



Ranked-choice voting and the spoiler effect: a supplementary note

Nicholas R. Miller¹

Received: 30 June 2023 / Accepted: 16 September 2023 / Published online: 29 September 2023
© The Author(s), under exclusive licence to Springer Science+Business Media, LLC, part of Springer Nature 2023

Abstract

This note supplements the recent work of McCune and Wilson (Public Choice 196(1–2):19–50, 2023) by providing a complete analysis of spoiler effects under both plurality voting and Ranked-Choice Voting in the case of three (potential) candidates. The trick for definitively identifying all spoiler possibilities under both voting rules in the three-candidate case is to partition the set of all three-candidate preference profiles into eight types by cross-classifying the candidates in terms of their plurality status and Condorcet relationships. The resulting typology allows us to identify the winners in all possible two-candidate and three-candidate elections under both voting rules and therefore suffices to identify all spoiler effects. It implies, among other things, that the set of profiles that are vulnerable to spoilers under Ranked-Choice Voting is a proper subset of those vulnerable to spoilers under plurality rule.

Keywords Spoiler effects · Ranked-choice voting · Plurality voting

Spoiler effects imply that who wins an election depends in an unreasonable way on the slate of candidates appearing on the ballot. The vulnerability of voting rules to spoiler effects is distinctly bothersome as it encourages various kinds of electoral mischief. It is well known that the familiar (in the US, UK, and Canada) plurality voting rule frequently entails spoiler effects. Indeed, the susceptibility of US elections to spoilers—most notably, Nader in 2000—is the dominant and recurring complaint in William Poundstone’s book on *Gaming the Vote: Why Elections Aren’t Fair* (2008).

While its most enthusiastic advocates sometimes claim that Ranked-Choice Voting (RCV)—also known as Instant Runoff Voting, among many other names—precludes the spoiler problem, other RCV advocates and most commentators more cautiously claim that, while RCV mitigates spoiler effects relative to plurality voting, it does not entirely eliminate them. This more modest claim has received powerful support in the recent paper by McCune and Wilson (2023), who employ a variety of techniques and data sources to

✉ Nicholas R. Miller
nmiller@umbc.edu

¹ Department of Political Science, University of Maryland Baltimore County (UMBC), Baltimore, MD 21250, USA

compare the frequency of spoiler effects under the two voting rules and find such effects occur considerably less frequently under RCV than plurality.

However, McCune and Wilson's analysis, lucid and comprehensive though it is, does not provide a complete analysis of the simplest case with just three (potential) candidates. The purpose of this brief note is to supplement their paper by setting out the social choice foundations in the three-candidate case.¹ The note is motivated by the principle that a complete understanding of the simplest case is a helpful, if not always necessary, first step toward a more thorough understanding of the general case.

It is in order first to provide an explicit definition of a 'spoiler' candidate within the standard social choice framework in which *voting rules* map voter *preference profiles* into *winners*. My definition aligns with that of McCune and Wilson though, given the scope of this note, it is stated in terms of three candidates only. The definition is equivalent to the expansion and contraction consistency condition in rational choice theory often called 'Sen's condition α ' (see Sen, 1970, p. 17).

A spoiler candidate is most commonly described in terms of the effect of a third candidate X entering what had been a two-candidate 'straight fight' between Y and Z —that is, in terms of 'agenda expansion'. In this context, third candidate X is *not* a spoiler if (i) the same candidate (Y or Z) wins both with X out and with X in, or (ii) X wins when X enters (while Y or Z wins otherwise). However, third candidate X is a *spoiler* if Y (resp. Z) wins with X out of the race but Z (resp. Y) wins with X in. A spoiler candidate can also be described in terms of the effect of candidate X exiting what had been a three-candidate contest—that is, in terms of 'agenda contraction'. Candidate X is not a spoiler if (i) the same candidate (Y or Z) wins both with X in and with X out or (ii) X wins when X is in and another candidate (Y or Z) necessarily wins when X is out. However, candidate X is a spoiler if Z (resp. Y) wins with X in and Y (resp. Z) wins with X out. (Within the formal social choice framework, the entry and exit descriptions are logically equivalent.) Furthermore, it is possible for two candidates to be *mutual spoilers*—that is, X is a spoiler to Y (i.e., Y wins if X stays out but Z wins if X enters) and at the same time Y is a spoiler to X (i.e., X wins if Y stays out but Z wins if Y enters).

The trick for definitively identifying all spoiler possibilities under both voting rules in the three-candidate case is to partition the set of all three-candidate preference profiles into eight *types* by cross-classifying the candidates in terms of their plurality status and Condorcet relationships.² In a three-candidate profile, the candidate with the most first preferences is the *Plurality Winner* (PW), the candidate with the second most first preferences is the *Plurality Runner-Up* (P2), and the candidate with the fewest first preferences is the *Plurality Loser* (PL). Likewise in each profile, if a majority of voters rank candidate X over Y , we say that X *beats* Y , and this defines the *pairwise majority rule* or *Condorcet* relationship between each pair of candidates. A *Condorcet Winner* (CW) beats both other candidates, and a *Condorcet Loser* (CL) is beaten by both other candidates. If X beats Y , Y beats Z , and Z beats X , or if Y beats X , X beats Z , and Z beats Y , a *Condorcet cycle* exists and neither a Condorcet Winner nor a Condorcet Loser exists.

¹ This supplement might be incorporated into Sect. 3 of McCune and Wilson's paper as it elaborates on their Remark 1.

² We follow McCune and Wilson, as well as Graham-Squire and McCune (2023) and many others, by ignoring the possibility of ties—a defensible assumption given a large (electorate-sized) number of voters. Essentially the same definitions and results apply if ties are allowed but all statements become considerably more complicated. Moreover, there is no standard tie-breaking rule for elections. We also assume that there are no ties in individual preference orderings.

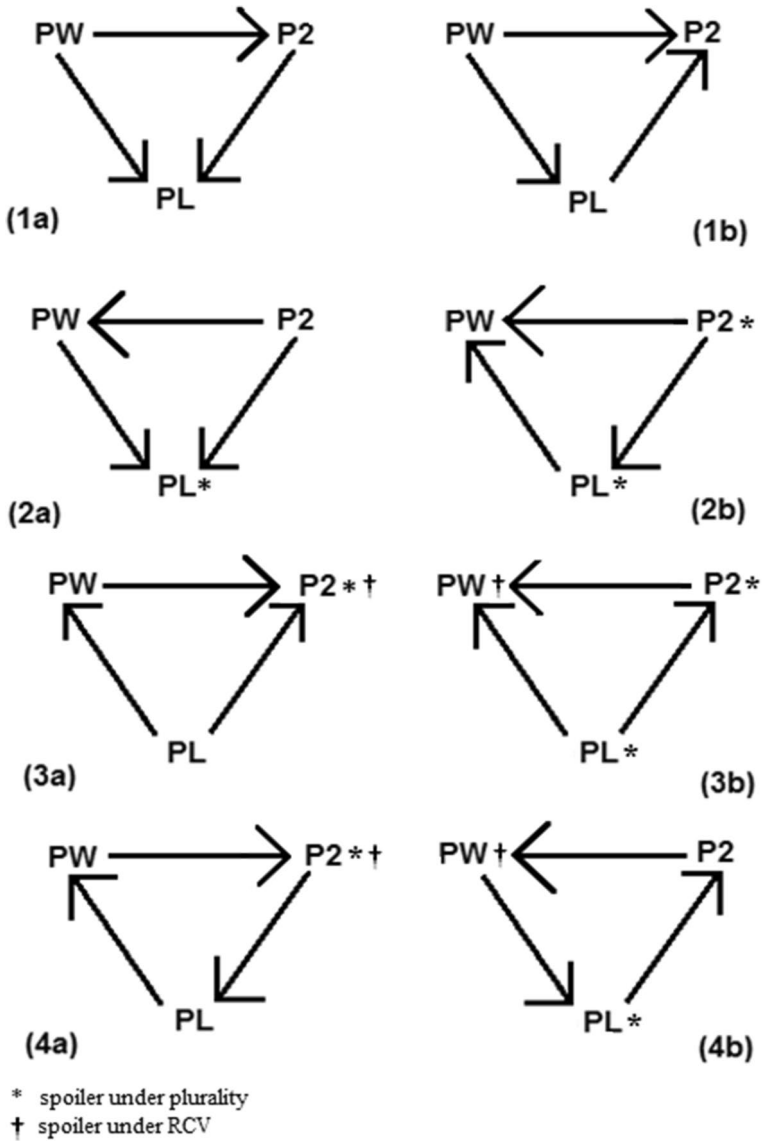


Fig. 1 Typology of all possible three candidate preference profiles

The typology resulting from this cross-classification is shown in Fig. 1, where candidates are labeled by their plurality status and an arrow from X to Y means that X beats Y . There are eight distinct types of profiles: the six possible transitive orderings over PW, P2, and PL, i.e., (1a) through (3b), and the ‘forward’ and ‘backward’ cycles over PW, P2, and PL, i.e., (4a) and (4b). It is straightforward to show that plurality status and Condorcet relationships are completely independent—that is, profiles of all eight types may logically occur (see Miller, 2014). In particular, the Plurality Loser may be a Condorcet Winner (as when a centrist candidate is ‘squeezed’ between left-of-center and right-of-center

candidates), and the Plurality Winner may be a Condorcet Loser (as in McCune and Wilson's Table 4). That all types are logically possible of course does not mean that they are equally likely to occur. The distribution of simulated profiles over types depends on the profile generating rule (e.g., IC versus IAC). And the distribution of profiles from empirical election results depends on the ideological or other structuring of these elections.³

The typology in Fig. 1 allows us to identify winners in all possible two-candidate and three candidate elections under both voting rules and therefore suffices to identify all spoiler effects. Plurality status determines (i) the winner of three-candidate contests under plurality rule and (ii) the elimination order under RCV; Condorcet relationships determine (iii) the winner of every two-candidate contest under both voting rules including (iv) the winner of an 'instant runoff' under RCV. Note that if a *majority winner* (i.e., a candidate supported by an absolute majority of first preference) exists, it is both a Plurality Winner and a Condorcet Winner, and the profile belongs to type (1a) or (1b); however, such profiles may lack majority winners.

We first look for spoilers under plurality rule. For example, given profiles of type (2a) in Fig. 1, PW cannot be a spoiler because PW wins whenever PW is in the race and P2 cannot be a spoiler because PW wins regardless of whether P2 is in the race or not; however, PL is a spoiler because P2 wins if PL is out while PW wins if PL is in. Spoilers in profiles of other types may be identified in similar manner. In Fig. 1, spoilers under plurality rule are identified as P2* and PL*. Examination of Fig. 1 leads to the following observations.

- (1) Plurality voting is vulnerable to spoiler effects if and only if the Plurality Winner fails to be a Condorcet Winner.
- (2) The Plurality Winner is never a spoiler under plurality voting (since PW by definition wins every three-candidate contest).
- (3) The Plurality Runner-Up is a spoiler under plurality rule if and only if the Plurality Loser beats the Plurality Winner.
- (4) The Plurality Loser is a spoiler under plurality rule if and only if the Plurality Runner-Up beats the Plurality Winner.
- (5) Plurality voting is vulnerable to mutual spoiler effects if and only if the Plurality Winner is also a Condorcet Loser.
- (6) A Condorcet Winner is a spoiler under plurality rule if and only if it is one of two mutual spoilers.
- (7) A Condorcet Loser is a spoiler under plurality rule if and only if, with respect to the plurality status of the other two candidates, the weaker beats the stronger.

We next look for spoilers under RCV. For example, given profiles of type (3a) in Fig. 1, PW cannot be a spoiler because, if PW is in the race, PW wins by beating P2 in the runoff, and PL cannot be a spoiler because PW wins regardless of whether PL is in the race or out; however, P2 is a spoiler because PL wins if P2 is out of the race while PW wins if P2 is in. Spoilers in profiles of other types may be identified in similar manner. In Fig. 1, spoilers under RCV are identified as PW† and P2†). Examination of Fig. 1 leads to the following observations.

- (8) RCV is vulnerable to spoiler effects if and only if (i) the Plurality Loser is a Condorcet Winner or (ii) there is no Condorcet Winner because majority rule is cyclical.

³ However, the cyclical profiles (4a) and (4b) evidently occur infrequently in empirical elections.

- (9) The Plurality Loser is never a spoiler under RCV (since PL by definition never makes the runoff making the runoff equivalent to the two-candidate race without PL's entry).
- (10) The Plurality Winner is a spoiler under RCV if and only if the Plurality Loser beats the Plurality Runner-Up and the Plurality Runner-Up beats the Plurality Winner.
- (11) The Plurality Runner-Up is a spoiler under RCV if and only if the Plurality Loser beats the Plurality Winner and the Plurality Winner beats the Plurality Runner-Up.
- (12) RCV is never vulnerable to mutual spoiler effects.
- (13) A Condorcet Winner is never a spoiler under RCV.
- (14) A Condorcet Loser is a spoiler under RCV if and only if the Plurality Loser is a Condorcet Winner.

Finally, comparison of spoiler effects under the two voting rules leads to the following observations.

- (15) Spoilers of any kind under either voting rule are precluded if and only if the Plurality Winner is also a Condorcet Winner.
- (16) Only the Plurality Runner-Up can simultaneously be a spoiler under plurality voting and RCV.
- (17) The Plurality Loser is often a spoiler under plurality voting (in four profile types) but (as previously noted) is never a spoiler under RCV.
- (18) A Condorcet Loser is a spoiler in two profile types under both voting rules.
- (19) In one sense, RCV does *not* 'dominate' plurality voting with respect to resisting spoiler effects—specifically, the Plurality Winner can be an RCV spoiler but is never a plurality spoiler.
- (20) But in a more important sense, RCV *does* 'dominate' plurality voting with respect to resisting spoiler effects—specifically, the set of profile types that are vulnerable to spoilers under RCV is a proper subset of those vulnerable to spoilers under plurality rule.

The last observation implies that, *regardless of how three-candidate preference profiles are generated* (e.g., arbitrarily, by simulation, from actual election results), *RCV spoilers cannot occur more frequently than plurality spoilers and almost certainly occur less frequently*. Thus, the *direction* of McCune and Wilson's results in Sect. 4 (pertaining to the frequency of spoilers the three-candidate IAC and IC cases) is *logically determined*, though the *magnitude* of RCV's advantage depends on the process that generates the profiles (as the different results for IAC and IC illustrate).

Figure 1 also supports the following observation, which summarizes McCune and Wilson's Remark 1.

- (21) Under both plurality voting and RCV, a profile has a spoiler if and only if the winner of the three-candidate race is not a Condorcet winner.

As McCune and Wilson note, popular concern focuses on spoilers that are 'weak' or 'nonviable' candidates, and they suggest that Condorcet Loser status may be an indicator of 'weakness'. In the three-candidate cases with a Condorcet Loser, that candidate is a spoiler in two profile types out of six under both voting rules. Plurality Loser status might be another indicator of 'weakness' (and one that is more evident under both voting rules

and pre-election polls); PL is a spoiler in four profile types out of eight under plurality voting but (as we have already noted) can never be a spoiler under RCV.

Perhaps the strongest indicator of a ‘weak’ candidate would be one who is *both* a Condorcet Loser and a Plurality Loser. Such a candidate is a spoiler under plurality voting given a type (2a) profile. This represents the classic ‘Nader effect’ situation, where PL is Nader, P2 is Gore, and PW is Bush in Florida in 2000.⁴ As presumably ‘nonviable’ candidates, Nader and similar PL candidates in type (2a) profiles represent the most bothersome type of spoiler under plurality voting, and RCV has the distinctive virtue of eliminating such spoiler effects. (Since PL does not make the runoff, the RCV runoff is the same contest as would occur if PL did not enter in the first place.)

Typical ‘vote splitting’ situations appear in profiles of type (2b) and (3b). While each of P2 and PL can beat PW in a straight fight, if both P2 and PL enter a plurality voting contest, they split their evidently shared majority support, thereby producing a PW victory and becoming mutual spoilers. As such, they may, but need not, be (near) ‘clones’ such that (almost) no voters have PW as their second preference. RCV precludes this spoiler problem also, as PL is eliminated in the first round and P2 then beats PW in the runoff. However, if the profile belongs to type (3b), there is a rub: PL wins if PW is not in the race but P2 wins if PW is in the race, so PW—an evidently ‘viable’ candidate—meets the definition of a spoiler.

While the ‘vote splitting’ phenomenon fits profile types (2b) and (3b), not all such profiles entail obvious vote splitting effects under plurality voting. Consider the much discussed 2009 Burlington mayoral election (Table 2 in McCune and Wilson), which fits type (3b) with Wright as PW, Kiss as P2, and Montroll as PL. Under plurality voting, Wright would win, making Kiss and Montroll mutual spoilers even though a substantial minority (about 18% of voters with complete rankings) place Wright between the other two). Under RCV, Montroll beats Kiss if Wright is out of the race but, with Wright in the race, Montroll is eliminated in the first round and Kiss (barely) wins the runoff against Wright, making Wright a spoiler.

In contrast, the 2022 U.S. House special election in Alaska (see Graham-Squire & McCune, 2022) fits profile type (3a), with Peltola as PW, Palin as P2, and Begich as PL. In the actual RCV election, Begich was eliminated in the first round, and Peltola beat Palin in the runoff (although a substantial majority of Begich votes transferred to Palin). But if Palin had not entered, Begich would have won the election, making Palin an RCV spoiler. Had the election been held under plurality voting, Peltola would also have won and Palin would also have been a spoiler. In further contrast, the 2022 U.S. House general election in Alaska evidently fits type (1b) with Pelota as PW, Palin as P2, and Begich as PL, and the 2022 Alaska Senate election fits type (1a), so neither election had a spoiler under either voting rule.

Figure 1 and its implications provide a complete analysis of spoiler effects under both plurality voting and Ranked-Choice Voting in elections with (potentially) three candidates. Apart from providing theoretical insights that may prove useful for examining spoiler effects with more candidates, the typology defined in Fig. 1 makes it straightforward to find spoilers under both voting rules in any set of three-candidate RCV election results or preference profiles. In fact this author has previously conducted such a search. A preliminary version of Miller (2014) used the analytics of Fig. 1 to identify spoilers in British general elections from 1992

⁴ In contrast, type (1a) represents a ‘Buchanan non-effect’ situation. (Buchanan won about 35,000 votes in Florida in 2000, presumably coming largely at Bush’s expense, but Bush remained the Plurality Winner even with Buchanan in the race.).

through 2010 under both plurality voting (which the British call First-Past-the-Post) and RCV (which the British call the Alternative Vote)⁵ Overall, spoiler effects occurred in 11.3% of constituency elections under plurality voting but only 4.6% under RCV. However, these rates varied considerably over elections and were exceptionally high in the 2010 election. Mutual spoiler effects occurred in 1.6% of the plurality elections, and the mutual spoilers were always ideologically adjacent candidates, mostly Labor and Liberal (88%) and secondarily Conservative and Liberal (12%). In party terms, under plurality voting Labor candidates were spoilers in 6.2% of all elections, Liberals in 3.6%, and Conservatives in 3.0%; under RCV, Labor candidates were spoilers in 1.7% of all elections, Liberals (often Plurality Losers) never, and Conservatives in 2.8%. Perhaps the most notable finding—something not implied by the Fig. 1 typology or suggested by McCune and Wilson’s results—was that the frequency of spoiler effects under both voting rules increased quite dramatically as elections became closer, as measured by the percent of first preferences for the Plurality Loser (PL%). Under plurality voting, no spoiler effects occurred until PL% reached about 7%, their frequency was about 5–15% for PL% of about 10–25%, and well above 50% for still closer elections. Under RCV, no spoiler effects occurred until PL% reached about 10%, their frequency was about 1–8% for PL% of about 10–25%, and upwards of 50% for still closer elections.

Acknowledgements I thank David McCune for helpful comments and corrections, particularly pertaining to the RCV elections in Alaska.

References

- Graham-Squire, A., & McCune, D. (2022). A mathematical analysis of the 2022 Alaska special election for U.S. House. arXiv preprint, [arXiv:2209.04764](https://arxiv.org/abs/2209.04764).
- Graham-Squire, A., & McCune, D. (2023). An examination of ranked-choice voting in the United States, 2004–2022. *Representation*. <https://doi.org/10.1080/00344893.2023.2221689>
- McCune, D., & Wilson, J. (2023). Ranked-choice voting and the spoiler effect. *Public Choice*, 196(1–2), 19–50.
- Miller, N. R. (2014). The alternative vote and Coombs rule versus first-past-the-post: A social choice analysis of simulated data based on English elections, 1992–2010. *Public Choice*, 158(3–4), 399–425.
- Sen, A. K. (1970). *Collective Choice and Social Welfare*. Holden Day.

Publisher’s Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Springer Nature or its licensor (e.g. a society or other partner) holds exclusive rights to this article under a publishing agreement with the author(s) or other rightsholder(s); author self-archiving of the accepted manuscript version of this article is solely governed by the terms of such publishing agreement and applicable law.

⁵ The section on spoilers was dropped before publication to reduce the paper to appropriate journal article length. The analysis in was restricted to English constituencies because virtually all were effectively three-candidate (Labour, Liberal Democrat, Conservative) contests, which was not the case in Wales, Scotland, or Northern Ireland. Since the resulting 2642 constituency elections were conducted under plurality voting, the electoral data provided only (what were taken to be) first preferences. Second preferences in each constituency were allocated in proportion to second preferences nationwide, as determined by surveys for each of the five elections. These surveys indicated that English voter preferences were ‘partially single-peaked’—that is, most but not all Labour (‘left-of-center’) voters had the Liberal Democrats (the ‘centrist’ party) as their second preference as did most but not all Conservative (‘right-of-center’) voters, while Liberal Democrat voters had more evenly divided second preferences.