Public Choice (2021) 186:141–148
https://doi.org/10.1007/s11127-019-00767-4

Will quadratic voting produce optimal public policy?

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Received: 4 April 2019 / Accepted: 6 December 2019 / Published online: 11 November 2020

Abstract

Under quadratic voting people are able to buy votes with money. The claims that rational voters will make efficient electoral choices rest on assumptions about how voters acquire and share information. Specifically, that all voters share common knowledge about the probability that any one of them will be the decisive voter, but do not (appear to) share knowledge in any specialized way within special interest groups. This paper asserts that quadratic voting is no more likely to promote efficiency than the current system of one-person-one-vote. Information costs are critical. If information is costly, organized interest groups on either side of an issue provide low-cost information to their members and sharing common knowledge across groups is less likely. Then, small differences lead to large welfare losses. If information is free, special-interest groups provide opportunities for collusion that undermines the efficiency of quadratic voting. Even if collusion could be prevented, the dual uses of money to buy votes and to disseminate information organizes interest groups as if their members were colluding. The role of information and the fact that voting is not costless create efficiency biases under quadratic voting that favor political organization and concentrated values. To the extent that these attributes are overrepresented in the present system, quadratic voting will only make it worse.

Keywords Quadratic voting · Equilibrium · Voting efficiency

JEL Classification H-10 · H-11 · D-61

1 Introduction

Quadratic voting has been proposed by several scholars, including in a recent book by Posner and Weyl (2019). It belongs to a class of voting models that allows voters to express the intensities of their preference along a continuum as opposed to one-person-one-vote. Perhaps the most popular is the probabilistic voting model in which voters are viewed by

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candidates as having probabilities of voting that are responsive to changes in the platforms they propose. Analogously, Becker (1983) considers pressure among interest groups as shaping political outcomes.

In a series of papers, Goodman and Porter (1985, 1988, 2004) summarize the many ways one could influence an election—casting a vote, making campaign contributions, influencing others, and so on—as a homogeneous, continuous variable. Common to all the continuous voting models is the concept of equilibrium. If certain concavity/convexity conditions hold, a unique platform will exist that can defeat all others in a majority vote and the equilibrium is stable. By treating effort per unit of benefit (effort/benefit ratio) as the "price" voters are willing to pay to secure a dollar's worth of gain, Goodman and Porter identify a host of equilibrium and comparative static properties of the model.

For purposes of this paper, one related finding is critical: In equilibrium, the marginal social benefit from a policy variable divided by its marginal social cost must equal the political price people are willing to pay for a dollar in benefits, divided by the political price people are willing to pay to avoid a dollar of cost. Optimality requires that the prices be equal.[1] In voting models, a sufficient condition for optimality is that the effort/benefit ratio be equal across all voters (Goodman and Porter 1985) or, alternatively, that the aggregate effort/benefit ratio be equal for opposed groups. Quadratic voting is said to mimic that optimality condition and it is claimed to promote efficiency relative to other voting mechanisms (Lalley and Weyl 2018; Posner and Weyl 2017).

Critical to the efficiency of quadratic voting is the role of information. Prominent among a set of necessary conditions for quadratic efficiency is that opposition groups have the same weighted average estimate of the probability that a single additional vote cast will determine the election outcome.[2] A sufficient condition for that result is that everyone have the same estimate. Goodman and Porter (2004) show that small differences in estimates can generate huge inefficiencies. If information is not costly then everyone should have the same estimate of the likelihood that a single vote cast will determine the outcome of an election. When information is not free, however, organized interest groups play critical roles in supplying information to voters.

A second critical assumption is of no collusion. Collusion skews the political outcome in favor of high-value voters. Collusion is problematic for quadratic voting because of the increasing marginal cost of a quadratic vote. It always is cheaper for high-value voters to pay lower-value voters to cast a vote than to cast an additional vote personally. Here, a low-cost information environment makes explicit collusion in the form of vote buying more likely by identifying potential collusion partners and reducing the cost of enforcing collusive agreements. As we demonstrate below, an organized special-interest group facilitates collusion.[3]

A third critical assumption is that the act of voting is costless. When voting is costly only high-value voters vote, skewing the outcome to favor those voters. Ironically, efforts to make voting less costly—mail-in ballots and internet voting—make collusion more likely by permitting vote sellers to demonstrate that they kept their part of the bargain, something that is not possible with secret ballots.

[1] A generalization of this finding is proposed by Tideman and Plassmann (2017), who argue that no social choice mechanism is likely to be efficient unless "all parties involved … bear the marginal social costs of their actions."

[2] Weights depend on the distribution of benefits within the group.

[3] Some of the problems, including collusion and voter misinformation are discussed by Weyl (2017).

In real world voting, money always has played a role in determining political outcomes by providing information, organization, and influence. Quadratic voting opens a second path for money's influence—the direct purchase of votes. Money's dual role serves to organize groups in ways that mimic collusion without individuals contracting for votes explicitly. That is, even if it were possible that everyone holds the same subjective probability of casting the decisive vote, that voting were costless, and that vote-buying agreements could be banned, quadratic voting still would provide more voting power to high-value voters relative to small-value voters than efficiency requires because money has two uses in quadratic politics.

The claim that quadratic voting promotes efficiency necessarily is a relative claim—e.g., it's better than one-person-one-vote systems—because the necessary conditions cannot be satisfied in the real world. What is called for is a comparison of the equilibrium positions under alternative voting systems.[4] Posner and Weyl (2013) do that in the market for corporate governance, suggesting that quadratic voting is a better means of reducing managerial shirking than is share voting. Quadratic voting is designed to assign greater political weight to high-value voters by permitting them to buy more votes. Such vote-buying promotes efficiency when high values are underrepresented in the present system. Posner and Weyl use the example of gay rights wherein a proponent with a value of \$100 can be defeated by two opponents each having a value of $-\$1$. However, when high values already are over-represented, quadratic voting will make the outcome worse.

In Sect. 2, we show how pure quadratic voting mimics the efficiency conditions introduced by Goodman and Porter. Section 3 demonstrates how small deviations can cause huge inefficiencies. Section 4 relaxes the assumptions of the quadratic model to reveal sources of inefficiency and concludes with the observation that the dual role of money in quadratic voting makes collusive-like behavior inevitable. Section 5 ends the paper.

2 Quadratic voting

In quadratic voting one can buy n votes for *\$n²* for or against an issue or candidate; the most votes wins. The marginal cost of n votes is $2n$. The expected benefit is pB, where p is the perceived probability that the nth vote will break a tie and B is the benefit to the vote buyer of a favorable political outcome. So, if people are rational maximizers,

$$pB = 2n, \text{ or } p/2 = n/B \text{ for each voter,} \tag{1}$$

where n/B is the quadratic model's equivalent of the effort/benefit ratio. If the subjective probability, p, is the same for everyone, the effort/benefit ratios also will be the same and optimality will be guaranteed.

Note that in Eq. (1) every voter is understating by a factor of $1-p$ the true value of the election to herself. Unlike the marketplace, in the political system people never are expected to reveal their preferences fully. We can still get optimal outcomes, however, if every voter understates her preferences to the same extent.

Note also that the probability of being the decisive voter quickly approaches zero as the number of voters becomes very large. Even so, small differences in perceptions of those probabilities can cause large welfare losses. Suppose that the beneficiaries of a public

[4] Without equilibria, the idea of efficiency is moot.

spending project believe that their probability of being the decisive voter is 1/1,000,000, on the average, while taxpayers who must bear the cost believe their probability to be 2/1,000,000. Equation (1) shows that the opposition group will buy twice as many votes per unit of cost as the proponent group buys per unit of benefit, defeating any issue for which the benefit is less than twice the cost.

3 Inefficiencies in political markets

Harberger (1954) calculated the deadweight welfare loss from monopoly in the private sector to be about 1/10th of one percent of private sector output. We adopt the same assumptions and methods of calculation here to measure the welfare consequences of misallocation in the public sector under a variety of voting models.

Let $B(Q)$ be the benefit of the group advocating for more public sector output, Q, and let $C(Q)$ be the cost to the group opposing that expansion. If a unique winning platform, Q^e, exists, it must satisfy

$$\lambda^P MB(Q^e) = \lambda^O MC(Q^e). \tag{2}$$

Here, the superscripts, P and O, represent the proponents and opponents of spending; MB and MC represent marginal benefit and marginal cost of Q for the two groups, respectively. The λs in Eq. (2) represent group effort per unit of benefit in the Goodman–Porter model. They are analogous to votes per unit of benefit in the quadratic voting model and group pressure in the Becker model.

Equation (2) says that in equilibrium, the marginal support the proponents offer for increasing Q is exactly equal to the marginal loss of support from the opposition. If the λs are the same, marginal social benefit will equal marginal social cost and Q^e will be the optimal level of output.

What if the λs are not the same? Harberger assumed constant marginal cost and unitarily elastic demands and approximated the deadweight welfare loss from monopoly by

$$WL = \left[(MB - MC) \times (Q^* - Q^e)\right]/2, \tag{3}$$

where Q^* is optimal output and Q^e is output determined by the political process.

In Fig. 1, we assume that $2\lambda^P = \lambda^O$.[5] The welfare loss from political misallocation as a fraction of spending, $MC \times Q^e$, can be expressed as a fraction of the effort/benefit ratios. In political equilibrium, $MB(Q^e) = (\lambda^O/\lambda^P)MC(Q^e)$ and, by the assumption of unitary elasticity, $Q^* = (\lambda^O/\lambda^P)Q^e$. Substituting those into Eq. (3) and dividing by spending reveals that the welfare loss per dollar of expenditure is $\theta^2/2$, where $\theta = (\lambda^O - \lambda^p)/\lambda^P$.

Thus, if the effort/benefit ratios differ by a factor of only two, the welfare loss from public spending is \$0.50 per dollar. If they differ by a factor of three, a dollar of waste will accompany every dollar spent.

[5] From Eq. (1) and consistent with free riding in large groups, we know that λ will be very small. For visual effect, we have assumed that$\lambda^P = 1/4$ and $\lambda^O = 1/2$, since only the ratio matters in determining the political outcome, Q^e.

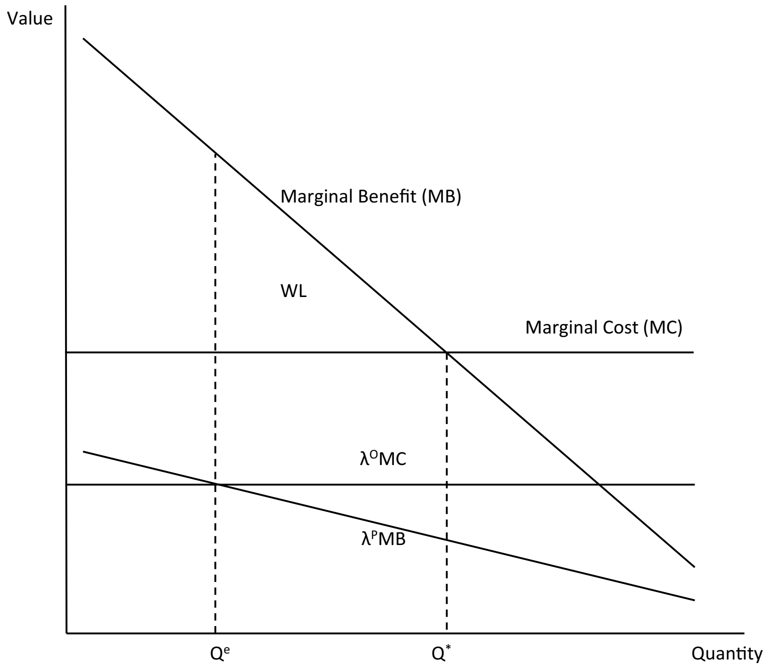

Fig. 1 Political distortions and welfare loss

4 Reasons for inefficient outcomes[6]

4.1 Costs of information

Why would voters have different estimates of the probability that one vote will be decisive? The answer is that their knowledge is not the same and the cost of obtaining information is not free. The real question is why anyone would expect different people's estimates to be the same. Organized groups (see below) provide their members with low-cost information—and also misinformation (exaggerating, for example, the importance of their vote). An equally serious problem, resolved the same way, is that people often do not have accurate information about the true benefit and the true cost to them of a candidate or a policy change. A source of information is a group they are members of or identify with.

[6] Lalley and Weyl (2018) address voting costs, collusion and different perceptions of the probability of casting the decisive vote. Their focus is on preserving the efficiency properties of quadratic voting and they demonstrate the robustness of quadratic voting over a wide range of assumption about the distribution of voters, their preferences and their perceptions of being the decisive voter. We have the opposite focus. We draw attention to the same phenomenon that leads to special interest dominance in voting models: concentrated versus dispersed benefits, collusion with heterogeneous versus homogeneous preferences and persuasion as well as the expanded role of money in determining the political outcome.

4.2 The cost of voting

Consider a group of 200 voters who each stand to lose \$1.01 if a particular proposal is passed and one individual who stands to gain \$100. For the moment, assume that all 201 voters agree that the probability of being the decisive voter is 1/25. If voting is costless, the 200 voters, whose expected cost is \$0.0404, each buy 0.0202 votes and the proposal receives 4.04 votes against. The one individual in favor of the proposal has an expected value of \$4.00 and buys 2.0 votes. The issue efficiently fails by a vote of 4.0 to 2.0. However, if the cost of voting is \$1.00, the 200 opponents will buy only 0.0002 votes each, for a total of 0.04 votes, while the single proponent will buy 1.98 votes in favor and the proposal inefficiently will pass. With one-person-one-vote, the same proposal would fail because the 200 voters still have a small incentive (\$0.01) to vote against. If the 200 voters' values were instead \$0.99 each, the proposal inefficiently would pass under one-person-one-vote because the incentive to vote against it disappears.

If voting is costly, one-person-one-vote systems favor concentrated interests over widely dispersed interests. However, quadratic voting intensifies that distortion.

4.3 Explicit collusion

Assume that the proponent group has four members in the example above: one who stands to gain \$100 if the proposal passes and three others who would gain \$1.00 if the proposal passes. If voting is costless and all voters perceive that the probability of being decisive is 1/25, the opponent group again buys 200 \$0.02 votes (4.0 vote total). Working independently, one proponent would buy one 2.0 vote (for \$4) and three would buy 0.02 votes. The issue efficiently would fail by 4.0 to 2.06.

However, the proponent group can do better. The \$4.00 that was to be spent on 2.0 votes could be distributed among the four proponents. One dollar would buy 1.0 votes for each opponent. Coupled with the 0.02 votes the three opponents would purchase, the opposition would now tally 4.06 opposing votes and the issue inefficiently would pass.

For any amount of money M that an interest group wishes to spend in total in the absence of collusion, the optimal distribution of M among N members with collusion is M/N each.[7] Moreover, the benefit of collusion increases with increases in variation in the amount each member of the coalition is willing to contribute to voting.[8] With vote buying, quadratic voting is biased toward coalitions of voters whose aggregated value largely is concentrated in a few people.

Note that redistribution among those with common political interests is prevalent even under one-person-one-vote regimes. An employer may give workers time off (thereby paying them to vote) to go vote against an issue or candidate that the company opposes. In trade associations and political action committees (PACs), members with larger stakes tend

[7] It can be shown that $X_i = M/N$ maximizes $\sum_{i=1}^{N} X_i^{\frac{1}{2}}$, subject to $M = \sum_{i=1}^{N} X_i$ for all i.

[8] Let $m = M/N$ be the optimal allocation of M within an interest group and consider any unequal distribution $m + \delta$ and $m - \delta$ $(0 < \delta < m)$ between two members. Collusive allocation of m yields $2\sqrt{m}$ votes for the two individuals and the alternative allocation yields $\sqrt{m+\delta} + \sqrt{m-\delta}$ votes. Collusion then gains $G = 2\sqrt{m} - \sqrt{m+\delta} - \sqrt{m-\delta} > 0$ votes and the amount of votes gained by collusion increases with increases in variation, δ, because $\partial G/\partial\delta = 2\delta(m^2 - \delta^2)^{-\frac{1}{2}} > 0$.

to contribute more. However, quadratic voting would appear to increase the return from political organization and collusive activity.

4.4 Implicit collusion

Even if vote buying were prohibited, the influence of money on other voters creates a legal form of "vote buying" that mimics explicit collusion. To demonstrate, let $n(I)$ $[n' > 0, n'' < 0]$ be the number of quadratic votes that can be generated by expending I to provide information. Under quadratic voting (absent money's role in providing information) a single voter with C to spend on votes can buy $\sqrt{C}$ votes. But because capital also provides information, C buys $\sqrt{C-i} + n(I_0 + i)$ votes, where I_0 is total information-allocated capital contributed by other supporters. Maximizing votes requires the proper allocation of i to each voter. Diversion of a supporter's dollar to information provision reduces quadratic votes by $1/2\sqrt{C}$ and increases information-driven votes by n'. That reallocation increases vote-buying potential only for high-value supporters—those for whom $C > 1/(2n')^2$. Low-value supporters always should buy quadratic votes. Large-value supporters should allocate capital to i until $C - i = 1/(2n')^2$, leaving each supporter to spend the same amount, $1/(2n')^2$, on quadratic votes.

What is more important, a candidate controls the allocation and she will seek capital to be used for informational purposes only from high-value supporters—leading each of them to spend the same amount for quadratic votes. Low-value supporters will be encouraged to specialize in buying quadratic votes. For the high-value voters, that outcome looks exactly like explicit collusion: they each spend the same amount on quadratic votes and use the remainder to buy votes (indirectly) from low-value voters. Moreover, only high-value voters have that opportunity—they first use their money to buy cheap quadratic votes and then spend the remainder providing information to influence others (just as they would with conventional voting).

4.5 Costs of collective action: special interests

Quadratic voting requires information about the probability of being the decisive voter; groups with superior information about the voting behavior of others and lower costs of disseminating information are likely sources of such information. Groups that are better organized, have superior channels of communication and enjoy more trust among their members can strengthen members' beliefs about the closeness of a race than can a disorganized opposition group. Better organization will generate more votes per unit of benefit. Quadratic voting would appear to increase the returns from such behavior.

5 Conclusions

Quadratic voting inefficiencies are likely for the reasons discussed herein and those inefficiencies can be quite large. To know whether quadratic voting is more efficient than conventional voting systems requires comparing equilibriums; the evidence is mixed. When the current system favors high concentrations of value (e.g., rent seeking industries or government contractors) or organized political interest groups (e.g., labor unions or Congress itself), quadratic voting will make the system worse. In policy areas for which concentrated values currently are underrepresented (e.g., gay rights, rent controls) quadratic voting can make the outcome more efficient. That is because quadratic voting imparts greater weight to value in voting.

While quadratic voting is not likely to be adopted, the recent literature on quadratic voting has served an important function. It draws our attention to how difficult it is to get optimal public policies, even if people can buy and sell votes. Studying voting inefficiencies has increased our understanding of "government failure", which should always be considered in conjunction with market failure in deciding on the best role for government in the economy.

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