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Efficient entanglement channel construction schemes for a theoretical quantum network model with d-level system

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Abstract Quantum entanglement plays an essential role in the field of quantum information and quantum computation. In quantum network, a general assumption for many quantum tasks is that the quantum entanglement has been prior shared among participants. Actually, the distribution of entanglement becomes complex in the network environment. We present a theoretical quantum network model with good scalability. Then, three efficient and perfect schemes for the entanglement channel construction are proposed. Some general results for d-level system are also given. Any two communication sites can construct an entanglement channel via Bell states with the assistance of the intermediate sites on their quantum chain. By using the established entanglement channel, n sites can efficiently and perfectly construct an entanglement channel via an n-qudit cat state. More importantly, an entanglement channel via an arbitrary n-qudit state can also be constructed among any n sites, or even among any t sites where $1 \leq t \leq n$. The constructed multiparticle entanglement channels have many useful applications in quantum network environment.

Keywords Quantum network model · Entanglement channel construction · Quantum d-level system · Multiparticle entanglement

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1 Introduction

Based on the basic principles of quantum mechanics, great progresses have been made in information science in the past few decades. For example, the quantum key distribution (QKD) protocols [1–3] can establish an unconditionally secure shared key between two parties; quantum algorithms can solve certain problems much faster than any best-known classical algorithms, such as Shor's algorithm for factoring large integers [4] and Grover's algorithm for accelerating data searches [5].

For large scale quantum information processing and quantum computation, it is necessary to consider the network environment. Generally, a quantum network contains quantum nodes and quantum channels. A node represents a quantum system that can store or process quantum information in the network and spatially separated nodes are connected by quantum channels which can transmit quantum states. A typical implementation of quantum channel is fiber optic. For long distance quantum states transmission, quantum repeaters [6–8] are proposed to resolve the problem of fiber attenuation. There are also entanglement channel used for quantum tasks in quantum network. Entanglement channel is entangled states that each node holds part of the particles in order to complete the tasks and it is also our subject in this paper. Quantum network has attracted wide attention in recent years and many quantum network models have been proposed [9–13].

Quantum network is originated from the QKD network which deals with distribution of the secret keys in network environment. The DARPA quantum network [14,15] is the world's first quantum encrypted functional network that has been running since 2004. In Europe, the SECOQC QKD network [16,17] has been put into operation in 2008, which aims at developing a global network for unconditionally secure key distribution. In 2010, the Tokyo QKD Network was reported [18]. Some researchers are concentrating on interconnection of quantum computers and come to the concept of quantum internet. A complete architecture of quantum internet was reported in Ref. [19]. Lloyd et al. [20] proposed a robust scheme for constructing a quantum internet which allowed the reliable transmission of quantum information between spatially separated quantum computers. Kimble [21] pointed out that the quantum network could be fulfilled by optical interactions of single photons and atoms, thereby achieved entanglement distribution and quantum teleportation between nodes. Metwally [13] proposed a theoretical scheme to generate entanglement network via Dzyaloshinskii-Moriya (DM) interaction. The entanglements between qubits dynamically change via DM interaction and it is possible to generate entangled channel between two different nodes.

Entanglement is a central notion in quantum information science [22]. However, in the previous proposals [23–33] using the entanglement channel, it is assumed that the entanglement has been prior shared among the participants. The problem of how the entanglement is distributed has been neglected. An intuitional solution to this problem is that one party prepares the entanglement states and then sends them to the others. However, it should be noticed that a series of relay transmission is needed in network environment; thus, the distribution of the entangled particles among the participants becomes inefficient and unreliable. In quantum network, it is necessary to consider the transmission of quantum states in an efficient and reliable way. In this paper, we present a theoretical quantum network model in d-level system using quantum entanglement. Accordingly, three entanglement channels are constructed.

By utilizing the rules of entanglement swapping (ES) of generalized Bell states, we first show a scheme of how to construct a generalized entanglement channel via Bell states between any two sites. The established Bell state can also be used for two party quantum communication schemes like quantum cryptography [23], quantum dense coding [24] or quantum teleportation [25,26], etc. Cat states are commonly used as multiparticle entanglements in multiparty quantum information processing and quantum computation [27–30,32]. The construction of an entanglement channel via an n-qudit cat state among any n spatially separated sites is proposed. And the scheme is further extended to construct an entanglement channel via an arbitrary n-qudit state. Unlike the relay transmission of particles from the start site to the end site where a series of sending and receiving processes are involved, the scheme provides an efficient and perfect way of entanglement channel construction. Based on the established multiparticle entanglement, multiparty quantum information processing and multiparty quantum computation, such as quantum secret sharing [27], reduction of classical communication complexity [28], secure multiparty quantum computation [29], controlled teleportation [30,31] and joint remote state preparation [32,33], etc., can be implemented in our network model.

The rest of this paper is outlined as follows. In Sect. 2, some basic concepts for d-level system are given. In Sect. 3, we present the quantum network model. Then the entanglement channel constructing scheme between any two sites is introduced in Sect. 4. An efficient construction scheme of entanglement channel via an n-qudit cat state among any n sites is shown in Sect. 5. In Sect. 6, we first show some basic rules of distribute an arbitrary qudit state by Bell state entanglement channel. Then a more general construction scheme of entanglement channel via an arbitrary n-qudit state among any n sites is proposed. We discuss and conclude the paper in Sect. 7.

2 Preliminaries

2.1 Generalized d-level states

The generalized two-qudit Bell state in d-level system is

$$|\phi\,(u_1, u_2)\rangle = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} \omega^{ju_1}\, |j,\ \ j + u_2\rangle, \tag{1}$$

where $\omega = e^{2\pi i/d}$ and $u_1,\ u_2 \in \{0, 1, \ldots,\ d-1\}$. For simplicity, we use the notation (u_1, u_2) in Z_d to represent $|\phi(u_1, u_2)\rangle$ in the following context. The computational basis can also be expanded in terms of Bell states

$$|u_1,\ u_2\rangle = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} \omega^{-ju_1}\, |\phi(j, u_2 - u_1)\rangle. \tag{2}$$

The symbol "+" and "−" mean the adder and the subtractor modulo d, respectively.

For an n-qudit cat state in d-level system, the cat state and its inverse form can be written as

$$|\phi(u_1,\ u_2,\ldots,u_t,\ldots,u_n)\rangle = \frac{1}{\sqrt{d}}\sum_{j=0}^{d-1}\omega^{ju_1}\,|j,\ j+u_2,\ldots,j+u_t,\ldots,j+u_n\rangle, \tag{3}$$

$$|u_1,u_2,\ldots,u_t,\ldots,u_n\rangle = \frac{1}{\sqrt{d}}\sum_{j=0}^{d-1}\omega^{-ju_1}\,|\phi(j,u_2-u_1,\ldots,u_t-u_1,\ldots,u_n-u_1)\rangle, \tag{4}$$

where $u_t \in \{0, 1, \ldots,\ d-1\}$ with $t \in \{1, 2, \ldots,\ n\}$. There are a set of d^n maximally entangled cat states in Hilbert space $\mathbb{H}^{d^n}$ which form a complete orthonormal basis.

In the d-level system, the generalized Pauli operators X_d and Z_d are defined as [34]

$$(Z_d)^b(X_d)^a;\quad a, b \in \{0, 1, \ldots, d-1\}, \tag{5}$$

where $X_d\,|j\rangle = |j+1\rangle$ and $Z_d\,|j\rangle = \omega^j\,|j\rangle$.

A well-known set of quantum states are the four two-qubit Bell states in the two-level system which can be written as

$$|\phi\,(u_1,u_2)\rangle = \frac{1}{\sqrt{2}}\left(|0,u_2\rangle + (-1)^{u_1}\,|1,u_2\oplus 1\rangle\right);\quad u_1,u_2\in\{0,1\}, \tag{6a}$$

with $\oplus$ means module 2 adder. Accordingly, the three-qubit GHZ states are

$$|\phi(u_1,u_2,u_3)\rangle = \frac{1}{\sqrt{2}}\left(|0,u_2,u_3\rangle + (-1)^{u_1}\,|1,u_2\oplus 1,u_3\oplus 1\rangle\right); \quad u_1,u_2,u_3\in\{0,1\}. \tag{6b}$$

2.2 Rules of entanglement swapping

Entanglement swapping deals with the basis transformation of two cat states between the cat states basis and the computational basis, where the states are expanded in the computational basis, then a subset of particles are swapped and the resulting states are reexpanded in the cat states. Here, we revise the equation of ES of two generalized Bell states in Ref. [35] to

$$\left|\phi(v,v')\right\rangle_{12}\left|\phi(u,u')\right\rangle_{34} = \frac{1}{d}\sum_{k,l}^{d-1}\omega^{kl}\left|\phi(v+k,\ u'+l)\right\rangle_{14}\left|\phi(u-k,\ v'-l)\right\rangle_{32}, \tag{7}$$

where the initial state of particles (1, 2) and (3, 4) are $(v,\ v')$ and $(u,\ u')$. For the ES of a generalized Bell state and a generalized cat state, there is

$$|\phi\,(v_1,\ v_2,\ldots,v_t,\ldots,\ v_n)\rangle_{12\ldots t\ldots n}\left|\phi(u,u')\right\rangle_{ss'}$$
$$=\frac{1}{d}\sum_{k,l}^{d-1}\omega^{kl}\left|\phi\left(v_1+k,\ v_2,\ldots,u'+l,\ldots,v_n\right)\right\rangle_{12\ldots s'\ldots n}|\phi(u-k,\ v_t-l)\rangle_{st},\tag{8}$$

where $1 < t \leq n$. The detailed deducing can be found in "Appendix 1". It is clearly that Eq. (7) can be viewed as a specific case of Eq. (8). Notice that the ES is achieved by using only the Bell basis measurement.

3 Quantum network model

In our quantum network model, each node is a part of the quantum system and represents a communication site in the network. Every two adjacent sites in the quantum network share a set of Bell states as the entanglement channel connecting them. The Bell state can be either in a maximally entangled or in a partially entangled state, which will be discussed later. The network model can be viewed as an undirected graph, and there exists at least one path between any two vertices, that is to say, the graph is connected according to graph theory. We call a path between two sites as a quantum chain with the chain length L.

A specific case of the network model is shown in Fig. 1, where the quantum network contains two subnets, i.e., subnet-1 and subnet-2. Node A in subnet-1 and node B in Subnet-2 act as gateways that interconnect the two subnets. Apparently, the quantum network is connected. One path from site 1 to site 3 is ($1 \rightarrow 2 \rightarrow \text{A} \rightarrow \text{B} \rightarrow 3$) which is represented by the bold line.

The presented quantum network model has good scalability since new node or new subnet can be added to the network easily by connecting it to a node of an existed network. The network model can be used to build up a large scale quantum network since it can be extended to any scale or any structure theoretically.

In order to communicate with each other in the model, entanglement channel should be constructed among communication sites. The constructions of three types of entanglement channel are presented in our model: the two-qudit Bell state, the n-qudit cat

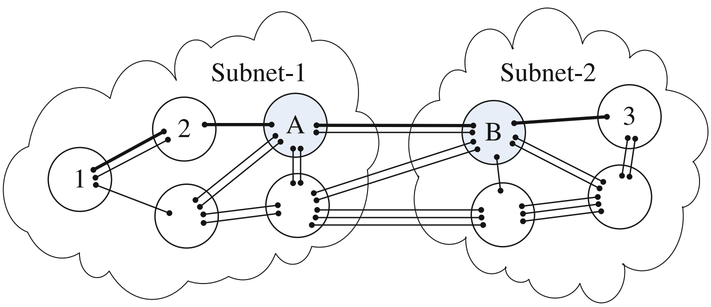


Fig. 1 A quantum network model with two subsystems. Site is represented by a *circle*. Adjacent sites in network are linked by a set of Bell states which is represented by *lines*

state and an arbitrary n-qudit state. The entanglement channel between any two sites in the network model can be constructed with the Bell states. Two new schemes of constructing a multiparticle entanglement channel among multiparty sites are introduced.

4 Entanglement channel construction between two sites via Bell states

An entanglement channel between any two sites can be constructed by using ES successively in quantum chain with the assistance of the intermediate sites if they do not have direct connection beforehand. Here, we will give some general results of constructing an entanglement channel in d-level system via generalized Bell states.

4.1 Ideal situation

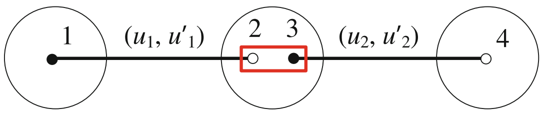
For each two adjacent sites in the network, suppose they share a series of maximally entangled Bell states. Figure 2 shows that the chain length between two sites is $L = 2$. Each site is represented by a big circle. The qudit is represented by a dot and entanglement qudits are connected by solid lines. The first qudit of a cat state is represented by black solid dot while the others by hollow dot. The rectangle represents the Bell basis measurements. The initial states of particles are $(u_1, u_1')_{12}$ and $(u_2, u_2')_{34}$, respectively.

According to the rules of ES of d-level Bell states, the entangled states shared among those three sites can be written as

$$\begin{aligned}&\left|\phi\left(u_1, u_1'\right)\right\rangle_{12}\left|\phi\left(u_2, u_2'\right)\right\rangle_{34}\\&=\frac{1}{d}\sum_{k_1,l_1}^{d-1}\omega^{k_1 l_1}\left|\phi\left(u_1+k_1,\ u_2'+l_1\right)\right\rangle_{14}\left|\phi\left(u_2-k_1,\ u_1'-l_1\right)\right\rangle_{32}.\end{aligned} \tag{9}$$

If the intermediate site performs a projective measurement on his/her two particles in Bell basis, the state of particles (1, 4) will collapse into one of the d^2 Bell states with equal probability. We denote the measurement result of particles (i, j) by M_{ij}, the remaining state of particles (i, j) by S_{ij}, where $M_{ij}, S_{ij} \in \{0, 1, \ldots, d-1\}^2$. Let the measurement outcomes on the qudits (3, 2) be represented by $M_{32} = (u_2 - k_1, u_1' - l_1) = (m_1, m_1')$, then the remaining state of particles (1, 4) will collapse into $S_{14} = (u_1 + k_1,\ u_2' + l_1) = (u_1 + u_2 - m_1,\ u_2' + u_1' - m_1')$. Thus, if one gets M_{32} on qudits (3, 2) labeled by (m_1, m_1'), one also gets the initial states of (1, 2) and (3, 4), then S_{14} will be deduced simply.

Fig. 2 Quantum chain with length $L = 2$



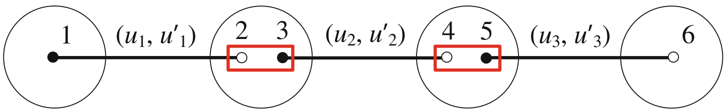


Fig. 3 Quantum chain with length $L = 3$ where four sites are involved

Suppose $L = 3$ shown in Fig. 3. If every intermediate site of the chain measures two particles in his/her possession in Bell basis orderly, the state of particles (1, 6) will still be one of the d^2 Bell states. By reusing the result of case $L = 2$, one can get

$$\begin{aligned}&\left|\phi(u_1 + u_2 - m_1,\ u'_2 + u'_1 - m'_1)\right\rangle_{14} \left|\phi(u_3, u'_3)\right\rangle_{56}\\ &\quad= \frac{1}{d}\sum_{k_2,l_2}^{d-1} \omega^{k_2 l_2} \left|\phi(u_1 + u_2 - m_1 + k_2, u'_3 + l_2)\right\rangle_{16}\\ &\qquad\otimes \left|\phi\left(u_3 - k_2,\ u'_2 + u'_1 - m'_1 - l_2\right)\right\rangle_{54}. \qquad (10)\end{aligned}$$

If M_{54} is set as $M_{54} = \left(u_3 - k_2,\ u'_2 + u'_1 - m_1 - l_2\right) = \left(m_2, m'_2\right)$, then S_{16} will be

$$\begin{aligned}S_{16} &= \left(u_1 + u_2 - m_1 + k_2, u'_3 + l_2\right)\\ &= \left(u_1 + u_2 + u_3 - m_1 - m_2, u'_3 + u'_2 + u'_1 - m'_1 - m'_2\right).\end{aligned}$$

There will be $d^2 \times d^2$ different measurement results in the intermediate sites.

An entanglement channel between two remote sites will be established by performing the above ES operations successively. For n Bell states ($n + 1$ sites, $2n$ particles) that form a quantum chain with length n, if each $n - 1$ intermediate site measures two particles in his/her possession in Bell basis, there will be $d^{2(n-1)}$ different measurement results. And the state of particles $(1, 2n)$ will collapse into one of the d^2 Bell states and satisfy

$$S_{1,2n} = \left(\sum_{t=1}^{n} u_t - \sum_{t=1}^{n-1} m_t,\ \sum_{t=1}^{n} u'_t - \sum_{t=1}^{n-1} m'_t\right), \qquad (11)$$

where (u_t, u'_t) is the initial state of qudits $(2t - 1, 2t)$ in the chain and (m_t, m'_t) is the measurement results of $(2t + 1, 2t)$ in the intermediate site. Here, we give a simple proof of Eq. (11) by induction.

Proof **Initial step:** Equation (11) is correct for $n = 2$ and $L = 2$ because of $S_{14} = \left(u_1 + u_2 - m_1,\ u'_2 + u'_1 - m'_1\right)$.

Inductive step: Assume that Eq. (11) is true for $n = x$ (x Bell states among $x + 1$ sites form a quantum chain with length $L = x$); that is, if we measure particles (3, 2), (5, 4), …, $(2x - 1, 2x - 2)$ in Bell basis with results $M_{32}, M_{54} \ldots M_{2x-1,2x-2}$, then $S_{1,2x}$ will be one of the Bell states and matches:

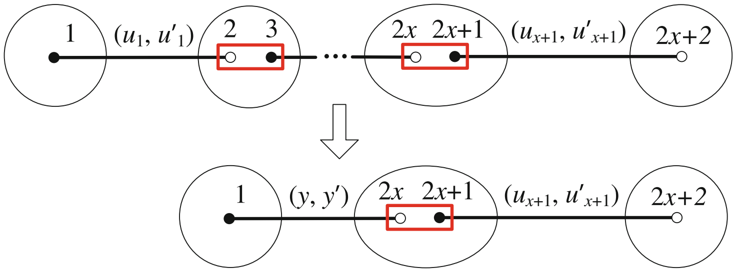


Fig. 4 Quantum chain with length $L = x + 1$ where $x + 2$ sites are involved

$$S_{1,2x} = \left(\sum_{t=1}^{x} u_t - \sum_{t=1}^{x-1} m_t,\ \sum_{t=1}^{x} u'_t - \sum_{t=1}^{x-1} m'_t\right).$$

The quantum chain with $n = x + 1$ is shown in Fig. 4.

If one performs ES orderly in first $x - 1$ intermediate site, particles $(1, 2x)$ will collapse into an entangled Bell state. Let $S_{1,2x} = (y, y')$, then the state of quantum chain can be written as

$$\begin{aligned}&\left|\phi(y, y')\right\rangle_{1,2x} \left|\phi\left(u_{x+1}, u'_{x+1}\right)\right\rangle_{2x+1,2x+2}\\&= \frac{1}{d}\sum_{k_x,l_x}^{d-1} \omega^{k_x l_x} \left|\phi\left(y + k_x, u'_{x+1} + l_x\right)\right\rangle_{1,2x+2} \left|\phi\left(u_{x+1} - k_x,\ y' - l_x\right)\right\rangle_{2x+1,2x}\end{aligned} \tag{12}$$

Again, if one performs a Bell basis measurement on $(2x + 1, 2x)$ and get outcomes $M_{2x+1,2x} = (u_{x+1} - k_x,\ y' - l_x) = (m_x, m'_x)$, then the state of $(1, 2x + 2)$ will be

$$\begin{aligned}S_{1,2x+2} &= \left(y + k_x, u'_{x+1} + l_x\right) = \left(y + u_{x+1} - m_x, u'_{x+1} + y' - m'_x\right)\\&= \left(\sum_{t=1}^{x} u_t - \sum_{t=1}^{x-1} m_t + u_{x+1} - m_x,\ \sum_{t=1}^{x} u'_t - \sum_{t=1}^{x-1} m'_t + u'_{x+1} - m'_x\right),\\&= \left(\sum_{t=1}^{x+1} u_t - \sum_{t=1}^{x} m_t,\ \sum_{t=1}^{x+1} u'_t - \sum_{t=1}^{x} m'_t\right)\end{aligned} \tag{13}$$

which means the Eq. (11) is true for $n = x + 1$. This completes the inductive step. □

For the two-level system, Eq. (11) can be rewritten as

$$S_{1,2n} = \left(\left(\bigoplus_{t=1}^{n} u_t\right) \oplus \left(\bigoplus_{t=1}^{n-1} m_t\right), \left(\bigoplus_{t=1}^{n} u'_t\right) \oplus \left(\bigoplus_{t=1}^{n-1} m'_t\right)\right). \tag{14}$$

An entanglement channel via Bell states between two communication sites is established with the above process and each site will hold half of the entangled particles. For the chain with length L and each pair of adjacent sites share q Bell states, the resources needed to construct q entanglement channels between two remote sites are qL pairs of Bell states, $2q(L-1)$ labels information in Z_d (one label means one cbit in the two-level system) for measurement outcomes and $2qL$ labels for quantum chain's initial state which is unnecessary if the initial states are all in (0, 0).

This scheme is also the basis for achieving the other two schemes. To check the perfect connection of the established entanglement channel, the two sites can verify the validity of the Bell state by performing measurement with random Z basis and X basis.

4.2 Practical situation

In a practical situation, the imperfect Bell state entanglement, the error introduced by local unitary operation and measurement will necessarily affect the fidelity of the constructed entanglement. Due to decoherence or noise, the bipartite entanglement ρ^{ij} shared between neighboring sites becomes a noisy mixed state

$$\rho^{ij} = \lambda \left|\phi(i,j)\right\rangle \left\langle\phi(i,j)\right| + \frac{1-\lambda}{d^2} I. \tag{15}$$

where $0 \leq \lambda \leq 1$ is the reliability of entanglement between adjacent nodes. The fidelity of the bipartite entanglement is

$$F_0 = \left\langle\phi(i,j)\right| \rho^{ij} \left|\phi(i,j)\right\rangle = \lambda + \frac{1-\lambda}{d^2} \leq 1. \tag{16}$$

Here, error model of one and two qubit operations are described by [6]

$$U\rho U^\dagger = p_1 U\rho U^\dagger + \frac{1-p_1}{2} tr_1(\rho) \otimes I_1, \tag{17a}$$

$$U\rho U^\dagger = p_2 U\rho U^\dagger + \frac{1-p_2}{4} tr_{12}(\rho) \otimes I_{12}. \tag{17b}$$

And single qubit measurement are represented as

$$P_0 = \delta \left|0\right\rangle \left\langle 0\right| + (1-\delta) \left|1\right\rangle \left\langle 1\right|, \tag{18a}$$

$$P_1 = (1-\delta) \left|0\right\rangle \left\langle 0\right| + \delta \left|1\right\rangle \left\langle 1\right|, \tag{18b}$$

where p_1, p_2 and δ are the reliabilities of the local operations and measurements. It is well-known that the Bell basis measurement can be represented by a CNOT operation and two single qubit measurements. In a quantum chain with length L, the fidelity of the entanglement generated between the start site and the end one after performing the ES in intermediate sites becomes

$$F_1 = \frac{1}{4} + \frac{3\lambda^L}{4} \left(\frac{p_1^2 p_2 (4\delta^2 - 1)}{3} \right)^{L-1}. \tag{19}$$

Entanglement purification [36] should be performed to increase the fidelity of the entanglement. The fidelity of bipartite entanglement at the end of the purification procedure can be written as

$$F_2 = f(m, L, \lambda, p_1, p_2, \delta), \tag{20}$$

where m is the number of purification steps.

In this case, the resources needed to construct a reliable Bell state entanglement channel between two remote sites are m times as much as the ideal situation. If the chain length L is too long to cause F_1 less than the minimum value required for purification, we can divide the quantum chain into several segments and perform ES and purification in each segment respectively. After that, the same procedures can be performed among those segments to construct the entanglement between two sites.

5 Entanglement channel construction among n sites via an n-qudit cat state

Multiparticle entanglements, for example, n-qudit cat states, are crucial quantum resources for quantum network communications. A basic issue in network model is how to construct a multiparticle entanglement channel among n spatially separated sites. An intuitional solution is that one of the sites, i.e., the primary site, prepares the n-qudit state initially; then he/she keeps one qudit in his/her possession and sends the others qudits to each other site, namely the secondary site. Each site will hold one qudit in the end of the process. However, it should be noticed that if the primary site and the secondary one don't have direct connection beforehand, a series of qudit transmission in the intermediate sites is needed.

5.1 Entanglement channel construction among n sites via an n-qudit cat state

Cat states are important kind of multiparticle entanglements with wide applications [27–30,32]. Suppose the primary site A wants to construct an n-qudit cat state entanglement channel with other $n-1$ secondary sites $\mathrm{B}_t$where $t \in \{1, 2, \ldots, n-1\}$. Initially, the primary site prepares a cat state labeled by $(v_1,\ v_2, \ldots,\ v_n)_{12\ldots n}$; while the primary site also constructs a Bell state entanglement channel $(u_t, u'_t)_{s_t s'_t}$ with each secondary site B_t by using the scheme shown in Sect. 4. Here, s_t is A's qudit and s'_t is B_t's qudit, while (u_t, u'_t) are two labels in Z_d (see Fig. 5). Each site will hold one particle of the cat state after the construction.

If the primary site performs Bell basis measurement sequentially, the cat state will be distributed among other $n-1$ sites. The following are details of the process.

Step 1: The primary site performs a Bell basis measurement on $(s_1,\ 2)$ which satisfy

$$\begin{aligned}
&|\phi(v_1, v_2, v_3, \ldots, v_n)\rangle_{12\ldots n} \left|\phi(u_1, u'_1)\right\rangle_{s_1 s'_1}\\
&\quad = \frac{1}{d}\sum_{k_1,l_1}^{d-1} \omega^{k_1 l_1} \left|\phi(v_1+k_1, u'_1+l_1, v_3, \ldots, v_n)\right\rangle_{1 s'_1 3\ldots n} |\phi(u_1-k_1, v_2-l_1)\rangle_{s_1 2}.
\end{aligned} \tag{21}$$

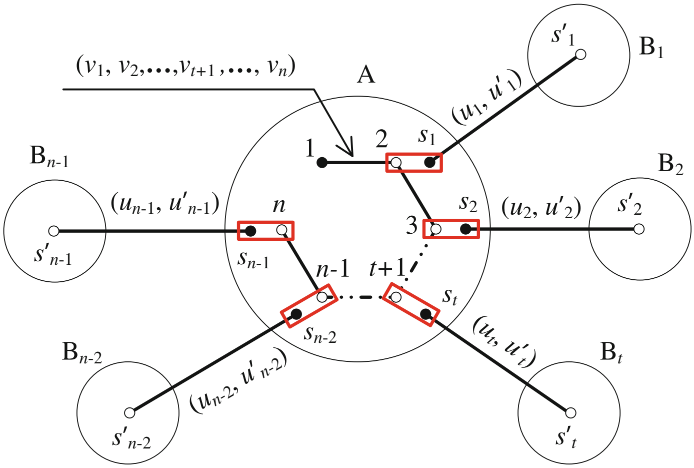


Fig. 5 Entanglement channel construction via an n-qudit cat state among n sites in the network. The cat state $(1, 2, 3, \ldots, n)$ is prepared by the primary site and marked by labels $(v_1, v_2, \ldots, v_n)$. Between the primary site A and the t-th secondary site B$_t$, they have established a entanglement channel (s_t, s'_t) whose initial state is marked by (u_t, u'_t) with t from 1 to $n-1$

Let $M_{s_12} = (u_1 - k_1,\ v_2 - l_1) = (m_1, m'_1)$, then the state of $(1,\ s'_1, 3, \ldots, n)$ becomes

$$\begin{aligned} S_{1s'_13\ldots n} &= (v_1 + k_1, u'_1 + l_1, v_3, \ldots, v_n) \\ &= (v_1 + u_1 - m_1,\ u'_1 + v_2 - m'_1,\ v_3, \ldots, v_n) \\ &= (x_1^{(1)},\ x_2, v_3, \ldots, v_n). \end{aligned}$$

Step 2: The primary site performs a Bell basis measurement on $(s_2,\ 3)$ where

$$\begin{aligned} &\left|\phi(x_1^{(1)},\ x_2, v_3, \ldots, v_n)\right\rangle_{1s'_13\ldots n} |\phi(u_2, u'_2)\rangle_{s_2s'_2} \\ &= \frac{1}{d}\sum_{k_2,l_2}^{d-1} \omega^{k_2l_2} \left|\phi(x_1^{(1)} + k_2, x_2, u'_2 + l_2, \ldots, v_n)\right\rangle_{1s'_1s'_2\ldots n} |\phi(u_2 - k_2,\ v_3 - l_2)\rangle_{s_23} \end{aligned} \tag{22}$$

Let $M_{s_23} = (u_2 - k_2,\ v_3 - l_2) = (m_2, m'_2)$, then the state of $(1,\ s'_1,\ s'_2,\ \ldots,\ n)$ becomes

$$\begin{aligned} S_{1s'_1s'_2\ldots n} &= (x_1^{(1)} + k_2, x_2, u'_2 + l_2, \ldots,\ v_n) \\ &= (x_1^{(1)} + u_2 - m_2,\ x_2,\ u'_2 + v_3 - m'_2, \ldots,\ v_n) \\ &= (x_1^{(2)},\ x_2,\ x_3, \ldots,\ v_n). \end{aligned}$$

Step t: The primary site performs a Bell basis measurement on $(s_t,\ t+1)$, then

$$\begin{aligned}&\left|\phi(x_1^{(t-1)},\ x_2,\ \ldots,\ x_t,\ v_{t+1}\ ,\ \ldots,\ v_n)\right\rangle_{1s_1'\ldots s_{t-1}',t+1\ldots n}\left|\phi(u_t,u_t')\right\rangle_{s_ts_t'}\\&=\frac{1}{d}\sum_{k_t,l_t}^{d-1}\omega^{k_tl_t}\left|\phi(x_1^{(t-1)}+k_t,\ x_2,\ \ldots,\ x_t,\ u_t'+l_t,\ldots,\ v_n)\right\rangle_{1s_1'\ldots s_{t-1}'s_t'\ldots n}\\&\quad\otimes\left|\phi(u_t-k_t,\ v_{t+1}-l_t)\right\rangle_{s_t,t+1}.\end{aligned}\tag{23}$$

Let $M_{s_t,t+1}=(u_t-k_t,\ v_{t+1}-l_t)=(m_t,m_t')$, then the state of $(1,\ s_1',\ldots,s_{t-1}',s_t',\ldots,n)$ becomes

$$\begin{aligned}S_{1s_1'\ldots s_{t-1}'s_t'\ldots n}&=(x_1^{(t-1)}+k_t,\ x_2,\ldots,\ x_t,\ u_t'+l_t,\ldots\ ,\ v_n)\\&=(x_1^{(t-1)}+u_t-m_t,\ x_2,\ldots,\ x_t,\ u_t'+v_{t+1}-m_t',\ldots\ ,\ v_n)\\&=(x_1^{(t)},\ x_2,\ldots,\ x_t,\ x_{t+1},\ldots,\ v_n).\end{aligned}$$

Step $n-1$(the last step): The primary site performs a Bell basis measurement on $(s_{n-1},\ n)$ and gets

$$\begin{aligned}&\left|\phi(x_1^{(n-2)},\ x_2,\ldots,\ x_{n-1},\ v_n)\right\rangle_{1s_1'\ldots s_{n-2}'n}\left|\phi(u_{n-1},u_{n-1}')\right\rangle_{s_{n-1}s_{n-1}'}\\&=\frac{1}{d}\sum_{k_{n-1},l_{n-1}}^{d-1}\omega^{k_{n-1}l_{n-1}}\left|\phi(x_1^{(n-2)}+k_{n-1},\ x_2,\ldots,x_{n-1},u_{n-1}'+l_{n-1})\right\rangle_{1s_1'\ldots s_{n-2}'s_{n-1}'}\\&\quad\otimes\left|\phi(u_{n-1}-k_{n-1},\ v_n-l_{n-1})\right\rangle_{s_{n-1}n}.\end{aligned}\tag{24}$$

Let $M_{s_{n-1},n}=(u_{n-1}-k_{n-1},\ \ v_n-l_{n-1})=(m_{n-1},m_{n-1}')$, then the state of $(1,\ s_1',\ldots\ ,s_{n-2}',s_{n-1}')$ becomes

$$\begin{aligned}S_{1s_1'\ldots s_{n-2}'s_{n-1}'}&=(x_1^{(n-2)}+k_{n-1},\ x_2,\ \ldots,\ x_{n-1},\ u_{n-1}'+l_{n-1})\\&=(x_1^{(n-2)}+u_{n-1}-m_{n-1},x_2,\ldots,x_{n-1},\ u_{n-1}'+v_n-m_{n-1}')\\&=(x_1^{(n-1)},\ x_2,\ldots,\ x_{n-1},\ x_n).\end{aligned}$$

Here, we conclude that the state of the distributed cat state becomes

$$\begin{aligned}S_{1s_1's_2'\ldots s_{n-1}'}=\Bigg(&v_1+\sum_{t=1}^{n-1}u_t-\sum_{t=1}^{n-1}m_t,\ v_2+u_1'-m_1',\ v_3\\&+u_2'-m_2',\ldots,v_n+u_{n-1}'-m_{n-1}'\Bigg),\end{aligned}\tag{25}$$

where $(v_1,v_2,\ldots,v_n)_{12\ldots n}$ is the initial state of the cat state, $(u_t,u_t')_{s_is_i'}$ is the initial states of the entanglement channel, $(m_t,m_t')_{s_t,t+1}$ is the measurement outcome of

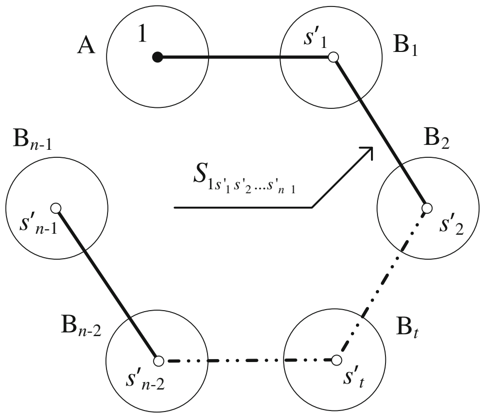


Fig. 6 The established n-qudit entanglement channel among n sites where each site holds one qudit. The entangled cat state is labeled by $S_{1s'_1s'_2\ldots s'_{n-1}}$

qudits with t from 1 to $n-1$. And the recursion equation of intermediate variable after step t satisfies

$$x_1^{(t)} = x_1^{(t-1)} + u_t - m_t = v_1 + \sum_{i=1}^{t} u_i - \sum_{i=1}^{t} m_i,$$
$$x_{i+1} = v_{i+1} + u'_i - m'_i, \quad i \in \{1, 2, \ldots, t\}. \tag{26}$$

After the above $n-1$ steps, an entanglement channel via an n-qudit cat state is established among n spatially separated sites where each site holds one particle and the cat state is labeled by $S_{1s'_1s'_2\ldots s'_{n-1}}$ (Fig. 6). The primary site can calculate the collapsed state according to Eq. (25). The cat state entanglement channel can also be transformed into other maximally entangled cat states by performing local operators.

5.2 Example of entanglement channel construction via a three-qubit GHZ state

The three-qubit GHZ state can be used for quantum secret sharing (QSS) [27]. To construct this kind of channel, Alice (in subnet A) prepares the GHZ state and intends to send the two qubits 2, 3 to remote sites Bob (in subnet B) and Charlie (in subnet C). Alice, Bob and Charlie will hold one qubit of the triplet for the QSS task finally. In two-level system and for $n = 3$, Eq. (25) can be reduced to

$$S_{1s'_1s'_2} = \left(v_1 \oplus \left(\bigoplus_{t=1}^{2} u_t\right) \oplus \left(\bigoplus_{t=1}^{2} m_t\right),\ v_2 \oplus u'_1 \oplus m'_1,\ v_3 \oplus u'_2 \oplus m'_2\right). \tag{27}$$

Alice needs to perform the Bell state construction scheme presented in Sect. 4 to establish entanglement channels with two remote sites Bob and Charlie. Let the Bell state established between Alice and Bob be $(u_1,\ u'_1)_{s_1s'_1} = (1,\ 1)$, and between Alice and Charlie be $(u_2,\ u'_2)_{s_2s'_2} = (0,\ 1)$. Here, assume the GHZ state Alice prepared be $(v_1,\ v_2,\ v_3)_{123} = (1,\ 0,\ 0)$. Then Eq. (27) can be rewritten as

$$S_{1s'_1s'_2} = \left(\bigoplus_{t=1}^{2} m_t,\ 1 \oplus m'_1,\ 1 \oplus m'_2\right). \tag{28a}$$

In the following, Alice performs two Bell basis measurements to distribute the GHZ state. The measurement results are $M_{s_1 2} = (m_1, m_1')$ and $M_{s_2 3} = (m_2, m_2')$. It can be seen from Eq. (28a) that the remaining state will be $S_{1s_1's_2'} = (0, 0, 0)$ if $m_1 = m_2, m_2' = 1$ and $m_1' = 1$; which means the desired entanglement channel has been constructed among three participants for QSS task. For others measurements, the local Pauli operator should be performed to convert the remaining state to $|\phi(0, 0, 0)\rangle$. The QSS task can be performed among subnets A, B and C by using the constructed channel. These results can also be concluded if we rewrite the quantum system as

$$\begin{aligned}
&|\phi(1,0,0)\rangle_{123}\,|\phi(1,1)\rangle_{s_1s_1'}\,|\phi(0,1)\rangle_{s_2s_2'} \\
&= \frac{1}{4}(|\phi(0,0)\rangle\,|\phi(0,0)\rangle\,|\phi(0,1,1)\rangle + |\phi(0,0)\rangle\,|\phi(1,0)\rangle\,|\phi(1,1,1)\rangle \\
&\quad + |\phi(1,0)\rangle\,|\phi(0,0)\rangle\,|\phi(1,1,1)\rangle + |\phi(1,0)\rangle\,|\phi(1,0)\rangle\,|\phi(0,1,1)\rangle \\
&\quad + |\phi(0,0)\rangle\,|\phi(0,1)\rangle\,|\phi(0,1,0)\rangle - |\phi(0,0)\rangle\,|\phi(1,1)\rangle\,|\phi(1,1,0)\rangle \\
&\quad + |\phi(1,0)\rangle\,|\phi(0,1)\rangle\,|\phi(1,1,0)\rangle - |\phi(1,0)\rangle\,|\phi(1,1)\rangle\,|\phi(0,1,0)\rangle \\
&\quad - |\phi(0,1)\rangle\,|\phi(0,0)\rangle\,|\phi(0,0,1)\rangle - |\phi(0,1)\rangle\,|\phi(1,0)\rangle\,|\phi(1,0,1)\rangle \\
&\quad + |\phi(1,1)\rangle\,|\phi(0,0)\rangle\,|\phi(1,0,1)\rangle + |\phi(1,1)\rangle\,|\phi(1,0)\rangle\,|\phi(0,0,1)\rangle \\
&\quad - |\phi(0,1)\rangle\,|\phi(0,1)\rangle\,|\phi(0,0,0)\rangle + |\phi(0,1)\rangle\,|\phi(1,1)\rangle\,|\phi(1,0,0)\rangle \\
&\quad + |\phi(1,1)\rangle\,|\phi(0,1)\rangle\,|\phi(1,0,0)\rangle - |\phi(1,1)\rangle\,|\phi(1,1)\rangle\,|\phi(0,0,0)\rangle)_{s_12s_2231s_1's_2'}.
\end{aligned} \tag{28b}$$

It can be viewed that Eq. (28a) is equivalent to Eq. (28b) in a more concise and compact form.

6 Entanglement channel construction among n sites via an arbitrary n-qudit state

Besides the cat states, other types of useful entanglement resources, like the W states [37,38], the cluster states [39,40] and the χ-type states [41,42], have been widely used in multiparty quantum information processing and multiparty quantum computation.

Suppose a primary site wants to construct an arbitrary n-qudit state entanglement among n sites in the network model where each site will hold one particle of the state after the construction. Using the established Bell state between the primary site and the secondary site, an arbitrary n-qudit state entanglement channel can be constructed among n sites. The arbitrary n-qudit state has the following form

$$|\gamma_n\rangle_{12\ldots n} = \sum_{k_1,k_2,\ldots,k_n=0}^{d-1} \alpha_{k_1,k_2,\ldots,k_n}\,|k_1, k_2, \ldots, k_n\rangle_{12\ldots n}, \tag{29}$$

where $\alpha_{k_1,k_2,\ldots,k_n}$ are complex coefficients and satisfy the normalized condition $\sum_{k_1,k_2,\ldots,k_n=0}^{d-1} |\alpha_{k_1,k_2,\ldots,k_n}|^2 = 1$, and the subscribes $1, 2, \ldots, n$ denote the qudit of the state.

Before discussing the details of the scheme, we give some basic rules of distribution one qudit of the n-qudit state by a Bell state entanglement channel. For an arbitrary n-qudit state and a Bell state, the following equation works (see "Appendix 2")

$$|\gamma_n\rangle_{12\ldots t\ldots n} \left|\phi(u,u')\right\rangle_{ss'} = \frac{1}{d} \sum_{k_1,k_2,\ldots,k_n,j,j'}^{d-1} \alpha_{k_1,k_2,\ldots,k_n} \omega^{j(u-j')} \left|\phi(j', k_t - j)\right\rangle_{st} \otimes \left|k_1, k_2, \ldots, j+u', \ldots, k_n\right\rangle_{12\ldots s'\ldots n}. \tag{30}$$

It is clearly that if we measure the qudits (s, t) in Bell basis and get the result $(m, m')_{st}$ which will happen with equal probability $1/d^2$ for each labels$m, m' \in Z_d$, then the remaining state will collapse into

$$\begin{aligned}
&{}_{st}\left\langle\phi(m,m')\right| \sum_{k_1,k_2,\ldots,k_n,j,j'}^{d-1} \alpha_{k_1,k_2,\ldots,k_n} \omega^{j(u-j')} \left|\phi(j', k_t - j)\right\rangle_{st} \\
&\quad \otimes \left|k_1, k_2, \ldots, j+u', \ldots, k_n\right\rangle_{12\ldots s'\ldots n} \\
&= \sum_{k_1,k_2,\ldots,k_n,j,j'}^{d-1} \alpha_{k_1,k_2,\ldots,k_n} \omega^{j(u-j')} \delta(m, j')\delta(m', k_t - j) \\
&\quad \cdot \left|k_1, k_2, \ldots, j+u', \ldots, k_n\right\rangle_{12\ldots s'\ldots n} \\
&= \sum_{k_1,k_2,\ldots,k_n}^{d-1} \alpha_{k_1,k_2,\ldots,k_n} \omega^{(k_t-m')(u-m)} \left|k_1, k_2, \ldots, k_t - m' + u', \ldots, k_n\right\rangle_{12\ldots s'\ldots n}
\end{aligned} \tag{31}$$

The qudit state can be recovered by performing an appropriate operator based on the measurement outcome. It means one of the qudit in the n-qudit state is distributed faithfully from the primary site in position t to the remote site in position s'.

6.1 Entanglement channel construction among n sites via an arbitrary n-qudit state

Initially, the primary site prepares the arbitrary n-qudit state $|\gamma_n\rangle_{12\ldots n}$ and also constructs a channel with each secondary site via Bell states. The established Bell state entanglement channel between the primary site and the t-th secondary site is marked by $(u_t, u_t')_{s_t,s_t'}$ with $1 \le t \le n-1$ (see Fig. 5). Based on Eq. (31), the primary site needs to perform one-qudit state distribution successively to construct the entanglement channel via an n-qudit state.

Step 1: Distribute the second qudit of the n-qudit state to the first secondary site.

$$|\gamma_n\rangle_{123\ldots n}\left|\phi(u_1,u_1')\right\rangle_{s_1,s_1'}=\frac{1}{d}\sum_{k_1,k_2,\ldots,k_n,j_1,j_1'}^{d-1}\alpha_{k_1,k_2,\ldots,k_n}\omega^{j_1(u_1-j_1')}\left|\phi(j_1',k_2-j_1)\right\rangle_{s_12}$$
$$\otimes\left|k_1,\,j_1+u_1',\;k_3,\ldots,k_n\right\rangle_{1s_1'3\ldots n}. \tag{32}$$

Let the measurement outcomes be $M_{s_12}=(j_1',k_2-j_1)=(m_1,m_1')$, then the state of $(1,s_1',3,\ldots,n)$ will collapse into

$$S_{1s_1'3\ldots n}=\sum_{k_1,k_2,\ldots,k_n}^{d-1}\alpha^{(2)}_{k_1,k_2,\ldots,k_n}\left|k_1,x_1,\;k_3,\ldots,k_n\right\rangle_{1s_1'3\ldots n}=\left|\gamma_n^{(2)}\right\rangle_{1s_1'3\ldots n},$$

where $\alpha^{(2)}_{k_1,k_2,\ldots,k_n}=\alpha_{k_1,k_2,\ldots,k_n}\omega^{(k_2-m_1')(u_1-m_1)}$ and $x_1=k_2-m_1'+u_1'$.

Step 2: Distribute the third qudit of the n-qudit state to the second secondary site.

$$\left|\gamma_n^{(2)}\right\rangle_{1s_1'3\ldots n}\left|\phi(u_2,u_2')\right\rangle_{s_2,s_2'}=\frac{1}{d}\sum_{k_1,k_2,\ldots,k_n,j_2,j_2'}^{d-1}\alpha^{(2)}_{k_1,k_2,\ldots,k_n}\omega^{j_2(u_2-j_2')}\left|\phi(j_2',k_3-j_2)\right\rangle_{s_23}$$
$$\otimes\left|k_1,x_1,\,j_2+u_2',\ldots,k_n\right\rangle_{1s_1's_2'\ldots n}. \tag{33}$$

Let $M_{s_23}=(j_2',k_3-j_2)=(m_2,m_2')$, then state of $(1,s_1',s_2',\ldots,n)$ will be

$$S_{1s_1's_2'\ldots n}=\sum_{k_1,k_2,\ldots,k_n}^{d-1}\alpha^{(3)}_{k_1,k_2,\ldots,k_n}\left|k_1,x_1,x_2,\ldots,k_n\right\rangle_{1s_1's_2'\ldots n}=\left|\gamma_n^{(3)}\right\rangle_{1s_1's_2'\ldots n},$$

where $\alpha^{(3)}_{k_1,k_2,\ldots,k_n}=\alpha_{k_1,k_2,\ldots,k_n}\omega^{\sum_{i=1}^{2}(k_{i+1}-m_i')(u_i-m_i)}$ and $x_2=k_3-m_2'+u_2'$.

Step *t*: Distribute the $(t+1)$-th qudit of the n-qudit state to the t-th secondary site where $1\le t\le n-1$. After the priori $t-1$ step, the n-qudit will become

$$\left|\gamma_n^{(t)}\right\rangle_{1s_1'\ldots s_{t-1}',t+1,t+2\ldots n}=\sum_{k_1,k_2,\ldots,k_n}^{d-1}\alpha^{(t)}_{k_1,k_2,\ldots,k_n}$$
$$\cdot\left|k_1,x_1,\ldots,x_{t-1},k_{t+1},k_{t+2},\ldots,k_n\right\rangle_{1s_1'\ldots s_{t-1}',t+1,t+2\ldots n}. \tag{34}$$

So one obtains

$$\left|\gamma_n^{(t)}\right\rangle_{1s'_1\ldots s'_{t-1},t+1,t+2\ldots n}\left|\phi(u_t,u'_t)\right\rangle_{s_t,s'_t}=\frac{1}{d}\sum_{k_1,k_2,\ldots,k_n,j_t,j'_t}^{d-1}\alpha_{k_1,k_2,\ldots,k_n}^{(t)}\omega^{j_t(u_t-j'_t)}$$
$$\cdot\left|\phi(j'_t,k_{t+1}-j_t)\right\rangle_{s_t,t+1}$$
$$\otimes\left|k_1,x_1,\ldots,x_{t-1},j_t+u'_t,k_{t+2},\ldots,k_n\right\rangle_{1s'_1\ldots s'_{t-1},s'_t,t+2\ldots n}. \tag{35}$$

Let $M_{s_t t+1}=(j'_t,k_{t+1}-j_t)=(m_t,m'_t)$, then the state of $(1,s'_1\ldots s'_{t-1},s'_t,t+2\ldots n)$ will be

$$S_{1s'_1\ldots s'_{t-1},s'_t,t+2\ldots n}=\sum_{k_1,k_2,\ldots,k_n}^{d-1}\alpha_{k_1,k_2,\ldots,k_n}^{(t+1)}$$
$$\cdot\left|k_1,x_1,\ldots,x_{t-1},x_t,k_{t+2},\ldots,k_n\right\rangle_{1s'_1\ldots s'_{t-1},s'_t,t+2\ldots n}=\left|\gamma_n^{(t+1)}\right\rangle,$$

where $\alpha_{k_1,k_2,\ldots,k_n}^{(t+1)}=\alpha_{k_1,k_2,\ldots,k_n}\omega^{\sum_{i=1}^{t}(k_{i+1}-m'_i)(u_i-m_i)}$ and $x_t=k_{t+1}-m'_t+u'_t$.

Step *n*−1(the last step): Distribute the last qudit of the n-qudit state to the last secondary site. The n-qudit after the priori $n-2$ step is

$$\left|\gamma_n^{(n-1)}\right\rangle_{1s'_1s'_2\ldots s'_{n-2}n}=\sum_{k_1,k_2,\ldots,k_n}^{d-1}\alpha_{k_1,k_2,\ldots,k_n}^{(n-1)}\left|k_1,x_1,x_2,\ldots,x_{n-2},k_n\right\rangle_{1s'_1s'_2\ldots s'_{n-2}n}. \tag{36}$$

So one gets

$$\left|\gamma_n^{(n-1)}\right\rangle_{1s'_1s'_2\ldots s'_{n-2}n}\left|\phi(u_{n-1},u'_{n-1})\right\rangle_{s_{n-1},s'_{n-1}}$$
$$=\frac{1}{d}\sum_{k_1,k_2,\ldots,k_n,j_{n-1},j'_{n-1}}^{d-1}\alpha_{k_1,k_2,\ldots,k_n}^{(n-1)}\omega^{j_{n-1}(u_{n-1}-j'_{n-1})}\left|\phi(j'_{n-1},k_n-j_{n-1})\right\rangle_{s_{n-1}n}$$
$$\otimes\left|k_1,x_1,x_2,\ldots,x_{n-2},j_{n-1}+u'_{n-1}\right\rangle_{1s'_1s'_2\ldots s'_{n-2}s'_{n-1}} \tag{37}$$

Let $M_{s_{n-1}n}=(j'_{n-1},k_n-j_{n-1})=(m_{n-1},m'_{n-1})$. Here we get the conclusion that the ***n***-qudit state will collapse into

$$S_{1s'_1s'_2\ldots s'_{n-2}s'_{n-1}}$$
$$=\sum_{k_1,k_2,\ldots,k_n}^{d-1}\alpha_{k_1,k_2,\ldots,k_n}^{(n)}\left|k_1,x_1,x_2,\ldots,x_{n-2},x_{n-1}\right\rangle_{1s'_1s'_2\ldots s'_{n-2}s'_{n-1}}=\left|\gamma_n^{(n)}\right\rangle, \tag{38}$$

where $\alpha^{(n)}_{k_1,k_2,\ldots,k_n} = \alpha_{k_1,k_2,\ldots,k_n}\omega^{\sum_{t=1}^{n-1}(k_{t+1}-m'_t)(u_t-m_t)}$ and $x_t = k_{t+1} - m'_t + u'_t$ with $1 \le t \le n-1$. And the intermediate state after step t is

$$\left|\gamma_n^{(t+1)}\right\rangle_{1s'_1\ldots s'_{t-1},s'_t,t+2\ldots n} = \sum_{k_1,k_2,\ldots,k_n}^{d-1} \alpha^{(t+1)}_{k_1,k_2,\ldots,k_n} \left|k_1, x_1, \ldots, x_{t-1}, x_t, k_{t+2}, \ldots, k_n\right\rangle_{1s'_1\ldots s'_{t-1},s'_t,t+2\ldots n}, \quad (39)$$

where $\alpha^{(t+1)}_{k_1,k_2,\ldots,k_n} = \alpha^{(t)}_{k_1,k_2,\ldots,k_n}\omega^{(k_{t+1}-m'_t)(u_t-m_t)} = \alpha_{k_1,k_2,\ldots,k_n}\omega^{\sum_{i=1}^{t}(k_{i+1}-m'_i)(u_i-m_i)}$ and $x_t = k_{t+1} - m'_t + u'_t$ with $\alpha^{(1)}_{k_1,k_2,\ldots,k_n} = \alpha_{k_1,k_2,\ldots,k_n}$ and $\left|\gamma_n^{(1)}\right\rangle = |\gamma_n\rangle$.

6.2 Recovery operators

The n-qudit state can also be recovered to its initial state if necessary. In this case, the above scheme can be viewed as a distributed teleportation scheme for d-level system. Each secondary site performs the recovery operator in reverse order, that is, the last secondary site performs the recovery operator firstly, and so on. Then after the last $(n-t-1)$ secondary sites perform their operator, the n-qudit state becomes $\left|\gamma_n^{(t+1)}\right\rangle_{1s'_1\ldots s'_t\ldots s'_{n-1}}$. It is clearly seen that the t-th secondary site can recover the state from $\left|\gamma_n^{(t+1)}\right\rangle$ to $\left|\gamma_n^{(t)}\right\rangle$ by performing an appropriate operator, in other words, it makes the changes $|x_t\rangle \to |k_{t+1}\rangle$ and $\alpha^{(t+1)} \to \alpha^{(t)}$ in position s'_t.

Recovery operator *X*

If the t-th secondary site performs $X^{m'_i-u'_i}$ on his/her qudit, it will make the reverse change $|x_t\rangle \to |k_{t+1}\rangle$ since

$$X^{m'_t-u'_t}\,|x_t\rangle = \left|x_t + m'_t - u'_t\right\rangle = |k_{t+1}\rangle\,. \quad (40)$$

Recovery operator *Z*

After performing the above X recovery operator, the t-th secondary site performs $\omega^{p_t}Z^{r_t}$ to achieve the effect $\alpha^{(t+1)} \to \alpha^{(t)}$ on his/her qudit, then we have

$$\begin{aligned}\omega^{p_t}Z^{r_t}\left(\alpha^{(t+1)}_{k_1,k_2,\ldots,k_n}|k_{t+1}\rangle\right) &= \alpha^{(t+1)}_{k_1,k_2,\ldots,k_n}\omega^{p_t}\left(Z^{r_t}|k_{t+1}\rangle\right)\\ &= \alpha^{(t)}_{k_1,k_2,\ldots,k_n}\omega^{(k_{t+1}-m'_t)(u_t-m_t)}\omega^{p_t}\omega^{r_t k_{t+1}}|k_{t+1}\rangle\\ &= \alpha^{(t)}_{k_1,k_2,\ldots,k_n}(\omega^{m'_t m_t - m'_t u_t}\omega^{p_t})(\omega^{k_{t+1}(u_t-m_t)}\omega^{r_t k_{t+1}})|k_{t+1}\rangle\\ &= \alpha^{(t)}_{k_1,k_2,\ldots,k_n}|k_{t+1}\rangle\,,\end{aligned} \quad (41)$$

which means $\omega^{p_t}Z^{r_t} = \omega^{m'_t u_t - m'_t m_t}Z^{m_t-u_t}$.

So, in the t-th secondary site, the recovery operator to be performed on qudits'_t should be

$$R_t = \omega^{m'_t u_t - m'_t m_t} Z^{m_t - u_t} X^{m'_t - u'_t}. \tag{42}$$

The recovery process can be explained as follows,

$$\begin{aligned}
& I \otimes \cdots \otimes R_t \otimes \cdots \otimes I \left(|\gamma_n^{(t+1)}\rangle_{1s'_1 \ldots s'_{t-1}, s'_t, s'_{t+1} \ldots s'_{n-1}} \right) \\
&= \sum_{k_1, k_1, \ldots, k_n}^{d-1} \alpha^{(t+1)}_{k_1, k_2, \ldots, k_n} |k_1, x_1, \ldots, x_{t-1}\rangle_{1s'_1 \ldots s'_{t-1}} \\
&\quad \otimes \omega^{p_t} Z^{r_t} X^{m'_t - u'_t} |x_t\rangle_{s'_t} \otimes |k_{t+2}, \ldots, k_n\rangle_{s'_{t+1} \ldots s'_{n-1}} \\
&= \sum_{k_1, k_1, \ldots, k_n}^{d-1} \alpha^{(t+1)}_{k_1, k_2, \ldots, k_n} |k_1, x_1, \ldots, x_{t-1}\rangle_{1s'_1 \ldots s'_{t-1}} \\
&\quad \otimes \omega^{p_t} Z^{r_t} |k_{t+1}\rangle_{s'_t} \otimes |k_{t+2}, \ldots, k_n\rangle_{s'_{t+1} \ldots s'_{n-1}} \\
&= \sum_{k_1, k_1, \ldots, k_n}^{d-1} \alpha^{(t)}_{k_1, k_2, \ldots, k_n} |k_1, x_1, \ldots, x_{t-1}, k_{t+1}, k_{t+2}, \ldots, k_n\rangle_{1s'_1 \ldots s'_{t-1}, s'_t, s'_{t+1} \ldots s'_{n-1}} \\
&= \left| \gamma_n^{(t)} \right\rangle_{1s'_1 \ldots s'_{t-1}, s'_t, s'_{t+1} \ldots s'_{n-1}}.
\end{aligned}$$

From a global point of view, the entire recovery process can be written as

$$I \otimes R_1 \otimes R_2 \otimes \cdots \otimes R_t \otimes \cdots \otimes R_{n-1} \left(\left| \gamma_n^{(n)} \right\rangle_{1s'_1 s'_2 \ldots s'_t \ldots s'_{n-1}} \right) = |\gamma_n\rangle_{1s'_1 s'_2 \ldots s'_t \ldots s'_{n-1}}, \tag{43}$$

where R_t is the recovery operator of the t-th secondary site to be performed on qudit s'_t with $1 \leq t \leq n-1$. It can be seen that the reverse order is unnecessary and the recovery operator in each secondary site can be performed in other orders. When all the recovery operations are done, the entire n-qudit has been distributed faithfully among n sites where each site holds one qudit.

The arbitrary n-qudit state can also be distributed to a set of secondary sites less than $n-1$ where some secondary sites or the primary site holds more than one qudit. In this case, an entanglement channel with n-qudit states can be constructed among t sites with $1 \leq t \leq n$.

6.3 Example of entanglement channel construction via a three-qubit W state

For the two-level system, after performing the distribution process the state among n site becomes

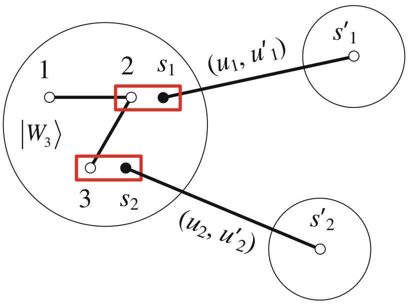


Fig. 7 An example of entanglement channel construction via a three-qubit W state

$$\left|\gamma_n^{(n)}\right\rangle = \sum_{k_1,k_1,\ldots,k_n=0}^{1} \alpha_{k_1,k_2,\ldots,k_n}^{(n)} \left|k_1, x_1, x_2, \ldots, x_{n-2}, x_{n-1}\right\rangle_{1s'_1s'_2\ldots s'_{n-2}s'_{n-1}}, \quad (44)$$

where ${\alpha^{(n)}}_{k_1,k_2,\ldots,k_n} = \alpha_{k_1,k_2,\ldots,k_n}(-1)^{\oplus_{t=1}^{n-1}(k_{t+1}\oplus m'_t)(u_t\oplus m_t)}$ and $x_t = k_{t+1} \oplus m'_t \oplus u'_t$ with $1 \le t \le n-1$. And the related recovery operator is

$$R_t = (-1)^{m'_t(u_t\oplus m_t)} Z^{m_t\oplus u_t} X^{m'_t\oplus u'_t}. \quad (45)$$

Here, an example of entanglement channel construction via a three-qubit W state between the primary site and two secondary sites is shown in Fig. 7. The three-qubit W state is

$$|W_3\rangle = \alpha_{001}\,|001\rangle + \alpha_{010}\,|010\rangle + \alpha_{110}\,|100\rangle, \quad (46)$$

with complex coefficients subject to $|\alpha_{001}|^2 + |\alpha_{010}|^2 + |\alpha_{100}|^2 = 1$. The entanglement channel shared between the primary site and the secondary one is a Bell state established by using the scheme related to Section 4.

The primary site measures particles $(s_1, 2)$ and $(s_2, 3)$ in Bell basis and gets result (m_1, m'_1) and (m_2, m'_2), respectively. The remaining state can be written as

$$\begin{aligned}
\left|W_3^{(3)}\right\rangle_{1s'_1s'_2} &= \sum_{k_1,k_2,k_3}^{1} \alpha_{k_1,k_2,k_3}^{(3)} \left|k_1, x_1, x_2\right\rangle \\
&= \sum_{k_1,k_2,k_3}^{1} \alpha_{k_1,k_2,k_3} (-1)^{\oplus_{t=1}^{2}(k_{t+1}\oplus m'_t)(u_t\oplus m_t)} \\
&\quad \cdot \left|k_1, k_2 \oplus m'_1 \oplus u'_1, k_3 \oplus m'_2 \oplus u'_2\right\rangle \\
&= \alpha_{001}(-1)^{((m'_1)(u_1\oplus m_1))\oplus((1\oplus m'_2)(u_2\oplus m_2))} \left|0, m'_1 \oplus u'_1, 1 \oplus m'_2 \oplus u'_2\right\rangle \\
&\quad + \alpha_{010}(-1)^{((1\oplus m'_1)(u_1\oplus m_1))\oplus((m'_2)(u_2\oplus m_2))} \left|0, 1 \oplus m'_1 \oplus u'_1, m'_2 \oplus u'_2\right\rangle \\
&\quad + \alpha_{100}(-1)^{((m'_1)(u_1\oplus m_1))\oplus((m'_2)(u_2\oplus m_2))} \left|1, m'_1 \oplus u'_1, m'_2 \oplus u'_2\right\rangle. \quad (47)
\end{aligned}$$

Let the states be $(u_1, u_1')_{s_1 s_1'} = (1, 1)$ and $(u_2, u_2')_{s_2 s_2'} = (0, 0)$, then

$$\begin{aligned}\left|W_3^{(3)}\right\rangle_{1s_1's_2'} &= \alpha_{001}(-1)^{((m_1')(1\oplus m_1))\oplus((1\oplus m_2')(m_2))}\left|0, m_1' \oplus 1, 1 \oplus m_2'\right\rangle \\ &\quad + \alpha_{010}(-1)^{((1\oplus m_1')(1\oplus m_1))\oplus(m_2' m_2)}\left|0, m_1', m_2'\right\rangle \\ &\quad + \alpha_{100}(-1)^{((m_1')(1\oplus m_1))\oplus(m_2' m_2)}\left|1, m_1' \oplus 1, m_2'\right\rangle \end{aligned} \tag{48a}$$

Here, a new W state entanglement channel is generated among three parties for quantum information tasks, such as joint remote preparation proposed in Ref. [33]. If the task requires the W state to be exact the same state as Alice prepared, a specific recovery operator $I \otimes R_1 \otimes R_2$ can be performed on secondary sites where

$$I \otimes R_1 \otimes R_2 = I \otimes (-1)^{m_1'(1\oplus m_1)} Z^{m_1 \oplus 1} X^{m_1' \oplus 1} \otimes (-1)^{m_2' m_2} Z^{m_2} X^{m_2'}. \tag{48b}$$

Now we consider two cases of the measurement outcomes:

1. Suppose the measurement results are $(m_1, m_1') = (0, 0)$ and $(m_2, m_2') = (0, 0)$, the resulting state becomes

$$\left|W_3^{(3)}\right\rangle_{1s_1's_2'} = \alpha_{001}\left|011\right\rangle - \alpha_{010}\left|000\right\rangle + \alpha_{100}\left|110\right\rangle.$$

   The corresponding recovery operator is $I \otimes ZX \otimes I$.
2. Suppose the measurement results are $(m_1, m_1') = (1, 0)$ and $(m_2, m_2') = (0, 1)$, then

$$\left|W_3^{(3)}\right\rangle_{1s_1's_2'} = \alpha_{001}\left|010\right\rangle + \alpha_{010}\left|001\right\rangle + \alpha_{100}\left|111\right\rangle.$$

The recovery operator is $I \otimes X \otimes X$.

7 Discussions and conclusions

Our major concern is the perfect distribution of the entanglement among spatially separated sites in network environment. A theoretical network model with perfect connection and good scalability is presented. In the network model, each node can store and locally manipulate qudits. Each pair of adjacent node is connected by a set of Bell states. No direct qudit transmissions are involved in our model except for the priori shared Bell states between two adjacent sites. The entanglements dynamically change in Metwally's scheme, while our scheme utilizes ES to generate not only the bipartite entanglements, but also the multipartite ones.

Three efficient and perfect entanglement channel construction schemes are proposed. Any two sites can construct an entanglement channel via Bell states with the help of the intermediate sites in quantum chain. The two sites can also verify the perfection of the Bell state to prevent outside attacks. This scheme is expressly different from the way of quantum repeaters. The quantum repeater was designed to resolve the problem of fiber attenuation over long distances, while our scheme is for node address and perfect connection in the network model. Furthermore, the general d-level system is considered and some general results about constructing entanglement channel are given in details.

Any n sites in the network model can construct an entanglement channel via an n-qudit cat state. The n-qudit entanglement channel could be constructed in an efficient and perfect manner with the established entanglement channel between the primary site and the secondary one. Compared with relay transmission of the qudit from the primary site to the secondary one in quantum chain, this scheme only involves Bell states and Bell basis measurements and it provides a secure transmission of the qudit states since the qudits are impossible to be intercepted.

More importantly, we extend the scheme to a more general form where an entanglement channel via an arbitrary n-qudit state can be constructed among n sites. This can also be viewed as a scheme of secret sharing of quantum information among n sites. The state can be recovered to its initial state or transformed to other state by performing appropriate generalized Pauli operators if necessary. The number of the secondary sites can also be less than $n-1$, in other words, an arbitrary n-qudit state entanglement channel, like W states, cluster states, and χ-type states, can be constructed among any t sites where $1 \leq t \leq n$.

The network topology determines how the communication sites are connected and communicated. Our first scheme using generalized Bell state entanglement channel between two sites could be viewed as a point to point network. If the scheme is performed successively, any type of network topologies can be constructed. In the others two proposed schemes, the primary site builds a star topology network where the primary site locates in the center of the model, shares Bell states with each secondary site and distributes the n-qudit states among n sites which will form a mesh topology network depending on the entanglement of the n-qudit states.

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Appendix 1: Detailed deduction of Eq. (8)

$$|\phi(v_1,\ v_2,\ \ldots,\ v_t,\ldots,\ v_n)\rangle_{12\ldots t\ldots n}\left|\phi(u,u')\right\rangle_{ss'}$$

$$=\left(\frac{1}{\sqrt{d}}\sum_{j=0}^{d-1}\omega^{jv_1}\,|j,\ j+v_2,\ \ldots,\ j+v_t,\ \ldots,\ j+v_n\rangle_{12\ldots t\ldots n}\right)$$

$$\otimes\left(\frac{1}{\sqrt{d}}\sum_{j'=0}^{d-1}\omega^{j'u}\left|j',\ j'+u'\right\rangle_{ss'}\right)$$

$$= \frac{1}{d}\sum_{j,j'}^{d-1}\omega^{jv_1}\omega^{j'u}\,|j,\ j+v_2,\ldots,\ j+v_t,\ldots,\ j+v_n)\rangle_{12\ldots t\ldots n}\left|j',\ j'+u'\right\rangle_{ss'}$$

$$= \frac{1}{d}\sum_{j,j'}^{d-1}\omega^{jv_1+j'u}\left|j,\ j+v_2,\ldots,\ j'+u',\ldots,\ j+v_n)\right\rangle_{12\ldots s'\ldots n}\left|j',\ j+v_t\right\rangle_{st}$$

$$= \frac{1}{d^2}\sum_{j,j',w,w'}^{d-1}\omega^{jv_1+j'u}\omega^{-wj-w'j'}\left|\phi(w,\ v_2,\ldots,\ j'+u'-j,\ldots,\ v_n)\right\rangle_{12\ldots s'\ldots n}$$
$$\otimes\left|\phi(w',\ j+v_t-j')\right\rangle_{st}$$

$$\xrightarrow{j'-j=l}$$

$$= \frac{1}{d^2}\sum_{j,l,w,w'}^{d-1}\omega^{jv_1+(j+l)u}\omega^{-wj-w'(j+l)}\left|\phi(w,\ v_2,\ldots,\ u'+l,\ldots,\ v_n)\right\rangle_{12\ldots s'\ldots n}$$

$$\otimes\left|\phi(w',\ v_t-l)\right\rangle_{st}$$

$$= \frac{1}{d^2}\sum_{j,l,w,w'}^{d-1}\omega^{j(v_1+u-w-w')}\omega^{lu-lw'}\left|\phi(w,\ v_2,\ldots,\ u'+l,\ldots,\ v_n)\right\rangle_{12\ldots s'\ldots n}$$

$$\otimes\left|\phi(w',\ v_t-l)\right\rangle_{st}$$

$$= \frac{1}{d}\sum_{l,w,w'}^{d-1}\left(\frac{1}{d}\sum_{j}^{d-1}\omega^{j(v_1+u-w-w')}\right)\omega^{lu-lw'}\left|\phi(w,\ v_2,\ldots,\ u'+l,\ldots,\ v_n)\right\rangle_{12\ldots s'\ldots n}$$

$$\otimes\left|\phi(w',\ v_t-l)\right\rangle_{st}$$

$$= \frac{1}{d}\sum_{l,w,w'}^{d-1}\delta(v_1+u-w-w',0)\omega^{lu-lw'}\left|\phi(w,\ v_2,\ldots,\ u'+l,\ldots,\ v_n)\right\rangle_{12\ldots s'\ldots n}$$

$$\otimes\left|\phi(w',\ v_t-l)\right\rangle_{st}\quad\blacktriangle$$

$$\xrightarrow{u-w'=k}$$

$$= \frac{1}{d}\sum_{l,k}^{d-1}\omega^{lk}\left|\phi(v_1+k,\ v_2,\ldots,\ u'+l,\ldots,\ v_n)\right\rangle_{12\ldots s'\ldots n}|\phi(u-k,\ v_t-l)\rangle_{st}$$

▲ Here we use the identity $\frac{1}{d}\sum_{j=0}^{d-1}\omega^{jn}=\delta(n,0)$.

Appendix 2: Detailed deduction of Eq. (30)

$$
\begin{aligned}
&|\gamma_n\rangle_{12\ldots t\ldots n}\left|\phi(u,u')\right\rangle_{ss'}\\
&=\sum_{k_1,k_2,\ldots,k_n}^{d-1}\alpha_{k_1,k_2,\ldots,k_n}|k_1,k_2,\ldots,k_t,\ldots,k_n\rangle_{12\ldots t\ldots n}\frac{1}{\sqrt{d}}\sum_{j=0}^{d-1}\omega^{ju}\left|j,\ j+u'\right\rangle_{ss'}\\
&=\frac{1}{\sqrt{d}}\sum_{k_1,k_2,\ldots,k_n,j}^{d-1}\alpha_{k_1,k_2,\ldots,k_n}\omega^{ju}|k_1,k_2,\ldots,k_t,\ldots,k_n\rangle_{12\ldots t\ldots n}\left|j,\ j+u'\right\rangle_{ss'}\\
&=\frac{1}{\sqrt{d}}\sum_{k_1,k_2,\ldots,k_n,j}^{d-1}\alpha_{k_1,k_2,\ldots,k_n}\omega^{ju}|j,k_t\rangle_{st}\left|k_1,k_2,\ldots,j+u',\ldots,k_n\right\rangle_{12\ldots s'\ldots n}\\
&=\frac{1}{\sqrt{d}}\sum_{k_1,k_2,\ldots,k_n,j}^{d-1}\alpha_{k_1,k_2,\ldots,k_n}\omega^{ju}\left(\frac{1}{\sqrt{d}}\sum_{j'=0}^{d-1}\omega^{-j'j}\left|\phi(j',k_t-j)\right\rangle_{st}\right)\\
&\quad\otimes\left|k_1,k_2,\ldots,j+u',\ldots,k_n\right\rangle_{12\ldots s'\ldots n}\\
&=\frac{1}{d}\sum_{k_1,k_2,\ldots,k_n,j,j'}^{d-1}\alpha_{k_1,k_2,\ldots,k_n}\omega^{j(u-j')}\left|\phi(j',k_t-j)\right\rangle_{st}\\
&\quad\otimes\left|k_1,k_2,\ldots,j+u',\ldots,k_n\right\rangle_{12\ldots s'\ldots n}
\end{aligned}
$$

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