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Information is important to Condorcet jurors

RUTH BEN-YASHAR
Department of Economics, Bar-Ilan University, 52900 Ramat-Gan, Israel;
E-mail: benyasr@mail.biu.ac.il

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Abstract. Group decision making is very significant in a broad variety of settings. This paper deals with committees that make binary decisions and addresses the question of whether informative decisions can be assumed within this framework. We show that when using the optimal decision rule, informative decision making is a Nash equilibrium. Thus we justify the assumption of informative decision making and provide support for the relevance of assumptions such as independent decision making, when using the optimal decision rule.

1. Introduction

Group binary decision making is very significant in economics, medicine, law and other disciplines. For example, juries deciding whether to acquit or convict a defendant, committees considering job candidates or projects, populations choosing between two policies, and specialists deciding whether to carry out a medical procedure. In all these cases, the committee members share a common task. However, the members can each make different decisions, since they receive different information and their abilities to absorb, process and interpret this information are not identical.

This paper considers a jury framework with two states (guilty or innocent), each with an *a priori* probability and two possible decisions (conviction or acquittal). There are therefore four possible final decisions, each associated with specific utilities: two correct decisions (conviction of guilty and acquittal of innocent defendants) and two incorrect ones (acquittal of guilty and conviction of innocent defendants). The committee consists of several decision-makers with the same preferences who seek to maximize the expected utility. Each decision-maker receives a signal from the same distribution function and makes a binary decision, i.e., whether to convict or acquit the defendant. The final decision is reached by a decision rule, and is based on all the individual decisions and environmental information that depends on the *a priori* probabilities and utilities.

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This paper follows Condorcet's (1785) approach. Condorcet developed a formal framework for efficient aggregation of all information from all individuals (Piketty, 1999). The Condorcet Jury Theorem defines the conditions under which the majority rule is an efficient aggregation of all information from all individuals. This theorem can be extended to more general cases, which include decision-makers receiving different quality signals and multiple-stage decisions. Condorcet's theoretical results and calibrations are simple, yet they provide the basis for modern economic modelling, which aims at finding the optimal decision rule, i.e., the rule which affords efficient information aggregation.

Nitzan and Paroush (1982) defined the optimal decision rule, i.e., the decision rule that maximizes the expected utility, in the context of dichotomous symmetric choice. Ben-Yashar and Nitzan (1997) defined the optimal decision rule in a more general framework, which allows asymmetric choice. Other studies analyzed the optimal decision rule under constraints (Ben-Yashar, Khuller and Kraus, 2001; Ben-Yashar and Kraus, 2002) and other aspects of collective decision-making.[1] All the above studies assume that each individual decides exclusively according to his/her signal, i.e., informatively. Several studies have taken strategic decisions into consideration generalizing Condorcet's approach (Austen Smith and Banks, 1996; Feddersen & Presendorfer, 1996, 1997,1998; Myerson, 1998).[2] These studies demonstrated that informative decisions can be inefficient. For example, it can be shown that jurors make strategic decisions, i.e., non-informative decisions, in order to increase the expected utility (e.g. see Piketty, 1999).

In this paper we justify informative decision making, by showing that when using the optimal decision rule, informative decisions are Nash equilibria. An individual would not find it worthwhile to deviate from his/her signal if all the other committee members decide informatively. Note that, although informative decisions can be Nash equilibria, it is possible that a group of committee members can increase the expected utility by deciding non-informatively. In other words, it is possible that while being Nash equilibria, informative decisions are inefficient. (This discussion however, is beyond the scope of this paper).

The question of whether informative decisions are Nash equilibria has been discussed in Austin-Smith and Banks (1996). They show that in certain cases informative decisions may not be Nash equilibria, even when individuals share common preferences. They also address specifically the relationship between the optimal decision rule and informative decisions that are Nash equilibria. The contribution of our paper is to describe explicitly the conditions under which informative decisions are not Nash equilibria, finding that with the optimal decision rule these conditions do not apply. Thus, we show that when using the optimal decision rule, informative decisions are in fact Nash equilibria. This result provides an answer to the citisizm raised against the assumption of informative decisions, when using the optimal decision rule. Such citisizm has lead authors to consider non-informative decisions which include dependent decisions and sincere decisions. Dependent decisions imply that individuals may be influenced by decisions made by others (e.g., Berg, 1993, 1994; Ladha 1992, 1993, 1995). Sincere decisions imply that each individual makes his/her decision as if he/she is solely responsible for the outcome.[3] Our result suggests that these efforts to apply non-informative decisions are not required in the case of the optimal decision rule. Furthermore, by justifying informative decision making when using the optimal decision rule, this paper supports the assumption that decision-makers decide independently.

Finally, we show that if the optimal rule is not used, a juror may choose to deviate from his/her signal. He/she will do so if he/she thinks that this can improve the expected utility, increasing the expected utility to above the level that can be achieved by applying informative decisions. Thus, in the case where the optimal rule is not used, informative decisions may not be an equilibrium.

2. The Model

An n-member jury must decide whether to convict or acquit a defendant. The defendant is either guilty (1) or innocent (-1). The final decision is based on the individual jurors' decisions. There are therefore two possible correct decisions (1/1) (convicting a guilty defendant), and $(-1/-1)$ (acquitting an innocent defendant) and two corresponding incorrect ones ($(-1/1)$ and $(1/-1)$).[4]

Let $B(1/1)$ and $B(-1/1)$ be the utilities of convicting and acquitting a guilty defendant, respectively, where $B(1/1) > B(-1/1)$. Similarly, the utilities of the other two possiblilities are $B(-1/-1)$ and $B(1/-1)$, where $B(-1/-1) > B(1/-1)$. The positive net utilities of convicting a guilty defendant and acquitting an innocent one are: $B(1) = B(1/1) - B(-1/1)$ and $B(-1) = B(-1/-1) - B(1/-1)$, respectively. Let α be the *a priori* probability that a defendant is guilty, where $0 < \alpha < 1$. If $\alpha = 1/2$, the probabilities that defendants are guilty or innocent are equal. Let $x_i, x_i, \in \{-1, 1\}$ denote the juror i's decision, $x_i = 1$ and $x_i = -1$ then represent conviction and acquittal, respectively. The vector $x = (x_1, \ldots, x_n)$ describes the decision profile of n jurors making decisions simultaneously.

Assuming that each juror receives the signal 1 or -1, reflecting the true state, the vector $s = (s_0, \ldots, s_n)$ describes the jurors' signal profile. Let the probability of receiving signal $1(-1)$ in the state $1(-1)$ be $\Pr(1/1)(\Pr(-1-1))$. Specifically, assuming $\Pr(1/1) = \Pr(-1/-1) = p$, where $p \in (0.5, 1)$, which is the probability that a juror receives the "correct" signal: 1 if the defendant is guilty (1), and -1 if the defendant is innocent (-1). Note that $(1 - p)$ is the probability that the juror receives the "incorrect" signal: -1 if the defendant is guilty (1), and 1 if the defendant is innocent (-1).

The final decision regarding the defendant is made using a decision rule, whereby the function f assigns 1 or -1 (conviction or acquittal) to a decision profile x in $\Omega = \{1, -1\}^n$. That is, $f : \Omega \to \{1, -1\}$.

Supposing a qualified majority rule is used:[5]

$$f = \begin{cases} 1 & N(1) \geq kn \\ -1 & \text{otherwise} \end{cases},$$

where, $N(1)$ is the number of jurors who decide to convict the defendant, n is the total number of jurors and k is the minimum proportion of jurors deciding to convict that will lead to a verdict of guilty ($1/n \leq k \leq 1$ and kn is an integer). The parameter k represents the decision rule f. Thus, if $k = \frac{1}{2}$, the result is conviction by the simple majority rule. If $kn = n$ (hierarchy), only a unanimous verdict will lead to conviction, whereas if $kn = 1$ (polyarchy), the final decision is conviction even if only one juror decides to convict the defendant.

3. The Optimal Decision Rule and Different Types of Decisions

The purpose of this section is to describe the different types of decisions and to demonstrate the expected utilities that can be reached under these types of decisions. The various types of decisions are informative decisions and non-informative decisions. The latter includes dependent decisions, sincere decisions and strategic decisions. We show that when using the optimal decision rule the expected utility is higher with informative decisions than with any other type of decision. We also demonstrate that if the optimal rule is not used, non-informative decisions improve the expected utility.

Informative decision

Informative decision making means that decision-makers decide exclusively according to the signals they receive. Thus, although not stated explicitly, studies based on the Condorcet approach assume informative decisions.

Dependent decisions

In dependent decision making, indivduals may be influenced by the decisions of others. However, it should be noted that independent decision making, in which indivduals are not influenced by the decisions of others, does not preclude collective learning.

Sincere decisions

In sincere decisions the decision-maker decides as if he/she alone determines the final decision, i.e., chooses the alternative yielding the highest expected utility for his/her signal. In formal terms, assuming $B(1) = B(-1)$, an individual who receives the signal 1 would decide 1 if $\alpha p > (1 - \alpha)(1 - p) \Leftrightarrow p > 1 - \alpha$, and -1 otherwise. An individual receiving a signal -1 would decide 1 if $(1 - \alpha)p < \alpha(1 - p) \Leftrightarrow p < \alpha$, and -1 otherwise.

Thus, sincere decisons fall into several sub-types.

Sub-type 1: If $p > \alpha$ and $p > 1 - \alpha$, the decision is informative, because the probability of an individual making a correct decision based on his/her signal is greater than the *a priori* probability of both states. The probabilities of making the correct decision are: $\Pr(1/1) = p$ and $\Pr(-1/-1) = p$.

Sub-type 2: If $p > 1 - \alpha$ and $p < \alpha$, the decision is always 1, because the *a priori* probability for state 1 is greater than that of the correct decision based on the signal. In this case, the probabilities of making a correct decision are: $\Pr(1/1) = 1$ and $\Pr(-1/-1) = 0$.

Sub-type 3: If $p < 1 - \alpha$ and p $< \alpha$, the individual decides 1 if he/she receives the signal -1, and -1 if the signal is 1. However, sub-type 3 is of no interest, since the probability of making a correct decision is less than 1/2.

Sub-type 4: If $p > \alpha$ and $p < 1-\alpha$, the individual decides -1, and the probabilities of making a correct decision are: $\Pr(1/1) = 0$ and $\Pr(-1/-1) = 1$.

Strategic decisions

Individuals with the same preferences may also decide strategically to reach a noninformative decision, which is not necessarily made in a dependent and sincere manner. For example, an individual who always ignores his/her signal and decides -1 expects to increase his or her utility.

Example Consider a company committee consisting of three members: the director (1), the manager (2) and the deputy-manager (3), who must decide whether to approve or reject a particular project. All three members have the same probability of receiving the correct signal, say $p = 0.6$. The *a priori* probability that this project is good, α, is 0.65. The utilities are assumed to be the same for both correct decisions (approving a good project or rejecting a bad one) or both incorrect ones (rejecting a good project or approving a bad one), so $B(1) = B(-1)$. Therefore, the utilities may be ignored.

The optimal decision rule and informative decision making

Assuming informative decision making, the optimal decision rule gives equal weights to the three committee members, 0.4054 (In 0.6/0.4), with a fixed bias as a result of the *a priori* probability of 0.6190 ($\ln 0.65/0.35$) (Ben-Yashar and Nitzan, 1997).

$$\hat{f} = \text{sign}(0.6190 + 0.4054\chi_1 + 0.4054\chi_2 + 0.4054\chi_3),$$

where $\hat{f}$ is the optimal decision rule.

The final decision is -1 only if all members decide -1, and 1 otherwise. According to the optimal decision rule, the expected utility[6] is 0.684:

$$0.65(1 - 0.4^3) + 0.35(0.6^3) = 0.684.$$

We now demonstrate that the expected utility is higher with informative decision making than with any other type of decision making when using the optimal decision rule.

The optimal decision rule and dependence between decision-makers

Suppose the manager and the deputy manager are dependent on the committee's director. Although the decision-makers are homogenous (they have the same probability of receiving the correct signal), dependence may exist between the decision-makers if they have different roles. The expected utility would then be given by the probability that the director received the correct signal, i.e., $0.6 < 0.684$ (the expected utility is lower with dependent than with informative decision making, see above). Under such dependence, it would be preferable to use a different rule, such as one that always gives the decision 1, thereby increasing the expected utility to 0.65 ($0.684 > 0.65 > 0.6$). The expected utility is lower with dependent than with informative decisions, even if only one decision-maker decides dependently and the others decide informatively. If we suppose that the deputy-manager (3) is dependent on the director, the expected utility would be $0.672 < 0.684$.

$$0.65(1 - 0.4^3 - 0.4^2 \times 0.6) + 0.35(0.6^3 + 0.6^2 \times 0.4) = 0.672.$$

Such dependence would affect the final decision only if both others decide -1. In this case, if member (3) reverts to the director's decision, the final decision would also change from 1 to -1, decreasing the expected utility (because, according to the optimum rule, 1 is the preferred result in this case). An analysis of the dependence between decision-makers appears in Nitzan and Paroush(1984).

The optimal decision rule and sincere decision making

Assuming sincere decisions, each committee member considers the *a priori* probability to be higher than that of reaching the correct decision based on his/her signal, and therefore always decides 1 (sub-type 2). In this case, the expected utility would be $0.65 < 0.684$ (the expected utility is lower with sincere than with informative decision making, see above). This result is also valid if only one of the decision-makers decides sincerely and the others decide informatively, since the final decision is always 1 and the expected utility is 0.65 when using the optimal decision rule.

The optimal decision rule and strategic decision making

If we suppose that member (3) decides to ignore his/her signal and always decides -1 (which is not dependent or sincere decision making), such a strategy would not be worthwhile since the expected utility is lower, i.e, $0.672 < 0.684$:

$$0.65(1 - 0.4^3 - 0.4^2 \times 0.6) + 0.35(0.6^3 + 0.6^2 \times 0.4) = 0.672.$$

This strategy would only affect the final decision if both other decision-makers decide -1. In this case, if member (3) changes his/her decision from 1 to -1, the final decision would also change from 1 to -1, thus decreasing the expected utility. In this example, when using the optimal decision rule, it is not worthwhile for an individual to deviate from his or her signal, assuming all the others decide informatively.

An example of worthwhile strategic decision making is presented below. If we suppose that the decision rule is not the optimal decision rule, e.g., the decision rule gives -1 only if all individuals decide 1, otherwise the final decision is 1. Such a decision rule only differs from the optimal decision rule in two cases: (1) If all individuals decide 1, the final decision is -1 (as opposed to using the optimal decision rule, which gives 1). (2) If all individuals decide -1, the final decision is 1 (as opposed to using the optimal decision rule). With such a rule, the expected utility is 0.532:

$$0.65(1 - 0.6^3) + 0.35(0.4^3) = 0.532.$$

If member (1) deviates from his or her signal, and always decides -1, this would increase the expected utility assuming all the other individuals decide informatively. The final decision is only affected by a strategic decision if all the members receive the signal 1, and member (1) deviates from his/her signal and decides -1. In this case, the final decision would be 1 (the same as with the optimal decision rule). The expected utility is 0.65, and although such a strategic decision increases the expected utility, it is still lower than with an informative decision and the optimal decision rule.

4. Informative Decision Making is a Nash Equibrium When Using the Optimal Decision Rule

This section shows that when using the optimal decision rule informative decision making is a Nash equilibrium. Thus, there is no reason to take strategic decisions into account, since individual deviation from the received signal is not worthwhile. We first describe conditions under which informative decision making is not a Nash equilibrium. This applies to all decision rules, k, where kn is the minimum number of jurors who must decide to convict in order to lead to a verdict of guilty. For example, with the simple majority rule, $k = (n + 1)/2$; and with the unanimity (hierarchy) rule, $k = 1$, a jury can reach a verdict of guilty only if all jurors decide to convict.

In informative decision making a jury decides to convict if and only if they are given the signal 1. A juror who believes his/her colleagues are deciding informatively may then decide to base his/her decision not only on private information, but also on the decisions he/she believes the other jurors are making.

A juror's decision is pivotal if he/she received the signal -1 (1) but knows that $kn-1$ ($n - kn$) out of n jurors were given the signal $1(-1)$.[7] Clearly, only pivotal decisions are significant. If a pivotal decision-maker believes the other jurors are making their decisions informatively, he/she would rationally base his/her decision on the expected utility. A juror would thus ignore his or her signal if this is expected to improve the utility. This leads to the following lemma.

Lemma

Condition (1): In the case of a signal -1, informative decision making is not a Nash equilibrium if:

$$\left(\frac{1-p}{p}\right)^{2(kn-1)-n} < \frac{\alpha}{1-\alpha}\frac{B(1)}{B(-1)}. \tag{1}$$

Condition (2): In the case of a signal 1, informative decision making is not a Nash equilibrium if:

$$\frac{\alpha}{1-\alpha}\frac{B(1)}{B(-1)} < \left(\frac{1-p}{p}\right)^{2kn-n}. \tag{2}$$

Proof of this lemma is presented in Appendix A.

Condition (1) refers to a pivotal juror receiving the signal -1, who believes that the other jurors are deciding informatively, and decides to ignore his/her signal. Condition (2) refers to a pivotal juror receiving the signal 1, who believes that the other jurors are deciding informatively, and decides to ignore his/her signal.

These conditions hold for all possible rules, k. A rational juror, whose decision is pivotal and who believes that all the other jurors are deciding informatively, would compare the expected utilities from informative and non-informative decision making. A juror would ignore his/her signal if this is expected to increase the expected utility.

Proposition If the optimal decision rule is used, then informative decision making is a Nash equilibrium.

Proof [8] According to Ben-Yashar and Nitzan (1997) the description of the optimal decision rule, kn, is as follows:[9]

$$\frac{n}{2} - \frac{\gamma + \delta}{2\beta} + 1 \geq kn > \frac{n}{2} - \frac{\gamma + \delta}{2\beta} \tag{3}$$

where

$$\gamma = \ln \frac{\alpha}{1-\alpha}, \qquad \delta = \ln \frac{B(1)}{B(-1)}, \qquad \beta = \ln \frac{p}{1-p}.$$

The conditions under which informative decision making is not a Nash equilibrium (see lemma) may be written as follows:

$$kn > \frac{n}{2} - \frac{\gamma + \delta}{2\beta} + 1 \quad \text{or} \quad kn < \frac{n}{2} - \frac{\gamma + \delta}{2\beta}. \tag{4}$$

With the optimal decision rule (3), the inequalities in (4) do not apply. □

This result implies that with the optimal decision rule, strategic considerations can be ignored.

In the lemma, we stated the conditions under which informative decisions are not Nash equilibria for all possible rules, k. This is now applied to the unanimity (hierarchy) rule, $kn = n$. With this rule, the verdict is guilty only if all jurors decide to convict. This rule often applies in economic organizations or systems.

With the unanimity (hierarchy) rule, i.e., $kn = n$, assuming $\alpha = 1/2$, $B(1/1) = B(-1/-1) = 0$, $B(-1/1) = -(1-q)$ and $B(1/-1) = -q$, the conditions under which informative decision making is not a Nash equilibrium (see lemma) may be written as follows:

$$q < \frac{p^{n-1}(1-p)}{p^{n-1}(1-p) + (1-p)^{n-1}p} \quad \text{or} \quad q > \frac{p^n}{p^n + (1-p)^n} \tag{5}$$

The condition for the optimality of the unanimity rule (hierarchy), Ben-Yashar & Nitzan (1997, 2001), assuming that $\alpha = 1/2$, $B(1/1) = B(-1/-1) = 0$, $B(-1/1) = -(1-q)$ and $B(1/-1) = -q$, is:

$$\frac{p^{n-1}(1-p)}{p^{n-1}(1-p)+(1-p)^{n-1}p} \leq q < \frac{p^n}{p^n+(1-p)^n}. \tag{6}$$

For proof, see Appendix B.

Clearly with the optimal decision rule (6) the inequalities in (5) do not apply.[10]

5. Conclusion

In this paper we have addressed the debate on whether informative decision making can be an underlying assumption. In order to do so we described explicitly the conditions under which informative decisions are not Nash equilibria, and this has allowed us to determine that when using the optimal decision rule, informative decisions are Nash equilibria. The conclusion is therefore that in the case where the optimal decision rule is used, there is no justification to consider any decisions other than informative decisions. More specifically, attemps to apply non-informative decision making, out of concern that informative decisions should not be assumed, are redundant in the case where the optimal decision rule is used. It is however justified to consider non-informative decision making, such as strategic decisions when the optimal decision rule is not used. In this case strategic decisions can improve utility.

Appendix A

Partitioning the set of all decision profiles into $X(1, f)$ and $X(-1, f)$, where $X(1, f) = \{x \in \Omega : f(x) = 1\}$ and $X(-1, f) = \{x \in \Omega : f(x) = -1\}$, for a given rule *f*, the jury convicts a guilty defendant and acquits an innocent one with probability $\Pi(f/1)$ and $\Pi(f/-1)$, where $\Pi(f/1) = \Pr\{x \in X(1, f) : 1\}$ and $\Pi(f/-1) = \Pr\{x \in X(-1, f) : -1\}$. Since f is a decision rule,

$$\Pr\{x \in X(-1, f) : -1\} = 1 - \Pi(f/-1)$$

and

$$\Pr\{x \in X(1, f) : -1\} = 1 - \Pi(f/-1).$$

The expected utility E is:

$$E = \alpha[B(1/1)\Pi(f/1) + B(-1/1)(1-\Pi(f/1))] + (1-\alpha)$$
$$[B(-1/-1)\Pi(f/-1) + B(1/-1)(1-\Pi(f/-1))] \tag{A1}$$

or

$$E = \alpha B(1)\Pi(f/1) + (1-\alpha)B(-1)\Pi(f/-1)$$
$$+(\alpha B(-1/1) + (1-\alpha)B(1/-1)).$$

Let $g(x/1)$ and $g(x/-1)$ denote the probabilities of x given states 1 and -1. i.e., the defendant is guilty (should be convicted) or innocent (should be acquitted). Hence,

$$\Pi(f/1) = \sum_{x\in X(1,f)} g(x/1) \quad \text{and} \quad \Pi(f/-1) = \sum_{x\in X(-1,f)} g(x/-1).$$

Then E, the expected utility is:

$$\begin{aligned} E &= \alpha B(1)\Pi(f/1) + (1-\alpha)B(-1)\Pi(f/-1) + C \\ &= \alpha B(1) \sum_{x\in X(1,f)} g(x/1) + (1-\alpha)B(-1) \sum_{x\in X(-1,f)} g(x/-1) + C \end{aligned} \quad \text{(A2)}$$

where C is a constant independent of f and x.

Assuming informative decision making, the vector $x = (x_1 \ldots, x_n)$ is the same as the signal profile $s = (s_1 \ldots, s_n)$, where, $\forall i$, $x_i = s_i$. Thus, the jurors may be partitioned into $A(x)$ and $R(x)$ for any signal profile, x or s, such that $i \in A(x)$, if $x_i = 1$ and $i \in R(x)$, if $x_i = -1$, and $g(x/1)$ and $g(x/-1)$ is:

$$g(s/1) = g(x/1) = \prod_{i\in A(x)} p_i \qquad \prod_{i\in R(x)} (1-p_i)$$

and

$$g(s/-1) = g(x/-1) = \prod_{i\in R(x)} p_i \qquad \prod_{i\in A(x)} (1-p_i).$$

Proof A pivotal juror who believes his/her colleagues are making decisions informatively may base his/her decision not only on private information, but also on the decisions he/she believes the others are making. Note that only pivotal decisions are significant.

For every rule k, a decision is only pivotal if $(kn-1)$ jurors decided to convict and $(n-kn)$ decided to acquit. Thus, a pivotal juror i receiving the signal -1, who believes that the other jurors are making their decisions informatively, would rationally base his/her decision on the expected utility. More formally, assume $s_i = -1$ and x is the decision profile with informative decision making, such that $|A(x)| = kn-1$ and $|R(x)| = n-(kn-1)$.

The final decision with the given rule would then be to acquit the defendant. The expected utility of such a decision profile would therefore be $(1-\alpha)B(-1)g(s/-1)$. If juror i ignores his/her private information and decides on conviction, this is the final decision, and the expected utility is $\alpha B(1)g(s/1)$. The individual juror thus decides to ignore his/her signal if $\alpha B(1)g(s/1) > (1-\alpha)B(-1)g(s/-1)$. This condition ensures that E is maximized in equation (A2). Since $g(s/-1) = p(1-p)^{kn-1}p^{n-1-(kn-1)}$ and $g(s/1) = (1-p)p^{kn-1}(l-p)^{n-1-(kn-1)}$, the above condition may be rewritten as:

$$\frac{\alpha}{1-\alpha}\frac{B(1)}{B(-1)} > \frac{(1-p)^{kn-1}p^{n-(kn-1)}}{p^{kn-1}(1-p)^{n-(kn-1)}}$$

or

$$\frac{\alpha}{1-\alpha}\frac{B(1)}{B(-1)} > \left(\frac{1-p}{p}\right)^{2(kn-1)-n}$$

Similarly, a pivotal juror receiving the signal 1, who believes that the other jurors are deciding informatively, would act rationally, and will therefore make a decision based on the expected utility. This juror would therefore ignore his/her private signal and decide to acquit (-1), if:

$$\frac{\alpha}{1-\alpha}\frac{B(1)}{B(-1)} > \left(\frac{1-p}{p}\right)^{2kn-n}$$

□

Appendix B

A necessary and sufficient condition for the optimality of the unanimity rule (hierarchy) is:

$$n-1 < kn \leq n \Leftrightarrow -\frac{(\gamma+\delta)}{\beta} < n \leq 2 - \frac{(\gamma+\delta)}{\beta}\ .$$

This can be derived from the general results of Ben Yashar & Nitzan (1997, 2001), giving the necessary and sufficient condition for the optimality of the unanimity rule (i.e., hierarchy). Assuming that $\alpha = 1/2$, $B(1/1) = B(-1/-1) = 0$, $B(-1/1) = -(1-q)$ and $B(1/-1) = -q$, the condition for the optimality of the unanimity rule (hierarchy) is:

$$\frac{p^{n-1}(1-p)}{p^{n-1}(1-p)+(1-p)^{n-1}p} \leq q < \frac{p^n}{p^n+(1-p)^n}.$$

Proof.

$$q < \frac{p^n}{p^n + (1-p)^n} \Leftrightarrow q < \frac{1}{1 + \left(\frac{1-p}{p}\right)^n} \Leftrightarrow \tag{B1}$$
$$\left(\frac{1-p}{p}\right)^n < \frac{1-q}{q} \Leftrightarrow n \ln \frac{1-p}{p} < \ln \frac{B(1)}{B(-1)} \Leftrightarrow$$
$$n > \frac{-\ln \frac{B(1)}{B(-1)}}{\ln \frac{p}{1-p}} \Leftrightarrow n > -\frac{\delta}{\beta}$$

$$q \geq \frac{p^{n-1}(1-p)}{p^{n-1}(1-p) + (1-p)^{n-1}p} \Leftrightarrow q \geq \frac{1}{1 + \left(\frac{1-p}{p}\right)^{n-2}} \Leftrightarrow \tag{B2}$$
$$\left(\frac{1-p}{p}\right)^{n-2} \geq \frac{1-q}{q} \Leftrightarrow n \ln \frac{p}{1-p} \leq -\ln \frac{B(1)}{B(-1)} + 2 \ln \frac{p}{1-p} \Leftrightarrow$$
$$n \leq \frac{2 \ln \frac{p}{1-p} - \ln \frac{B(1)}{B(-1)}}{\ln \frac{p}{1-p}} \Leftrightarrow n \leq 2 - \frac{\delta}{\beta}$$

□

Notes

1. Other papers on collective decision making include: Nitzan & Paroush (1980, 1985), Groliman et al. (1983), Kleveorick et al. (1984), Shapely & Grofman (1984), Sah & Stiglitz (1988), Sah (1990, 1991) and Ben-Yashar and Paroush (2001).
2. Other studies on stategic decisions include Ladha et al. (1996), Mclennan (1988), Wit (1998), Dekel & Piccione (2000) & Persico (2000).
3. An individual makes his/her decision as if he/she is solely responsible for the outcome means that he/she chooses the alternative with the highest expected utility based on the signal received and environmental factors, i.e., information that depends on the *a priori* probabilities and utilities.
4. Note that each possibility consists of two terms: the right hand one describing the state (guilty or innocent), and the left-hand one the collective decision (conviction or acquittal).
5. This assumption is plausible because the optimal decision rule is a qualified majority rule, as shown by Ben-Yashar and Nitzan (1997).
6. More accurately, the collective probability of making the correct decision is 0.684. However, because of the specific assumption regarding utilities in this example, it may be regarded as the expected utility.
7. Two other cases in which a juror is pivotal are: If he/she received signal $1(-1)$ and knows that $kn - 1$ $(n - kn)$ out of n jurors have been given the signal $1(-1)$, which is equivalent to $n - kn(kn - 1)$ out of n jurors receiving the signal $-1(1)$. These two cases are in essence the same as those discussed above.
8. This proof makes explicit use of the optimal decision rule, as opposed to the indirect approach of others (Austen-Smith & Banks, 1996).
9. Note that kn in this paper corresponds to $(n - kn)$ in Ben-Yashar and Nitzan (1997).

10. This specific case has been described by Fedderson and Pesendorfer (1998). In Feddersen and Pesendorfer $\beta(n-1,n)$ denotes $\frac{p^{n-1}(1-p)}{p^{n-1}(1-p)+(1-p)^{n-1}p}$; and the parameter $q \in (0,1)$ characterizes the juror's threshold of reasonable doubt.

References

Austen-Smith, D., & Banks, J. S. (1996). Information aggregation, rationality and the Condorcet jury theorem. *American Political Science Review, 90*, 34–45.

Ben-Yashar, R., & Nitzan, S. (1997). The optimal decision rule for fixed-size committees in dichotomous choice situations: The general result. *International Economic Review, 38*, 175–186.

Ben-Yashar, R., & Nitzan, S. (2001). The robustness of optimal organizational architectures: A note on hierarchies and polyarchies. *Social Choice and Welfare, 18*, 155–163.

Ben-Yashar, R., & Paroush, J. (2001). Optimal decision rules for fixed-size committees in polychotomous choice situations. *Social Choice and Welfare, 18*, 737–746.

Ben-Yashar, R., & Kraus, S. (2002). Optimal collective dichotomous choice under quota constraints. *Economic Theory, 19*, 839–852.

Ben-Yashar, R., Khuller, S., & Kraus S. (2001). Optimal collective dichotomous choice under partial order constraints. *Mathematical Social Science, 41*, 349–364.

Berg, S. (1993). Condorcet's jury theorem, dependency among voters. *Social Choice and Welfare, 10*, 87–96.

Berg, S. (1994). Evaluation of some weighted majority decision rules under dependent voting. *Mathematical Social Science, 28*, 71–83.

Condorcet, NC de (1785). Essai sur l'Application de l'Analyse à la Probabilitè des Dècisions Rendues à la Pluralitè des Voix, Paris. See I. McLean and F. Hewitt, translators, 1994.

Dekel, E., & Piccione, M. (2000). Sequential voting procedures in symmetric binary elections. *Journal of Political Economy, 108*, 34–55.

Feddersen, T., & Pesendorfer, W. (1996). The swing voter's curse. *American Economic Review, 86*, 408–424.

Feddersen, T., & Pesendorfer, W. (1997). Voting behavior and information aggregation in elections with private information. *Econometrica, 65*, 1029–1058.

Feddersen, T., & Pesendorfer, W. (1998). Convicting the innocent: The inferiority of unanimous jury verdicts under strategic voting. *American Political Science Review, 92*, 23–35.

Grofman, B., Owen, G., & Feld, S.L. (1983). Thirteen theorems in search of the truth. *Theory and Decision, 15*, 261–278.

Klevorick, A. K., Rotschild, M., & Winship, C. (1984). Information processing and jury decision making. *Journal of Public Economics, 23*, 245–278.

Ladha, K. K. (1992). The Condorcet jury theorem, free speech and correlated votes. *American Political Science, 36*, 617–634.

Ladha, K. K. (1993). Condorcet's jury theorem in light of de Finetti's theorem, Majority voting with correlated votes. *Social Choice and Welfare, 10*, 69–86.

Ladha, K. K. (1995). Information pooling through majority-rule voting: Condorcet's jury theorem with correlated votes. *Journal of Economic Behavior and Organization, 26*, 353–372.

Ladha, K. K., Miller, G., & Oppenheimer, J. (2003). Information aggregation by majority rule: Theory and Experiments, mimeo.

McLennan, A. (1998). Consequences of the Condorcet jury theorem for beneficial information aggregation by rational agents. *American Political Science Review, 92*, 413–418.

Myerson, R. (1998). Extended Poisson games and the Condorcet jury theorem. *Games and Economic Behavior, 25*, 111–131.

Nitzan, S., & Paroush, J. (1980). Investment in human capital and social self protection under uncertainty. *International Economic Review, 21*, 547–557.

Nitzan, S., & Paroush, J. (1982). Optimal decision rules in uncertain dichotomous choice situation. *International Economic Review, 23*, 289–297.

Nitzan, S., & Paroush, J. (1984). The significance of independent voting under uncertain dichotomous choice situations. *Theory and Decision, 17*(1), 47–60.

Nitzan, S., & Paroush, J. (1985). *Collective decision making: An economic outlook*. Cambridge University Press.

Persico, N. (2004). Committee design with endogenous information. *The Review of Economic Studies*, *71*(1), 165–194.

Piketty, T. (1999). The information-aggregation approach to political institutions. *European Economic Review, 43*, 791–800.

Sah, R. K. (1990). An explicit closed-form formula for profit-maximizing k-out of$-n$ systems subject to two kinds of failures. *Microelectronics and Reliability, 30*, 1123–1130.

Sah, R. K. (1991). Fallibility in human organizations and political systems. *The Journal of Economic Perspectives, 5*, 67–88.

Sah, R. K., & Stiglitz, J. E. (1988). Qualitative properties of profit-maximizing k-out of-N systems subject to two kinds of failures. *IEEE Transactions on Reliability, 37*, 515–520.

Shapley, L., & Grofman, B. (1984). Optimizing group judgmental accuracy in the presence of interdependencies. *Public Choice, 43*, 329–343.

Wit, J. (1998). Rational choice and the Condorcet jury theorem. *Games and Economic Behavior, 22*, 364–376.