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The Condorcet jury theorem and extension of the franchise with rationally ignorant voters

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Abstract The Condorcet Jury Theorem (CJT), which provides a justification for democracy, is based on voters who are imperfectly informed insofar as they know the correct policy with a probability of less than one but greater than one-half. We reassess the consequences of the CJT for democracy when extension of the franchise adds equal numbers of non-distinguishable informed and uninformed voters to the collective decision making group. Uninformed voters vote correctly with probability one-half. We show that adding equal numbers of informed and uninformed voters maintains the CJT conclusion that enlarging the group of decision makers increases the likelihood of a correct collective decision.

Keywords Condorcet jury theorem · Rational ignorance · Expressive voting · Franchise

JEL Classification H11 · P16

1 Introduction

In 1785, the Marquis de Condorcet[1] formulated what is now known as the Condorcet Jury Theorem (CJT), which states that a group of decision makers that utilizes a simple majority rule is more likely to make a correct collective decision than any one of its members taken alone, and that the likelihood of a correct decision becomes certain as group size tends to infinity (known as the asymptotic part of the theorem). The CJT provides a justification for broad democratic participation in collective decision making when all voters seek the same objective but differ in beliefs regarding the correct means of achieving the objective (Hillman 2009). In the simplest form of the CJT, a group of decision makers votes independently on a binary choice with each voter having the same probability $p > 1/2$ of choosing the correct alternative. The Theorem indicates that, the larger is the group, the better the group performs in terms of the likelihood of making the correct decision by majority voting. The Theorem

[1] Condorcet (1785).

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has previously been generalized in various ways.[2] Our generalization in this paper focuses on the basic conclusion of the CJT regarding the justification for democracy. We prove a theorem indicating that extension of the franchise to include equal numbers of informed and rationally uninformed voters sustains the conclusion that an increase in the number of voters increases the probability of reaching the correct decision through majority voting. An informed individual is of the type described by the Theorem in knowing the correct alternative (for example an appropriate economic policy) with probability greater than one-half. An uninformed individual is rationally ignorant and votes randomly, making a correct decision with a probability of one-half. Rational ignorance is therefore not an impediment to efficiency through extension of the franchise, provided that rationally ignorant voters are matched by imperfectly informed voters.[3]

Formally, we enlarge the collective decision making group by adding an informed member whose probability of voting correctly is $p > 1/2$ and an uninformed member whose probability of choosing correctly is $p = 1/2$.[4] Within this framework, we show that larger groups continue to perform better in being more likely to reach correct collective decisions. Our conclusion is valid regardless of the probability that informed members make the correct decision and regardless of the proportion of informed and uninformed members in the original group. Our result is interesting analytically because generally the addition of uninformed members decreases the probability that the group will choose the correct alternative. This is the reason for the basic assumption of the Condorcet Jury Theorem that all members of a collective-decision making group have a probability of more than one-half of choosing the correct alternative—that is, that all are informed in this sense.[5] We proceed now to set out the model and prove our result.

[2]Early expositions and generalizations were by Hoeffding (1956), Grofman (1975), Grofman et al. (1983), Feld and Grofman (1984), Nitzan and Paroush (1982, 1985), Young (1988), Owen et al. (1989), Boland (1989). Ladha (1992, 1993, 1995) and Berg (1993) relaxed the independence assumption; Austen-Smith and Banks (1996) and Ben-Yashar (2006) generalized to a strategic voting model; Louis and Ching (1996) investigated a polychotomous setting; and Miller (1986) explored the case of conflicting interests in a two party electorate. Kanazawa (1998) showed that heterogeneous groups perform better than homogeneous groups. Paroush (1998) emphasized the importance of boundedness away from one-half; Berg and Paroush (1998) investigated hierarchical voting; and Berend and Paroush (1998) formulated necessary and sufficient conditions for outcomes in heterogeneous groups. Ben-Yashar and Paroush (2000) generalized the non-asymptotic part of the Theorem. Karotkin and Paroush (2003) modeled a trade-off between quality and quantity when members are added to the group. Berend and Sapir (2005, 2007) extended the analysis of Ben-Yashar and Paroush to further generalize the non-asymptotic part of the Theorem, and Baharad and Ben-Yashar (2009) investigated the validity of the CJT under subjective probabilities. For an overview of decision theory to which the CJT is central, see Gerling et al. (2005).

[3]On extensions of the franchise, see for example Husted and Kenny (1997), Lizzeri and Persico (2004), and Aidt et al. (2006). Much of the literature on franchise extension has focused on extension of the franchise to women and the different objectives that women seek from government compared to men (see, for example, Aidt and Dallal 2008). The CJT does not apply when objectives differ. In the case that we study, as in the original theorem, there is a common objective but people are uncertain about the appropriate means of achieving the objective.

[4]Other papers have considered the addition of two members to a heterogeneous group. Feld and Grofman (1984) asked whether, when two groups are combined, the probability of the enlarged group reaching the correct decision is a function of the average, median, or majority competence of the original group in cases where individual competence can be greater than or less than one-half. Karotkin and Paroush (2003) provided sufficient conditions for the addition of two members to a group increasing group competence when the probability of each member in the new group making the correct decision is q and the corresponding probability of each member in the original group is p, where $p > q$.

[5]Paroush (1998) showed that the asymptotic part of the CJT is valid if the probability of each individual in a heterogeneous group choosing the correct alternative satisfies $p > 1/2 + \varepsilon$ ($\varepsilon > 0$). It is certainly possible that the addition of two new members will decrease the probability of the group making the correct decision even

2 The model and the result

There are $n = 2k + 1$ individuals in a collective-decision making group; n is an odd number greater than one. The choice is between two alternatives, one of which is preferred by all individuals $i \in N = \{1, \dots, n\}$. The identity of the preferred alternative is however unknown. An informed voter i chooses the preferred alternative with probability $p_i = \lambda$ where $1/2 < \lambda < 1$. An uninformed voter makes the correct choice with probability $p_i = 1/2$. The vector $\underline{P} = \{\lambda, \lambda, \dots, \lambda, 1/2, 1/2, \dots, 1/2\}$ defines the group's collective competence. Preferences are identical and all individuals therefore want the same correct collective decision to be made. Individuals' probabilities of voting for the correct alternative are not statistically correlated and simple majority rule is used to aggregate votes to determine the collective decision. There are n_1 informed voters and n_2 uninformed voters, making total group size $n = n_1 + n_2$.

Following Ben-Yashar and Paroush (2000), we first present the probability that a group with n members and competency vector $\underline{P}$ will choose the correct alternative under the simple majority rule $\pi(\underline{P}, n)$. Let $\pi(\underline{P}, n)$ be the product of probabilities summed over cases that result in correct majority voting. With $n = 2k + 1$:

$$\pi(\underline{P}, 2k+1) = \sum_{S \in S^N_{k\uparrow}} \prod_{i \in S} p_i \prod_{j \notin S} (1 - p_j),$$

where

$$S^T_{a\uparrow} = \{S | S \subseteq T, |S| > a\}.$$

We use the following notation:

$$N' = N \backslash \{i, j\} = \{1, \dots, i-1, i+1, \dots, j-1, j+1, \dots, n\}$$

$$S^T_a = \{S | S \subseteq T, |S| = a\}.$$

By isolating members p_i, p_j from the original group, we can rewrite $\pi(\underline{P}, 2k+1)$ as:

$$\pi(\underline{P}, 2k+1) = p_i p_j A + (1 - (1 - p_i)(1 - p_j))B + C,$$

where A is the sum of all products of $k - 1$ probabilities p_t $(t \neq i, j)$ and the remaining $(1 - p_l)$ terms $(l \neq i, j, t)$:

$$A = \sum_{S \in S^{N'}_{k-1}} \prod_{t \in S} p_t \prod_{l \in N' \backslash S} (1 - p_l),$$

B is the sum of all products of k probabilities p_t $(t \neq i, j)$ and the remaining $(1 - p_l)$ terms $(l \neq i, j, t)$:

$$B = \sum_{S \in S^{N'}_{k}} \prod_{t \in S} p_t \prod_{l \in N' \backslash S} (1 - p_l),$$

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if their competencies satisfy this condition. Our paper does not allow for full heterogeneity. We consider only two types of group members, informed and uninformed, and ask how the probability of the group choosing the correct alternative changes with the addition of a pair composed of one of each of these types, when the original group contains at least one uninformed member. It would certainly be interesting to investigate the validity of the asymptotic part of the CJT within our framework in future research.

C is the sum of products of more than k, up to $n-2$, probabilities p_t $(t \neq i, j)$ and the remaining $(1-p_l)$ terms $(l \neq i, j, t)$:

$$C = \sum_{S \in S_{k\uparrow}^{N'}} \prod_{t \in S} p_t \prod_{l \in N' \setminus S} (1-p_l).$$

Note that A, B, and C include neither p_i nor p_j.

The probability that the group will choose the correct alternative without members p_i, p_j (i.e., a group of $2k-1$ members) is $B+C$. Therefore, adding members p_i, p_j improves the performance of the group *iff*

$$p_i p_j A + (1-(1-p_i)(1-p_j))B + C > B + C. \tag{1}$$

We wish to determine whether, when the franchise is extended by adding an imperfectly informed and an uninformed voter, the probability increases that majority voting will result in the correct decision. When $p_j = 1/2$ and $p_i = \lambda$, condition (1) reduces to:

$$\frac{\lambda}{1-\lambda} A > B. \tag{2}$$

Condition (2) determines whether the addition of the imperfectly informed and uninformed voters improves the voting performance of the group. In order to demonstrate the meaning of condition (2), we need to express the terms A and B explicitly for the case where the original group contains informed and uninformed voters. In order to do so (see Ben-Yashar and Danziger 2010), we denote the function g as the probability that, of I members each of whom has probability p of making the correct assessment, exactly i make the correct decision:

$$g(i, I, p) \equiv \binom{I}{i} p^i (1-p)^{I-i}. \tag{3}$$

The function Γ that now follows is the probability that, in a decision-making group with n_1 informed members and n_2 uninformed members, exactly M make the correct assessment:

$$\Gamma(n_1, n_2, M) \equiv \sum_{i=0}^{M} \left[g(i, n_1, \lambda) g\left(M-i, n_2, \frac{1}{2}\right)\right]. \tag{4}$$

We use (3) and (4) to rewrite the terms A and B as:

$$A = \Gamma\left(n_1-1, n_2-1, \frac{n-3}{2}\right) \equiv \sum_{i=0}^{\frac{n-3}{2}} \left[g(i, n_1-1, \lambda) g\left(\frac{n-3}{2} - i, n_2-1, \frac{1}{2}\right)\right], \tag{5}$$

$$B = \Gamma\left(n_1-1, n_2-1, \frac{n-1}{2}\right) \equiv \sum_{i=0}^{\frac{n-1}{2}} \left[g(i, n_1-1, \lambda) g\left(\frac{n-1}{2} - i, n_2-1, \frac{1}{2}\right)\right]. \tag{6}$$

Lemma *The term B can be expressed as*

$$B = \sum_{i=0}^{\frac{n-1}{2}} \left[g(i, n_1 - 1, \lambda) g\left(\frac{n-1}{2} - i, n_2 - 1, \frac{1}{2}\right)\right]$$

$$= \sum_{i=0}^{\frac{n-3}{2}} \left[g(i, n_1 - 1, \lambda) g\left(\frac{n-3}{2} - i, n_2 - 1, \frac{1}{2}\right) X_i\right],$$

where

$$X_i = \left[(n_1 - 1 - i)\frac{\lambda}{1-\lambda} + \left(n_2 - 1 - \frac{n-3}{2} + i\right)\right] \frac{2}{n-1}. \tag{7}$$

The proof to this Lemma is shown in Ben-Yashar and Danziger (2010).
Denote:

$$m_i = g(i, n_1 - 1, \lambda) g\left(\frac{n-3}{2} - i, n_2 - 1, \frac{1}{2}\right).$$

Rewriting inequality (2), which is the condition that guarantees improved (i.e., more accurate) group performance in our setting, and using the Lemma, results in:

$$\frac{\lambda}{1-\lambda} \sum_{i=0}^{\frac{n-3}{2}} m_i > \sum_{i=0}^{\frac{n-3}{2}} m_i X_i. \tag{8}$$

We now state our basic conclusion as a theorem.

Theorem *Adding an informed member* ($p_i = \lambda$) *and an uninformed member* ($p_i = 1/2$) *simultaneously to a group that contains either informed or uninformed members* (*with at least one uninformed member*) *increases the probability that the group will choose the correct alternative.*

Proof A sufficient condition for inequality (8) to hold is that $\forall i, 0 \le i \le \frac{n-3}{2}$ whenever $m_i \neq 0$, the following inequality holds:

$$\frac{\lambda}{1-\lambda} \ge X_i,$$

where, for at least one index of i, this inequality is strict. We will now show that this inequality holds:

$$\frac{\lambda}{1-\lambda} \ge X_i$$

$$\Leftrightarrow \frac{\lambda}{1-\lambda} \ge \left[(n_1 - 1 - i)\frac{\lambda}{1-\lambda} + \left(n_2 - 1 - \frac{n-3}{2} + i\right)\right] \frac{2}{n-1}$$

$$\Leftrightarrow \frac{n-1}{2} \ge (n_1 - 1 - i) + \left(n_2 - 1 - \frac{n-3}{2} + i\right)\frac{1-\lambda}{\lambda}$$

$$\Leftrightarrow \frac{n-1}{2} - (n_1 - 1 - i) \ge \left(n_2 - 1 - \frac{n-3}{2} + i\right)\frac{1-\lambda}{\lambda}.$$

Since $\frac{n-1}{2} - (n_1 - 1 - i) = n_2 - 1 - \frac{n-3}{2} + i$ we can rewrite the last inequality as:

$$\left(n_2 - 1 - \frac{n-3}{2} + i\right)\left(1 - \frac{1-\lambda}{\lambda}\right) \geq 0. \tag{9}$$

Since $\frac{1-\lambda}{\lambda} < 1$, inequality (9) holds only if $i \geq \frac{n_1 - n_2 - 1}{2}$.

Note that $i \geq \frac{n_1 - n_2 - 1}{2}$ is a necessary (not sufficient) condition for $m_i \neq 0$.

In order to finalize the proof, we need to show that there exists at least one index of i for which the inequality is strict, i.e.,

$$i > \frac{n_1 - n_2 - 1}{2}.$$

We therefore need to show that there exists an i that ensures $m_i \neq 0$ and therefore $i \leq n_1 - 1$ and that such i is in the relevant range, i.e.,

$$i \leq \frac{n-3}{2}.$$

These conditions are satisfied whenever $n > 1$, which is our assumption, and whenever $n_2 > 1$, that is, when there is at least one uninformed member in the original group. □

3 Intuition

Feld and Grofman (1984) conjectured that adding pairs of persons with competence λ and 1/2 cannot diminish group competence. We shall provide a formal proof consistent with their intuition. Although this proof is more intuitive and simple than the proof in the previous section, the former is more general and allows us to derive further results. The intuition for our conclusion is provided by a two-step decomposition: In the first step we add an informed person (with probability λ) to an odd numbered group that contains voters characterized only by probabilities λ and 1/2: the competence of the group must then increase. In the second step we add an uninformed member to an even numbered group and the competence of the group does not change.

Proceeding with the first step, we have:

Proposition *Adding an informed person to an odd numbered group consisting of informed and uninformed members with at least one uninformed member necessarily increases the competence of the group.*

Proof Note that $\pi(\underline{P}, 2k+1)$ is the probability that an odd numbered group with $2k+1$ members reaches the correct decision, and denote by $\pi(\underline{P}^{+\lambda}, 2k+2)$ the probability of a correct decision by the same group subsequent to adding an informed member. One can write $\pi(\underline{P}^{+\lambda}, 2k+2)$ as the sum of probabilities of a correct and incorrect decision made by the added informed member, i.e.,

$$\pi(\underline{P}^{+\lambda}, 2k+2) = \lambda\left[\pi(\underline{P}, 2k+1) + \frac{1}{2}\Gamma(n_1, n_2, k)\right] + (1-\lambda)\left[\pi(\underline{P}, 2k+1) - \frac{1}{2}\Gamma(n_1, n_2, k+1)\right].$$

Note that $n_1 + n_2 = 2k + 1$. Hence:

$$\pi(\underline{P}^{+\lambda}, 2k+2) = \pi(\underline{P}, 2k+1) + \frac{1}{2}[\lambda\Gamma(n_1, n_2, k) - (1-\lambda)\Gamma(n_1, n_2, k+1)]. \tag{10}$$

Since

$$\frac{\lambda}{1-\lambda} > \frac{\Gamma(n_1, n_2, k+1)}{\Gamma(n_1, n_2, k)}, \tag{11}$$

it follows that

$$\pi(\underline{P}^{+\lambda}, 2k+2) > \pi(\underline{P}, 2k+1). \tag{12}$$

Note that inequality (11) holds under the same conditions as condition (2), which we have proved is valid. □

Proposition *Adding an uninformed member to an even numbered group consisting of informed and uninformed members does not change the performance of the group.*

Proof We again express the performance of an odd numbered group with $2k + 3$ members in reaching the correct decision as the sum of probabilities of correct and incorrect decisions made by the added uninformed member:

$$\frac{1}{2}\left[\pi(\underline{P}^{+\lambda}, 2k+2) + \frac{1}{2}\Gamma(n_1+1, n_2, k+1)\right] + \frac{1}{2}\left[\pi(\underline{P}^{+\lambda}, 2k+2) - \frac{1}{2}\Gamma(n_1+1, n_2, k+1)\right].$$

One can easily see that the latter equals:

$$\pi(\underline{P}^{+\lambda}, 2k+2).$$ □

The explanation to this result is that adding an uninformed member to an even numbered group will not change the probability that the group chooses the correct alternative, because the addition of an individual to an even numbered group can alter the outcome only if half the group members vote for one alternative, while the other half votes for the second alternative. In this case the alternative is chosen in accord with the decision of the added member who has a probability of one half of being correct. This probability is similar to the probability of an even numbered group (without this member) reaching the correct alternative since, when half the group chooses one alternative, while the other chooses a different alternative, one of the alternatives is randomly chosen. According to our result, one can see that, if $\lambda = 1/2$, i.e., we add two uninformed members to the group, the probability that the group decides correctly necessarily decreases.

4 Conclusions

The result that we have demonstrated is not found in previous literature on the Condorcet Jury Theorem. The addition of two informed members always increases a group's competence: we do not need to prove this statement, since, if the addition of informed and uninformed members always improves the competence of the group, then it is straightforward that adding two informed members will also improve the group's competence. Correspondingly, adding two uninformed members will necessarily diminish the competence of the collective-decision making group.[6] It is therefore not trivial to determine whether or not the addition of one informed member and one uninformed member improves a group's competence. We have shown that enlarging a collective decision-making group by adding an imperfectly informed and uninformed voter sustains the basic conclusion of the CJT that a larger number of voters is more likely to make the correct collective decision through majority voting. The validity of our conclusion is independent of the proportion of the pre-existing number of informed and uninformed members in the collective decision making group. In practical terms, democracy is not encumbered by rational ignorance on a part of members of a population of voters, provided that rationally ignorant voters are matched by imperfectly informed voters.

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[6] In this case, with $p_i = p_j = 1/2$, if we use condition (1), we find that the competence of the group will improve only if $A > B$ but it is known that $A < B$ is always true. See Ben-Yashar and Danziger (2010).

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