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A note on entanglement swapping of atomic states through the photonic Faraday rotation

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Abstract We propose two schemes of entanglement swapping of atomic states confined in cavities QED useful for quantum computation and quantum communication via a photonic Faraday rotation. They employ a source of linearly-polarized photon, a single photodetector, a quarter-wave plate, and three or four cavities, respectively.

Keywords Entanglement swapping · Photonic Faraday rotation · QED cavity

1 Introduction

In quantum computation and quantum communication [1] the entanglement of states constitutes a fundamental resource for many protocols, such as quantum teleportation [2], quantum dense coding [1], and distributed quantum computation [3]. In this context, entanglement swapping plays an important role in several protocols of quantum information transfer; in particular, it is arguably one of the most important ingredients for quantum repeaters and quantum relays [4–6], as well as in teleportation of entangled states. For the entanglement swapping protocol, two pairs of particles are usually employed with each pair previously entangled, and a Bell-state measurement made upon a particle of each pair leads the remaining particles to an entangled state, even if they have never interacted previously with each other.

Due to the importance of entanglement swapping various experimental results have been presented recently [7–14]. In Ref. [7] two pairs of polarized entangled photons are employed and, by making a Bell-state measurement upon the photon of each pair,

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they displayed an entanglement of freely propagating particles that have no previous interaction any other means. Entanglement swapping operations were reported in [8] via nuclear magnetic resonance quantum-information processing, over long distances in optical fibers [10], and unconditionally for continuous variables [9]. The entanglement swapping for continuous-variable has been used to obtain quantum teleportation beyond the no-cloning limit [11]. Also, multistage entanglement swapping in photonic system [12], an ion-trap quantum processor through entanglement swapping [13], and the first experimental demonstration of the GHZ entanglement swapping [14] have been reported.

A lot of theoretical schemes have also appeared in the recent literature [15–22] generalizing the standard entanglement swapping for: multiparticle systems [15]; multiqudit systems [16], where the authors have extended the scheme originally proposed for two pairs of qubits to an arbitrary number of systems composed by an arbitrary number of qudits; d-level systems in a generalized cat state [17], useful for protocol of secret sharing. Concerning with secret sharing in cavity QED [20], multiparty secret sharing of quantum information [18] and classical information [21], secure multiparty quantum communication by Bell states [19] and by entangled qutrits [22] based on entanglement swapping have also been proposed.

In addition to the list of entanglement swapping applications, the purification of entangled states by local actions, using a variant of entanglement swapping, was studied in Refs. [23–25]. This issue was extended for continuous variables in [26]. The quantum key distribution schemes [27] and teleportation of a two-particle entangled state [28,29] employing entanglement swapping have been reported, as well as entanglement swapping without joint measurement [30]. In the cavity-QED context, a scheme based on two atoms and two cavities initially prepared in two pairs of atom-photon nonmaximally entangled states, was considered in [31] to create maximally entangled photon–photon and atom-photon states via entanglement swapping, with atomic states in either a three-level cascade or lambda configuration in [32], with resonant interaction of a two-mode cavity with a λ-type three-level atom involving only a single measurement in [33] and, in [34], an alternative scheme to implement the entanglement swapping. More recently, an entanglement swapping using the two-photon Jaynes-Cummings model was proposed in [35].

Here, as employed in Ref. [36] for quantum information processing, we take advantage of the quantum regime of strong interactions between single atoms and photons present in a microtoroidal resonator [37] to propose an entanglement swapping of states of atoms confined in distant low-Q cavities using photonic Faraday rotations. The main idea is to make use of the Faraday rotation produced by single-photon-pulse input and output process with regard to low-Q cavities [38]. In view of our applications, we revisited the input–output relation for a cavity coherently interacting with a trapped three-level atom, recently considered in Ref. [36,39]. We consider a three-level atom interacting with two degenerate cavity-modes of a low-Q cavity pumped by photonic emission of a single photon source via optical fibers. Figure 1 shows the atomic levels of each atom trapped inside one of the cavities. Each transition is governed by the Jaynes-Cummings model.

2 Theoretical model

The Hamiltonian that describes the system of a three-level atom (Fig. 1) interacting with two degenerate cavity-modes of a low-Q cavity is given by [40,41]

$$H = H_0 + \hbar\lambda \sum_{j=\mathrm{L,R}} (a_j^\dagger \sigma_{j-} + a_j \sigma_{j+}) + H_\mathrm{R}, \tag{1}$$

with

$$H_0 = \sum_{j=\mathrm{L,R}} \left[\frac{\hbar\omega_0}{2} \sigma_{jz} + \hbar\omega_c a_j^\dagger a_j \right], \tag{2}$$

and

$$H_\mathrm{R} = H_{\mathrm{R}0} + i\hbar \left[\int_{-\infty}^{\infty} d\omega \sum_{j=\mathrm{L,R}} \alpha(\omega) \left(b_j^\dagger(\omega) a_j + b_j(\omega) a_j^\dagger \right) \right.$$
$$\left. + \int_{-\infty}^{\infty} d\omega \sum_{j=\mathrm{L,R}} \overline{\alpha}(\omega) \left(c_j^\dagger(\omega)\sigma_{j-} + c_j(\omega)\sigma_{j+} \right) \right], \tag{3}$$

where λ is the atom–field coupling constant, $a_j^\dagger(a_j)$ is the creation (annihilation) operator of the field-mode into the cavity with $j = \mathrm{L, R}$, $\omega_0(\omega_c)$ is the atomic (field) frequency, and $\sigma_{\mathrm{L}-}$ and $\sigma_{\mathrm{L}+}(\sigma_{\mathrm{R}-}$ and $\sigma_{\mathrm{R}+})$ are the lowering and raising operators of the transition L (R), respectively. The L and R transitions are shown in Fig. 1. $H_{\mathrm{R}0}$ stands for the Hamiltonian of the free reservoirs, where the field and atomic reservoirs are given by $H_{\mathrm{R}c} = \hbar \int_{-\infty}^{\infty} d\omega\omega b_j^\dagger b_j$ and $H_{\mathrm{R}A} = \hbar \int_{-\infty}^{\infty} d\omega\omega c_j^\dagger c_j$ $(j = \mathrm{L, R})$, respectively. The reservoirs couple with field and atomic systems independently, at different values of frequency ω, with coupling amplitudes $\alpha = \sqrt{\kappa/2\pi}$ and $\overline{\alpha} = \sqrt{\gamma/2\pi}$, respectively. κ and γ are the cavity-field and atomic damping rates, b_j and c_j ($b_j^\dagger$ and $c_j^\dagger$) are the annihilation (creation) operators of the reservoirs.

Next, due to the presence of a pumping field into the cavity by an optical fiber one can conveniently change to a rotating frame with respect to the pumping field frequency ω_p using the following transformation:

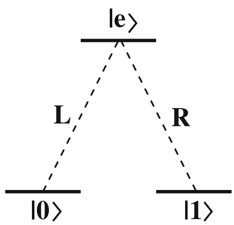


Fig. 1 Atomic configuration of the three-level atom trapped in the low-Q cavities

$$H_{eff} = U^{\dagger} H U - \sum_{j=\mathrm{L,R}} \left[\hbar\omega_p a_j^{\dagger} a_j + \frac{\hbar\omega_p}{2}\sigma_{jz} \right], \tag{4}$$

where $U = \exp\{-i \sum_{j=\mathrm{L,R}}[\omega_p(a_j^{\dagger}a_j + b_j^{\dagger}b_j + c_j^{\dagger}c_j) + \frac{\omega_p}{2}\sigma_{jz}]t\}$. At this point, using the Heisenberg equations for the operators a_j and σ_{j-} (consequently for $a_j^{\dagger}$ and σ_{j+}), with $j = \mathrm{L, R}$, we get

$$\dot{a}_j(t) = -\left[i(\omega_c - \omega_{\mathrm{p}}) + \frac{\kappa}{2}\right] a_j(t) - g\sigma_{j-}(t) - \sqrt{\kappa}a_{\mathrm{in},j}(t), \tag{5}$$

$$\dot{\sigma}_{j-}(t) = -\left[i(\omega_0 - \omega_{\mathrm{p}}) + \frac{\gamma}{2}\right] \sigma_{j-}(t) - g\sigma_{jz}(t)a_j(t) + \sqrt{\gamma}\sigma_z(t)b_{\mathrm{in,j}}(t). \tag{6}$$

The relation between the input and output fields reads $a_{\mathrm{out},j}(t) = a_{in,j}(t) + \sqrt{\kappa}a_j(t)$, $j = \mathrm{L, R}$. Here we assume the reservoirs at zero temperature, in a way that $b_{in,j} \simeq 0$. Now, the use of adiabatic approximation for the above evolution equations allows us to obtain a single relation between the input and output field states in the form [36,39]

$$r(\omega_{\mathrm{p}}) = \frac{[i(\omega_c - \omega_{\mathrm{p}}) - \frac{\kappa}{2}][i(\omega_0 - \omega_{\mathrm{p}}) + \frac{\gamma}{2}] + g^2}{[i(\omega_c - \omega_{\mathrm{p}}) + \frac{\kappa}{2}][i(\omega_0 - \omega_{\mathrm{p}}) + \frac{\gamma}{2}] + g^2}, \tag{7}$$

where $r(\omega_{\mathrm{p}}) \equiv a_{\mathrm{out},j}(t)/a_{\mathrm{in},j}(t)$ is the reflection coefficient of the photon system. On the other hand, considering the case $g = 0$ and an empty cavity we have [40,41]

$$r_0(\omega_{\mathrm{p}}) = \frac{i(\omega_c - \omega_{\mathrm{p}}) - \frac{\kappa}{2}}{i(\omega_c - \omega_{\mathrm{p}}) + \frac{\kappa}{2}}. \tag{8}$$

We can see in Eq. 8 that $r_0(\omega)$ can be written as a pure phase shift, $r_0(\omega) = e^{i\phi_0}$, the same being not valid for $r(\omega)$ in Eq. 7, where $|r(\omega)| \neq 1$. However, take advantage of the words of Ref. [39]: "the photon experiences a very weak absorption, and thereby we may approximately consider that the output photon only experiences a pure phase shift without any absorption". Indeed, due to the strong κ and weak γ and g, the absolute values of the coefficients above are close to unity. So, the form of a pure phase shift $r(\omega) = e^{i\phi}$ is a good approximation.

According to [36,39] the transitions $|e\rangle \leftrightarrow |0\rangle$ and $|e\rangle \leftrightarrow |1\rangle$ are due to the coupling between two degenerate cavity modes a_{L} and a_{R} with left (L) and right (R) circular polarization, respectively. For the atom initially prepared in $|0\rangle$, the transition $|0\rangle \rightarrow |e\rangle$ will occur only if the L circularly polarized single-photon pulse $|\mathrm{L}\rangle$ enters the cavity. Hence Eq. 7 leads the input pulse to the output one as $|\Psi_{\mathrm{out}}\rangle_{\mathrm{L}} = r(\omega_{\mathrm{p}})|\mathrm{L}\rangle \approx e^{i\phi}|\mathrm{L}\rangle$ with ϕ the corresponding phase shift being determined by the parameter values. Note that an input R circularly polarized single-photon pulse $|\mathrm{R}\rangle$ would only sense the empty cavity; as a consequence the corresponding output governed by Eq. 8 is $|\Psi_{\mathrm{out}}\rangle_{\mathrm{R}} = r_0(\omega_{\mathrm{p}})|\mathrm{R}\rangle = e^{i\phi_0}|\mathrm{R}\rangle$ with ϕ_0 a phase shift different from ϕ. Therefore, for an input linearly polarized photon pulse $|\Psi_{\mathrm{in}}\rangle = \frac{1}{\sqrt{2}}(|\mathrm{L}\rangle + |\mathrm{R}\rangle)$, the output pulse is

$$|\Psi_{\text{out}}\rangle_- = \frac{1}{\sqrt{2}}(e^{i\phi}|\text{L}\rangle + e^{i\phi_0}|\text{R}\rangle). \tag{9}$$

This also implies that the polarization direction of the reflected photon rotates an angle $\Theta_F^- = (\phi_0 - \phi)/2$ with respect to that of the input one, called Faraday rotation [38]. If the atom is initially prepared in the state $|1\rangle$, then only the R circularly polarized photon could sense the atom, whereas the L circularly polarized photon only interacts with the empty cavity. So we have,

$$|\Psi_{\text{out}}\rangle_+ = \frac{1}{\sqrt{2}}(e^{i\phi_0}|\text{L}\rangle + e^{i\phi}|\text{R}\rangle), \tag{10}$$

where the Faraday rotation is $\Theta_F^+ = (\phi - \phi_0)/2$.

3 Entanglement swapping protocols

Case # 1—Firstly, we assume the previously entangled state of the atoms confined in the cavities A and B, given by

$$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|01\rangle_{AB} + |10\rangle_{AB}). \tag{11}$$

The preparation of this state can be done using an additional polarized photon carried by optical fibers that interacts with these atoms one after the other, as described in Ref. [36]. In another spatial position, an entanglement has been previously prepared: it describes an atom and a photon, both confined in the cavity C, the photon being named 1 and is in a Faraday rotated state. This entanglement is written in the form

$$|\psi_{C1}\rangle = \frac{1}{\sqrt{2}}(|0\rangle_C|\eta_1\rangle_- + |1\rangle_C|\eta_1\rangle_+), \tag{12}$$

where $|\eta_1\rangle_- = (e^{i\phi}|L\rangle_1 + e^{i\phi_0}|\text{R}\rangle_1)/\sqrt{2}$ and $|\eta_1\rangle_+ = (e^{i\phi_0}|L\rangle_1 + e^{i\phi}|\text{R}\rangle_1)/\sqrt{2}$. The state in (12) is constructed via the interaction of the polarized photon (in the state $|\psi\rangle_1 = (|L\rangle_1 + |\text{R}\rangle_1)/\sqrt{2})$ with a three-level atom (previously prepared in $|\psi\rangle_C = (|0\rangle_C + |1\rangle_C)/\sqrt{2}$). Figure 2 shows the entire procedure of the case 1.

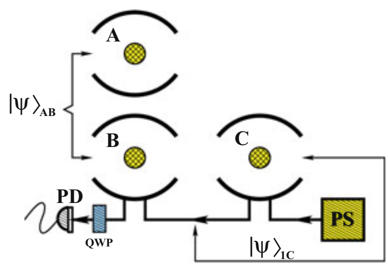


Fig. 2 (Color online) Schematic diagram of the entanglement swapping procedure for the case 1 using three three-level atoms trapped in cavities A, B, and C, a single photon source (PS), a quarter-wave plate (QWP), and a photodetector of polarization (PD)

Table 1 Atomic rotations completing the entanglement swapping procedure for the case 1

MAPS	RES	AO
$\|L0\rangle_{1B}$	$\|00\rangle_{AC} + \|11\rangle_{AC}$	σ_x
$\|L1\rangle_{1B}$	$\|00\rangle_{AC} - \|11\rangle_{AC}$	$i\sigma_y$
$\|R0\rangle_{1B}$	$\|01\rangle_{AC} - \|10\rangle_{AC}$	σ_z
$\|R1\rangle_{1B}$	$\|01\rangle_{AC} + \|10\rangle_{AC}$	$\mathbb{I}$

The first column represents the measurement in the atom B and the photon states (MAPS), second column is the result of entanglement swapping (RES), and the third is the atomic operation (AO) considering the atom A (local) represented by Pauli operators with $\mathbb{I}$ being the identity operator

The entanglement swapping is realized through a Bell-state measurement on the system composed by the photon and the atom confined in the cavity B. To this end, the photon is sent through the cavity B to interact with the atom, leading the whole atom-photon system to the state

$$|\psi'\rangle = \frac{1}{2\sqrt{2}}\Big[|00\rangle_{AC}(e^{i(\phi+\phi_0)}|\mathrm{L}\rangle_1 + e^{i(\phi+\phi_0)}|\mathrm{R}\rangle_1)|1\rangle_B \\ +|01\rangle_{AC}(e^{2i\phi_0}|\mathrm{L}\rangle_1 + e^{2i\phi}|\mathrm{R}\rangle_1)|1\rangle_B \\ +|10\rangle_{AC}(e^{2i\phi}|\mathrm{L}\rangle_1 + e^{2i\phi_0}|\mathrm{R}\rangle_1)|0\rangle_B \\ + |11\rangle_{AC}(e^{i(\phi+\phi_0)}|\mathrm{L}\rangle_1 + e^{i(\phi+\phi_0)}|\mathrm{R}\rangle_1)|0\rangle_B\Big]. \tag{13}$$

Next, assuming $\phi = \pi$ and $\phi_0 = \pi/2$, plus the application of a Hadamard operation upon the state of the atom B and the photon via an external laser beam and a quarter-wave plate (QWP), respectively, the state of the entire system evolves to

$$|\psi''\rangle = \frac{1}{2\sqrt{2}}[-i|\mathrm{L0}\rangle_{1B}(|00\rangle_{AC} + |11\rangle_{AC}) + i|\mathrm{L1}\rangle_{1B}(|00\rangle_{AC} - |11\rangle_{AC}) \\ - |\mathrm{R0}\rangle_{1B}(|01\rangle_{AC} - |10\rangle_{AC}) + |\mathrm{R1}\rangle_{1B}(|01\rangle_{AC} + |10\rangle_{AC})]. \tag{14}$$

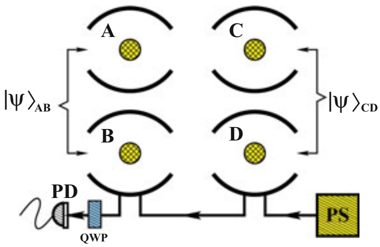
Finally, appropriate detections of the state of the atom B and the polarized photon state conclude the entanglement swapping. Table 1 summarizes the atomic rotations to complete the entanglement swapping.

Case # 2—Here, we start by considering two pairs of atoms, trapped inside the cavities A, B, C, and D, and previously entangled in the following states

$$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|01\rangle_{AB} + |10\rangle_{AB}), \\ |\psi\rangle_{CD} = \frac{1}{\sqrt{2}}(|01\rangle_{CD} + |10\rangle_{CD}). \tag{15}$$

The scheme for entanglement swapping is summarized in Fig. 3.

Fig. 3 (Color online) Schematic diagram of the entanglement swapping procedure for de case 2, using the same notation of Fig. 2 plus an additional atom confined in cavity D



Firstly, an auxiliary photon is sent to interact with the atom confined in the cavity D, leading the state of the entire atom-photon system to the form

$$|\varphi\rangle = \frac{1}{2}(|01\rangle_{AB} + |10\rangle_{AB}) \otimes (|01\rangle_{CD}|\eta_1\rangle_+ + |10\rangle_{CD}|\eta_1\rangle_-). \tag{16}$$

Next, considering a Hadamard operation upon the atom D, we have

$$\begin{aligned}|\varphi'\rangle = \frac{1}{4}\Big[&e^{i\phi_0}|\mathrm{L}010\rangle_{ABC}(|0\rangle_D - |1\rangle_D) + e^{i\phi}|\mathrm{L}011\rangle_{ABC}(|0\rangle_D + |1\rangle_D)\\ &+e^{i\phi}|\mathrm{R}010\rangle_{ABC}(|0\rangle_D - |1\rangle_D) + e^{i\phi_0}|\mathrm{R}011\rangle_{ABC}(|0\rangle_D + |1\rangle_D)\\ &+e^{i\phi_0}|\mathrm{L}100\rangle_{ABC}(|0\rangle_D - |1\rangle_D) + e^{i\phi}|\mathrm{L}101\rangle_{ABC}(|0\rangle_D + |1\rangle_D)\\ &+ e^{i\phi}|\mathrm{R}100\rangle_{ABC}(|0\rangle_D - |1\rangle_D) + e^{i\phi_0}|\mathrm{R}101\rangle_{ABC}(|0\rangle_D + |1\rangle_D)\Big].\end{aligned} \tag{17}$$

In sequence, the photon emerging form the cavity D is sent to interact with the atom in the cavity B and, soon after, a Hadamard operation upon the atom B and upon the photon states, transforms the state of the whole system to the form

$$\begin{aligned}|\varphi''\rangle = \frac{1}{8}\Big\{&[(e^{2i\phi} + e^{2i\phi_0})|\mathrm{L}\rangle_1 - (e^{2i\phi} - e^{2i\phi_0})|\mathrm{R}\rangle_1]|00\rangle_{AC}\\ &\times(|00\rangle_{BD} - |01\rangle_{BD} - |10\rangle_{BD} + |11\rangle_{BD})\\ &+[(e^{2i\phi} + e^{2i\phi_0})|\mathrm{L}\rangle_1 + (e^{2i\phi} - e^{2i\phi_0})|\mathrm{R}\rangle_1]|11\rangle_{AC}\\ &\times(|00\rangle_{BD} + |01\rangle_{BD} + |10\rangle_{BD} + |11\rangle_{BD})\\ &+2e^{i(\phi+\phi_0)}|\mathrm{L}\rangle_1|01\rangle_{AC}(|00\rangle_{BD} + |01\rangle_{BD} - |10\rangle_{BD} - |11\rangle_{BD})\\ &+ 2e^{i(\phi+\phi_0)}|\mathrm{L}\rangle_1|10\rangle_{AC}(|00\rangle_{BD} - |01\rangle_{BD} + |10\rangle_{BD} - |11\rangle_{BD})\Big\},\end{aligned} \tag{18}$$

and with the single choice $\phi = \pi$ and $\phi_0 = \pi/2$, we obtain

$$\begin{aligned}|\varphi'''\rangle = \frac{1}{4}\,[&|\mathrm{R}00\rangle_{BD}(|11\rangle_{AC} - |00\rangle_{AC}) + |\mathrm{R}01\rangle_{BD}(|11\rangle_{AC} + |00\rangle_{AC})\\ &+|\mathrm{R}10\rangle_{BD}(|11\rangle_{AC} + |00\rangle_{AC}) + |\mathrm{R}11\rangle_{BD}(|11\rangle_{AC} - |00\rangle_{AC})\\ &-i|\mathrm{L}00\rangle_{BD}(|01\rangle_{AC} + |10\rangle_{AC}) - i|\mathrm{L}01\rangle_{BD}(|01\rangle_{AC} - |10\rangle_{AC})\\ &+ i|\mathrm{L}10\rangle_{BD}(|01\rangle_{AC} - |10\rangle_{AC}) + i|\mathrm{L}11\rangle_{BD}(|01\rangle_{AC} + |10\rangle_{AC})].\end{aligned} \tag{19}$$

Table 2 Atomic rotations completing the entanglement swapping procedure for the case 2

MAPS	RES	AO
$\|R00\rangle_{BD}$	$\|11\rangle_{AC} - \|00\rangle_{AC}$	$-i\sigma_y$
$\|R01\rangle_{BD}$	$\|11\rangle_{AC} + \|00\rangle_{AC}$	σ_x
$\|R10\rangle_{BD}$	$\|11\rangle_{AC} + \|00\rangle_{AC}$	σ_x
$\|R11\rangle_{BD}$	$\|11\rangle_{AC} - \|00\rangle_{AC}$	$-i\sigma_y$
$\|L00\rangle_{BD}$	$\|01\rangle_{AC} + \|10\rangle_{AC}$	$\mathbb{I}$
$\|L01\rangle_{BD}$	$\|01\rangle_{AC} - \|10\rangle_{AC}$	σ_z
$\|L10\rangle_{BD}$	$\|01\rangle_{AC} - \|10\rangle_{AC}$	σ_z
$\|L11\rangle_{BD}$	$\|01\rangle_{AC} + \|10\rangle_{AC}$	$\mathbb{I}$

The first column represents the measurement in the atoms BD and the photon states (MAPS); second column is the result of entanglement swapping (RES), and the third is the atomic operation (AO) considering the atom A (local) represented by Pauli operators with $\mathbb{I}$ being the identity operator

So, with a detection of the photon polarization plus separated atomic measurements on the atoms B and D, one concludes the entanglement swapping. Table 2 presents the atomic operation to reconstruct the initial state.

4 Conclusion

In conclusion, we presented two protocols for entanglement swapping of atomic states confined in cavities QED using photonic Faraday rotations. It only involves virtual excitations of the atoms and considers low-Q cavities, ideal photodetectors, and fibers without absorption. On the other hand, the practical experimental imperfections due to photon loss and inefficiency in the detectors turn the protocol as being probabilistic. In this respect, we can estimate the success probability of the scheme take into account the losses mentioned above, based in Ref. [42], e.g., considering the coupling and transmission of the photon through the single-mode optical fiber given by $T_f = 0.2$, the transmission of each photon through the other optical components by $T_o = 0.95$, the fraction of photons with the correct polarization $p_\pi = 0.5$, the quantum efficiency of the single-photon detector as $\eta = 0.15$, $\Delta\Omega/4\pi = 0.02$ as the solid angle of light collection, and a single-photon rate by source given by 75 kHz. So, we estimate the success probability as $P = p_{Bell} \times T_f \times T_o \times p_\pi \times \eta \times \Delta\Omega/4\pi \simeq 7.125 \times 10^{-5}$ (considering $p_{Bell} = 0.25$ as the probability of the ideal Bell-state measurement without necessity of additional rotations), which results in one successful entanglement swapping event every $\simeq$5.34 s. The same estimative is obtained for the preparation of the two atoms entangled state. These schemes can also be modeled in a quantum-dot system, by simply substituting the atoms by excitons [43].

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