general a state to be teleported is unknown, it is more useful to calculate the average fidelity given by

$$F_{\text{av}} = \frac{1}{4\pi} \int_0^{2\pi} d\phi \int_0^\pi \sin \theta F(\theta, \phi) d\theta, \quad (11)$$

where 4π is the solid angle.

By virtue of Eqs. (10) and (11), we obtain the time-dependent average fidelity of quantum teleportation for the channel state (7) as

$$F_{av}(t) = \frac{2}{3}(\rho_{11}(t) + \rho_{44}(t)) + \frac{1}{3}(\rho_{22}(t) + \rho_{33}(t)) + \frac{1}{3}(\rho_{14}(t) + \rho_{41}(t)), \quad (12)$$

in which the matrix elements $\rho_{11}(t), \dots, \rho_{44}(t)$ and $\rho_{14}(t), \rho_{41}(t)$ are given in (8). To make a comparison between the dynamics of average fidelity and that of the entanglement, we adopt Wootters' concurrence [37] \mathcal{C} as the entanglement measure. The concurrence \mathcal{C} for any (reduced) density matrix ρ of two qubits is defined as

$$\mathcal{C}(\rho) = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}, \quad (13)$$

where $\lambda_i (\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4)$ are the eigenvalues of the matrix $\zeta = \rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$, with σ_y a Pauli matrix and ρ^* the complex conjugation of ρ in the standard basis. The time-dependent concurrence $\mathcal{C}_{AB}(t)$ of the quantum channel AB can be expressed as

$$\mathcal{C}_{AB}(t) = 2 \max\{0, |\rho_{14}(t)| - \sqrt{\rho_{22}(t)\rho_{33}(t)}\}. \quad (14)$$

In Fig. 1, we plot the average fidelity $F_{av}(t)$ as functions of rescaled time $\gamma_0 t$ and initial preparation of the channel state (7) in terms of α^2 for the Markovian regime. For clarity, the range of the average fidelity is plotted from $2/3$ to 1 since the teleportation loses its quantum advantage when the fidelity is less than the critical value $2/3$ [6]. The figure clearly shows that for $\alpha^2 < 1/2$, the average fidelity $F_{av}(t)$ abruptly drops to the value $2/3$ in a finite time, implying the teleportation suddenly loses the quantum advantage. By contrast, for $\alpha^2 \geq 1/2$, the average fidelity $F_{av}(t)$ asymptotically approaches to the limiting value $2/3$ when $\gamma_0 t \rightarrow \infty$. Since the channel state (7) suffers from ESD for $\alpha^2 < 1/2$ in the present model [24], it is reasonable for one to attribute the abrupt variation of the fidelity to the occurrence of ESD of the used channel. To verify this point and show the exact relation between them, we plot in Fig. 2 the dynamics of the fidelity $F_{av}(t)$ as well as the entanglement of the quantum channel. It can be seen from Fig. 2 that the time at which the fidelity drops to $2/3$ precisely corresponds to the moment of the appearance of ESD. By contrast, in the absence of ESD, such as for $\alpha^2 = 1/2$, the fidelity only asymptotically reaches $2/3$ without a sudden variation. Therefore, ESD of the quantum channel has a direct and detrimental effect on quantum teleportation.

3 Non-Markovian effect on quantum teleportation

It is known that the memory effect of non-Markovian reservoir may give rise to a revival of entanglement after it has been terminated in a finite time [32–34]. Therefore, it is interesting to know if the average fidelity of quantum teleportation can revive after it has dropped down the critical value in the non-Markovian reservoir. For this purpose, in Fig. 3, we plot the average fidelity $F_{av}(t)$ as functions of rescaled time $\gamma_0 t$

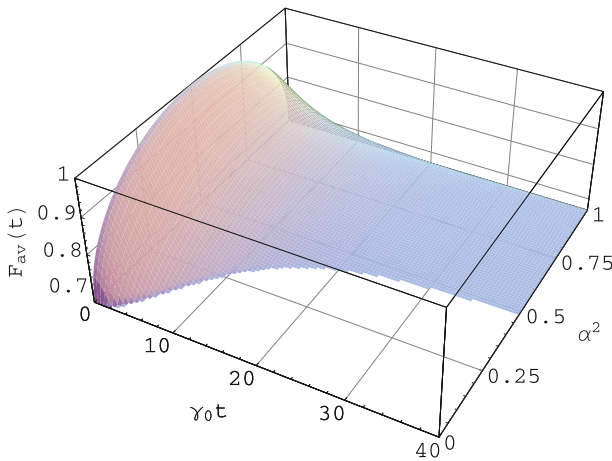


Fig. 1 The average fidelity $F_{av}(t)$ of teleportation as functions of rescaled time $\gamma_0 t$ and initial entanglement degree of the quantum channel in terms of α^2 for the Markovian reservoirs with $\lambda = 10\gamma_0$. The plot range of $F_{av}(t)$ is from $2/3$ to 1

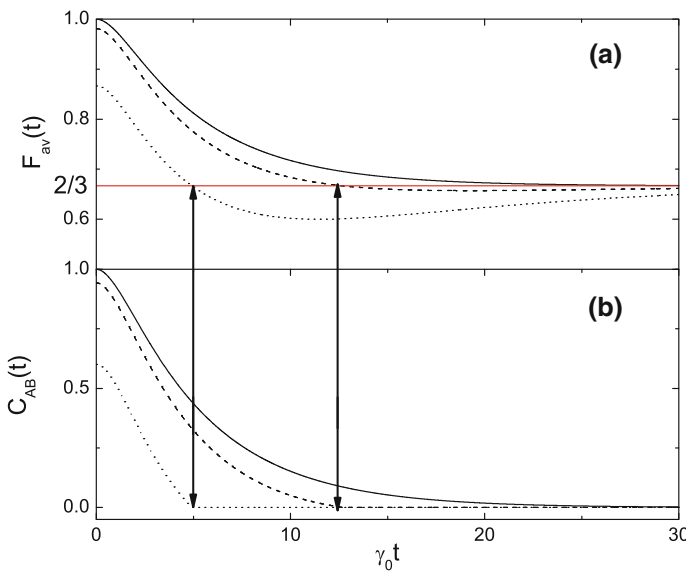


Fig. 2 The average fidelity $F_{av}(t)$ of the teleportation (a) and concurrence of the quantum channel (b) as a function of rescaled time $\gamma_0 t$ for $\alpha^2 = 1/2$ (solid line), $1/3$ (dashed line) and $1/10$ (dotted line) for the Markovian reservoirs with $\lambda = 10\gamma_0$. The red line represents the classical fidelity $2/3$

and the parameter α^2 for the non-Markovian regime with $\lambda = 0.01\gamma_0$. The range of the average fidelity is still plotted from $2/3$ to 1. From the figure, one can see the damped oscillations of the fidelity in the whole range of α^2 , which implies the revivals of the fidelity after dropping to the value $2/3$. In particular, we note that when $\alpha^2 < 1/2$ the fidelity can recover again even after disappearing for a finite interval of time. Here,

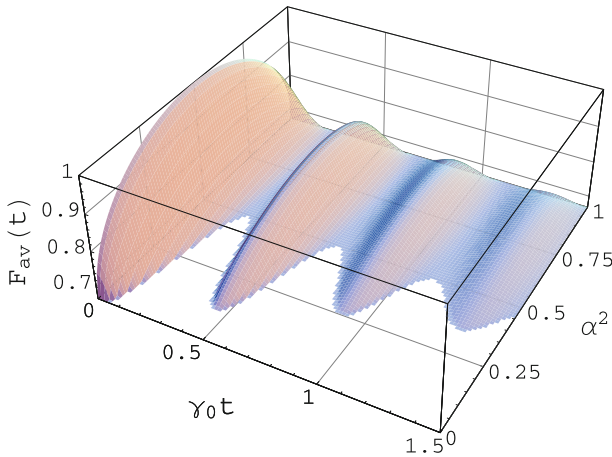


Fig. 3 The average fidelity $F_{av}(t)$ of the teleportation as functions of rescaled time $\gamma_0 t$ and initial entanglement degree of the quantum channel in terms of α^2 for the non-Markovian reservoirs with $\lambda = 0.01\gamma_0$. The plot range of $F_{av}(t)$ is from $2/3$ to 1

the revival of the average fidelity should also be attributed to the recovery of entanglement of the quantum channel. To make a comparison, we plot in Fig. 4 the dynamics of the average fidelity $F_{av}(t)$ and the concurrence $C_{AB}(t)$ of the quantum channel for the non-Markovian reservoirs with $\lambda = 0.01\gamma_0$. From the figure, we can observe that the time variations of the fidelity are in step with that of the concurrence of the quantum

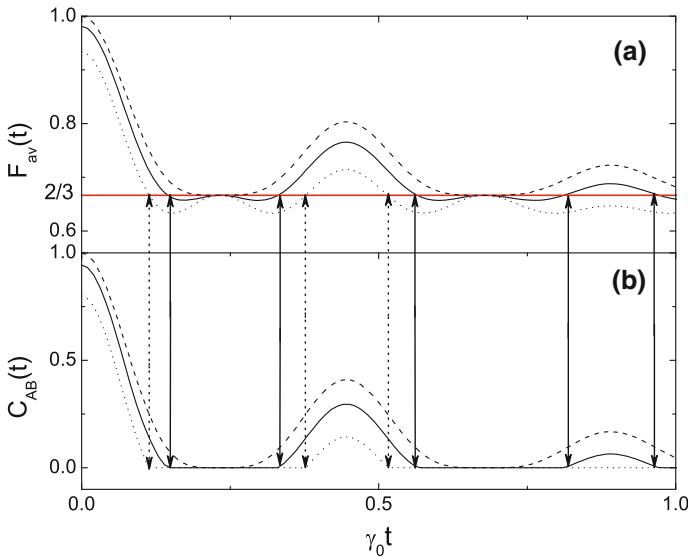


Fig. 4 The average fidelity $F_{av}(t)$ of the teleportation (a) and concurrence $C_{AB}(t)$ of the quantum channel (b) as a function of rescaled time $\gamma_0 t$ for $\alpha^2 = 1/2$ (dashed line), $1/3$ (solid line) and $1/5$ (dotted line) for the non-Markovian reservoirs with $\lambda = 0.01\gamma_0$. The red line represents the classical fidelity $2/3$

channel. The same as shown in Markovian regime, the moments at which the average fidelity drops to $2/3$ precisely correspond to the vanishing moments of the concurrence of the quantum channel. Moreover, an exactly corresponding relation exists between the revival of the average fidelity from $2/3$ and the recovery of entanglement from zero. In other words, during the intervals of null entanglement the average fidelity can not exceed the classical value $2/3$.

Since the sudden loss of quantum advantage of the teleportation is related to ESD of the used channel, one should try to avoid the occurrence of ESD in practical implementation of quantum teleportation. Fortunately, a lot of efficient strategies [38–41] that can protect entanglement from sudden death have been proposed. Besides, one can also choose to utilize the channel state that does not suffer from ESD. In comparison to the channel state (7) considered in this paper, the state in the form $|\Phi\rangle \sim |01\rangle + |10\rangle$ does not experience ESD in the vacuum reservoir [42] and thus may be more applicable in quantum teleportation.

4 Conclusion

In conclusion, we have studied quantum teleportation of a single-qubit state for the situation in which both qubits of the entangled channel are subjected to local structured reservoirs. It is shown the average fidelity of quantum teleportation can abruptly drop to the classical value $2/3$ implying the teleportation loses its quantum advantage in a finite time. An explicit link is constructed between the sudden variation of the fidelity and ESD of the used quantum channel. It is shown the time at which the fidelity drops to $2/3$ precisely corresponds to the moment of the occurrence of ESD of the quantum channel. The fact shows a detrimental effect of ESD on quantum teleportation. We have also demonstrated the memory effect of non-Markovian reservoir on the revival of the fidelity after it has dropped to $2/3$. Our study suggests that one should take into account the influence of ESD of the quantum channel and try to avoid it in practical implementation of quantum teleportation.

Acknowledgments The authors thank the anonymous referee for his or her suggestions to improve the paper. This work was supported by National Natural Science Foundation of China under Grant Nos.10947006 and 61178012, and the Specialized Research Fund for the Doctoral Program of Higher Education under Grant No. 20093705110001, and Scientific Research Foundation of Qufu Normal University for Doctors.

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