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Classical communication cost and probabilistic remote two-qubit state preparation via POVM and W-type states

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Abstract I present a new scheme for probabilistic remote preparation of a general two-qubit state by using two W-type states as the shared quantum channel and a proper POVM instead of the usual positive measurement. Also I explore the scheme's applications to five special ensembles of two-qubit states. The success probability and the classical communication cost in different cases are calculated minutely, respectively, which show that the remote two-qubit preparation can be realized with higher probability after consuming some more classical bits provided that the two-qubit state to be prepared is chosen from the special ensembles.

Keywords Remote state preparation · W-type state · Positive operator-valued measure · Unitary operation · Success probability · Classical communication cost

1 Introduction

Entanglement is one of the most counterintuitive features in quantum mechanics: assisted with entangled state one can complete many impossible tasks within the classical world. One of the most striking applications of entanglement is quantum teleportation (QT) which was first proposed by Bennett et al. [1] in 1993. It is a method for interchanging quantum resources between different places. Later, Lo [2], Pati [3], Bennett et al. [4] presented another interesting novel method to transmit pure quantum state also using a prior shared entanglement and some classical communication when the sender actually knows the transmitted state. This communication protocol is called remote state preparation (RSP) and viewed as "teleportation of a known state". In RSP,

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the state to be prepared is assumed to be completely known by the sender. In contrast, the teleported state is not required to be known by the sender in QT. Moreover, due to the prior knowledge about the original state, to some extent, the classical communication and entanglement cost can be reduced in RSP process. For an example, Pati [3] has shown that for a qubit chosen from equatorial or polar great circles on a Bloch sphere, RSP requires only 1 forward classical bit, exactly half that of QT. However, for general states, RSP procedure requires as much communication cost as QT. The detailed trade-off between the classical communication cost and the required entanglement in RSP procedure can be studied distinctly in the protocol proposed by Bennett et al. [4].

Up to now, RSP has already attracted many attentions [5–20], e.g., low-entanglement RSP [5], higher-dimensional RSP [6], optimal RSP [7], oblivious RSP [8], RSP without oblivious conditions [9], generalized RSP [10], faithful RSP [11], RSP for multi-parties [12,13], and continuous variable RSP in phase space [14,15], etc. Some RSP schemes have already been experimentally implemented, e.g., Peng et al. presented an RSP scheme with the technique of NMR (nuclear magnetic resonance) [21], Xiang et al. [22] and Peters et al. [23] proposed two other RSP schemes using spontaneous parametric down-conversion. Also some RSP schemes are investigated using different entangled states as quantum channel [24–54]. In terms of entanglements in quantum channels, these RSP schemes can be classified into two types. One uses maximally entangled states [24,25,30,38,40,42,49,50,52–54] while another utilizes non-maximally entangled states [26–29,31,34–36,41,48]. In the latter case, usually one or more auxiliary qubits need to be incorporated and entangled with the original qubits. After this, a proper measurement on qubits including the ancillas should be executed such that the original-qubit state is collapsed to one of the eligible states. Subsequently, the prepared state is retrieved from the eligible state by performing appropriate unitary operations that correspond to the measurement outcomes. Note that the so-called proper measurement is usually projective measurement (PM) [55] in the latter type of existing RSP schemes [27,28,34,35]. As a matter of fact, there lies another type of measurement named positive operator-valued measure (POVM) [56,57], which has already attracted many attentions and been employed in various quantum information processing [26,29,41,44–46,57–59].

Besides, it is well known that multipartite qubits can be entangled in different inequivalent ways. For tripartite entangled quantum system, it falls into two classes of irreducible entanglements [60–62], that is, Greenberger–Horne–Zeilinger (GHZ) state and W state or their modified version. The motivation of classifying entangled state is that if the entanglement of two states is equivalent, then both of the two states can be used to perform the same task, although the probability of successful performance of the task may depend on the amount of entanglement of the state. However, to my best knowledge in RSP, most of the previous schemes utilize the GHZ class of entangled state [31,36,48,49,53,54] and there has been few proposals for investigating the application of W state. As a matter of fact, W state is also a promising candidate in implementing quantum communication and other tasks in the realm of quantum information processing. For example, Joo et al. [63] presented a novel scheme for secure quantum communication via W state: quantum key distribution, probabilistic QSS of classical information and their synthesis, and so on [28,29,50].

However, to my best knowledge, there have been no proposals for how to generate RSP of a general two-qubit state with both the method of POVM and W states or their modified version so far. In view of that, in this paper, using two non-maximally entangled W states (referred to as W-type states hereafter) as the quantum channel, I attempt to propose a scheme to address the question raised above, in which a proper POVM is employed instead of the usual PM, and the corresponding success probabilities in different cases as well as the total classical communication cost are also calculated in detail.

This paper is organized as follows, in Sect. 2, a probabilistic RSP scheme is amply presented with two W-type states and the method of POVM. Also its applications to some special ensembles of two-qubit states are investigated in this section. Then some discussions regarding the comparisons between this scheme and the previous two-qubit RSP protocols as well as the implement feasibility of the scheme are given in Sect. 3, together with the summary.

2 Probabilistic RSP scheme and the exploration of its applications

Suppose Alice is the state preparer, Bob and Charlie are her two remote ministrants. The quantum channel linking Alice, Bob and Charlie is composed of two W-type states

$$\begin{aligned} |\psi_1\rangle_{123} &= a_1|001\rangle_{123} + b_1|010\rangle_{123} + c_1|100\rangle_{123} \quad (|a_1|^2 + |b_1|^2 + |c_1|^2 = 1), \\ |\psi_2\rangle_{456} &= a_2|001\rangle_{456} + b_2|010\rangle_{456} + c_2|100\rangle_{456} \quad (|a_2|^2 + |b_2|^2 + |c_2|^2 = 1). \end{aligned} \tag{1}$$

where a_j, b_j and c_j are nonzero real numbers and satisfy $|a_j| \geq |b_j| \geq |c_j| (j = 1, 2)$. Qubit pair (1, 4) belongs to Alice while qubit pairs (2, 5) and (3, 6) to Bob and Charlie, respectively (shown in Fig. 1a). Alice wants to prepare remotely a state in either Bob or Charlie's place via their collaboration. The state to be prepared is $|V\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$, where $\alpha, \beta, \gamma, \delta$ are arbitrary complex numbers and satisfy $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$. Alice knows it exactly while Bob and Charlie do not. Owing to the channel symmetry for Bob and Charlie, each of them has the chance to construct the state $|V\rangle$ with another one's assistance. Specifically, Charlie can construct it with Bob's help and vice versa. Without loss of generality, suppose Charlie is assigned by Alice to retrieve $|V\rangle$ hereafter. Then the RSP protocol can be realized as follows:

(i) To fulfill the state preparation, Alice carries out a two-qubit projective measurement on her qubit pair (1, 4) in a set of mutually orthonormal basis vectors $\{|\lambda_1\rangle, |\lambda_2\rangle, |\lambda_3\rangle, |\lambda_4\rangle\}$ (shown in Fig. 1b), which are given as

$$\begin{pmatrix} |\lambda_1\rangle \\ |\lambda_2\rangle \\ |\lambda_3\rangle \\ |\lambda_4\rangle \end{pmatrix} = \begin{pmatrix} \alpha & \beta & \gamma & \delta \\ \eta\alpha & \eta\beta & -\eta^{-1}\gamma & -\eta^{-1}\delta \\ \beta^* & -\alpha^* & \delta^* & -\gamma^* \\ \eta\beta^* & -\eta\alpha^* & -\eta^{-1}\delta^* & \eta^{-1}\gamma^* \end{pmatrix} \begin{pmatrix} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{pmatrix}, \tag{2}$$

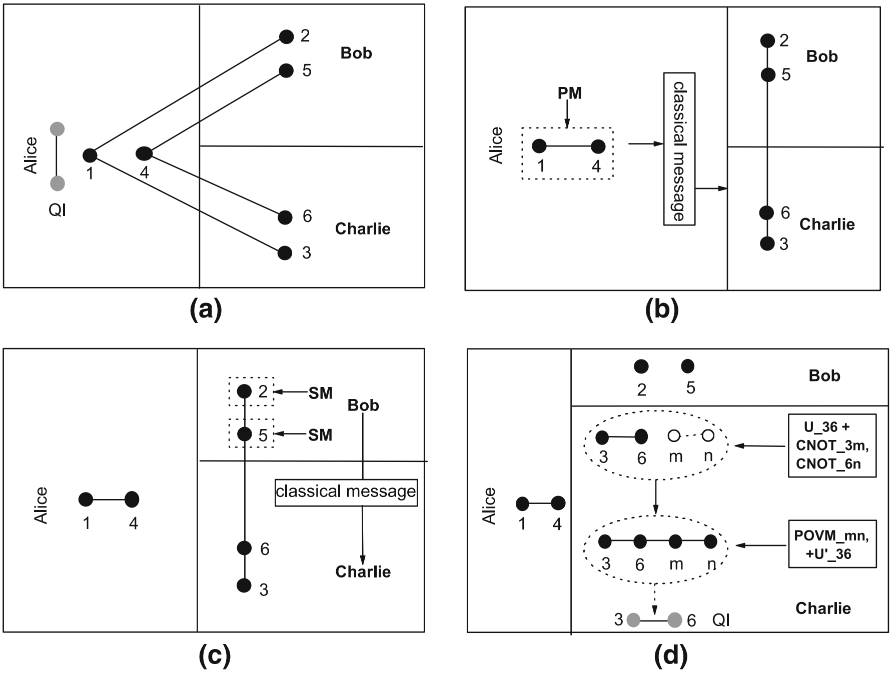

Fig. 1 Classical communication cost and remote preparation of a two-qubit state (QI) via POVM and two W-type states. **a** Alice, Bob and Charlie share the two W-type states. **b** Alice makes a two-qubit projective measurement (PM) on her qubit pair (1,4) and informs Charlie of her measurement result. **c** Bob performs a single-qubit measurement (SM) on his qubits 2 and 5, respectively, and then tells Charlie his measurement results via a classical channel. **d** Charlie constructs the prepared state by incorporating two auxiliary qubits (m, n) and executing some appropriate unitary operations (U, CNOT, U$'$) including a proper POVM. See text for more details

where $\eta = \sqrt{(|\gamma|^2 + |\delta|^2)/(|\alpha|^2 + |\beta|^2)}$. These four non-maximally entangled basis states are related to the computation basis vectors $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ and form a complete orthonormal basis set in a four-dimensional Hilbert space, i.e., $\langle\lambda_i|\lambda_j\rangle = \delta_{ij}$.

Thus, the state of the whole system in the basis $\{|\lambda_1\rangle, |\lambda_2\rangle, |\lambda_3\rangle, |\lambda_4\rangle\}$ can be written as

$$|\psi_1\rangle_{123}|\psi_2\rangle_{456} = |\lambda_1\rangle_{14}|\Gamma_1\rangle_{2356} + |\lambda_2\rangle_{14}|\Gamma_2\rangle_{2356} + |\lambda_3\rangle_{14}|\Gamma_3\rangle_{2356} + |\lambda_4\rangle_{14}|\Gamma_4\rangle_{2356}, \tag{3}$$

where

$$\begin{aligned}|\Gamma_1\rangle_{2356} &= \alpha^* a_1 a_2|0101\rangle_{2356} + \alpha^* a_1 b_2|0110\rangle_{2356} + \alpha^* b_1 a_2|1001\rangle_{2356}\\ &\quad+\alpha^* b_1 b_2|1010\rangle_{2356} + \beta^* a_1 c_2|0100\rangle_{2356} + \beta^* b_1 c_2|1000\rangle_{2356}\\ &\quad+\gamma^* c_1 a_2|0001\rangle_{2356} + \gamma^* c_1 b_2|0010\rangle_{2356} + \delta^* c_1 c_2|0000\rangle_{2356};\\ |\Gamma_2\rangle_{2356} &= \eta\alpha^* a_1 a_2|0101\rangle_{2356} + \eta\alpha^* a_1 b_2|0110\rangle_{2356} + \eta\alpha^* b_1 a_2|1001\rangle_{2356}\end{aligned}$$

$$
\begin{aligned}
&+\eta\alpha^* b_1 b_2|1010\rangle_{2356} + \eta\beta^* a_1 c_2|0100\rangle_{2356} + \eta\beta^* b_1 c_2|1000\rangle_{2356}\\
&-\eta^{-1}\gamma^* c_1 a_2|0001\rangle_{2356} - \eta^{-1}\gamma^* c_1 b_2|0010\rangle_{2356} - \eta^{-1}\delta^* c_1 c_2|0000\rangle_{2356};\\
|\Gamma_3\rangle_{2356} &= \beta a_1 a_2|0101\rangle_{2356} + \beta a_1 b_2|0110\rangle_{2356} + \beta b_1 a_2|1001\rangle_{2356}\\
&+\beta b_1 b_2|1010\rangle_{2356} - \alpha a_1 c_2|0100\rangle_{2356} - \alpha b_1 c_2|1000\rangle_{2356}\\
&+\delta c_1 a_2|0001\rangle_{2356} + \delta c_1 b_2|0010\rangle_{2356} - \gamma c_1 c_2|0000\rangle_{2356};\\
|\Gamma_4\rangle_{2356} &= \eta\beta a_1 a_2|0101\rangle_{2356} + \eta\beta a_1 b_2|0110\rangle_{2356} + \eta\beta b_1 a_2|1001\rangle_{2356}\\
&+\eta\beta b_1 b_2|1010\rangle_{2356} - \eta\alpha a_1 c_2|0100\rangle_{2356} - \eta\alpha b_1 c_2|1000\rangle_{2356}\\
&-\eta^{-1}\delta c_1 a_2|0001\rangle_{2356} - \eta^{-1}\delta c_1 b_2|0010\rangle_{2356} + \eta^{-1}\gamma c_1 c_2|0000\rangle_{2356}.
\end{aligned}
$$

After Alice's measurement, she broadcasts the measurement result via a classical channel. According to the Eq. 3, one can see that Alice's measurement result should be one of the four states defined in the Eq. 2. Without loss of generality, suppose Alice measures $|\lambda_3\rangle_{14}$, then the collapsed state of qubit pairs (2, 3) and (5, 6) will be $|\Gamma_3\rangle_{2356}$.

(ii) As proposed before, Charlie is assigned to construct the state $|V\rangle$. Then to realize the two-qubit preparation in his place, Charlie cooperates with Bob to get his help. Provided Bob agrees to help Charlie, he then performs a single-qubit measurement on his qubits 2 and 5 in the basis $\{|0\rangle, |1\rangle\}$, respectively (shown in Fig. 1c). In this way, it can be noted

$$
\begin{aligned}
|\Gamma_3\rangle_{2356} = {}&|00\rangle_{25}|\Lambda_3\rangle_{36} + |01\rangle_{25}(\beta a_1 b_2|10\rangle_{36} + \delta c_1 b_2|00\rangle_{36})\\
&+|10\rangle_{25}(\beta b_1 a_2|01\rangle_{36} - \alpha b_1 c_2|00\rangle_{36}) + |11\rangle_{25}\beta b_1 b_2|00\rangle_{36},
\end{aligned} \tag{4}
$$

where

$$
|\Lambda_3\rangle_{36} = \beta a_1 a_2|11\rangle_{36} - \alpha a_1 c_2|10\rangle_{36} + \delta c_1 a_2|01\rangle_{36} - \gamma c_1 c_2|00\rangle_{36}.
$$

After Bob's single-qubit measurements, he informs his measurement results to Charlie via a classical channel. According to the above equation, one can see it may be possible for the RSP process to be successful only when Bob measures $|00\rangle_{25}$, otherwise the RSP scheme fails.

(iiii) After having received Bob's classical message of the measurement result $|00\rangle_{25}$ in a certain interval, Charlie knows that his qubit pair (3, 6) is left in $|\Lambda_3\rangle_{36}$. Then to construct the state $|V\rangle$, Charlie performs $\sigma_3^x \otimes \sigma_6^z$ on qubits 3 and 6, which transforms $|\Lambda_3\rangle_{36}$ into

$$
|T\rangle_{36} = \alpha a_1 c_2|00\rangle_{36} + \beta a_1 a_2|01\rangle_{36} + \gamma c_1 c_2|10\rangle_{36} + \delta c_1 a_2|11\rangle_{36}. \tag{5}
$$

Next, Charlie introduces two auxiliary qubits m and n in the initial state $|00\rangle_{mn}$ and performs two controlled-not (CNOT) operations with qubits 3 and 6 as the controlled qubits while the auxiliary qubits m and n as the target ones, respectively. These two CNOT operations transform the joint state $|T\rangle_{36}|00\rangle_{mn}$ into the following form

$$\begin{aligned}|K\rangle_{36mn} &= \alpha a_1 c_2|0000\rangle_{36mn} + \beta a_1 a_2|0101\rangle_{36mn} + \gamma c_1 c_2|1010\rangle_{36mn} \\ &\quad + \delta c_1 a_2|1111\rangle_{36mn} \\ &= \frac{1}{4}(|R_1\rangle_{36}|H_1\rangle_{mn} + |R_2\rangle_{36}|H_2\rangle_{mn} + |R_3\rangle_{36}|H_3\rangle_{mn} + |R_4\rangle_{36}|H_4\rangle_{mn}), \quad (6)\end{aligned}$$

where

$$\begin{aligned}|R_1\rangle_{36} &= \alpha|00\rangle_{36} + \beta|01\rangle_{36} + \gamma|10\rangle_{36} + \delta|11\rangle_{36} \equiv |V\rangle, \\ |H_1\rangle_{mn} &= a_1c_2|00\rangle_{mn} + a_1a_2|01\rangle_{mn} + c_1c_2|10\rangle_{mn} + c_1a_2|11\rangle_{mn}, \\ |R_2\rangle_{36} &= \alpha|00\rangle_{36} + \beta|01\rangle_{36} - \gamma|10\rangle_{36} - \delta|11\rangle_{36}, \\ |H_2\rangle_{mn} &= a_1c_2|00\rangle_{mn} + a_1a_2|01\rangle_{mn} - c_1c_2|10\rangle_{mn} - c_1a_2|11\rangle_{mn}, \\ |R_3\rangle_{36} &= \alpha|00\rangle_{36} - \beta|01\rangle_{36} + \gamma|10\rangle_{36} - \delta|11\rangle_{36}, \\ |H_3\rangle_{mn} &= a_1c_2|00\rangle_{mn} - a_1a_2|01\rangle_{mn} + c_1c_2|10\rangle_{mn} - c_1a_2|11\rangle_{mn}, \\ |R_4\rangle_{36} &= \alpha|00\rangle_{36} - \beta|01\rangle_{36} - \gamma|10\rangle_{36} + \delta|11\rangle_{36}, \\ |H_4\rangle_{mn} &= a_1c_2|00\rangle_{mn} - a_1a_2|01\rangle_{mn} - c_1c_2|10\rangle_{mn} + c_1a_2|11\rangle_{mn}.\end{aligned}$$

From the Eq. 6, one can see that‘ Charlie can get the states $|R_i\rangle_{36}(i = 1, 2, 3, 4)$ provided that the states $|H_i\rangle_{mn}(i = 1, 2, 3, 4)$ are distinguished via an appropriate measurement. Note that $|R_1\rangle$ is exactly the prepared state. Readily, the prepared state can be further retrieved from $|R_2\rangle$, $|R_3\rangle$ and $|R_4\rangle$. Unfortunately, the four states $|H_i\rangle_{mn}(i = 1, 2, 3, 4)$ are not orthonormal in general. As a consequence, they cannot be differentiated deterministically by using a usual *PM*. Nevertheless, the discrimination can be achieved in a probabilistic manner by making an optimal *POVM* measurement [56,57]. Forasmuch, Charlie then performs an optimal POVM measurement [46,56–58] on the auxiliary qubits m and n as follows,

$$\begin{aligned}Q_1 &= \frac{1}{x}|M_1\rangle\langle M_1|, \quad Q_2 = \frac{1}{x}|M_2\rangle\langle M_2|, \\ Q_3 &= \frac{1}{x}|M_3\rangle\langle M_3|, \quad Q_4 = \frac{1}{x}|M_4\rangle\langle M_4|, \\ Q_5 &= I - \frac{1}{x}\sum_{i=1}^{4}|M_i\rangle\langle M_i|, \quad (7)\end{aligned}$$

where

$$\begin{aligned}|M_1\rangle &= \frac{1}{\sqrt{\xi}}\left(\frac{1}{a_1c_2}|00\rangle + \frac{1}{a_1a_2}|01\rangle + \frac{1}{c_1c_2}|10\rangle + \frac{1}{c_1a_2}|11\rangle\right)_{mn}, \\ |M_2\rangle &= \frac{1}{\sqrt{\xi}}\left(\frac{1}{a_1c_2}|00\rangle + \frac{1}{a_1a_2}|01\rangle - \frac{1}{c_1c_2}|10\rangle - \frac{1}{c_1a_2}|11\rangle\right)_{mn}, \\ |M_3\rangle &= \frac{1}{\sqrt{\xi}}\left(\frac{1}{a_1c_2}|00\rangle - \frac{1}{a_1a_2}|01\rangle + \frac{1}{c_1c_2}|10\rangle - \frac{1}{c_1a_2}|11\rangle\right)_{mn},\end{aligned}$$

$$|M_4\rangle = \frac{1}{\sqrt{\xi}}\left(\frac{1}{a_1c_2}|00\rangle - \frac{1}{a_1a_2}|01\rangle - \frac{1}{c_1c_2}|10\rangle + \frac{1}{c_1a_2}|11\rangle\right)_{mn},$$
$$\xi = \frac{1}{(a_1c_2)^2} + \frac{1}{(a_1a_2)^2} + \frac{1}{(c_1c_2)^2} + \frac{1}{(c_1a_2)^2},$$

I is an identity operator, x is a coefficient relating to a_j and $c_j (j = 1, 2)$ and should be able to assure Q_5 to be a positive operator. To exactly determine x, I would like to rewrite the five elements Q_1, Q_2, Q_3, Q_4 and Q_5 in the matrix form

$$Q_1 = \frac{1}{x\xi}\begin{pmatrix} \frac{1}{(a_1c_2)^2} & \frac{1}{a_1a_2a_1c_2} & \frac{1}{a_1c_2c_1c_2} & \frac{1}{a_1a_2c_1c_2} \\ \frac{1}{a_1a_2a_1c_2} & \frac{1}{(a_1a_2)^2} & \frac{1}{a_1c_2c_1a_2} & \frac{1}{a_1a_2c_1a_2} \\ \frac{1}{a_1c_2c_1c_2} & \frac{1}{a_1c_2c_1a_2} & \frac{1}{(c_1c_2)^2} & \frac{1}{c_1a_2c_1c_2} \\ \frac{1}{a_1a_2c_1c_2} & \frac{1}{a_1a_2c_1a_2} & \frac{1}{c_1c_2c_1a_2} & \frac{1}{(c_1a_2)^2} \end{pmatrix},$$

$$Q_2 = \frac{1}{x\xi}\begin{pmatrix} \frac{1}{(a_1c_2)^2} & \frac{1}{a_1a_2a_1c_2} & \frac{-1}{a_1c_2c_1c_2} & \frac{-1}{a_1a_2c_1c_2} \\ \frac{1}{a_1a_2a_1c_2} & \frac{1}{(a_1a_2)^2} & \frac{-1}{a_1c_2c_1a_2} & \frac{-1}{a_1a_2c_1a_2} \\ \frac{-1}{a_1c_2c_1c_2} & \frac{-1}{a_1c_2c_1a_2} & \frac{1}{(c_1c_2)^2} & \frac{1}{c_1a_2c_1c_2} \\ \frac{-1}{a_1a_2c_1c_2} & \frac{-1}{a_1a_2c_1a_2} & \frac{1}{c_1c_2c_1a_2} & \frac{1}{(c_1a_2)^2} \end{pmatrix},$$

$$Q_3 = \frac{1}{x\xi}\begin{pmatrix} \frac{1}{(a_1c_2)^2} & \frac{-1}{a_1a_2a_1c_2} & \frac{1}{a_1c_2c_1c_2} & \frac{-1}{a_1a_2c_1c_2} \\ \frac{-1}{a_1a_2a_1c_2} & \frac{1}{(a_1a_2)^2} & \frac{-1}{a_1c_2c_1a_2} & \frac{1}{a_1a_2c_1a_2} \\ \frac{1}{a_1c_2c_1c_2} & \frac{-1}{a_1c_2c_1a_2} & \frac{1}{(c_1c_2)^2} & \frac{-1}{c_1a_2c_1c_2} \\ \frac{-1}{a_1a_2c_1c_2} & \frac{1}{a_1a_2c_1a_2} & \frac{-1}{c_1c_2c_1a_2} & \frac{1}{(c_1a_2)^2} \end{pmatrix},$$

$$Q_4 = \frac{1}{x\xi}\begin{pmatrix} \frac{1}{(a_1c_2)^2} & \frac{-1}{a_1a_2a_1c_2} & \frac{-1}{a_1c_2c_1c_2} & \frac{1}{a_1a_2c_1c_2} \\ \frac{-1}{a_1a_2a_1c_2} & \frac{1}{(a_1a_2)^2} & \frac{1}{a_1c_2c_1a_2} & \frac{-1}{a_1a_2c_1a_2} \\ \frac{1}{a_1c_2c_1c_2} & \frac{-1}{a_1c_2c_1a_2} & \frac{-1}{(c_1c_2)^2} & \frac{1}{c_1a_2c_1c_2} \\ \frac{-1}{a_1a_2c_1c_2} & \frac{1}{a_1a_2c_1a_2} & \frac{1}{c_1c_2c_1a_2} & \frac{-1}{(c_1a_2)^2} \end{pmatrix},$$

$$Q_5 = \begin{pmatrix} A & 0 & 0 & 0 \\ 0 & B & 0 & 0 \\ 0 & 0 & C & 0 \\ 0 & 0 & 0 & D \end{pmatrix}, \text{ where } \begin{cases} A = 1 - \frac{4}{x\xi(a_1c_2)^2}, & B = 1 - \frac{4}{x\xi(a_1a_2)^2}, \\ C = 1 - \frac{4}{x\xi(c_1c_2)^2}, & D = 1 - \frac{4}{x\xi(c_1a_2)^2}. \end{cases} \tag{8}$$

Evidently, to let Q_5 be a positive operator, the coefficient x should be chosen such that all the diagonal elements A, B, C and D are nonnegative. So it should be an appropriate value within the scope $1 \leq x \leq 4$, as is strongly relative to a_j and $c_j (j = 1, 2)$.

After this POVM operation, Charlie can positively conclude the states $|H_i\rangle_{mn} (i = 1, 2, 3, 4)$ of qubits m and n in terms of the POVM's values. The probability in each case is

$$p = {}_{36mn}\langle K|Q_i|K\rangle_{36mn} = {}_{mn}\langle H_i|Q_i|H_i\rangle_{mn}/16 = \frac{1}{x\xi}, \quad (i = 1, 2, 3, 4). \tag{9}$$

However, if Charlie gets the value of the POVM's element Q_5 (such probability is $1 - 4 \times p = 1 - \frac{4}{x\xi}$), he cannot infer which state the qubits m and n are in.

In this case, it means the scheme fails. As proposed before, only if Charlie determines the states $|H_i\rangle_{mn}(i = 1, 2, 3, 4)$, he can construct the prepared state $|V\rangle$ on his qubit pair (3, 6) by performing an appropriate unitary operation. To be specifical, if $|H_1\rangle_{mn}, |H_2\rangle_{mn}, |H_3\rangle_{mn}$ or $|H_4\rangle_{mn}$ is determined, it means the state of qubit pair (3, 6) is $|R_1\rangle_{36}, |R_2\rangle_{36}, |R_3\rangle_{36}$, or $|R_4\rangle_{36}$), then Charlie needs only to perform the corresponding unitary operation $I_3 \otimes I_6, \sigma_z^3 \otimes I_6, I_3 \otimes \sigma_z^6$, or $\sigma_z^3 \otimes \sigma_z^6$ on his qubit pair (3, 6), respectively. In this way, the total success probability of the RSP scheme is

$$\begin{aligned} 4 \times p &= \frac{4}{x\xi} = \frac{4}{x} \times \left[\frac{1}{(a_1c_2)^2} + \frac{1}{(a_1a_2)^2} + \frac{1}{(c_1c_2)^2} + \frac{1}{(c_1a_2)^2}\right]^{-1} \\ &= \frac{4}{x} \times \frac{a_1^2a_2^2c_1^2c_2^2}{(a_1^2+c_1^2)(a_2^2+c_2^2)}. \end{aligned} \tag{10}$$

As depicted previously, Alice may measure $|\lambda_1\rangle_{14}, |\lambda_2\rangle_{14}$ or $|\lambda_4\rangle_{14}$ with a certain probability. In these cases, the collapsed state of the qubit pairs (2, 5) and (3, 6), according to the Eq. 3, will be $|\Gamma_1\rangle_{2356}, |\Gamma_2\rangle_{2356}$ and $|\Gamma_4\rangle_{2356}$, respectively. Then after step (ii), the state of qubit pair (3, 6) in Charlie's place will collapse to the following forms

$$\begin{aligned} |\Lambda_1\rangle_{36} &= \alpha^*a_1a_2|11\rangle_{36} + \beta^*a_1c_2|10\rangle_{36} + \gamma^*c_1a_2|01\rangle_{36} + \delta^*c_1c_2|00\rangle_{36}, \\ |\Lambda_2\rangle_{36} &= \eta\alpha^*a_1a_2|11\rangle_{36} + \eta\beta^*a_1c_2|10\rangle_{36} - \eta^{-1}\gamma^*c_1a_2|01\rangle_{36} - \eta^{-1}\delta^*c_1c_2|00\rangle_{36}, \\ |\Lambda_4\rangle_{36} &= \eta\beta a_1a_2|11\rangle_{36} - \eta\alpha a_1c_2|10\rangle_{36} - \eta^{-1}\delta c_1a_2|01\rangle_{36} + \eta^{-1}\gamma c_1c_2|00\rangle_{36}), \end{aligned}$$

respectively. Since Charlie has no knowledge of the four coefficients α, β, γ and δ, he cannot convert anyone of the above three states into the state $|V\rangle$ due to the involvement of an antiunitary operation [3,10,21]. Apparently, the RSP scheme fails in the latter three cases. Nonetheless, it should be noted that the coefficients α, β, γ and δ are assumed to be complex in the beginning. Then it is intriguing to ask whether the conversion can be unitarily realized provided that α, β, γ and δ are some special values. After my extensive investigations, I get the positive answer and find out some special ensembles, which are given as: *Ensemble I:* α, β, γ *and* δ *are real; Ensemble II:* α, β, γ *and* δ *satisfy* $\eta = 1$*; Ensemble III:* $\alpha, \beta, \gamma, \delta$ *are real and satisfy* $\eta = 1$*; Ensemble IV:* $|\alpha| = |\beta| = |\gamma| = |\delta| = \frac{1}{2}$ *and* $\alpha\gamma = \beta\delta$*;* and *Ensemble V:* $|\alpha| = |\beta| = |\gamma| = |\delta| = \frac{1}{2}$ *and* $\alpha\beta = \delta\gamma$. For each case, the treatment is similar in this paper. As enumerations, I will take *Ensemble I* and *Ensemble IV* to show the whole process of preparation, respectively. Incidentally, it is notable that in the whole RSP process, step (ii) would hold the line even if the state to be prepared is chosen from the special ensembles. In other words, some changes occur only in step (i) or step (iii), which are depicted as follows,

Ensemble I: α, β, γ *and* δ *are real* In this case, if Alice's measurement result is $|\lambda_1\rangle_{14}$, then according to the Eq. 3, the joint state of qubits 2, 3, 5 and 6 will be

$$\begin{aligned} |\Gamma_1'\rangle_{2356} = {}& \alpha a_1a_2|0101\rangle_{2356} + \alpha a_1b_2|0110\rangle_{2356} + \alpha b_1a_2|1001\rangle_{2356} \\ &+\alpha b_1b_2|1010\rangle_{2356} + \beta a_1c_2|0100\rangle_{2356} + \beta b_1c_2|1000\rangle_{2356} \\ &+\gamma c_1a_2|0001\rangle_{2356} + \gamma c_1b_2|0010\rangle_{2356} + \delta c_1c_2|0000\rangle_{2356}. \end{aligned} \tag{11}$$

After Bob getting the measurement results $|00\rangle_{25}$ in step (ii), the two qubits 3 and 6 in Charlie's place will be left in

$$|\Lambda_1'\rangle_{36} = \alpha a_1 a_2|11\rangle_{36} + \beta a_1 c_2|10\rangle_{36} + \gamma c_1 a_2|01\rangle_{36} + \delta c_1 c_2|00\rangle_{36}. \tag{12}$$

In step (iii), to fulfill the preparation, Charlie first performs $\sigma_3^x \otimes \sigma_6^x$ on his qubit pair (3, 6), which transforms $|\Lambda_1'\rangle_{36}$ into

$$|T'\rangle_{36} = \alpha a_1 a_2|00\rangle_{36} + \beta a_1 c_2|01\rangle_{36} + \gamma c_1 a_2|10\rangle_{36} + \delta c_1 c_2|11\rangle_{36}. \tag{13}$$

Then Charlie introduces two auxiliary qubits m and n in the initial state $|00\rangle_{mn}$, and performs two CNOT operations CNOT_{3m} and CNOT_{6n}, respectively. After this the joint state, $|T'\rangle_{36}|00\rangle_{mn}$ will be transformed into

$$\begin{aligned}|K'\rangle_{36mn} &= \alpha a_1 a_2|0000\rangle_{36mn} + \beta a_1 c_2|0101\rangle_{36mn} + \gamma c_1 a_2|1010\rangle_{36mn} \\ &\quad + \delta c_1 c_2|1111\rangle_{36mn}) \\ &= \frac{1}{4}(|R_1\rangle_{36}|H_1'\rangle_{mn} + |R_2\rangle_{36}|H_2'\rangle_{mn} + |R_3\rangle_{36}|H_3'\rangle_{mn} + |R_4\rangle_{36}|H_4'\rangle_{mn}),\end{aligned} \tag{14}$$

where

$$\begin{aligned}|H_1'\rangle_{mn} &= a_1 a_2|00\rangle_{mn} + a_1 c_2|01\rangle_{mn} + c_1 a_2|10\rangle_{mn} + c_1 c_2|11\rangle_{mn}, \\ |H_2'\rangle_{mn} &= a_1 a_2|00\rangle_{mn} + a_1 c_2|01\rangle_{mn} - c_1 a_2|10\rangle_{mn} - c_1 c_2|11\rangle_{mn}, \\ |H_3'\rangle_{mn} &= a_1 a_2|00\rangle_{mn} - a_1 c_2|01\rangle_{mn} + c_1 a_2|10\rangle_{mn} - c_1 c_2|11\rangle_{mn}, \\ |H_4'\rangle_{mn} &= a_1 a_2|00\rangle_{mn} - a_1 c_2|01\rangle_{mn} - c_1 a_2|10\rangle_{mn} + c_1 c_2|11\rangle_{mn}.\end{aligned}$$

Very similar to that proposed above, if $|H_i'\rangle_{mn}(i = 1, 2, 3, 4)$ are distinguished, the state $|V\rangle$ can be constructed via an appropriate unitary operation. According to the above equation, although the discrimination of the four states $|H_i'\rangle_{mn}(i = 1, 2, 3, 4)$ cannot be completed by performing the usual PM, it can be achieved in a probabilistic manner by making an optimal POVM measurement [46,56–58]. So Charlie then performs an optimal POVM measurement on the auxiliary qubits m and n, which takes the following matrix forms

$$W_1 = \frac{1}{x\xi}\begin{pmatrix} \frac{1}{(a_1a_2)^2} & \frac{1}{a_1c_2a_1a_2} & \frac{1}{a_1a_2c_1a_2} & \frac{1}{a_1c_2c_1a_2} \\ \frac{1}{a_1c_2a_1a_2} & \frac{1}{(a_1c_2)^2} & \frac{1}{a_1a_2c_1c_2} & \frac{1}{a_1c_2c_1c_2} \\ \frac{1}{a_1a_2c_1a_2} & \frac{1}{a_1a_2c_1c_2} & \frac{1}{(c_1a_2)^2} & \frac{1}{c_1c_2c_1a_2} \\ \frac{1}{a_1c_2c_1a_2} & \frac{1}{a_1c_2c_1c_2} & \frac{1}{c_1a_2c_1c_2} & \frac{1}{(c_1c_2)^2} \end{pmatrix},$$

$$W_2 = \frac{1}{x\xi}\begin{pmatrix} \frac{1}{(a_1a_2)^2} & \frac{1}{a_1c_2a_1a_2} & \frac{-1}{a_1a_2c_1a_2} & \frac{-1}{a_1c_2c_1a_2} \\ \frac{1}{a_1c_2a_1a_2} & \frac{1}{(a_1c_2)^2} & \frac{-1}{a_1a_2c_1c_2} & \frac{-1}{a_1c_2c_1c_2} \\ \frac{-1}{a_1a_2c_1a_2} & \frac{-1}{a_1a_2c_1c_2} & \frac{1}{(c_1a_2)^2} & \frac{1}{c_1c_2c_1a_2} \\ \frac{-1}{a_1c_2c_1a_2} & \frac{-1}{a_1c_2c_1c_2} & \frac{1}{c_1a_2c_1c_2} & \frac{1}{(c_1c_2)^2} \end{pmatrix},$$

$$W_3 = \frac{1}{x\xi}\begin{pmatrix} \frac{1}{(a_1a_2)^2} & \frac{-1}{a_1c_2a_1a_2} & \frac{1}{a_1a_2c_1a_2} & \frac{-1}{a_1c_2c_1a_2} \\ \frac{-1}{a_1c_2a_1a_2} & \frac{1}{(a_1c_2)^2} & \frac{-1}{a_1a_2c_1c_2} & \frac{1}{a_1c_2c_1c_2} \\ \frac{1}{a_1a_2c_1a_2} & \frac{-1}{a_1a_2c_1c_2} & \frac{1}{(c_1a_2)^2} & \frac{-1}{c_1c_2c_1a_2} \\ \frac{-1}{a_1c_2c_1a_2} & \frac{1}{a_1c_2c_1c_2} & \frac{-1}{c_1a_2c_1c_2} & \frac{1}{(c_1c_2)^2} \end{pmatrix},$$

$$W_4 = \frac{1}{x\xi}\begin{pmatrix} \frac{1}{(a_1a_2)^2} & \frac{-1}{a_1c_2a_1a_2} & \frac{-1}{a_1a_2c_1a_2} & \frac{1}{a_1c_2c_1a_2} \\ \frac{-1}{a_1c_2a_1a_2} & \frac{1}{(a_1c_2)^2} & \frac{1}{a_1a_2c_1c_2} & \frac{-1}{a_1c_2c_1c_2} \\ \frac{1}{a_1a_2c_1a_2} & \frac{-1}{a_1a_2c_1c_2} & \frac{-1}{(c_1a_2)^2} & \frac{1}{c_1c_2c_1a_2} \\ \frac{-1}{a_1c_2c_1a_2} & \frac{1}{a_1c_2c_1c_2} & \frac{1}{c_1a_2c_1c_2} & \frac{-1}{(c_1c_2)^2} \end{pmatrix}$$

and $W_5 = diag(B, A, D, C)$, respectively. After the manipulation, according to the POVM result, Charlie can conclude the corresponding state $|H_i'\rangle_{mn} (i = 1, 2, 3, 4)$, respectively, which happens with the probability

$$p = {}_{36mn}\langle K'|W_i|K'\rangle_{36mn} = {}_{mn}\langle H_i'|W_i|H_i'\rangle_{mn}/16 = \frac{1}{x\xi}, \quad (i = 1, 2, 3, 4). \tag{15}$$

Once Charlie determines $|H_i'\rangle_{mn} (i = 1, 2, 3, 4)$, it also means that the states $|R_i\rangle_{36} (i = 1, 2, 3, 4)$ are obtained, which can be readily seen from Eq. (14). Further, Charlie then constructs the prepared state by performing the corresponding unitary operation proposed above. Nevertheless, Charlie may get W_5's value with probability $1 - \frac{4}{x\xi}$. In this situation, he cannot infer which state the qubits m and n are in. Consequently, the remote preparation fails. Thus, the total success probability of the RSP scheme, in this case, is also $4p$.

If Alice's measurement result is $|\lambda_2\rangle_{14}$ or $|\lambda_4\rangle_{14}$, then after the step (ii), the joint state of qubits 3 and 6 will be

$$|\Lambda_2'\rangle_{36} = \eta\alpha a_1a_2|11\rangle_{36} + \eta\beta a_1c_2|10\rangle_{36} - \eta^{-1}\gamma c_1a_2|01\rangle_{36} - \eta^{-1}\delta c_1c_2|00\rangle_{36}$$

and $|\Lambda_4\rangle_{36}$, respectively. Due to Charlie's unawareness of the four coefficients α, β, γ and δ, these two states still cannot be unitarily converted into the state $|V\rangle$. So the total success probability of the RSP in *Ensemble I* is $8p$.

Ensemble IV: $|\alpha| = |\beta| = |\gamma| = |\delta| = \frac{1}{2}$ *and* $\alpha\gamma = \beta\delta$ In terms of $|\alpha| = |\beta| = |\gamma| = |\delta| = \frac{1}{2}$ and $\alpha\gamma = \beta\delta$, it can be easily obtained $\eta = 1$, $(\alpha^*)^{-1} = 4\alpha$, $(\beta^*)^{-1} = 4\beta$, $(\gamma^*)^{-1} = 4\gamma$, $(\delta^*)^{-1} = 4\delta$ and $\alpha^*\gamma^* = \beta^*\delta^*$. Then if Alice gets $|\lambda_1\rangle_{14}$, as proposed above, the state of qubit pair (3, 6), after step (ii), will be

$$|\Lambda_1''\rangle_{36} = 4\beta^*\delta^*(\gamma a_1a_2|11\rangle_{36} + \delta a_1c_2|10\rangle_{36} + \alpha c_1a_2|01\rangle_{36} + \beta c_1c_2|00\rangle_{36}). \tag{16}$$

Then in step (iii), to construct the original state $|V\rangle$, the local unitary operation $I_3 \otimes \sigma_6^x$ is performed on qubit pair (3, 6), which transforms the joint state $|\Lambda_1''\rangle_{36}$ into

$$|T''\rangle_{36} = 4\beta^*\delta^*(\alpha c_1a_2|00\rangle_{36} + \beta c_1c_2|01\rangle_{36} + \gamma a_1a_2|10\rangle_{36} + \delta a_1c_2|11\rangle_{36}). \tag{17}$$

Charlie then introduces two auxiliary qubits m and n in the initial state $|00\rangle_{mn}$ and performs the two CNOT operations CNOT_{3m} and CNOT_{6n}, respectively. After this the joint state of qubits 3, 6, m and n will be transformed into

$$\begin{aligned}|K''\rangle_{36mn} &= 4\beta^*\delta^*(\alpha c_1a_2|0000\rangle_{36mn} + \beta c_1c_2|0101\rangle_{36mn} + \gamma a_1a_2|1010\rangle_{36mn}\\ &\quad + \delta a_1c_2|1111\rangle_{36mn})\\ &= \beta^*\delta^*(|R_1\rangle_{36}|H_1''\rangle_{mn} + |R_2\rangle_{36}|H_2''\rangle_{mn} + |R_3\rangle_{36}|H_3''\rangle_{mn}\\ &\quad + |R_4\rangle_{36}|H_4''\rangle_{mn}),\end{aligned} \tag{18}$$

where

$$\begin{aligned}|H_1''\rangle_{mn} &= c_1a_2|00\rangle_{mn} + c_1c_2|01\rangle_{mn} + a_1a_2|10\rangle_{mn} + a_1c_2|11\rangle_{mn},\\ |H_2''\rangle_{mn} &= c_1a_2|00\rangle_{mn} + c_1c_2|01\rangle_{mn} - a_1a_2|10\rangle_{mn} - a_1c_2|11\rangle_{mn},\\ |H_3''\rangle_{mn} &= c_1a_2|00\rangle_{mn} - c_1c_2|01\rangle_{mn} + a_1a_2|10\rangle_{mn} - a_1c_2|11\rangle_{mn},\\ |H_4''\rangle_{mn} &= c_1a_2|00\rangle_{mn} - c_1c_2|01\rangle_{mn} - a_1a_2|10\rangle_{mn} + a_1c_2|11\rangle_{mn}.\end{aligned}$$

Likewise, the state $|V\rangle$ can be constructed via an appropriate unitary operation provided that $|H_i''\rangle_{mn}(i = 1, 2, 3, 4)$ are distinguished. To discriminate the four states $|H_i''\rangle_{mn}(i = 1, 2, 3, 4)$, an optimal POVM measurement [46,56–58] is performed by Charlie on the auxiliary qubits m and n, which takes the following matrix forms

$$S_1 = \frac{1}{x\xi}\begin{pmatrix} \frac{1}{(c_1a_2)^2} & \frac{1}{c_1c_2c_1a_2} & \frac{1}{c_1a_2a_1a_2} & \frac{1}{c_1c_2a_1a_2}\\ \frac{1}{c_1c_2c_1a_2} & \frac{1}{(c_1c_2)^2} & \frac{1}{c_1a_2a_1c_2} & \frac{1}{c_1c_2a_1c_2}\\ \frac{1}{c_1a_2a_1a_2} & \frac{1}{c_1a_2a_1c_2} & \frac{1}{(a_1a_2)^2} & \frac{1}{a_1c_2a_1a_2}\\ \frac{1}{c_1c_2a_1a_2} & \frac{1}{c_1c_2a_1c_2} & \frac{1}{a_1a_2a_1c_2} & \frac{1}{(a_1c_2)^2} \end{pmatrix},$$

$$S_2 = \frac{1}{x\xi}\begin{pmatrix} \frac{1}{(c_1a_2)^2} & \frac{1}{c_1c_2c_1a_2} & \frac{-1}{c_1a_2a_1a_2} & \frac{-1}{c_1c_2a_1a_2}\\ \frac{1}{c_1c_2c_1a_2} & \frac{1}{(c_1c_2)^2} & \frac{-1}{c_1a_2a_1c_2} & \frac{-1}{c_1c_2a_1c_2}\\ \frac{-1}{c_1a_2a_1a_2} & \frac{-1}{c_1a_2a_1c_2} & \frac{1}{(a_1a_2)^2} & \frac{1}{a_1c_2a_1a_2}\\ \frac{-1}{c_1c_2a_1a_2} & \frac{-1}{c_1c_2a_1c_2} & \frac{1}{a_1a_2a_1c_2} & \frac{1}{(a_1c_2)^2} \end{pmatrix},$$

$$S_3 = \frac{1}{x\xi}\begin{pmatrix} \frac{1}{(c_1a_2)^2} & \frac{-1}{c_1c_2c_1a_2} & \frac{1}{c_1a_2a_1a_2} & \frac{-1}{c_1c_2a_1a_2}\\ \frac{-1}{c_1c_2c_1a_2} & \frac{1}{(c_1c_2)^2} & \frac{-1}{c_1a_2a_1c_2} & \frac{1}{c_1c_2a_1c_2}\\ \frac{1}{c_1a_2a_1a_2} & \frac{-1}{c_1a_2a_1c_2} & \frac{1}{(a_1a_2)^2} & \frac{-1}{a_1c_2a_1a_2}\\ \frac{-1}{c_1c_2a_1a_2} & \frac{1}{c_1c_2a_1c_2} & \frac{-1}{a_1a_2a_1c_2} & \frac{1}{(a_1c_2)^2} \end{pmatrix},$$

$$S_4 = \frac{1}{x\xi}\begin{pmatrix} \frac{1}{(c_1a_2)^2} & \frac{-1}{c_1c_2c_1a_2} & \frac{-1}{c_1a_2a_1a_2} & \frac{1}{c_1c_2a_1a_2}\\ \frac{-1}{c_1c_2c_1a_2} & \frac{1}{(c_1c_2)^2} & \frac{1}{c_1a_2a_1c_2} & \frac{-1}{c_1c_2a_1c_2}\\ \frac{1}{c_1a_2a_1a_2} & \frac{-1}{c_1a_2a_1c_2} & \frac{-1}{(a_1a_2)^2} & \frac{1}{a_1c_2a_1a_2}\\ \frac{-1}{c_1c_2a_1a_2} & \frac{1}{c_1c_2a_1c_2} & \frac{1}{a_1a_2a_1c_2} & \frac{-1}{(a_1c_2)^2} \end{pmatrix}$$

and $S_5 = diag(D, C, B, A)$, respectively. Then according to the different POVM result, Charlie can conclude the corresponding state $|H_i''\rangle_{mn}(i = 1, 2, 3, 4)$, respectively. The probability in each case is

$$p = {}_{36mn}\langle K''|S_i|K''\rangle_{36mn} = {}_{mn}\langle H_i''|S_i|H_i''\rangle_{mn}/16 = \frac{1}{x\xi}, \quad (i = 1, 2, 3, 4). \quad (19)$$

After $|H_i''\rangle_{mn}(i = 1, 2, 3, 4)$ being determined, it means that the state $|R_i\rangle_{36}(i = 1, 2, 3, 4)$ is obtained. Then with the same analysis method proposed above, Charlie constructs the state to be prepared in his place. It is also possible for Charlie to get S_5's value, which happens with the probability $1 - \frac{4}{x\xi}$. In this case, Charlie cannot infer which state the qubits m and n are in. Consequently, the state of qubit pair (3, 6) cannot be determined, too, and it will end up with the RSP scheme's failure. So the total success probability, in this case, is also $4 \times p$.

If Alice's measurement result is $|\lambda_2\rangle_{14}$, then the joint state of the four qubits 2, 3, 5 and 6 will be

$$\begin{aligned} |\Gamma_2''\rangle_{2356} = {} & \alpha^* a_1 a_2|0101\rangle_{2356} + \alpha^* a_1 b_2|0110\rangle_{2356} + \alpha^* b_1 a_2|1001\rangle_{2356} \\ & + \alpha^* b_1 b_2|1010\rangle_{2356} + \beta^* a_1 c_2|0100\rangle_{2356} + \beta^* b_1 c_2|1000\rangle_{2356} \\ & - \gamma^* c_1 a_2|0001\rangle_{2356} - \gamma^* c_1 b_2|0010\rangle_{2356} - \delta^* c_1 c_2|0000\rangle_{2356}. \end{aligned} \quad (20)$$

Under Bob's help proposed in step (ii), the qubit pair (3, 6) will be converted into

$$\begin{aligned} |\Lambda_2''\rangle_{36} &= 4\beta^*\delta^*(\gamma a_1 a_2|11\rangle_{36} + \delta a_1 c_2|10\rangle_{36} - \alpha c_1 a_2|01\rangle_{36} - \beta c_1 c_2|00\rangle_{36}) \\ &= 4\beta^*\delta^* \sigma_3^z \otimes \sigma_6^x |T''\rangle_{36}. \end{aligned} \quad (21)$$

Compared with the Eqs. 16 and 17, it is direct to know that applying the similar analysis method proposed just above, the state $|V\rangle$ can be retrieved with the same probability $4 \times p$ in Charlie's place except for replacing $I_3 \otimes \sigma_6^x$ by $\sigma_3^z \otimes \sigma_6^x$ before introducing the two auxiliary qubits in step (iii).

If Alice measures $|\lambda_4\rangle_{14}$, then the four qubits 2, 3, 5 and 6 are left in

$$\begin{aligned} |\Gamma_4''\rangle_{2356} = {} & \beta a_1 a_2|0101\rangle_{2356} + \beta a_1 b_2|0110\rangle_{2356} + \beta b_1 a_2|1001\rangle_{2356} \\ & + \beta b_1 b_2|1010\rangle_{2356} - \alpha a_1 c_2|0100\rangle_{2356} - \alpha b_1 c_2|1000\rangle_{2356} \\ & - \delta c_1 a_2|0001\rangle_{2356} - \delta c_1 b_2|0010\rangle_{2356} + \gamma c_1 c_2|0000\rangle_{2356}. \end{aligned} \quad (22)$$

After the step (ii), the joint state of Charlie's qubits 3 and 6 collapses to

$$\begin{aligned} |\Lambda_4''\rangle_{36} &= \beta a_1 a_2|11\rangle_{36} - \alpha a_1 c_2|10\rangle_{36} - \delta c_1 a_2|01\rangle_{36} + \gamma c_1 c_2|00\rangle_{36} \\ &= \sigma_3^x \sigma_3^z \otimes \sigma_6^z |T\rangle_{36}. \end{aligned} \quad (23)$$

Similarly, it can also be direct to see that with the similar analysis method proposed before, the preparation can be realized with the probability $4 \times p$ in Charlie's place. Therefore, the success probability of the RSP protocol in *Ensemble IV* is $16 \times p$.

So far I have depicted the cases that the state to be prepared belongs to *Ensemble I* and *Ensemble IV*, respectively. While the prepared state belongs to other ensembles of states, applying the similar analysis method, the remote preparation can also be realized in a probabilistic manner in the latter three cases. All possible cases are summarized in Table 1 and here I do not depict them anymore. In short, in general, the

Table 1 ES shows which ensemble the state to be prepared belongs to, AS denotes the state to be prepared is a general two-qubit state, AMR stands for Alice's measurement result, U denotes the unitary operation performed on qubit pair (3, 6) after step (ii), $P_i(i = 1, 2, 3, 4, 5) \to U'$ signifies the elements of POVM and the other corresponding local unitary operation Charlie needs to perform after his POVM operation in step (iii)

ES	AMR	U	$P_i(i = 1, 2, 3, 4, 5) \to U'$
AS	$\|\lambda_3\rangle_{14}$	$\sigma_3^x \otimes \sigma_6^z$	$Q_1 \to I_3 \otimes I_6, Q_2 \to \sigma_3^z \otimes I_6, Q_3 \to I_3 \otimes \sigma_6^z, Q_4 \to \sigma_3^z \otimes \sigma_6^z$
I($\ni$AS)	$\|\lambda_1\rangle_{14}$	$\sigma_3^x \otimes \sigma_6^x$	$W_1 \to I_3 \otimes I_6, W_2 \to \sigma_3^z \otimes I_6, W_3 \to I_3 \otimes \sigma_6^z, W_4 \to \sigma_3^z \otimes \sigma_6^z$
II($\ni$AS)	$\|\lambda_4\rangle_{14}$	$-i\sigma_3^y \otimes \sigma_6^z$	$Q_1 \to I_3 \otimes I_6, Q_2 \to \sigma_3^z \otimes I_6, Q_3 \to I_3 \otimes \sigma_6^z, Q_4 \to \sigma_3^z \otimes \sigma_6^z$
III($\ni$AS, I, II)	$\|\lambda_2\rangle_{14}$	$i\sigma_3^y \otimes \sigma_6^x$	$W_1 \to I_3 \otimes I_6, W_2 \to \sigma_3^z \otimes I_6, W_3 \to I_3 \otimes \sigma_6^z, W_4 \to \sigma_3^z \otimes \sigma_6^z$
IV($\ni$AS, II)	$\|\lambda_1\rangle_{14}$	$I_3 \otimes \sigma_6^x$	$S_1 \to I_3 \otimes I_6, S_2 \to \sigma_3^z \otimes I_6, S_3 \to I_3 \otimes \sigma_6^z, S_4 \to \sigma_3^z \otimes \sigma_6^z$
	$\|\lambda_2\rangle_{14}$	$\sigma_3^z \otimes \sigma_6^x$	$S_1 \to I_3 \otimes I_6, S_2 \to \sigma_3^z \otimes I_6, S_3 \to I_3 \otimes \sigma_6^z, S_4 \to \sigma_3^z \otimes \sigma_6^z$
V($\ni$AS, II)	$\|\lambda_1\rangle_{14}$	$\sigma_3^x \otimes I_6$	$Q_1 \to I_3 \otimes I_6, Q_2 \to \sigma_3^z \otimes I_6, Q_3 \to I_3 \otimes \sigma_6^z, Q_4 \to \sigma_3^z \otimes \sigma_6^z$
	$\|\lambda_2\rangle_{14}$	$\sigma_3^x \otimes \sigma_6^z$	$Q_1 \to I_3 \otimes I_6, Q_2 \to \sigma_3^z \otimes I_6, Q_3 \to I_3 \otimes \sigma_6^z, Q_4 \to \sigma_3^z \otimes \sigma_6^z$

See text for more details
$Q_5 = diag(A, B, C, D)$; $W_5 = diag(B, A, D, C)$; $S_5 = diag(D, C, B, A)$

probabilistic RSP can be fulfilled via the two ministrants' collaboration. The success probability in the RSP scheme is $4 \times p = \frac{4}{x} \times \frac{a_1^2 a_2^2 c_1^2 c_2^2}{(a_1^2+c_1^2)(a_2^2+c_2^2)}$. If the state to be prepared is chosen from some special ensembles, the success probability can be enhanced to $8 \times p$ (*Ensemble I–II*) or even to $16 \times p$ (*Ensemble III–V*) after consuming some extra classical bits, respectively. Under the condition of $|a_i| = |b_i| = |c_i| = \frac{1}{\sqrt{3}}(i = 1, 2, 3)$ and $x = 1$, i.e., the quantum channel consists of two W states and the so-called POVM becomes the usual PM, the total success probability will be $\frac{1}{9}$. For the special ensembles of two-qubit states, it can come up to $\frac{2}{9}$ (*Ensemble I–II*) or $\frac{4}{9}$ (*Ensemble III–V*), respectively.

On the other hand, as we known, the classical message plays an important role in RSP processes [2–4]. In this scheme, two kinds of classical information transmitted processes are involved. One is to transmit the two-qubit joint measurement result performed by the sender Alice, another is to transfer the results of the two single-qubit measurements. In this section, it is interested to know the amount of the classical communication required in the whole RSP process, and my discussion is taken in the case that the original state is restored at Charlie's side.

As to the first classical communication process in this protocol, based on the Eq. 3, it can be noticed that after Alice's two-qubit projective measurement, she can obtain the result $|\lambda_3\rangle_{2356}$ with the measurement probabilities $p_2 = |\beta|^2(1 - |c_1|^2)(1 - |c_2|^2) + |\alpha|^2|c_2|^2(1 - |c_1|^2) + |\delta|^2|c_1|^2(1 - |c_2|^2) + |\gamma|^2|c_1|^2|c_2|^2$. If Alice's measurement result is $|\lambda_1\rangle_{14}$, $|\lambda_2\rangle_{14}$ or $|\lambda_4\rangle_{14}$, the RSP scheme fails due to Charlie's unawareness of the four coefficients α, β, γ and δ. In this way, Alice needs not to send any classical bits to Charlie in the above three cases. So the amount of the classical information needed in this transmitted process is $S_1 = -p_2 \log_2[p_2]$ bits.

With regard to the second classical communication process, it has already been proposed that when Alice's measurement result is $|\lambda_1\rangle_{2356}$, $|\lambda_2\rangle_{2356}$ or $|\lambda_4\rangle_{2356}$, the

Table 2 The relationship among the state to be prepared, the probabilities of Alice and Bob's measurement results and the amount of required classical bits

ES	AMR	P	P′	S
AS	$\|\lambda_3\rangle_{14}$	p_2	p'_2	$-p_2\log_2[p_2]-p_2p'_2\log_2[p'_2]$
I(∍AS)	$\|\lambda_1\rangle_{14}, \|\lambda_3\rangle_{14}$	p_1+p_2	$p'_1+p'_2$	$1-\sum_{k=1}^{2}(p_k\log_2[p_k]+p_kp'_k\log_2[p'_k])$
II(∍AS)	$\|\lambda_4\rangle_{14}, \|\lambda_3\rangle_{14}$	$2p_2$	$2p'_2$	$1-2\times(p_2\log_2[p_2]+p_2p'_2\log_2[p'_2])$
III(∍AS, I, II)	$\|\lambda_i\rangle_{14}(i=1,2,3,4)$	1	$2p'_1+2p'_2$	$2-2\times\sum_{k=1}^{2}(p_k\log_2[p_k]+p_kp'_k\log_2[p'_k])$
IV(∍AS, II)	$\|\lambda_i\rangle_{14}(i=1,2,3,4)$	1	4ν	4-$\nu\log_2[\nu]$
V(∍AS, II)	$\|\lambda_i\rangle_{14}(i=1,2,3,4)$	1	4ν	4-$\nu\log_2[\nu]$

P and P′ indicate the respective probability of Alice and Bob's measurement results corresponding to the successful RSP case. S denotes the total amount of classical bits required in the whole RSP process including ones used to show which ensemble the state to be prepared belongs to. See text for more details
$p_1=|\alpha|^2(1-|c_1|^2)(1-|c_2|^2)+|\beta|^2|c_2|^2(1-|c_1|^2)+|\gamma|^2|c_1|^2(1-|c_2|^2)+|\delta|^2|c_1|^2|c_2|^2$
$p'_1=(|\alpha|^2|a_1a_2|^2+|\beta|^2|a_1c_2|^2+|\gamma|^2|c_1a_2|^2+|\delta|^2|c_1c_2|^2)/p_1$; $\nu=\frac{1}{4}\times(1-|b_1|^2)(1-|b_2|^2)$

scheme fails. So in these cases, Bob needs not to implement the two single-qubit measurements. Further, after Bob's single-qubit measurements, there are four possible measurement results, the states $|00\rangle_{25}, |01\rangle_{25}, |10\rangle_{25}$ and $|11\rangle_{25}$. As depicted above, if Bob gets $|01\rangle_{25}, |10\rangle_{25}$ or $|11\rangle_{25}$, the RSP scheme fails, too. So it is also unnecessary for Bob to send any classical bits to Charlie in these three cases. Only when Bob measures $|00\rangle_{25}$, the RSP scheme may be realized in a probabilistic manner. The probability for Bob's measurement result $|00\rangle_{25}$ varies with Alice's bipartite joint measurement results $|\lambda_3\rangle_{2356}$ as

$$p'_2=(|\beta|^2|a_1a_2|^2+|\alpha|^2|a_1c_2|^2+|\delta|^2|c_1a_2|^2+|\gamma|^2|c_1c_2|^2)/p_2. \tag{24}$$

So the amount of the classical information required in the second transmitted process is $S_2=-p_2\times p'_2\log_2[p'_2]$ bits.

Therefore, the total classical communication cost required in this RSP scheme is

$$S=S_1+S_2=-p_2\log_2[p_2]-p_2\times p'_2\log_2[p'_2]\quad \text{bits.} \tag{25}$$

As proposed above, if the coefficients of the state to be prepared are some special values, the RSP protocol can be realized with higher probability. However,in the cases, it will consume some more classical bits. For the sake of clarity during applications, the relationships among the state to be prepared and the probabilities of Alice and Bob's measurement results as well as the amount of required classical bits are concisely summarized in Table 2.

Obviously, the classical communication cost is not only dependent on the parameters of the state taken as the quantum channel, but also related to the original state $|V\rangle$. For the special states chosen from *Ensemble IV–V*, if $|a_i|=|b_i|=\frac{1}{\sqrt{2}}, |c_i|=\frac{1}{2}$, then the success probability (SP) can be enhanced to $\frac{4}{9}$, and the total amount of classical communication cost (CCC) will equal 4.25 bits.

3 Discussion and summary

This scheme is along the line of An's idea [50] by virtue of controlled RSP (CRSP) instead of JRSP, and a proper POVM instead of the usual PM, too. Moreover, this scheme not only considered the general case, but also other five special ensembles of two-qubit state. Comparing with the previous RSP schemes [27,35,41,43,49,50, 53,54], the present one has the following features. First, this protocol has not only investigated the probabilistic RSP of an *arbitrary* two-qubit state, but also explored its applications to five special ensembles of states, together with the SP and the total CCC in all these cases, while the previous RSP schemes considered only one case, and no SP or CCC have been evaluated [27,35,41,43,50]. Second, the quantum channels are different in forms. In this scheme, I exploit two W-type states as the shared quantum channel, which are robust against decoherence, and so their use is a good choice for quantum information processing, whereas the quantum channels in the previous schemes were composed of Einstein–Podolsky–Rosen (EPR) pairs, GHZ states, or their modified versions [27,35,41,43,49,53,54]. Third, to realize the probabilistic RSP of a general two-qubit state in this protocol, I employ the method of POVM, instead of the usual PM used in the previous RSP schemes [27,35,43,49,50,53,54], to finally restore the state to be prepared.

As for feasibility, it is known for remote preparing a quantum state, the quantum source has to be an entangled quantum system so that the transmission of quantum information can be completed based on entanglement swapping. In this protocol, to realize the remote two-qubit preparation, two W-type states are taken as the quantum channel. To my best knowledge, nowadays there are various theoretical and experimental schemes for generating the W state [64–68]. Therefore, I believe that this RSP protocol with three-qubit entanglements may be realized in the realm of current experimental technology, and it may be helpful to better understanding the possible potential application of W and W-type states.

To summarize, utilizing a proper POVM instead of the usual PM, I have explicitly presented a new scheme for probabilistic remote preparation of an arbitrary two-qubit state in either distant ministrant's place. The quantum channel employed in this scheme is composed of two W-type states. By the two ministrants' collaboration, it is shown that the RSP protocol can be realized in a probabilistic manner via incorporating two auxiliary qubits and executing appropriate unitary operations. Furthermore, I have also explored its applications to five special ensembles of two-qubit states while only one ensemble of two-qubit states with W-type pairs has been discussed in [50]. It means that the result in [50] is only a special case of this scheme. Besides, the CCC calculated in this scheme not only involves the transmitted communication from a sender to the receiver, but also contains one between two ministrants. Thus, from the point of view of communication cost, this scheme may be useful not only in understanding the essence of the classical communication in RSP process, but also expanding the applied field of classical information science.

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