

Risk lovers and the rent over-investment puzzle

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Abstract In this paper, we prove existence and uniqueness of equilibrium in a rent-seeking contest given a class of heterogeneous risk-loving players. We explore the role third-order risk attitude plays in equilibrium and find that imprudence is sufficient for risk lovers to increase rent-seeking investment above the risk-neutral outcome. Moreover, we show that rent can be fully dissipated in a standard Tullock contest played by a large number of risk-lovers.

Keywords Prudence · Risk lovers · Rent seeking · Contest

JEL Classification C72 · D81 · D72

1 Introduction

In a rent-seeking contest, individuals, groups, or institutions compete with one another to obtain a known prize or rent. Competition often manifests itself in the expenditure of resources designed to increase the likelihood of receiving the given rent. A large literature applies the theory to politics, sports and many other fields.¹ A notable strand of the relevant research centers on testing the theoretical predictions of rent-seeking models in controlled laboratory settings. However, the majority of these studies suffer from a similar yet perplexing result: subjects tend to spend more in rent-seeking effort than theory would suggest

¹ See Congleton et al. (2008) or Konrad (2009) for an extensive review.

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for risk-neutral participants. The outcome customarily is called over-investment.² Houser and Stratmann (2012) and Sheremeta (2013) provide thorough reviews of the experimental literature and the prevalence of such a seemingly irrational finding. In the survey by Sheremeta (2013), the causes of the over-investment phenomenon include bounded rationality, additional utility of winning, other-regarding preferences, and probability distortion.

Within a framework of expected utility (EU) theory, we derive conditions under which rent over-investment can occur in equilibrium. These conditions rely on both an agent's attitude towards risk in general and attitude towards downside risk, also known as prudence. Originally defined by Kimball (1990), prudence is a precautionary savings motive whereby saving increases when future income prospects become riskier. Within the context of EU theory, prudence is equivalent to a positive third derivative of the utility function.³ Eeckhoudt and Schlesinger (2006) use a concept of risk apportionment to show prudence as a preference over 50–50 lottery pairs without the EU assumptions. In their work, prudent agents demonstrate a risk location preference, effectively desiring a zero mean risk in a high income state rather than a low one. In this paper, we emphasize the counterpart of prudence, i.e., imprudence, and define it as a negative third derivative of a contestant's utility function.

In addition to the relationship between imprudence and rent over-investment, we also find that agents' risk-lovingness may cause rent over-dissipation, i.e., the sum of all players' investment exceeds the rent.⁴ The existing theories on risk attitudes and rent-seeking deal almost exclusively with risk aversion.⁵ Most studies conclude that rent-seeking expenditures decline as agents become more risk averse (relative to the risk-neutral outcome). Hillman and Katz (1984) derive this result only when the rent is "small". Without imposing a restriction on the rent's value, Cornes and Hartley (2003) find that risk aversion reduces rent-seeking expenditure under the assumption that the utility function exhibits constant absolute risk aversion (CARA). Konrad and Schlesinger (1997) propose a more generalized model that places no limits on either rent size or functional form. Considering only a symmetric solution with identical agents, the authors' ultimate conclusion suggests ambiguity in the relation between risk aversion and rent-seeking expenditure. It is only more recently that research has addressed higher-order risk effects in these types of games. While not a rent-seeking model per se, Eeckhoudt and Gollier (2005) present a loss-prevention model,⁶ which is equivalent to a non-strategic contest wherein prudent agents supply less effort to avoid a loss than their risk-neutral counterparts. Treich

² The term "over-investment" used in this paper is different from the winner's curse common to auction literature. See also footnote 4.

³ Menezes et al. (1980) make the initial reference to this third-order effect and name it downside risk aversion. Their description suggests that downside-risk-averse agents dislike a transfer of risk from higher to lower levels of wealth.

⁴ Note the difference between the terms "over-investment" and "over-dissipation". In this paper, over-investment refers to an individual's rent-seeking investment that exceeds the risk-neutral prediction, while over-dissipation refers to an outcome where participants spend more in the aggregate than the value of the available rent. Using simple numerical analysis, Tullock (1980) shows the conditions under which rents are over-dissipated, namely non-linear rent-seeking production functions and the number of risk-neutral players.

⁵ Recent experimental studies suggest that risk-lovingness may help explain over-bidding in auctions; see Gneezy and Smorodinsky (2006) and Platt et al. (2013).

⁶ Also known as the self-protection problem, first studied by Ehrlich and Becker (1972).

(2010) proves, using Eeckhoudt and Gollier's (2005) analogy, that risk aversion reduces rent-seeking effort in a symmetric contest under the condition of prudence.

Recently, Jindapon (2013) complements Eeckhoudt and Gollier's (2005) analysis by proving that risk lovers also invest in loss prevention and they will exert more effort than the risk-neutral solution if they are imprudent. Our intuition suggests that we might be able to obtain a parallel result for a rent-seeking contest, which is a motivation for this paper. However, to our knowledge, no previous contributions to the relevant literature discuss the existence of equilibrium under risk-loving preferences, although several papers do focus on the characterization of a unique equilibrium. Notably, Szidarowszky and Okuguchi (1997) find a unique equilibrium under risk neutrality, and Cornes and Hartley (2003) prove a similar result under risk aversion with CARA preferences. More recently, Treich (2010) shows the existence and uniqueness of a symmetric equilibrium under decreasing absolute risk aversion (DARA) as long as the rent is small enough.⁷

In Sect. 2, we prove that for a set of heterogeneous risk-loving players, if each player's Arrow–Pratt measure of risk aversion does not change too quickly in the relevant income domain, then a Nash equilibrium exists and it is unique. Since the risk-aversion measure is constant given a convex CARA function, $u(w) = e^{\rho w}$, with $\rho > 0$, an equilibrium always exists uniquely within that class of utility functions. Other classes of utility functions that satisfy this restriction for some initial income and rent include convex CRRA (i.e., constant relative risk aversion) functions, $u(w) = w^{\theta}$, with $\theta > 1$. Note that we neither need to assume a very small rent in the context of Taylor's approximation nor a certain kind of monotonicity of the measure of risk aversion with respect to wealth.

We compare a risk lover's rent-seeking investment in equilibrium to the risk-neutral solution in Sect. 3. We find that risk lovingness and imprudence jointly are sufficient for an increase in rent-seeking investment above the risk-neutral level. This result is important, especially for researchers who use experimental observations to estimate each individual's parametric utility function. As mentioned above, over-investment is very common in the laboratory; using a concave functional form to fit over-investment data will create discrepancies.⁸ We focus on two simple functional forms that exhibit risk-loving behavior, convex CARA and convex CRRA, and discuss how over-investment can be explained by risk lovingness and imprudence. We also present examples of rent seeking wherein the sum of all players' expenditures exceeds the rent, a phenomenon known as rent over-dissipation. Specifically, we prove that rent over-dissipation occurs when a large number of risk-loving players participate in a contest. To our knowledge, this is the first paper to show that rent over-dissipation can occur in a standard Tullock contest with EU maximizing players.⁹

We discuss our results and conclude in Sect. 4.

⁷ Cornes and Hartley (2012) conjecture that DARA players may induce a unique equilibrium. Yamazaki (2009) also considers the DARA case without the assumption of small rent value. However, we discover a flaw in his proof that restricts the existence of equilibrium to some special cases. See Appendix 2 for further discussion.

⁸ See Abbink et al. (2010) for an example.

⁹ In a standard Tullock contest, the production function is linear and the number of players and the value of the prize, which is common knowledge, are fixed. See Tullock (1980) and Higgins et al (1985) for a treatment of both linear and non-linear Tullock contests.

2 Existence of equilibrium

Consider the following n -player rent-seeking game with $n \geq 2$. Player i faces an initial endowment of income I_i for $i = 1, \dots, n$. Players compete for an exogenous rent of a fixed value $R \leq I_i$, for all i , by choosing a level of rent-seeking investment $x_i \geq 0$. For a representative player i , the probability of winning the rent follows the logistic contest success function:

$$p_i = \frac{f_i(x_i)}{\sum_{j=1}^n f_j(x_j)}, \quad (1)$$

where f_i is player i 's production function, transforming effort, here investment, into the chance of winning.

Assumption 1 $f_i(0) = 0$, $f_i'(x_i) > 0$, and $f_i''(x_i) \leq 0$ for all $x_i \geq 0$ and $i = 1, 2, \dots, n$.

The assumption above is quite intuitive and common to both the rent-seeking literature and contests in general.

If successful in winning the rent, player i 's final wealth is $I_i - x_i + R$; it otherwise will be $I_i - x_i$. Preferences over wealth are described by a utility function u_i and each player chooses x_i to maximize his expected utility,

$$Eu_i = p_i u_i(I_i - x_i + R) + (1 - p_i) u_i(I_i - x_i). \quad (2)$$

Given its significance in our results, we proceed using a methodology similar to that of Szidarowszky and Okuguchi (1997) and Cornes and Hartley (2003). If we let $y_i = f_i(x_i)$, and $Y_{-i} = Y - y_i$, the EU in (2) can now be written as

$$Eu_i = \left(\frac{y_i}{y_i + Y_{-i}} \right) u_i(I_i - g_i(y_i) + R) + \left(\frac{Y_{-i}}{y_i + Y_{-i}} \right) u_i(I_i - g_i(y_i)), \quad (3)$$

where g_i is defined as f_i^{-1} . Assumption 1 implies that $g_i(0) = 0$, $g_i'(y_i) > 0$, and $g_i''(y_i) \geq 0$ for all y_i . According to (3), player i chooses his optimal rent-seeking effort y_i , corresponding to the optimal rent-seeking investment x_i , to maximize his expected utility. We can equivalently redefine this optimization problem as one where player i chooses a pair of (y_i, p_i) to maximize

$$U_i(y_i, p_i) = p_i u_i(I_i - g_i(y_i) + R) + (1 - p_i) u_i(I_i - g_i(y_i)), \quad (4)$$

subject to the winning probability constraint

$$p_i = \frac{y_i}{y_i + Y_{-i}} =: p(y_i | Y_{-i}). \quad (5)$$

Note that $p(y_i | Y_{-i})$, player i 's probability of winning, is increasing and concave in y_i , and decreasing in Y_{-i} .

Now consider player i 's indifference curves in the (y_i, p_i) space.¹⁰ Let function q_i represent the probability of winning the rent such that, given y_i , player i 's EU is equal to a constant \bar{U} , that is,

$$q_i(y_i | \bar{U}) := \frac{\bar{U} - u_i(I_i - g_i(y_i))}{u_i(I_i - g_i(y_i) + R) - u_i(I_i - g_i(y_i))}. \quad (6)$$

¹⁰ For a graphical representation of this approach, see Tullock (1975).

Given an increasing and concave constraint $p(y_i|Y_{-i})$, if player i 's indifference curve $q_i(y_i|\bar{U})$ is strictly increasing and strictly convex in y_i , a unique best response, y_i^* , to the sum of all other player's rent-seeking production, Y_{-i} exists.¹¹ We state the following assumptions about each player's utility function to ensure this result. We denote player i 's Arrow–Pratt measure of risk aversion by $r_i(w) = -u_i''(w)/u_i'(w)$.

Assumption 2 u_i is thrice differentiable with

- (i) $u_i'(w), u_i''(w) > 0$ for all w and
- (ii) $r_i(w) \geq 2r_i(z)$ for all $w, z \in (I_i - R, I_i + R)$.

Assumption 2 (i) suggests two conditions of note. First, strictly increasing utility implies that player i 's optimal investment, x_i^* , must be less than R and thus $y_i^* < f_i(R)$. Otherwise the distribution of his final wealth after investing x_i^* will be first-order stochastically dominated by the distribution when he does not participate in the contest. Since $I_i \geq R$, as assumed at the beginning of this section and $R > x_i^*$, each player's optimal investment is not constrained by his budget. Second, we can identify the relevant income domain for each player. Since $x_i^* \in [0, R)$, each player's final wealth must be less than $I_i + R$ (if he wins the rent) and greater than $I_i - R$ (if he does not win). Thus, the relevant income domain for player i is $(I_i - R, I_i + R)$.

Assumption 2 (ii) requires that the Arrow–Pratt measure of risk aversion does not change too quickly within the relevant income domain. If we pick any pair of w and z from the interval $(I_i - R, I_i + R)$ and find that $r_i(w) \geq r_i(z)$, then $r_i(w) \geq 2r_i(z)$. On the other hand, if $r_i(w) < r_i(z)$, we will have $r_i(w) \geq 2r_i(z)$ if and only if the difference between $r_i(z)$ and $r_i(w)$, i.e., $r_i(z) - r_i(w)$, is smaller than $-r_i(z)$. Many functional forms of convex utility functions possess this property. Consider a convex CARA function with the form $u_i(w) = e^{\rho w}$, where $\rho > 0$. Note that $r_i(w) = r_i(z) = -\rho$ for all w and z so Assumption 2 (ii) always holds. Another example is a convex CRRA function with the form $u_i(w) = w^\theta$, where $\theta > 1$. Condition (ii) holds when the range of relevant income is not too large. For example, if $I_i = 4$ and $R = 1$, then $r_i(w) \in ((1 - \theta)/3, (1 - \theta)/5)$ for all $\theta > 1$ and $w \in (3, 5)$. It is not difficult to check whether a convex utility function satisfies this condition.

Lemma 1 *Suppose that Assumption 2 holds. Indifference curves $q_i(y_i|\bar{U})$ are strictly increasing and strictly convex in y_i for all $y_i \in [0, f_i(R)$.*

Proof See Appendix 1.

We adopt the approach by Cornes and Hartley (2003) to show existence and uniqueness of equilibrium when all players are risk loving. First, given an aggregate investment of other players Y_{-i} , we define each player's best response function $y_i^*(Y_{-i})$ which is the investment by player i that maximizes (3). Then we derive player i 's share function $s_i(Y^*)$, which is the probability that player i wins the contest as a function of total effort of all players in the contest given player i 's optimal decision. Thus, $Y^* = y_i^*(Y_{-i}) + Y_{-i}$. Next, we obtain the following results regarding each player's best response $y_i^*(Y_{-i})$ and share function $s_i(Y^*)$.

Lemma 2 *Suppose that Assumptions 1 and 2 hold.*

¹¹ In fact, convexity of indifference curves is not necessary for the existence of player i 's best response function given a concave constraint as in (5). For a risk averter with CARA utility functions, indifference curves are concave yet Cornes and Hartley (2003) prove that the objective function is strictly quasiconcave and therefore a global maximum exists. In the proof of Lemma 2, we show, despite the fact that u_i is strictly convex in wealth, that Eu_i is strictly quasiconcave in y_i and player i 's best response function exists.

- (i) Player i 's best-response function $y_i^*(Y_{-i})$ exists.
- (ii) There exists a threshold

$$\kappa_i := \frac{u_i(I_i + R) - u_i(I_i)}{u_i'(I_i)g_i'(0)} \tag{7}$$

such that player i 's best response has the following properties:

- (a) $y_i^*(Y_{-i}) > 0$ for all $Y_{-i} \in [0, \kappa_i)$ and
 - (b) $y_i^*(Y_{-i}) = 0$ for all $Y_{-i} \geq \kappa_i$.
- (iii) Each player's share function $s_i(Y^*)$ has the following properties:
- (a) continuous on $Y^* > 0$,
 - (b) $s_i(Y^*) \rightarrow 1$ as $Y^* \rightarrow 0$,
 - (c) $s_i(Y^*) = 0$ for all $Y^* \geq \kappa_i$, and
 - (d) strictly decreasing in Y^* for all $Y^* \in (0, \kappa_i)$.

Proof See Appendix 1.

To prove part (i) of Lemma 2, we show that, given Assumptions 1 and 2, player i 's objective in (3) is strictly quasiconcave in y_i so there can be at most one positive stationary point. If such a point exists, it is the global maximum of player i 's expected utility. Otherwise, the global maximum is at $y_i = 0$. Lemma 2 (ii) implies that, for player i , when the aggregate investment of all other players is larger than some threshold, he will no longer participate in the contest. Competitiveness in that sense reduces the probability of winning; hence, the expected marginal benefit of winning becomes prohibitively small. Consider the indifference curve that goes through the origin, i.e., $q_i(y_i|\bar{U} = u_i(I_i))$ where $u_i(I_i)$ is utility from not participating in the contest. When Y_{-i} becomes very large, the slope of the constraint $p(y_i|Y_{-i})$ at the origin will be less than the slope of the indifference curve $q_i(y_i|\bar{U} = u_i(I_i))$ at the same point. The player thus chooses $y_i = 0$, a corner solution, when $Y_{-i} \geq \kappa_i$.

It follows from part (iii) of Lemma 2 that, for any n , $\sum_{i=1}^n s_i(Y^*)$ is strictly decreasing given $Y^* \in (0, \max\{\kappa_i\})$. Moreover, $\sum_{i=1}^n s_i(Y^*) \rightarrow n$ as $Y^* \rightarrow 0$ and $\sum_{i=1}^n s_i(Y^*) \rightarrow 0$ as $Y^* \rightarrow \max\{\kappa_i\}$. Hence there exists a unique value of $Y^* \in (0, \max\{\kappa_i\})$ such that $\sum_{i=1}^n s_i(Y^*) = 1$. We call such level of aggregate effort the equilibrium level of aggregate investment, Y^e . Therefore, spending by player i in equilibrium is $y_i^e = s_i(Y^e)Y^e$. Given Lemma 2, we can state sufficient conditions for a unique equilibrium under risk-loving attitudes.

Proposition 1 *If Assumptions 1 and 2 hold, the rent-seeking game has a unique Nash equilibrium.*

3 Rent over-investment and over-dissipation

We now move to demonstrate the role third-order risk attitude plays in a rent-seeking contest. Following Kimball (1990), we say that player i is prudent (imprudent) if $u_i'''(w) > (<) 0$ for all w . We begin with an examination of a generalized model and then proceed with a more concrete example involving a standard Tullock contest. Consider again the optimization problem in Eq. (3) from the previous section. In a symmetric equilibrium (i.e., $g_i(y) = g(y)$, $u_i(w) = u(w)$, and $I_i = I$ for $i = 1, \dots, n$) each player selects

the same rent-seeking expenditure x , thereby selecting a corresponding investment y that solves¹²

$$\begin{aligned} & \left(\frac{n-1}{n^2y}\right)[u(I-g(y)+R)-u(I-g(y))] \\ & = \left[\left(\frac{1}{n}\right)u'(I-g(y)+R) + \left(\frac{n-1}{n}\right)u'(I-g(y))\right]g'(y). \end{aligned} \tag{8}$$

Therefore the optimal investment, y^* , given R and n , satisfies the following condition,

$$y^*g'(y^*) = \frac{(n-1)R}{n^2} \left[\frac{\left(\frac{1}{R}\right) \int_{I-g(y^*)}^{I-g(y^*)+R} u'(w)dw}{\left(\frac{1}{n}\right)u'(I-g(y^*)+R) + \left(\frac{n-1}{n}\right)u'(I-g(y^*))} \right]. \tag{9}$$

Under the assumption of risk neutrality, the term in brackets reduces to 1 and the solution easily simplifies to $\frac{(n-1)R}{n^2} = \tilde{y}g'(\tilde{y})$, where \tilde{y} results from the risk-neutral optimal investment \tilde{x} . We now address what role third-order risk attitude plays in equilibrium.

Proposition 2 *Suppose that Assumptions 1 and 2 hold and all players are identical. If players are imprudent, then each player’s rent-seeking investment in equilibrium exceeds the corresponding risk-neutral level.*

Proof See Appendix 1.

In the proof of Proposition 2, we show that the bracketed ratio in (9) is greater than 1 whenever $u'' > 0$ and $u''' < 0$. Since the left-hand side of (9) is strictly increasing in y , we can conclude that $y^* > \tilde{y}$. If we examine a two-player contest, the optimal rent-seeking condition in (9) can be written as

$$y^*g'(y^*) = \frac{R}{4} \left[\frac{\int_{I-g(y^*)}^{I-g(y^*)+R} u'(w)dw}{\left(\frac{1}{2}\right)[u'(I-g(y^*)+R) + u'(I-g(y^*))]}R \right] \tag{10}$$

where again, under risk neutrality, the right-hand side becomes $R/4$. However, we find that convexity of u is not relevant in determining the value of the right-hand side of (10). Specifically, a negative u''' is necessary and sufficient for the bracketed ratio in (10) to be greater than 1 so that $y^* > \tilde{y}$.¹³

Corollary 1 *Suppose that Assumptions 1 and 2 hold and all players are identical. Under $n = 2$, each player’s rent-seeking investment in equilibrium exceeds the corresponding risk-neutral level if and only if players are imprudent.*

If we consider the approach of Menezes et al. (1980) and interpret a positive third derivative of a utility function as a preference for a rightward skew, these results make intuitive sense. Prudent agents dislike additional rent-seeking expenditure because it reduces their income in the “bad” state (i.e., not winning the rent) and thereby skews the

¹² This is a special case of the first-order condition for each player’s maximization problem derived below in (26) with $s_i(Y^*) = 1/n$ for $i = 1, \dots, n$.

¹³ Consider (10). If we plot u' and draw a chord connecting $u'(I-g(y))$ and $u'(I-g(y)+R)$, we find that the area under the chord, i.e., the denominator in the brackets, is smaller than the area under u' , i.e., the numerator in the brackets for any y if and only if u' is concave, i.e., $u''' < 0$, regardless of the sign of u'' .

income distribution leftward. However, imprudent agents can tolerate a leftward skew, and therefore, spend more in rent-seeking efforts to win the “prize”. This intuition can, however, be somewhat ambiguous in the presence of risk lovingness.¹⁴ The second-order and third-order effects may impact a player in opposite directions. In particular, it is impossible to conclude whether a prudent risk-lover will invest more or less in rent seeking than a risk-neutral player when $n \geq 3$. However, in a two-player game, this is possible and, more notably, imprudence is all that is needed to induce over-investment because the second-order effect does not matter. While this is quite limiting theoretically, much of the existing experimental research focuses on two-player contests.

For further clarification, consider a symmetric contest with a standard Tullock lottery where $f(x) = x$. In equilibrium, optimal rent-seeking investment in an n -player contest derived in (9) can be re-expressed as

$$x^* = \frac{(n - 1)R}{n^2} \left[\frac{\left(\frac{1}{R}\right) \int_{I-x^*}^{I-x^*+R} u'(w)dw}{\left(\frac{1}{n}\right)u'(I - x^* + R) + \left(\frac{n-1}{n}\right)u'(I - x^*)} \right]. \tag{11}$$

Table 1 presents a summary of optimal investment in a Tullock contest with various specifications for risk preference. We assume CARA and CRRA risk-lovers, i.e., $u(w) = e^{\rho w}$ and $u(w) = w^\theta$, with $\rho = 1.5, 3$ and $\theta = 1.5, 3$. We compare each agent’s optimal investment to the risk-neutral level in symmetric contests given two, three, and six players. The values of individual investment greater than the risk-neutral level are shown in bold. In all of the examples, initial income and rent size are $I = 4$ and $R = 1$, respectively. As suggested in Proposition 2, the CRRA agent with $\theta = 1.5$ chooses to invest more than the risk-neutral player because the former is an imprudent risk-lover. Both imprudence and risk lovingness have a positive effect on the optimal investment level. On the other hand, the CRRA agent with $\theta = 3$ and the CARA agents with both $\rho = 1.5$ and $\rho = 3$ are prudent risk-lovers.¹⁵ When $n = 2$, as discussed above, risk-loving preference does not play a role, only prudence does. Since prudence has a negative effect on the optimal level of investment, these agents invest less than the risk-neutral player. However, as n increases, their risk lovingness outweighs prudence so they choose to invest more than the risk-neutral level.

Interestingly, when six players participate in the contest, each CARA player invests more than 0.2 and the corresponding aggregate investment will be greater than the rent itself. See aggregate investment in equilibrium given different number of players in Table 2.

Proposition 3 *Suppose that Assumption 2 holds, all players are identical, and $f(x) = x$. If the number of players is large enough, the aggregate investment of all players in equilibrium will exceed the value of the rent.*

The above proposition can be stated without a formal proof. Using (7) we find that in a symmetric Tullock contest, each player has the same threshold value of κ_i , which is

$$\kappa = \frac{u(I + R) - u(I)}{u'(I)}. \tag{12}$$

¹⁴ See Konrad and Schlesinger (1997) for a detailed discussion. Treich (2010) considers only prudent risk-aversers.

¹⁵ See Crainich et al. (2013) and Ebert (2013) for properties of prudent risk-lovers.

Table 1 Individual investment in symmetric Tullock contests with 2, 3, and 6 players given $I_i = 4$ and $R = 1$

Number of players (n)	CARA $\rho = 1.5$	CARA $\rho = 3$	CRRA $\theta = 1.5$	CRRA $\theta = 3$	Risk Neutral
2	0.212	0.151	0.251	0.248	0.250
3	0.239	0.192	0.231	0.239	0.222
6	0.204	0.211	0.145	0.162	0.139

The numbers in bold correspond to rent over-investment

Table 2 Aggregate investment in symmetric Tullock contests with 2, 3, and 6 players given $I_i = 4$ and $R = 1$

Number of players (n)	CARA $\rho = 1.5$	CARA $\rho = 3$	CRRA $\theta = 1.5$	CRRA $\theta = 3$
2	0.424	0.302	0.502	0.496
3	0.717	0.576	0.693	0.717
6	1.224	1.266	0.870	0.972
Minimum n for rent over-dissipation	5	5	20	7

Note that $g_i'(0) = 1$ because $f_i(x_i) = x_i$ for $i = 1, \dots, n$. Since a player will not participate in the contest if the total expenditures of all other players exceed κ , then the aggregate expenditure in equilibrium, Y^e , will be less than κ . Using the properties of each player’s share function in Lemma 2 part (ii), we find that rent over-dissipation, i.e., $Y^e > R$, may occur in equilibrium for some large n if $\kappa > R$. Specifically, we know that $s_i(Y^*)$ is strictly decreasing in Y^* and $s_i(\kappa) = 0$. If $\kappa > R$, then we can choose a large n where $1/n = s_i(Y^e)$ so that $Y^e \in (R, \kappa)$. Given (12), we find that $\kappa > R$ if and only if

$$\frac{u(I + R) - u(I)}{R} > u'(I). \tag{13}$$

Such an inequality holds for any strictly convex utility and hence the result in Proposition 3 is obtained.¹⁶

For each numerical example given in Table 2, we can calculate the minimum number of players necessary for rent over-dissipation. If we let n^* denote the smallest n such that $1/n < s_i(R)$, then we see rent over-dissipation in the same contest when the number of players is greater than n^* . In each column from left to right we find, $s_i(1) = 0.228, 0.211, 0.052$ and 0.151 ; the value of n^* for each example can be calculated and is shown in the last row of Table 2. Our theoretical prediction is consistent with Lim et al.’s (2014) experiment wherein aggregate investment increases as the number of players increases.

¹⁶ This result still holds in some generalized contests. If each player’s production function has $f_i'(0) > 1$, then we will find $g_i'(0) < 1$ and the corresponding threshold value κ will be larger than in (12).

4 Conclusion

In this paper, we prove existence and uniqueness of equilibrium in a rent-seeking contest given a class of heterogeneous risk-loving players and demonstrate conditions under which rent over-investment is possible. In a symmetric contest, we find that imprudent risk-lovers will always spend more than their risk-neutral counterparts. This conclusion can explain, within an EU framework, the “excessive” expenditures often found in the experimental literature. Moreover, we show that rent over-dissipation in equilibrium is possible given a set of risk-loving players. At the very least, future experiments may benefit from screening participants for their third-order risk attitudes. Early work from Tarazona-Gomez (2004) and more recent studies by Deck and Schlesinger (2010) and Noussair et al. (2014) develop methods to identify prudence (or imprudence) within experimental settings.

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Appendix 1: Proofs

Proof of Lemma 1

The slope of indifference curve $q_i(y_i | \bar{U})$ is given by

$$q'_i(y_i | \bar{U}) = [q_i \phi_i(I_i - g_i(y_i)) + (1 - q_i) \psi_i(I_i - g_i(y_i))] g'_i(y_i), \quad (14)$$

where

$$\phi_i = \frac{u'_i(w + R)}{u_i(w + R) - u_i(w)} \quad (15)$$

and

$$\psi_i = \frac{u'_i(w)}{u_i(w + R) - u_i(w)}. \quad (16)$$

Thus, under general conditions, the indifference curve is always upward sloping. By letting $v_i := I_i - g_i(y_i)$, we derive the second derivative of $q_i(y_i | \bar{U})$ with respect to y_i from (14) as

$$q''_i(y_i | \bar{U}) = [q_i \phi_i(v_i) + (1 - q_i) \psi_i(v_i)] g''_i(y_i) + [q_i \Phi_i(v_i) + (1 - q_i) \Psi_i(v_i)] [g'_i(y_i)]^2, \quad (17)$$

where

$$\Phi_i(w) = \phi_i(w) [\phi_i(w) - \psi_i(w)] - \phi'_i(w) \quad (18)$$

and

$$\Psi_i(w) = \psi_i(w) [\phi_i(w) - \psi_i(w)] - \psi'_i(w). \quad (19)$$

The first term on the right-hand side of (17) is positive; however, the sign of the second term is uncertain. To guarantee that it is positive we assume (i) $\varphi'_i(w) \leq \varphi_i(w)$ [$\varphi_i(w) - \psi_i(w)$] and (ii) $\psi'_i(w) \leq \psi_i(w)[\varphi_i(w) - \psi_i(w)]$ for all $w \in (I_i - R, I_i)$. Even though Conditions (i) and (ii) are stated in terms of φ_i and ψ_i instead of u_i , their intuition is quite simple. Using (15) and (16), we can write conditions (i) and (ii) as

$$\frac{u''_i(w + R)}{u'_i(w + R)} \leq 2 \frac{u'_i(w + R) - u'_i(w)}{u_i(w + R) - u_i(w)} \tag{20}$$

and

$$\frac{u''_i(w)}{u'_i(w)} \leq 2 \frac{u'_i(w + R) - u'_i(w)}{u_i(w + R) - u_i(w)} \tag{21}$$

for all $w \in (I_i - R, I_i)$ respectively. Thus, both inequalities can be satisfied if

$$\frac{u''_i(w)}{u'_i(w)} \leq 2 \frac{u''_i(z)}{u'_i(z)} \tag{22}$$

for all $w, z \in (I_i - R, I_i + R)$.

Proof of Lemma 2

Part (i) Given the optimization problem in (3), an interior solution y_i^* satisfies the first-order condition:

$$F(y_i^*) := \frac{Y_{-i}}{(y_i^* + Y_{-i})^2} [u_i(I_i - g_i(y_i^*) + R) - u_i(I_i - g_i(y_i^*))] - \left[\left(\frac{y_i^*}{y_i^* + Y_{-i}} \right) u'_i(I_i - g_i(y_i^*) + R) + \left(\frac{Y_{-i}}{y_i^* + Y_{-i}} \right) u'_i(I_i - g_i(y_i^*)) \right] g'_i(y_i^*) = 0. \tag{23}$$

For the equivalent constrained-optimization problem given by (4) and (5), an interior solution (y_i^*, p_i^*) satisfies the tangency condition and the constraint:

$$\frac{Y_{-i}}{(y_i^* + Y_{-i})^2} - [p_i^* \phi_i(v_i^*) + (1 - p_i^*) \psi_i(v_i^*)] g'_i(y_i) = 0 \tag{24}$$

$$p_i^* - \frac{y_i^*}{y_i^* + Y_{-i}} = 0, \tag{25}$$

where $v_i^* = I_i - g_i(y_i^*)$ and φ_i and ψ_i are defined in (15) and (16), respectively. Substituting p_i^* from (25) into (24) yields

$$G(y_i^*) := \frac{Y_{-i}}{(y_i^* + Y_{-i})^2} - \left[\left(\frac{y_i^*}{y_i^* + Y_{-i}} \right) \phi_i(v_i^*) + \left(\frac{Y_{-i}}{y_i^* + Y_{-i}} \right) \psi_i(v_i^*) \right] g'_i(y_i) = 0. \tag{26}$$

Note that $G(y_i)$ is the difference between the slope of the constraint and the slope of indifference curve at $(y_i, p(y_i|Y_{-i}))$. Since the former is decreasing (in fact, strictly decreasing when $Y_{-i} > 0$) and the latter is strictly increasing (see Lemma 1), $G(y_i)$ is strictly decreasing in y_i . If Y_{-i} is less than a threshold (derived in part (ii) below), $G(y_i) > 0$

when $y_i = 0$. Moreover, we find that $G(y_i) < 0$ when y_i is large enough. There consequently exists only one value y_i^* such that $G(y_i^*) = 0$.

Next, we define

$$\Delta(y_i^*) := u_i(I_i - g_i(y_i^*) + R) - u_i(I_i - g_i(y_i^*)). \tag{27}$$

Since u' and u'' are strictly positive, we know that $\Delta(y_i)$ is strictly positive and strictly decreasing in y_i . Given the definitions of $G(y_i)$ and $\Delta(y_i)$, we can write the first-order condition in (23) as

$$F(y_i^*) = G(y_i^*)\Delta(y_i^*) = 0. \tag{28}$$

The properties of $G(y_i)$ and $\Delta(y_i)$ described above imply that $F(y_i)$ is strictly decreasing for all $y_i \leq y_i^*$ and that y_i^* is the only value of y_i such that the first-order condition holds. Thus we can say that $F(y_i) \geq (\leq) 0$ if and only if $y_i \leq (\geq) y_i^*$; it follows that Eu_i in (3) is strictly quasiconcave in y_i . Since budgets do not constrain each player’s investment, the only possible corner solution would be at zero, which occurs only when Y_{-i} is so large that y_i^* satisfying the first-order condition is negative. If this is the case, then we know that a positive solution to the first-order condition in (3) does not exist because $F(y_i) < 0$ for all $y_i \geq 0$, so player i ’s optimal decision is setting $y_i = 0$.

Part (ii) We find the status quo level of player i ’s utility with no investment and no chance of winning to be $u_i(I_i)$. The constrained optimization problem yields an interior solution, i.e., $y^* > 0$ given Y_{-i} , whenever the slope of the indifference curve $q_i(y_i|\bar{U} = u_i(I_i))$ at $y_i = 0$ is less than the slope of the constraint $p(y_i|Y_{-i})$ at $y_i = 0$. Since $q'_i(y_i|\bar{U})$ is given by (14) and $p'(y_i|Y_{-i}) = \frac{Y_{-i}}{(y_i+Y_{-i})^2}$, we have $q'_i(y_i|\bar{U} = u_i(I_i)) < p'(y_i|Y_{-i})$ at $y_i = 0$ if and only if

$$\psi_i(I_i)g'_i(0) < \frac{1}{Y_{-i}}. \tag{29}$$

By letting $\kappa_i = 1/[\psi_i(I_i)g'_i(0)]$, we find that $y^* > 0$ if and only if $Y_{-i} < \kappa_i$.

Part (iii) Define player i ’s share function, the probability that player i wins the contest given his optimal investment as $s_i(Y^*)$ where $Y^* = (Y_{-i}) + Y_{-i}$. Since $y_i^*(0) > 0$, $Y^* = 0$ is never an equilibrium and we do not need to define $s_i(0)$. Using the constraint (25), we rewrite the tangency condition (24) as

$$\frac{1 - p_i^*}{Y^*} - [p_i^* \phi_i(v_i^*) + (1 - p_i^*) \psi_i(v_i^*)] g'_i(v_i^*) = 0, \tag{30}$$

which can be rearranged as

$$p_i^* = \frac{[1 - \psi_i(v_i^*) g'_i(v_i^*) Y^*]}{[1 - \psi_i(v_i^*) g'_i(v_i^*) Y^*] + \phi_i(v_i^*) Y^*}. \tag{31}$$

Therefore, we have

$$s_i(Y^*) = \max \left\{ \frac{[1 - \psi_i(v_i^*) g'_i(v_i^*) Y^*]}{[1 - \psi_i(v_i^*) g'_i(v_i^*) Y^*] + \phi_i(v_i^*) Y^*}, 0 \right\}. \tag{32}$$

We find that s_i is a continuous function of Y^* for $Y^* > 0$ and that 1 is its upper bound. In addition, as Y^* approaches 0, s_i converges to 1. From part (ii) we know that $y^* = 0$ if $Y_{-i} \geq \kappa_i$, where $\kappa_i = 1/[\psi_i(I_i)g'_i(0)]$. Thus we can say that $s_i = 0$ if $Y^* \geq \kappa_i$, which is consistent with the numerator of (32). To show (d) we totally differentiate the system of Eqs. (24) and (25) with respect to Y_{-i} .

$$\frac{dp_i^*}{dY_{-i}} = \frac{-\frac{Y_{-i}}{Y^{*4}} - \frac{Y_i^*}{Y^{*2}} [p_i^* \phi_i(v_i^*) + (1 - p_i^*) \psi_i(v_i^*)] g''_i(y_i^*) + \frac{Y_i^*}{Y^{*2}} [p_i^* \phi'_i(v_i^*) + (1 - p_i^*) \psi'_i(v_i^*)] [g'_i(y_i)]^2}{q_i''(y_i^* | u_i^*) - p''(y_i^* | Y_{-i})} \tag{33}$$

$$\frac{dy_i^*}{dY_{-i}} = \frac{\frac{Y_i^* - Y_{-i}}{Y^{*3}} + \frac{Y_i^*}{Y^{*2}} [\phi_i(v_i^*) - \psi_i(v_i^*)] g'_i(y_i^*)}{q_i''(y_i^* | u_i^*) - p''(y_i^* | Y_{-i})} \tag{34}$$

where u_i^* is the optimal EU corresponding to y_i^* . Since $q_i''(y_i^* | u_i^*) \geq 0$ and

$$p''(y_i^* | Y_{-i}) = \frac{-2Y_{-i}}{(y_i^* + Y_{-i})^3} < 0, \tag{35}$$

the denominator of each equation is always positive. Consider the numerator of (33). The middle term is nonnegative because u_i is convex and g_i is concave. Thus $\frac{dp_i^*}{dY_{-i}} < 0$ if

$$\frac{Y_i^*}{Y^{*2}} [p_i^* \phi'_i(v_i^*) + (1 - p_i^*) \psi'_i(v_i^*)] [g'_i(y_i)]^2 < \frac{Y_{-i}}{Y^{*4}}. \tag{36}$$

Substituting $g'_i(y_i)$ from (24) and $\frac{Y_i^*}{Y^*}$ from (24) in (36) yields

$$p_i^*(1 - p_i^*) [p_i^* \phi'_i(v_i^*) + (1 - p_i^*) \psi'_i(v_i^*)] < [p_i^* \phi_i(v_i^*) + (1 - p_i^*) \psi_i(v_i^*)]^2. \tag{37}$$

Conditions (i) and (ii) in the proof of Lemma 1 guarantee that (37) holds, so $\frac{dp_i^*}{dY_{-i}} < 0$ for all $Y_{-i} \in (0, \kappa_i)$. Now consider (34). Substituting $q_i''(y_i^* | u_i^*)$ from (17) and $p''(y_i^* | Y_{-i})$ from (35) into (34) yields $\frac{dy_i^*}{dY_{-i}} > -1$. Given the definition of Y^* above, it follows that $\frac{dY^*}{dY_{-i}} = \frac{dy_i^*}{dY_{-i}} + 1$.

Since $\frac{dy_i^*}{dY_{-i}} > -1$, then $\frac{dY^*}{dY_{-i}} > 0$. Given $\frac{dp_i^*}{dY_{-i}} < 0$, it follows that $\frac{ds_i}{dY_{-i}} < 0$ for all $Y^* \in (0, \kappa_i)$.

Proof of Proposition 2

Using a generalization of Hermite–Hadamard’s inequality given by Theorem 5.11 in Pěčarić et al. (1992), we have

$$\left(\frac{1}{n}\right) u'(I - x + R) + \left(\frac{n - 1}{n}\right) u'(I - x) < \frac{n}{2R} \int_{I-x}^{I-x+2R/n} u'(w) dw \tag{38}$$

when $u''' < 0$. Using Steffensen’s inequality given by Theorem 6.19 in Pěčarić et al. (1992), we find

$$\int_{I-x}^{I-x+2R/n} u'(w)dw < \frac{2}{n} \int_{I-x}^{I-x+R} u'(w)dw \tag{39}$$

when $u'' > 0$. Thus, (38) and (39) jointly imply

$$\left(\frac{1}{n}\right)u'(I-x+R) + \left(\frac{n-1}{n}\right)u'(I-x) < \frac{1}{R} \int_{I-x}^{I-x+R} u'(w)dw \tag{40}$$

and the bracketed term in (9) is greater than 1. Proposition 2 clearly follows.

Appendix 2: A note on equilibrium in Yamazaki (2009)

In the proof of Lemma 1 found in Yamazaki (2009), the author attempts to confirm the convexity of an indifference curve G by examining its first partial derivative with respect to y_i :

$$\frac{\partial G_i}{\partial y_i} = \frac{g'_i(y_i)}{u_i(z_i^H) - u_i(z_i^L)} \left[u'_i(z_i^L) + \frac{\bar{U} - u_i(z_i^L)}{u_i(z_i^H) - u_i(z_i^L)} (u'_i(z_i^H) - u'_i(z_i^L)) \right], \tag{41}$$

where z_i^H denotes income with the rent and z_i^L is income without it, with all other variables being defined previously. The author incorrectly claims $u_i(z_i^H) - u_i(z_i^L)$ to be non-increasing in y_i when the difference is actually increasing in y_i under the assumption of risk aversion. Therefore we cannot conclude that indifference curves are convex and the existence of a unique equilibrium under DARA breaks down.

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