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Risk aversion in two-period rent-seeking games

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Abstract
This work analyzes a two-period rent-seeking game, with the aim of studying the effect of risk aversion on the optimal choices made by the rent-seekers. We first prove that the equilibrium in two-period rent-seeking games always is unique. The analysis also shows that more risk aversion reduces the investment in the rent-seeking game in a two-period framework without introducing the additional condition of prudence, required in one-period models. Similarly, the introduction of a risky rent, instead of a given rent, implies, in the two-period framework, a reduction in investment under the condition that the rent-seekers are risk averse. Moreover, with risk aversion, larger first-period wealth increases investment in the rent-seeking game and larger second-period wealth reduces it. When both first-period and second-period wealth increase, investment in the rent-seeking game declines if the rent-seeker is risk averse and imprudent. Lastly, when a risky level of second-period wealth is introduced, the rent-seeker increases (reduces) investment in the rent-seeking game if he is risk averse and prudent (imprudent).

Keywords Rent-seeking games · Two-period framework · Risk aversion · Risky rent · Changes in wealth

JEL Classification C72 · D72 · D81

1 Introduction

Many activities in peoples' lives can be described by rent-seeking games in which different agents compete for obtaining a rent. Activities of that kind are, for instance, lobbying, R&D rivalry for innovating, sports competitions and competitions for obtaining grants.

All such activities have some common characters. First, all rent-seekers have an *ex-ante* probability of winning the game, whereas, *ex-post*, only one of them will win. Moreover, each individual rent-seeker can increase his probability of winning, simultaneously reducing competitors' probabilities, by investing resources in the game. The investment always is costly, but it can be of a different nature or dimension. For instance, it can be a financial

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cost in the case of lobbying or R&D competition, training in the case of sports competitions, or effort in preparing a project or application in the case of a competition for a grant.

Thanks to their very general formulation, rent-seeking games have been studied widely in the literature, starting from the seminal papers by Tullock (1967, 1980). Tullock's approach has be applied in many fields. Beyond the examples listed above, rent-seeking games have been shown to be relevant for studying the effects of entrepreneurial activity on economic growth (Baumol 1990; Murphy et al. 1993), military conflicts and election campaigns (Hirshleifer 1989), politicians' behavior (McChesney 1997) and, very recently, to compare development across countries (Acemoglu and Robinson 2019) and in its early stages (e.g., Carugati et al. 2019) as well as to analyze polycentric governance (Tarko and Farrant 2019).[1]

Since in rent-seeking games, each player has only the chances of winning or losing, the games describe a risky context. Despite that observation, the study of rent-seeking games initially was implemented in a framework wherein rent-seekers were assumed to be risk neutral. Starting from Konrad and Schlesinger (1997), a significant literature (e.g., Cornes and Hartley 2003, 2012; Yamazaki 2009) introduced risk aversion into the analysis. In particular, Konrad and Schlesinger (1997) examined the effect of risk aversion on optimal investments in a rent-seeking game, concluding that the direction of the effect is ambiguous. More recently, Treich (2010) found a clear negative effect of risk aversion on investment in the rent-seeking game, relative to risk neutrality, when the risk averse rent-seeker also is prudent, i.e., when the utility function of the rent-seeker has a positive third derivative.[2] Moreover, Liu et al. (2018) contributed new results to the model studied by Treich (2010) and derived the opposite result, namely that risk aversion has a positive effect on investments in rent-seeking games when we introduce a payment contingent on the outcome of the contest and paid only by the winner ("contingent payment"), instead of considering thepayment to be a fixed cost independent from the game's outcome ("up front payment"), as it had been assumed in all of the previous literature. It is worth noting that such heterogenous results suggest clearly that the effect of risk aversion on investments in rent-seeking depend heavily on the structure of the game analyzed.

An important, but substantially neglected element in the analysis of rent-seeking games is the role of the timing of investments in them. In fact, models introducing risk neutrality and those assuming risk aversion both usually study rent-seeking games in a one-period framework, such that rent-seeking effort is expended contemporaneously with the contest that assigns the rent. But, although that timing can be appropriate for some real world situations, it is unsuitable for many others, wherein the investment precedes the beginning of the contest.

Consider the examples mentioned above. It is clear that lobbying is carried out over a long time horizon before the relevant decision on rent attribution is taken. Similarly, R&D investment typically precedes the period in which the innovation is discovered. Training for

[1] For other recent and less recent applications, see Mitchell and Munger (1981), Congleton (2019) and Mitchell (2019).

[2] The relevance of prudence was first identified in the precautionary saving problem studied by Leland (1968) and then was formalized in the seminal paper by Kimball (1990). A subsequent and broad body of literature has made prudence a well-established concept in decision theory under risk, and it also has proved prudence to be significant for many economic issues, such as self-protection models (Eeckhout and Gollier 2005; Menegatti 2009), portfolio choice (Chiu et al. 2012) and stochastic dominance (Levy 2006).

a sports competition takes place for a long period before the competition starts and grant applications require investments of time and knowhow that precede the grant's assignment.

It is worth noting that issues associated with a game's time structure were not as salient in the literature assuming risk neutrality, since the models studied there, although formally one-period games, basically were "atemporal". That feature, which will be explained in more detail in Sect. 3, depends substantially on the fact that in a two-period framework and in the case of risk neutrality, the intertemporal allocation of wealth is irrelevant for the decision maker, meaning that one-period and two-period frameworks substantially are equivalent in the case of risk neutrality.

However, the issue of the time structure of the game becomes important in the case of risk aversion for at least two reasons. First, wealth allocation between periods matters for a risk averse rent-seeker. Second, the literature on other economic problems, such as self-protection models (see Eeckhoudt and Gollier 2005; Menegatti 2009), shows that, in the presence of risk aversion, the conclusions in one-period and in two-period frameworks can be very different.

Moreover, the analysis of a two-period framework for rent-seeking games likewise is important for a different reason. As emphasized in the recent survey by Dechenaux et al. (2015, p. 627) "it appears that ... the findings from experimental studies suggest that more risk-aversion leads to lower effort in contests",[3] while it has been noted that *"theoretically, the direction of the effect of risk aversion on effort depended on the third derivative of utility"*. The results obtained in the present paper provide new insights into that issue. A brief relevant discussion will be proposed in Sect. 4.

Starting from the foregoing premises, the aim of this paper is to study, for the first time, investment in rent-seeking games in a two-period framework, wherein investment occurs before the rent assignment.[4]

It will be shown that our two-period framework yields results that differ significantly from those obtained in the case of one-period games. In particular, the main results relate to the effects of risk aversion on optimal investment. We show that, once a constraint on contestants' wealth is introduced, risk aversion alone reduces optimal investment in the two-period framework, without requiring the additional condition of prudence. That result, which differs from Treich (2010), holds both when comparing the risk averse rent-seeker with the risk neutral rent-seeker and when comparing two rent-seekers with different levels of risk aversion.

The paper also addresses the case of risky rent. In that context, we show that, unlike the previous literature, introducing a risky rent in a two-period framework again yields clear effects of risk aversion without needing to introduce other preference requirements.

Lastly, as emphasized recently by Schroyen and Treich (2016), an important issue in the study of contests is the effect of changes in wealth. We examine different cases considering changes in first-period wealth, second-period wealth, wealth in both periods and the effects of shifting from non-random to risky wealth. For all of those cases we derive clear conclusions

[3] Some experimental economics papers obtaining results in this direction are, for instance, Anderson and Freeborn (2010), Sheremeta and Zhang (2010) and Mago et al. (2013).

[4] It is worth emphasizing that the issue of a two-period game studied in the present paper is different from that of two-stage games previously analyzed in the literature (e.g., Sheremeta 2010; Stracke et al. 2014). In fact, two-stage games consider one contest occurring over two periods but, instead, a kind of game comprisingof two contests wherein the winners of the first stage are admitted to the second.

relating to the effects of attitudes toward risk on players' investments in the game, providing for each of them a specific interpretation.

The paper proceeds as follows. Section 2 presents the rent-seeking game and its properties. Section 3 studies the benchmark case of risk neutrality. Section 4 examines optimal investment in the rent-seeking game under risk aversion. Section 5 analyzes the effect of changes in wealth. Section 6 concludes.

2 The game

The framework introduced is based on Konrad and Schlesinger (1997) and Treich (2010), except for the time structure of the model. We consider a contest with n identical utility maximizing agents competing for a rent $b > 0$. The probability of winning the contest for the representative rent-seeker i is given by $p_i = p_i(x_1, \dots x_n)$, where $x_j \geq 0$ for $j = 1, \dots, n$ is the investment made in winning the contest by rent-seeker j. The contest success function (CSF) p_i is assumed to be both differentiable and symmetric, such that for all $i = 1, \dots, n$, $p_i \in [0, 1]$ with $\sum_{i=1}^{n} p_i = 1$.

As in Konrad and Schlesinger (1997) and Treich (2010), we introduce the following assumptions for the contest success function:

Assumption A1 $\frac{\partial p_i}{\partial x_i}(x_1, ...x_n) \geq 0$ and $\frac{\partial p_i}{\partial x_j}(x_1, ...x_n) \leq 0$ for all x_i.

Assumption A2 $\frac{\partial^2 p_i}{\partial x_i^2}(x, ...x) < 0$ for all x.

Assumption A3 $\frac{\partial^2 p_i}{\partial x_i \partial x_j}(x, ...x) \leq 0$ for all x.

Assumption A4 $p_i(x, \dots, x) = \frac{1}{n}$ for all x

Assumption A1 states that the probability of success is a non-decreasing function of one's own investment and a non-increasing function of the investments of the other rent-seekers. Assumptions A2 and A3 introduce the usual requirements of decreasing marginal returns to investment. Assumption A4 states that, when all rent-seekers exert the same level of effort, they have the same probability of winning the contest.

Notice that our analysis is restricted to the case of symmetric games. That is the same restriction as in the contributions of Konrad and Schlesinger (1997), Treich (2010) and Liu et al. (2018), which provide the main starting point for the present paper.

Unlike Konrad and Schlesinger (1997) and Treich (2010), we assume that the investment in the game is made *before* the rent is assigned, which means that the investment is made in Period 0 and the contest for obtaining the rent takes place in Period 1. In the two periods, the representative rent-seeker is endowed with initial wealth w_0 and w_1, respectively.

In the specified framework, we consider the representative rent-seeker's choice of the optimal investment in the rent-seeking game, x_i, in order to maximize his intertemporal expected utility:

$$U(x_i) = u(w_0 - x_i) + \delta[p_i u(w_1 + b) + (1 - p_i)u(w_1)], \quad (1)$$

where u is the rent-seeker's one-period utility function and $\delta \leq 1$ is the subjective intertemporal discount factor.[5] We assume that intertemporal utility is such that at least one solution for the maximization problem exists.

Lastly, notice that Eq. (1) implicitly assumes that one-period utility is the same in the two periods. That assumption often is introduced in two-period problems but it is worth emphasizing that it is necessary for some of the results derived henceforth.

3 Risk neutrality

In order to analyze the effects of risk aversion on the optimal investment in the rent-seeking game, we first study the benchmark case of risk neutrality. In that case intertemporal utility (1) simply becomes

$$F(x_i) = w_0 - x_i + \delta[p_i(w_1 + b) + (1 - p_i)w_1] = w_0 - x_i + \delta[w_1 + p_i b]. \tag{2}$$

Given (2), the optimal level of x_i for the risk neutral agent (labelled x_n) satisfies the first-order condition:

$$p'_n = \frac{1}{\delta b} \tag{3}$$

(where $p'_n = \frac{\partial p_i}{\partial x_i}$ for $x_i = x_n$). Note that the maximum is unique since Assumption A2 ensures that $\frac{d^2 F}{dx_i^2} = F''(x_i) < 0$ for every x_i.

As anticipated in Sect. 1, under the assumption of no intertemporal discounting ($\delta = 1$), the formalization of the choice problem for the risk neutral agent essentailly is atemporal, since the two-period intertemporal utility in (2) is analytically equivalent to the corresponding utility in the one-period framework given by

$$p_i(w + b) + (1 - p_i)w = w - x_i + p_i b, \tag{4}$$

letting $w = w_0 + w_1$.

The foregoing implies that the first-order condition for Problem (4) is the same as for Problem (2), implying in turn that:

Proposition 1 *Under no intertemporal discounting, the optimal investment in the rent-seeking game is the same in the one-period and in the two-period frameworks with risk neutrality.*

The equivalence of the one-period and the two-period games in the case of risk neutrality has a straightforward interpretation. With risk neutrality, the allocation of wealth between the two periods is irrelevant because the rent-seeker wants only to maximize his total wealth. Choosing in a one-period framework or in a two period framework thus is the same for the rent-seeker.

[5] Notice the formal similarity between the present game and the "ability contest" of Schroyen and Treich (2016). That model, however, does not study an intertemporal framework but, rather, a one-period framework wherein the cost of participating in the contest is non-monetary.

The picture clearly changes in the case of positive intertemporal discounting. Here, in fact, the first-order conditions for the one-period and two-period frameworks differ because the two-period framework includes the term δ. Moreover, since p_n' is decreasing in x_i by assumption A2, we obtain immediately that

Proposition 2 *Heavier intertemporal discounting (i.e., a smaller δ) reduces the optimal investment in the rent-seeking game x_i with risk neutrality.*

The interpretation of that result again is straightforward. More discounting means that the rent seeker assigns lesser importance to future wealth. Since in a two-period setup the cost of investing in the game is in the present, while the potential benefit is in the future, the implication is that the potential benefit is less valuable for the rent-seeker and, hence, less is invested in the game.

It is worth emphasizing that a larger discount rate also can be interpreted as a circumstance in which the interval between the two periods is longer. Such an interpretation may open the model for applications to analyses comparing choices under different waiting times. For example, different firms may have different head-starts in their R&D projects or in building relationships with governments, implying that the time before they might gain the rent can vary for them. Those differences affect the extent of rent dissipation, since if a firm has to wait longer for projects to bear fruit, it invests less. For example, the previous literature has noted that restrictions on who is allowed or not allowed to seek rents may have the purpose of securing morer rents by preventing dissipation (Haber 2002; Aligica and Tarko 2014).

4 Risk aversion

4.1 Uniqueness of the equilibrium

We consider now a risk averse rent-seeker, whose preferences are represented by the utility function $u(.)$, with $\frac{du}{dx_i} = u'(.) > 0$ and $\frac{d^2u}{dx_i^2} = u''(.) < 0$. It is worth noting that the assumption of the utility function's concavity, which identifies risk aversion with reference to the rent-seeker's attitude toward risk, also has consequences for the optimal intertemporal allocation of wealth. In particular, in an intertemporal framework, the concavity of the utility function is related to both risk aversion and the elasticity of intertemporal substitution. A discussion of some of the implications of that for our analysis is provided in the interpretations of Propositions 7 and 8 in Sect. 4.2. Moreover, the restriction can be seen as a possible limitation of the present analysis and opens space for future development of the model in the direction of considering more complex kinds of preferences. An example, although not widely adopted in the literature, is the case of Kreps and Porteus (1978) preferences. .[6]Lastly, for a general discussion of the issue, see, for instance, Gollier (2001, chapter 20).

In order to clarify some of the following results and the comparison between the results in the present paper and those in the previous literature, we also recall that a risk-averse

[6] Kreps-Porteus preferences describe a setting which distinguish between risk aversion and intertemporal substitution. They are, however, not widely adopted because of their complexity from an analytical standpoint.

agent can be either prudent, imprudent or prudence-neutral, depending on whether the third derivative of the utility function is positive, negative or null.[7]

The risk-averse rent-seeker chooses the optimal level of investment, x_i, in order to maximize intertemporal expected utility (1). Given (1), the optimal level of x_i for the risk-averse rent-seeker (labelled x_a) satisfies the first-order condition:

$$U'(x_a) = -u'(w - x_a) + \delta p'_a[u(w + b) - u(w)] = 0, \tag{5}$$

where $p'_a = \frac{\partial p_i}{\partial x_i}$ for $x_i = x_a$.

One of the first issues with (5) is the uniqueness of the equilibrium. As such, we obtain that

Proposition 3 *The optimal investment in the rent-seeking game is unique in the two-period setup with risk aversion.*

Proof The result is straightforward since, given assumption A2 and risk aversion, we have $\frac{\partial p'_a}{x_i} < 0$ and $u'' < 0$ for every x_i which ensure that $U''(x_i) < 0$ for every x_i. □

The last result is important since to date the literature has shown that multiple equilibria may, in general, arise in one-period rent-seeking games (see Cornes and Hartley 2008; Yamazaki 2009; Treich 2010), with the exception of the case of contingent payment (Liu et al. 2018). Proposition 3 shows instead that, in the two-period framework, the optimal level of investment in the rent-seeking game always is unique.

Moreover, as in the case of risk neutrality, we can identify the effect of a larger intertemporal discounting (lower δ) on the optimal investment in the rent-seeking game:

Proposition 4 *Heavier intertemporal discounting (i.e., a smaller δ) reduces the optimal investment in the rent-seeking game under risk aversion.*

Proof By the implicit function theorem we have that

$$\frac{dx_a}{d\delta} = -\frac{\frac{\partial U'}{\partial \delta}}{\frac{\partial U'}{\partial x_a}}. \tag{6}$$

As shown above, $\frac{\partial U'}{\partial x_a} < 0$, while

$$\frac{\partial U'}{\partial \delta} = p'_a[u(w + b) - u(w)] > 0, \tag{7}$$

impling that $\frac{dx_a}{d\delta} > 0$ and proving the proposition. □

[7] For a more detailed description of prudence see, for instance, Kimball (1990). Also note that, as shown by Menegatti (2014), risk aversion and imprudence are compatible only when the utility function is defined over a bounded domain. That assumption is, however, suitable for the context studied herein.

4.2 Risk aversion and optimal investment

The main issue for the analysis of risk aversion in rent-seeking games is to study how it affects the optimal choice of investment x_i. To examine that problem, we first compare the optimal choices of risk averse and risk neutral rent-seekers. We start the analysis with the simplified case wherein rent-seeker wealth in the two periods is the same ($w_0 = w_1 = w$). We obtain the following results:

Proposition 5 *When rent-seeker wealth in the two periods is the same, the optimal investment in the rent-seeking game is smaller with risk aversion than with risk neutrality ($x_a < x_n$).*

Proof We evaluate $U'(x_i)$ in (5) for $x_i = x_n$, obtaining

$$U'(x_n) = -u'(w - x_n) + \delta p'_n[u(w + b) - u(w)], \tag{8}$$

which, by (3), is equivalent to

$$U'(x_n) = -u'(w - x_n) + \frac{1}{b}[u(w + b) - u(w)] \tag{9}$$

and, by the mean value theorem is, in turn, equivalent to

$$U'(x_n) = -u'(w - x_n) + u'(w + k), \tag{10}$$

where $k \in (0, b)$. Now, risk aversion implies that

$$u'(w - x_n) > u'(w + k), \tag{11}$$

implying in turn that $U'(x_n) < 0$. That result, together with $U''(.) < 0$, implies $x_a < x_n$, proving the proposition. □

Proposition 5 states clearly that risk aversion leads to less investment in the rent-seeking game than risk neutrality when rent seekers are endowed with the same wealth in both two periods. Although that case is simplified, it has a clear and relevant interpretation. In fact, assuming that wealth is the same in the two periods means removing the incentive for the rent-seeker to reallocate wealth from one period to the other for the purpose of consumption smoothing.[8] Therefore the problem analyzed here is "free" of consumption smoothing reallocation effects.

In the more general case (where $w_0 \neq w_1$), we have:

Proposition 6 *Ooptimal investment in the rent-seeking game is lower with risk aversion than with risk neutrality ($x_a < x_n$), when first-period wealth is not larger than second-period wealth (i.e., $w_0 \leq w_1$).*

[8] It should be noted, however, that, if saving were introduced in the model in the place of effort, it would not be zero because of the presence of risk and according to the so-called "precautionary motive for saving".

Proof The proof is similar to the proof of Proposition 5 until Equation (10). That equation is replaced here by

$$U'(x_n) = -u'(w_0 - x_n) + u'(w_1 + k). \tag{12}$$

Risk aversion now implies that

$$u'(w_0 - x_n) > u'(w_1 + k) \tag{13}$$

if $w_0 \leq w_1$. Given that result, the last steps of the proof are the same as in the proof of Proposition 5. □

The interpretation of Proposition 6 is related to that of Proposition 5. Proposition 5 showed that, under no incentive of intertemporal reallocation for consumption smoothing, risk aversion implies less investment in the rent-seeking game than risk neutrality. Proposition 6 shows that, if the incentive for intertemporal reallocation exists, the same result holds when the goal of consumption smoothing (related to the comparison between w_0 and w_1) pushes the rent-seeker to reallocate wealth to the present, reinforcing the incentive to reduce the investment in the game.

Propositions 5 and 6 reveal significant findings for the analysis of the effects of risk aversion on investment in rent-seeking games. In fact, in a one-period framework, Treich (2010) showed that a risk averse agent chooses to invest less in rent-seeking games than the risk neutral agent does under the assumption that the risk averse agent also is prudent (i.e., that his the third derivative of his utility function is positive). However, Liu et al. (2018) showed that, in a one-period game wherein the payment of the entry cost is contingent on winning the contest, risk aversion implies a larger investment than risk neutrality does. The present paper shows that, in a two-period game, when the incentive of consumption smoothing is not relevant, risk aversion implies less investment in rent-seeking games than risk neutrality, without requiring the assumption of prudence.

The next step in the analysis of the effects of risk aversion on the optimal investment in rent-seeking games is the comparison between two rent-seekers who both are risk averse, but one is more risk averse than the other. For that case, we consider two rent-seekers whose preferences are represented by the utility functions $u(.)$ and $v(.)$, both of which are increasing and concave. We also assume that rent-seeker u is more risk averse than rent-seeker v in the sense of Arrow and Pratt, which implies that function u can be written as an increasing and concave transformation of function v (i.e., that a function $h(.)$ exists such that $u(.) = h(v(.))$, where $h'(.) > 0$ and $h''(.) < 0$). In this case we obtain:

Proposition 7 *If rent-seeker u is more risk averse than rent-seeker v in the sense of Arrow and Pratt then he chooses less investment in the rent-seeking game, when first-period wealth is not larger than second-period wealth (i.e., $w_0 \leq w_1$).*

Proof We let $U(x_i) = u(w_0 - x_i) + \delta[p_i u(w_1 + b) + (1 - p_i)u(w_1)]$ and $V(x_i) = v(w_0 - x_i) + \delta[p_i v(w_1 + b) + (1 - p_i)v(w_1)]$. We also label by x_u the optimal investment in the rent-seeking game for rent-seeker u and by x_v the optimal investment in the rent-seeking game for rent-seeker v. The first-order condition for rent-seeker v requires

$$V'(x_v) = -v'(w_0 - x_v) + \delta p'_v[v(w_1 + b) - v(w_1)] = 0 \tag{14}$$

(where $p'_v = \frac{\partial p}{\partial x_i}$ for $x_i = x_v$), which implies

$$p'_v = \frac{v'(w_0 - x_v)}{\delta[v(w_1 + b) - v(w_1)]} \tag{15}$$

We now evaluate $U'(x_v)$ obtaining:

$$U'(x_v) = -u'(w_0 - x_v) + \delta p'_v[u(w_1 + b) - u(w_1)] \tag{16}$$

Since rent-seeker u is more risk averse than rent-seeker v in the sense of Arrow and Pratt, (16) can be rewritten as:

$$U'(x_v) = -h'(v(w_0 - x_v))v'(w_0 - x_v) + \delta p'_v[h(v(w_1 + b)) - h(v(w_1))] \tag{17}$$

By mean value Theorem, (17) is equivalent to

$$U'(x_v) = -h'(v(w_0 - x_v))v'(w_0 - x_v) + \delta p'_v h'(v(w_1 + k))[v(w_1 + b) - v(w_1)] \tag{18}$$

(where $k \in (0, b)$). Substituting now (15) into (18) we obtain

$$U'(x_v) = v'(w_0 - x_v)[-h'(v(w_0 - x_v)) + h'(v(w_1 + k))] \tag{19}$$

Now, since $h(.)$ is concave, the right-hand side of (19) is negative under the assumption $w_0 \leq w_1$, implying that $U'(x_v) < 0$. Since $U''(.) < 0$ and since $U'(x_u) = 0$ for the first-order condition for rent-seeker u, we get that $x_u < x_v$ proving the proposition. □

Proposition 7 generalizes Proposition 6 showing that more risk aversion in the sense of Arrow and Pratt implies less investment in the rent-seeking game when the incentive of consumption smoothing is excluded or acts in the same direction. Note that, while no results on this issue are derived by Treich (2010), Liu et al. (2018) showed that, in the case of one-period games and up front payment, a reduction in optimal investment is obtained when the rent-seeker has more risk aversion and more downside risk aversion in the sense of Ross. Also note that, like Proposition 6, this result is also the opposite to a further result derived by Liu et al. (2018) who show that, in a one-period game where the payment of the investment cost is contingent on winning the contest, more risk aversion in the sense of Arrow and Pratt implies more investment in the rent-seeking game.

4.3 Risky rent

Starting from Wärneryd (2003) a further issue in the analysis of rent-seeking games is the case where rent b is risky instead of being given. In this case we assume that the rent is represented by the random variable $\tilde{b}$ where $E[\tilde{b}] = b$. We now study the effect of the introduction of a random rent, by comparing the optimal choice of the risk averse rent-seeker when the rent is risky with optimal choice when the rent is given.

When the rent is risky, a risk averse rent-seeker chooses the optimal level of investment by maximizing

$$E[Ux_i)] = u(w - x_i) + \delta[p_i E[u(w + \tilde{b})] + (1 - p_i)u(w)] \tag{20}$$

The optimal level of x_i (labelled x_{aa}) thus satisfies the first-order condition:

$$E[U'(x_{aa})] = -u'(w - x_{aa}) + \delta p'_{aa}[E[u(w + \tilde{b})] - u(w)] = 0 \tag{21}$$

(where $p'_{aa} = \frac{\partial p}{\partial x_i}$ for $x_i = x_{aa}$). By comparing (5) and (21) we now obtain that:

Proposition 8 *The optimal investment in case of risky rent is lower than in case of given rent under risk aversion.*

Proof Given, $\tilde{b}$ where $E[\tilde{b}] = b$, risk aversion implies $E[u(w + \tilde{b})] < u(w + b)$, implying in turn that, evaluating $E[U'(.)]$ in (21) for $x_i = x_a$, we obtain $E[U'(x_a)] < 0$. Since $U''(.) < 0$, this implies $x_{aa} < x_a$. □

It is significant to compare the result above with those obtained by Treich (2010) for one-period rent-seeking games and Liu et al. (2018) for one-period games with contingent payment. In fact, both Treich (2010) and Liu et al. (2018) obtain that the risk averse rent-seeker chooses less investment with a risky rent only if he also is prudent. On the contrary, Proposition 8 shows that in the two-period framework the same behavior occurs without introducing the assumption of prudence.

4.4 Comparison of results

The results derived in Sects. 4.2 and 4.3 show that the effects of risk aversion on the optimal choice of investment in rent-seeking games differ in one-period and two-period games. Table 1 summarizes the main differences, comparing the results obtained herein with those derived in the previous literature. The main conclusions that can be drawn from the comparisons are the following.

First, in a two-period framework, more risk aversion tends to reduce investment in the rent-seeking game. That finding holds both when comparing a risk-averse agent with a risk-neutral one and when comparing two risk averse rent-seekers.

The result comparing risk averse with risk neutral rent-seekers is similar to that obtained by Treich (2010) in a one-period framework, with the significant difference that, in the one-period framework, the additional assumption of prudence is required, while it is not in the two-period framework.

Similarly, the result comparing two risk averse agents is consistent with that obtained by Liu et al. (2018) in the one-period framework, with the main difference being that, in the one-period framework, we find less investment in the rent-seeking game under the two conditions of more risk aversion and more downside risk aversion á la Ross (1981), which are stronger than the condition of more Arrow-Pratt risk aversion required in the two-period framework. The effect derived also is opposite to that obtained by Liu et al. (2018) in the one-period model with contingent payment.

Moreover, risk aversion likewise has a negative effect on investment in the rent-seeking game when the rent becomes risky. That conclusion holds both in the two-period and one-period frameworks with upfront and contingent payments. In the two-period framework, however, risk aversion alone is sufficient to obtain that results, while in both one-period models it must be accompanied by prudence.

Lastly, it is worth noting that the foregoing results also are important in light of the findings from experimental economics. As discussed in Sect. 1, experimental evidence strongly supports the existence of a negative effect of risk aversion on investment in rent-seeking games. In that regard, the conclusion obtained in the present paper, which confirms the same idea from a theoretical standpoint without requiring the additional condition

Table 1 A comparison of results in one-period and two-period rent-seeking games

	Risk aversion vs risk neutrality	More risk aversion	Risky rent
One-period game with up front payment (Treich 2010; Liu et al. 2018)	Risk averse rent-seeker invests less in the game under prudence (Treich 2010)	More risk aversion á la Ross and more downside risk aversion á la Ross implies less investment in the game (Liu et al. 2018)	Risk averse rent-seeker invests less in the game under prudence (Treich 2010)
One-period game with contingent payment (Liu et al. 2018)	Risk averse rent-seeker invests more in the game	More risk aversion á la Arrow and Pratt implies more investment in the game	Risk averse rent-seeker invests less in the game under prudence
Two-period game with $w_0 \leq w_1$ (This paper)	Risk averse rent-seeker invests less in the game	More risk aversion á la Arrow and Pratt implies less investment in the game	Risk averse rent-seeker invests less in the game

of prudence, provides a new theoretical foundation for experimental findings, potentially stronger than that provided by the existing literature.

5 Two-period games and changes in wealth

As emphasized recently by Schroyen and Treich (2016), a further significant issue in the analysis of contests relates to the effects of changes in wealth on optimal investments. In the two-period framework examined in the present paper, that issue has many dimensions. First, we consider the case when wealth in the two periods is different (i.e., when $w_0 \neq w_1$) and we study the effect of a change in first-period wealth and of a change in second-period wealth. We then consider cases when wealth in the two periods is the same (i.e., $w_0 = w_1 = w$) and of the effects of changes in it. In all such cases, we focus on the choice of the risk averse agent.

In the case of different wealth in the two periods, we obtain the following results:

Proposition 9 *Larger wealth in first period (w_0) increases optimal investment in the rent-seeking game with risk aversion.*

Proof By the implicit function theorem, we have that

$$\frac{dx_a}{dw_0} = -\frac{\frac{\partial U'}{\partial w_0}}{\frac{\partial U'}{\partial x_a}}. \tag{22}$$

As shown above, $\frac{\partial U'}{\partial x_a} < 0$, while

$$\frac{\partial U'}{\partial W_0} = -u''(w_0 - x_a) > 0, \tag{23}$$

whichimplies that $\frac{dx_a}{dw_0} > 0$, thus proving the proposition. □

Proposition 10 *Larger second-period wealth (w_1) reduces optimal investment in the rent-seeking game withr risk aversion.*

Proof By the implicit function theorem, we have that

$$\frac{dx_a}{dw_1} = -\frac{\frac{\partial U'}{\partial w_1}}{\frac{\partial U'}{\partial x_a}}. \tag{24}$$

As shown above, $\frac{\partial U'}{\partial x_a} < 0$, while

$$\frac{\partial U'}{\partial W_1} = \delta p'_a[u'(w_1 + b) - u'(w_1)] < 0, \tag{25}$$

implying that $\frac{dx_a}{dw_1} < 0$ and proving the proposition. □

The two effects obtained in Propositions 9 and 10 have straightforward interpretations. When first-period wealth increases, the rent seeker's first-period marginal utility declines, reducing the marginal cost of investment and, thus, incentivizing the rent-seeker to invest more. On the other hand, when second-period wealth increases, the rent sseeker's second-period marginal utility declines, reducing the marginal benefit of the potential rent and, hence, incentivizing the rent-seeker to invest less in the game.

It is worth emphasizing some possible applications of the result in Proposition 9. First, it implies that receiving a large inheritance should increase rent-seeking into bequests. That finding has a simple empirical implication: when comparing different generations of entrepreneurs we should observe second-generation wealthy people engaged in more rent-seeking than first-generation self-made entrepreneurs. Similarly, Proposition 9 also suggests that firms experiencing an increase in wealth should invest more in lobbying, possibly explaining the commonly observed pattern in which firms that ignore lobbying at first engage in it after they have become successful.[9]

A more complex situation arises when we assume that wealth in the two periods is the same. In that case we obtain

Proposition 11 *Larger wealth in both periods (i.e., when $w_0 = w_1 = w$) reduces the optimal investment in the rent-seeking game under risk aversion and imprudence.*

Proof By the implicit function theorem, we have that

$$\frac{dx_a}{dw} = -\frac{\frac{\partial U'}{\partial w}}{\frac{\partial U'}{\partial x_a}}. \tag{26}$$

As shown above, $\frac{\partial U'}{\partial x_a} < 0$, while

$$\frac{\partial U'}{\partial w} = -u''(w - x_a) + \delta p'_a[u'(w + b) - u'(w)], \tag{27}$$

which, by the mean value theorem, is equivalent to

$$\frac{\partial U'}{\partial w} = -u''(w - x_a) + \delta p'_a b[u''(w + k)] \tag{28}$$

(when $k \in (0, b)$).

Proposition 5 showed that, when $w_0 = w_1 = w$, $x_a < x_n$, which by assumption A2 implies that $p'_a > p'_n$. That result, together with (3) and $u''(.) < 0$, implies:

$$-u''(w - x_a) + \delta p'_a b[u''(w + k)] < -u''(w - x_a) + \delta p'_n b[u''(w + k)] = -u''(w - x_a) + [u''(w + k)]. \tag{29}$$

The foregoing implies that $\frac{\partial U'}{\partial w} < 0$ and, thus, $\frac{dx_a}{dw} < 0$ when

$$u''(w + k) < u''(w - x_a). \tag{30}$$

We have that (30) holds when the third derivative of the utility function is negative, i.e., with imprudence. Thus, risk aversion and imprudence together imply that an increase in w reduces x_a, proving the proposition. □

[9] The sam issue potentially could be studied in a framework wherein firms compete in a sequence of two games: an R&D game and a lobbying game. According to Proposition 9, firms succeeding in the first game should invest more in lobbying in the second game.

Proposition 11 shows that, if wealth increases in both periods, the rent-seeker reduces investment in the rent-seeking game if he is not only risk averse but also imprudent. The interpretation of that results is related to a possible interpretation of imprudence found in the literature. On the one hand, optimal wealth allocation with risk aversion requires that part of the additional wealth is reallocated to the first period (where expected wealth is smaller by reducing investment in the game. On the other hand, Eeckhoudt and Schlesinger (2006) and Menegatti (2007) show that imprudence can be seen as a desire to allocate less wealth to the period when uncertainty is faced.[10] Given that interpretation, it is clear that, when wealth in both periods increases, an imprudent rent-seeker is pushed to reallocate some of the additional wealth to the first period and to reduce second-period expected wealth (where uncertainty is faced) by lowering investment in the game. Doing so reinforces the effect of risk aversion, determining a clear direction of the change in rent-seeking investments.

The last change in wealth considered is when second-period wealth becomes risky.[11] For that purpose, we assume that second-period wealth to be a random variable $\tilde{w}$, with $E[\tilde{w}] = w$. As such, the rent-seeker's maximization problem becomes:

$$E[U(x_i)] = u(w - x_i) + \delta[p_i E[u(\tilde{w} + b)] + (1 - p_i)E[u(\tilde{w})]]. \tag{31}$$

The optimal level of x_i (labelled x_{aaa}) thus satisfies the first-order condition:

$$E[U'(x_{aaa})] = -u'(w - x_{aaa}) + \delta p'_{aaa}[E[u(\tilde{w} + b)] - E[u(\tilde{w})]] = 0 \tag{32}$$

(where $p'_{aaa} = \frac{\partial p}{\partial x_i}$ for $x_i = x_{aaa}$). By comparing (5) and (32) we now obtain

Proposition 12 *The optimal investment in the rent-seeking game is larger (smaller) with risky second-period wealth under both risk aversion and prudence (imprudence).*

Proof We evaluate $E[U'(.)]$ in (32) for $x_i = x_a$, which is equal to

$$E[U'(x_a)] = -u'(w - x_a) + \delta p'_a[E[u(\tilde{w} + b)] - E[u(\tilde{w})]]. \tag{33}$$

Since $U''(.) < 0$, we now have that $x_{aaa} > x_a$ if $E[U'(x_a)] > 0$. By (5), that occurs if

$$E[u(\tilde{w} + b)] - E[u(\tilde{w})] > u(w + b) - u(w). \tag{34}$$

By Jensen's Inequality, the relation holds when the function $u(w + b) - u(w)$ is convex in w, which occurs, in turn, when $u''(w + b) - u''(w) > 0$. The last inequality holds under prudence. Lastly, the proof in the case of imprudence is similar. □

Proposition 12 shows that, if second-period wealth becomes random, the risk averse rent-seeker increases (reduces) investment in the rent-seeking game if he also is prudent (imprudent). The same result likewise is related to the interpretation of prudence provided above. When we introduce a further source of uncertainty owing to the risky income in the

[10] Both papers provide interpretations for prudence. The interpretation of imprudence can, however, be derived easily. Eeckhoudt and Schlesinger (2006) relate the result to harm disaggregation, i.e to the desire to separate the harm of incurring a sure loss and the harm of facing a risk. Menegatti (2007) relates it to a reduction in the utility premium.

[11] We do not perform the same analysis for first-period wealth since it is plausible to assume that the values of all variables for the period wherein the rent-seeker makes a choice are known with certainty.

second-period, a prudent (imprudent) rent-seeker desires to raise (lower) expected wealth in that period since he will face uncertainty from it. For that reason, the rent-seeker is willing to invest more (less) in the first period in order to affect the probability of winning the rent in the second period so as to increase (reduce) expected second-period wealth. It is important to emphasize that the mechanism at work here is exactly the same as the traditional mechanism in which prudence affects optimal agent behavior when second-period income risk is introduced into saving models (e.g., Leland 1968).

6 Conclusions

The time structure of activities that can be described by rent-seeking games suggests that, in many cases, investment in the game precedes the time at which the rent is assigned. Such a structure implies that a two-period formalization, unlike the one-period formalization usually adopted in the literature, is appropriate for such situations.

Starting from those premises, the present article proposes the first formalization of a two-period rent-seeking game, with the aim of studying the effects of risk aversion on the optimal choices made by the rent seekers. The main results are the following.

We first show that, unlike one-period frameworks, the equilibrium level of investment is unique in a two-period framework.

The analysis also shows that risk aversion reduces investments in the rent-seeking game in a two-period framework with respect to the optimal choices of risk neutral agents. Unlike the traditional one-period framework with upfront payments, the same result holds in the two-period framework without introducing the additional condition of prudence. Moreover, the same result holds when comparing two risk averse rent-seekers after introducing more risk aversion á la Arrow and Pratt instead of introducing the stronger condition of more risk aversion and more downside risk aversion á la Ross that is required in the one-period framework. Lastly, introducing a risky rent instead of a given rent in the two-period framework implies less investment when the rent-seeker is risk averse, while the same effect occurs only when the rent-seeker is both risk averse and prudent in the one-period framework.

It is worth noting that the results presented herein imply that less is invested in the rent-seeking game in the two-period framework under a more parsimonious set of conditions on risk attitudes than in a one-period framework.

Examining different kinds of changes in wealth provides other noteworthy results. With risk aversion, larger first-period wealth raises investment in the rent-seeking game and larger second-period wealth reduces it. When both first-period and second-period wealth increase, investment in the rent-seeking game declines when the rent-seeker is risk averse and imprudent. Lastly, when a risky level of wealth in the second-period is introduced, the rent-seeker increases (reduces) investment in the contest when he is risk averse and prudent (imprudent).

It is worth noting that prudence/imprudence, which disappears as a requirement in comparisons between risk aversion and risk neutrality in a two-period framework, is again significant when changes in wealth are analyzed. The role of that feature of agent preferences can be interpreted in the same way that it is interpreted in the literature with respect to other problems, such as saving, and relates to a desire to manage the level of given wealth in the period wherein the rent-seeker faces risks.

The analysis of two-period rent-seeking games proposed in this study also paves the way for future extensions in different directions. One of the most significant extensions would be to explore the optimal rent-seeking investment in contexts where different individual or household choices are made at the same time. In particular, the intertemporal framework studied in this paper implies that rent-seeking effort plays two roles: it changes the probability of winning the contest in the second period and it changes the allocation of wealth over the two periods. That conclusion suggests that one of the next analytical steps could be to examine joint choices of optimal investments in the game and of variables affecting the intertemporal allocation of wealth, such as saving.[12]

Moreover, as mentioned in Sect. 1, the results derived in the present paper's two-period setup usefully could be extended to other rent-seeking models, usually studied in one-period frameworks. It also is important to emphasize that new possible fields of application for rent-seeking analysis specifically related to the time structure introduced in the present paper are possible. In some cases, in fact, rent-seeking processes necessarily occur over time. That is what happens, for instance, in the case of "regime uncertainty" (see Higgs 1997), wherein future changes in regimes may produce uncertainty about the sizes of future rents. A similar effect likewise is generated by creative destruction, which makes future rents uncertain too.[13] Other applications may involve issues relating to economic history, such as the potential effects of changes in state capacity in creating rent availability uncertainty.

Lastly, notice that a future research strand stemming from the present paper also could explore Tullock's paradox. In fact, the present paper shows that risk aversion pushes agents to reduce investment in rent seeking in a two-period framework. Risk aversion and risky rents in a context of rentseeking across different periods may imply a mitigation of rent dissipation and potentially resolve the paradox at least in part. A specific research agenda in that direction may be promising.

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[12] The spirit of such possible future analyses would be similar to that of work examining in a two-period framework the interaction between saving and self-protection (e.g., Menegatti and Rebessi 2011; Steinorth 2011; Peter 2017), starting from models wherein each instrument is first studied on its own.

[13] For instance, the creation of ride-sharing apps generates uncertainty about taxicab rents. Many other applications to the so-called gig economy are possible

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