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Coalition governments versus minority governments: Bargaining power, cohesion and budgeting outcomes

GERALD PECH*
Department of Economics, National University of Ireland, Galway; e-mail: GPech@nuigalway.ie

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Abstract. Recent empirical work investigating the role of minority governments in the selection of fiscal policies has shown that the majority status does not affect the budget size. This paper presents an analytical framework which accounts for this result. It combines a government formation game and a budget game involving cabinet and parliament. A general indifference result applies. An exogenous shock to the bargaininig environment which absorbes the cohesion of the government increases the demand for expenditures. At the same time the conditions for the formation of a minority government are fulfilled. If the formateur is strong, a minority government can be a device for cutting expenditures.

1. Introduction

Political instability is often believed to result in inferior fiscal policies. Several studies have found indicators of political instability to be significant in explaining fiscal policies: De Haan and Sturm (1994) found that a higher frequency of government change results in more rapid growth of debt. Perotti and Kontopoulos (1999) and Volkerink and De Haan (2001) relate size fragmentation – measured variably as the effective number of parties in the government, the parliament or the number of spending ministries – to higher public expenditures and higher deficits. Roubini and Sachs (1989a, 1989b) and Corsetti and Roubini (1993) contributed some early articles which systematically related fiscal policies to political variables. Their indicator of power dispersion within the government gives the lowest score to single party majority governments, intermediate scores to coalition governments and the highest score to minority governments. The indicator was found to explain

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differences in debt policies (Roubini and Sachs 1989a, 1989b) and expenditure policies (Roubini and Sachs 1989b). The intuition that was offered for this result is that minority and majority coalition governments would point to various levels of political instability. Edin and Ohlsson (1991) argued that the result of Roubini and Sachs (1989a) could be explained by the presence of minority governments alone. They hypothesized that negotiations in the parliament, not in the government, could be the main obstacle in cutting deficits. De Haan and Sturm (1997), after making some corrections in the assignment of the indicator value, dismiss the Roubini-Sachs hypothesis completely. Other articles come to the same conclusion. These are Borelli and Royed (1995), Hahm, Kamlet and Mowery (1996) and De Haan, Sturm, and Beekhuis (1999) with respect to debt policies and De Haan and Sturm (1994) and Perotti and Kontopoulos (1999) with respect to debt and expenditure policies. It is, therefore, fair to say that the overwhelming evidence is that the majority status of the government does not affect fiscal policies.

This paper lays out a formal model which explains this result. It integrates a bargaining game over the expenditure budget and a game over the formation of the government. In the government formation stage political actors form anticipations over the outcome which is attained in the budgeting game. The budget has to be separately approved in the cabinet and the parliament. We say there is a government premium when there is an advantage of holding a cabinet post over freely negotiating the budget in the parliament. As the government premium relates pay offs inside and outside the government, it is intuitive that it should play a central role in a rational theory of government formation.

The idea that the bargaining environment and the pay offs for the parties under the different arrangements are driving forces behind the formation and termination of governments has been explored in a number of studies. Strom (1990) reports that countries in which minority governments regularly emerge typically have institutions which give a strong influence to opposition parties, for example in parliamentary committees. On the benefit side, if the electoral benefits from joining the government are doubtful, a party is more likely to support the government without entering the cabinet. If a party receives an extra benefit over its next best alternative this makes the cabinet as an organizational form not only more desirable, but also more stable: An increase in the value of the outside option for at least one party in the cabinet is a necessary condition for cabinet instability. [1]The implied hypothesis that cabinets more easily dissolve later in life – when most of the available benefits have been consumed – to give way for new elections has been successfully tested by Diermeier and Stevenson (2000). This suggests that the size of the cake which is available inside the government is a main determinant of the stability – or cohesion – of the government.

Warwick (1992), on the other hand, has found ideological coherence of the parties involved in a government to be the main explanatory variable of cabinet stability. He argues that indicators of an unstable bargaining situation like the number of parties in the parliament – found to be explanatory of cabinet survival in King, Alt, Burns and Laver (1990) – are only a surrogate for ideological diversity of the cabinet. Volkerink and De Haan (2001) found that both, an unstable bargaining environment and ideological fragmentation of the government raises government expenditures. This latter result is consistent with a view that expenditure benefits and ideological coherence are alternative means of securing government stability.[2]

While the political science literature discusses a party's pay off mostly in terms of office benefits and policy objectives, the model developed in this paper focuses on expenditure benefits, i.e., it assumes that political actors receive benefits from outlays directed towards their constituency minus general taxes. The "ideological pay off" is assumed to be equal across all coalitions. If securing a government premium is an aim for a coalition party in the government formation stage, it remains to show what the circumstances are under which a party succeeds in seizing this benefit. In our basic model of a three party legislature, the party which is asked to support the government is pivotal and, therefore, able to demand a cabinet post when the government premium is positive. We also derive sufficient conditions under which a party commands over bargaining power in the general version of the model.[3]The model predicts minority governments to form either in the absence of a government premium or when there is a government premium but the formateur is relatively strong in the government formation stage. Hereby it takes a different view from other approaches which explain the emergence of minority governments. In Diermeier and Merlo (2001), (super-) majority governments form when the formateur is favored by the status quo in policy negotiations while minority governments form when she is disadvantaged by the status quo. Laver and Shepsle (1996) and Baron (1998) predict the emergence of minority governments only in cases where the formateur is in a strong position in the government formation stage. The only theoretical model in the literature which relates expenditures to cohesion is Persson, Roland, and Tabellini (2000). They do not focus, however, on minority governments but on the relationship between cohesion of legislative coalitions and government expenditures. Their model, designed to explain behavioral differences between the European-style cabinet system and the congressional system, predicts expenditures to be high when the cohesion of legislative coalitions is high. This paper reverses their argument in saying that expenditures are a means of securing government stability.

If majority governments correspond to a situation where the bargaining power of the supporting party in the cabinet is high and minority governments tend to emerge when its bargaining power in the parliament is high, the overall relationship between majority status and expenditures may just be neutral: With no bias in the distribution of bargaining powers in cabinet or parliament the effects of the presence of minority governments and majority governments with excessive expenditures should just average out. This neutrality result holds if all parameters of the model are common knowledge. Exogenous shocks which raise the bargaining power of the supporting party and/or the ideological pay off in the government, tend to consume the cohesion of the government and render the option to leave the government more attractive. If the head of government – who always wants to accommodate a justified demand by the supporting coalition party – cannot perfectly monitor the effect of such a shock on the position of its coalition partner, the government could be brought down and succeeded by a minority government with higher expenditures. There are a few cases in the sample of Roubini and Sachs where a coalition government had been replaced by a minority government. Of those cases, household data indicates that at least the Moro minority government in Italy after 1974 and the Swedish Fälldin minority government after 1978 as well as the early Jörgensen government after 1981 faced severe budget problems with a time lag of about one year after gaining power.

If the position of the formateur is strong in the government formation stage, she would form a minority government when it is less expensive in terms of expenditures. An example of such a government is the Danish minority government run by Schlüter. Schlüter initiated a policy of budget consolidation in 1982 but resigned in 1984 to call for new elections. Although he remained head of a minority government, the election outcome improved his position and allowed him to continue with his austerity measures. In our model, the bargaining power of the supporting party is only checked if it is not pivotal like it is in the three party legislature. In those cases, a minority government can be effective in cutting expenditures.

Section 2.1 gives an overview of the model. Section 2.2 derives equilibrium expenditure policies. Section 2.3 determines bargaining outcomes in the parliament. Section 2.4 addresses bargaining in the cabinet. The government formation game for a three party legislature is analyzed in Section 2.5. Section 3 states the neutrality result. Section 4 relaxes previous assumptions and discusses extensions. Section 5 concludes.

2. Bargaining outcomes with deterministic institutions

2.1. *Overview of the model*

There are $n \geq 3$ parties in the legislature. Party i has a seat share of π^i. No two parties have the same seat share and no party has a simple majority of seats. The set of all minimum winning coalitions including i is M^i. Each party i perfectly represents the interests of its constituency. Its objective function is quasi-linear in per head expenditures of a group specific good g^i, the tax payment per head, τ, and a term B^i which captures the preference for belonging to a certain government coalition, C:

$$u^i = h(g^i) - \tau + B^i(C).$$

This utility function pertains to the standard model of special interest politics (see Persson and Tabellini, 2002). Subutility from consumption of the publicly provided good for the constituency, $h(g^i)$, is concave in the per head outlay g^i with $h(0) = 0$, $\lim_{g\to 0} h_g = \infty$ and $\lim_{g\to 0} h_{gg} = -\infty$. τ is the tax payment per head. The utility imputation space is bounded, i.e., $\lim_{g\to\infty} u(g, g) = -\infty$. The precise value of the preference parameter B is unknown when the game starts and has an expected value of zero for any coalition into which the party may enter. The budget is an expenditure-taxation policy $\{g^k, G\}_{k=1}^{k=n}$ satisfying budget balance, i.e.,

$$G = \sum_k g^k \leq \tau.$$

Substituting the budget constraint into the objective function, denoting aggregate expenditures per head G and taking expectations one can write

$$U^i = U^i(g^i, G).$$

It follows, that the utility of the zero budget is $U(0, 0) = 0$.

My model of government formation and budgeting consists of three main stages (2.1.1–2.1.3).

2.1.1. *Stage 3: Budget negotiations in the parliament*

In the final stage, a government budget bill is submitted to the parliament. It either accepts the government bill with a simple majority of votes or rejects it. In the latter case bargaining in parliament starts with the selection of a parliamentary proposal maker. The bargaining horizon is either long or short. It is short if it turns out that the president who selects the proposal maker for the parliament strongly backs the candidate she selected in the first place while in a situation of political instability the selection process is more opportunistic. The conditional expected outcome – or continuation value – if an agent enters bargaining and the horizon is long is $V^i|L$. The continuation value if the horizon is short is $V^i|S$. A long bargaining horizon admits a more equal distribution of pay offs, so typically for a small party $V^i|S < V^i|L$ is fulfilled. Agents are uncertain about which bargaining situation applies and attach a probability P to the event that the bargaining horizon is long. In the case of complete information all agents agree on P before they enter the parliamentary stage. The expected pay off which a party receives in the parliament is

$$EV^i = PV^i|L + (1 - P)V^i|S. \qquad (1)$$

In Section 4.1 we discuss the implications of incomplete information where the government bill is written under uncertainty over the precise demand of legislators.

2.1.2. *Stage 2: Cabinet negotiations over the budget bill*

In the cabinet the supporting party can ensure that it receives an exogenously given pay off v^C from the budget bill.

2.1.3. *Stage 1: Government formation stage*

A formateur is selected to propose either a majority coalition government or a minority government headed by herself. We assume that the bargaining horizon is open and parties make proposals in a fixed order according to the ranking of their seat share π^i. The process stops if a proposal is accepted by a majority of the legislature.

If v^C exceeds EV^j for j, it enjoys a premium p when in the government:

$$p = \text{Max}(v^C - EV^j, 0)$$

If the party which is invited to support the government has bargaining power, a majority coalition government is proposed whenever the premium is positive. In the case where $p = 0$, randomization between majority and minority governments occurs. We can interpret p as a proxy for the cohesion of the government. In the case where $p = 0$, the supporting coalition party is indifferent between being inside the government or being asked in the parliament to support the government bill. Any event which renders the utility of participating in this precise coalition government B(C) negative, for example a scandal which tarnishes the head of the government, would make it a preferred alternative for the party to leave the government. Before we go on to sequentially solving the game in the cabinet and the parliamentary stage, we have to determine the equilibrium expenditure proposal for a proposal maker at either stage.

2.2. *Expenditure policies*

Throughout the analysis we assume that an expenditure policy $\{g^i, i \in N\}$ results from the submission of a proposal maker ω. The proposal maker wishes to give to as many responders as are necessary for passage their minimum pay off R^j they require to accept the proposal. So she chooses from the set of all minimum winning coalitions including herself, M^ω, the coalition $C \in M^\omega$ which gives her the highest utility. Then ω picks a proposal which maximizes her own pay off under the restraint that the demands of all other coalition members $j \in C\backslash\omega$ are satisfied:

$$\max_{C \in M^\omega, \{g^i\}} U^\omega \text{ s.t. } U^j \geq R^j; j \in C\backslash\omega. \tag{2}$$

We write $F(\{R^i, i \in C^\omega\backslash\omega\})$ for the maximal value of U^ω as a function of the reservation prices. The first order conditions are necessary and sufficient conditions for an optimum of problem (2). With respect to responder $j \in C\backslash\omega$ they require

$$\frac{h_g(g^\omega)}{h_g(g^j)} = \lambda^j. \tag{3}$$

λ^j is the shadow price which ω attaches to the utility of a junior coalition party j. λ^j might exceed one in the case where a small party makes an offer to a large party. $\lambda^j = 0$ corresponds to an allocation where j gets nothing and $\lambda^i = 1$ corresponds to an equal split between ω and j.

We now want to determine the effect of an increase in the reservation price of a responder on public outlays. For this we define

$$\mu^i = \frac{1 + \sum_{j \in C\backslash\{\omega,i\}} \lambda^j}{\lambda^i},$$

the rate at which the proposal maker trades taxes on herself and everybody else for taxes on i. The more everybody else's utility decreases relative to i, the greater is the second round effect on aggregate expenditures of an initial increase in expenditures on i's behalf. For the party with the lowest reservation price, μ^i always exceeds 1**.** We find:

Lemma 1. If i is sufficiently disadvantaged in the distribution within C and the marginal evaluation schedule for public expenditures is sufficiently inelastic, i.e., if

$$\sum_{k \in C\backslash i} \frac{h_g^k}{-h_{gg}^k} < \mu^i \frac{h_g^i}{-h_{gg}^i}, \tag{4}$$

aggregate spending increases with a rise in R^i.

Proof. See Appendix 1. □

If (4) is fulfilled,[4] an increase in the demand of i is met by expenditure increases rather than a tax reduction. For this to be the case, the marginal expenditure benefit for i has to be sufficiently high which implies that expenditures on i's behalf have to be sufficiently low.[5]

2.3. *Negotiations in the parliament*

Now consider the final stage of the budget process where the parliament considers the government bill. If the bill is rejected, a new proposal maker ω' for the parliament is selected. The recognition probability is proportional to a party's seat share, π^i. Such an assumption is reasonable, because parliamentary decisions are often prepared in committees where the composition of the committees roughly reflects the composition of the overall assembly. A proposal is accepted if it gives their reservation price R^j to as many parties as are necessary to achieve a majority in the parliament. A party's reservation price is its expected pay off from rejecting a proposal and moving to the next bargaining round.

The bargaining horizon is either short or long. If it is short, ω' makes a take it or leave it offer. In that case she has to offer the default utility $U(0, 0)$ to the parties included in a legislative coalition. Because reservation prices coincide, ω' randomizes between making offers to those minimum winning coalitions which involve a minimum number of parties: If all parties have the same reservation price, it is cheaper to include fewer parties in a coalition which commands over a majority of seats in the legislature. This property of the model is in line with empirical evidence presented by Volkerink and De Haan (2001) and Perotti and Kontopoulos (1999) that an increase in the number of parties in the legislature raises government expenditures. For a party i, the probability of being included in a proposal submitted by j is $r^{ji} = m^{ji}/m^j$ where m^j is the number of minimum winning coalitions of minimum size considered by j and m^{ji} is the number of those coalitions of which i is a member. With this we can write the expected pay off for any party i from rejecting a government proposal under a short bargaining horizon as

$$V^i|S = \pi^i F(\{U^k(0,0)\}, k \in C^{\omega'} \backslash \omega') + \sum_{j \neq i} \pi^j \left[\frac{m^{ji}}{m^j} U(0,0) - (1 - \frac{m^{ji}}{m^j}) G^j \right]. \tag{5}$$

The first term on the right hand side refers to the pay off of i if it is selected to submit a proposal for a typical coalition $C^{\omega'}$. The sum refers to the expected outcome if i is not selected. If another party j is recognized, i receives its reservation price with probability m^{ij}/m^{j} and pays the taxes involved with a proposal by j, G^{j}, with probability $1 - m^{ji}/m^{j}$. Because a larger party can always replace a smaller party in a proposal, m^{ji}/m^{j} is monotonically increasing in the seat share. The first term in (5) is strictly increasing in the seat share π^{i}, so that a party with a higher seat share has always a higher expected pay off in the bargaining game with a short horizon.

In the second scenario the bargaining horizon is open and the legislators adopt stationary strategies. Now, in each bargaining round a proposal maker ω' is randomly selected among all parties. In a stationary equilibrium reservation prizes and the randomization probabilities r^{ji} have to be determined from the equilibrium conditions of the model (see Baron and Ferejohn, 1989). To see this, consider two parties which compete for a place in every other party's preferred coalition and assume for the moment that the bargaining horizon is two periods, so that a party's reservation price in the first period is simply $V^{i}|S$. Now, the party with the lower seat share would attract every other party's offer. This raises its reservation price relative to the reservation prize of the other party. If it eventually exceeds the reservation price of its competitor, it loses all its offers. An equilibrium in which both competitors have the same reservation prize R^{*} can be sustained with appropriate randomization strategies of the other parties where some party j is prepared to randomize between i and k if $R^{i*} = R^{k*}$. Now, the continuation value, i.e., the expected value for party i from moving one stage forward in the bargaining game is

$$V^{i}|L = \pi^{i}F(\{R^{k*}\}, k \in C^{\omega'}) + \sum_{j} \pi^{j}\left[r^{ji*}R^{i*} + (1 - r^{ji*})G^{j}\right] \qquad (6)$$

with r^{ij*} and R^{i*} referring to equilibrium values. In an equilibrium of the open horizon bargaining game reservation prizes and randomization probabilities have to satisfy the recursive system of equations

$$R^{i*} = V^{i}|L(\{R^{k*}\}, k \in N), \forall i \in N.$$

Because $V^{i}|L$ is stationary, it is also party i's expected value from entering negotiations in the parliament in the first place, given that the bargaining horizon is long. While it is possible that reservation prices are equated with an opening of the bargaining horizon, the rank of reservation prices never changes. Therefore, a party with a larger seat share never has a lower reservation price. But then we can make a precise statement about the rank of continuation values from entering the parliamentary stage, EV^i. By (1), the expected pay off from rejecting the government bill in the first place, EV^i, is determined as the expected value of conditional expected pay offs under a short and under a long bargaining horizon. Given that $V^i|S$ strictly increases in π^i, it follows that EV^i also strictly increases in π^i.

Lemma 2. With $P < 1$, the order of continuation values from entering the parliamentary stage preserves the rank of the seat shares, that is for $\pi^i > \pi^{i+1}$ we have $EV^i > EV^{i+i}$.
Proof. See Appendix. □

It is intuitive that at least the party with the lowest $V^i|S$ should gain with a move to an open bargaining horizon. Formally, this can be proved for the case $n = 3$:

Lemma 3. For $n = 3$, the expected pay of for the smallest party 3 increases with a move to an open bargaining horizon, i.e., $V^3|L > V^3|S$.

Proof. See Appendix. □

The reason why this lemma does not generalize without further restrictions to $n > 3$ is that unlike in the case $n = 3$ the smallest party need not necessarily get an offer by every other party even if it is cheapest. For example, consider the case $n = 4$ and the constellation $\pi^1 + \pi^4 < 0.5$. Here, the only minimum winning coalition of which party 4 becomes a member is (2, 3, 4).

2.4. *The cabinet stage*

In the cabinet stage, each junior coalition party of a government coalition $\overline{C}$ can realize an exogenously given pay off v^C.[6]For any party k, v^C may or may not exceed the pay off from bargaining in the parliament, EV^k. The assumption that v^C is exogenously given reflects that cabinet jurisdictions are not arbitrarily divisible. The head of government receives a pay off $F(\{v^C, k \in \overline{C}\}) > v^C$.

In the end, it is the necessary approval in parliament which justifies expenditures on behalf of the junior coalition partners. However, the ability to extract higher pay offs in the cabinet than in the parliament can be justified by the institutional power which a cabinet minister holds independently of his parliamentary backing. For example, consider a decentralized budget game where each cabinet minister holds jurisdiction over the provision of the good g^k which his clientele prefers and for which she submits a budget proposal $g^{k\prime}$. In that case, the budget proposal on the floor consists of the decentralized proposal $\{g^{k\prime}, k \in \overline{C}\}$ and yields a pay off $v^{C\prime}$ (see Laver and Shepsle, 1996; or Austen-Smith and Banks, 1990). As these proposals give rise to inefficiencies due to a common pool problem, there is scope for budget coordination. If the head of head of government coordinates that budget[7] and, by formal or informal budget laws, she is restricted to submit a budget which is accepted by all ministers in a vote against the uncoordinated budget proposal, she gives $v^{C\prime}$ to every member of the cabinet. As she wants to ensure passage by the parliament as well, the head of a majority coalition government picks a budget bill $\{g^{\omega}, g^{i}, i \in N\backslash\omega\}$ which offers the greater value of v^{C} and EV^{k} to all members of her government:[8]

$$R^{k} = \mathrm{Max}(v^{C}, EV^{k}), k \in \overline{C}\backslash\omega. \quad (7)$$

If ω is the head of a minority government, she has to satisfy the demand of a parliamentary majority coalition, $C^{\omega}\backslash\omega$. A party outside the coalition receives no expenditures.

2.5. *The government formation game*

In this section we derive the equilibrium of the government formation game for the case $n = 3$. In the government formation game the party with the largest seat share makes the first offer which has to be accepted by a majority of votes.[9] An offer implies giving another party a seat in the cabinet or ask it to tolerate a minority government. There is a pay off configuration uniquely associated with each offer. Because the formateur cannot commit herself to anything unless she offers a cabinet seat, we know that the bid to support a budget bill of a minority government goes to the cheapest party in the parliament. This is party 3 if party 1 or 2 are heading the minority government.

If an offer is rejected, the proposal right goes to the party with the next highest seat share. If the offers of all parties have been rejected the game starts from the beginning. This is the government formation game analyzed in Baron (1991). Like him, we only consider stationary strategies, i.e., strategies which prescribe the same action-profile in all structurally identical subgames. In this case, every subgame commencing after the rejection of the proposal of certain proposer is structurally identical. Therefore, a response only depends on the proposal and on the rank of the proposer.

2.5.1. *The case of a government premium*

In the case where there is a government premium, party 3 is offered a cabinet post by party 1. This equilibrium is supported because party 1 and 2 have a common interest to prevent party 3 from making a proposal which is the worst proposal from party 1's view.[10] Because 3 knows that it eventually gets an offer of a minority government headed by party 2 in the budgeting stage if party 1's proposal is rejected, party 1 has to make a strictly better offer than the minority pay off in the first stage.

Proposition 1. In the case of $n = 3$, if $v^C > EV^3$ there is a government premium for party 3 and the outcome of the government formation game is a majority coalition government in which party 3 joins a government with party 1.

Proof. See Appendix. □

2.5.2. *The case of no government premium*

Here, 1 cannot offer 3 a strictly better pay off than under a minority government, because 3 receives the same pay off in a majority coalition government as under a minority government. We have the following proposition:[11]

Proposition 2. If $v^C \leq EVP^3$, i.e., if there is no government premium for party 3, the outcome in the government formation game is a minority government of party 1 or a majority government formed by party 1 with support of party 3.

Proof. See Appendix. □

The reason why 3 does not reject this offer is slightly more subtle, however. It is the fact that 2 now credibly threatens to offer 1 a cabinet post after party 3 has rejected 1's offer which leaves 3 in an unfavorable position. Here, 1 is spared the worst case scenario: party 3 finds no one to accept its own proposal if acceptance does not occur in the round where 1 proposes.

3. A neutrality result

The prediction of the model so far is that governments with a positive government premium have majority status and expenditures rise with the premium: for a fixed reversion level EV^3expenditures vary directly with v^C. For a minority government, expenditures vary with EV^3 for a fixed v^C. If we want to derive conditional expenditures for each type of government, we have to make assumptions on the probability distribution of EV^3 and v^C in a sample. Having no further information, it is reasonable to hold Laplace expectations, i.e., that any value has the same probability measure. In that case, it is easy to see that the effect of majority status on expenditures is neutral on average:

Proposition 3. For $n = 3$, suppose that G is given by $G := f(Max(v^C, EV^3))$ and let v^C and EV^3 be equally and independently distributed on [0, 1]. Then average expenditure for all governments (minority and majority) is the same as average expenditure for the group of governments where randomization between majority and minority governments occurs.

Proof. To save notation we write e instead of EV^3. $Pr(e \geq v^C)$ gives the probability that e exceeds v^C. Let f(e) and $f(v^C)$ denote expenditure levels corresponding to e and v^C, respectively. Conditional expenditures for the group where randomizing over minority and majority governments occurs is

$$EG(e \geq v^C) = \frac{\int_0^1 f(e)\, Pr(e \geq v^C) de}{\int_0^1 Pr(e \geq v^C) de} = \frac{\int_0^1 f(e) \left(\int_0^e dv^C\right) de}{\int_0^1 e de} = 2 \int_0^1 ef(e)de$$

and average expenditure across all governments is

$$EG = \int_0^1 f(e)\, Pr(e \geq v^C) de + \int_0^1 f(v^C)\, Pr(v^C \geq e) dv^C = 2 \int_0^1 ef(e)de. \square$$

In the simple case where $G = Max(v^C, EV^3)$ average expenditure is $2/3$ in both cases. While this neutrality result holds under rather specific assumptions, the following section presents generalizations. We find that departures from neutrality may go in either direction, so neutrality emerges as the natural focal point of an analysis of government behavior.

4. Extensions of the model

4.1. *Incomplete information*

A midterm shock which results in a rise of P, the probability that the bargaining horizon is long, raises the bargaining power of the supporting party and consumes cohesion of the government. However, this does not threaten the stability of the government if information is complete: The head of government would like to offer a budget amendment to satisfy any verifiable demand of the other coalition party rather than having an offer rejected. But a government bill may fail in the parliament if information is incomplete. To see this, assume that the supporting party, when entering the parliamentary stage assumes that the bargaining horizon is long or short with certainty while the head of government only has a prior distribution $(P, 1 - P)$ over these events. If the bargaining institution were common knowledge or if the other party could truthfully signal what its claim is, the head of government offers v^C unless $V^3|L$ is actually claimed. Under incomplete information, however, she has to weigh the risks of overpaying and government failure. But even if the bargaining institution were common knowledge, a reduction of the government premium would make the government more vulnerable to shocks in preferences B(C) which are private information. Therefore, an increase in EV^3 always increases the probability of a government replacement.

I now construct a case where a coalition government's budget is voted down if the bargaining horizon turns out to be long, that is if the political environment is unexpectedly unstable. Consider the following strategies:

(a) The head of government proposes a bill giving v^C to the junior coalition partner, which is accepted in the cabinet.
(b) 3 rejects the bill in the parliament, realizes its reservation price $V^3|L$ and spending increases.

We assume that $F(v^C) > V^3|L > v^C > V^3|S$. So the junior coalition party votes for the cabinet proposal when the institution in the parliament is the simple closed rule but prefers to negotiate in the parliament when the bargaining horizon is long. 2 always votes against the budget.

Proposition 4. Strategy configuration $\langle(a), (b)\rangle$ is an equilibrium if the prior P that the parliament acts under a long bargaining horizon is sufficiently small.

Proof. First, P must be sufficiently low so that $v^C > EV^3$ and the government premium is positive ex ante. Secondly, the head of government must find it advantageous to propose v^C instead of $V^3|L$ to 3.[12] If the bill is rejected, the proposal-maker himself expects only $V^3|L$. If she proposes $V^3|L$ for her junior partner, she herself gets $F(V^3|L)$, which by construction exceeds $V^3|L$ but falls short of $F(v^C)$. Therefore, the head of government proposes v^C, if $(1 - P)F(v^C) + PV^3|L > F(V^3|L)$ which happens if P is sufficiently small. □

A midterm shock which consumes the government premium can bring down a coalition government, a possibility more fully discussed in Pech (2001). By proposition 2, a replacing government would be a minority government with positive probability. At the same time, it would have higher expenditures compared to the failed government.

4.2. *Ideology and office rents*

To the extend that the supporting party is willing to trade expenditures for office rents or ideological benefits which are achieved only when in the government, coalition governments would have lower expenditures on average. The existence of such benefits lowers the critical expenditure pay off in the cabinet which renders the government premium positive. This argument applies only as far as the head of state can perfectly observe the extent to which such an implicit advantage lowers the demand of the other party. Otherwise she may find it too risky to actually underbid the parliamentary value once the game arrives at the budgeting stage. Furthermore, ideological benefits need not always be positive. A party may perceive the membership in a certain coalition government as jeopardizing its reelection prospects [13]in which case the critical expenditure pay off in the cabinet is higher.

4.3. *More than three parties*

Here we give a set of sufficient conditions under which the results on the government formation stage in section II.2.5 generalize to $n > 3$. Let L^i be the set of the parties which are asked to support a minority government headed by party i in the budgeting stage (i.e., they are cheapest) and let $\overline{M}^i$ set of all coalitions which party i would like to form a cabinet government with and let m be the number of parties in a cabinet government.

1. $L^1 \subseteq L^2$, *i.e., parties which support a minority government headed by* 1 *in the budgeting stage are also asked for support by* 2.
2. $L^1 \in \overline{M}^1$ *and* $L^2 \in \overline{M}^2$, *i.e., a majority coalition government needs the support of as many parties* m *as a minority government.*
3. *Parties* 1 *and* 2 *have a majority in the chamber.*

Because of condition 3 it is sufficient that after its own proposal has failed 2 is prepared to cooperate with 1 to end bargaining. It is obvious that in the case $n = 3$ these conditions are satisfied.

Proposition 5. (a) Assume that conditions 1 through 3 hold. If $v^C(m) > EV^i$ for all $j \in L^1$, the unique outcome of the government formation game is a majority coalition government. If the relationship is violated for at least one party $j \in L^1$, the outcome is either a majority government or a minority government supported by j. (b) If in violation of condition 1 $j \in L^1$ but $j \notin L_2$, the outcome is a minority government supported by j whenever $v^C(m)$ exceeds EV^j.

The logic behind this proposition is the same as in the proof of propositions 1 and 2. In the case where condition 1 is violated because a party is only included in a minority government headed by 1, it lacks bargaining power in the government formation game so it cannot demand a government premium. In that case 1 can make the offer which is cheapest in terms of expenditures. If conditon 2 is violated, we have to compare pay offs under completely different coalitions. However, as we demonstrate in Section 5 of the appendix, one can construct an example, where one party is pivotal in the sense that it is the preferred party with which 1 and 2 want to form their government. In that case, its pay off is decisive for the decision to run a minority or majority government even if it is not included in a minority government headed by 1.

If proposition 5 (a) holds we can easily analyze the case of a government which relies on the support of more than one party. Now a government will exclusively form as a majority coalition government if for every party potentially supporting a minority government the value EV^i is below v^C. For this to be the case, v^C has to be relatively higher than in the case of one supporting party if the values of EV^i are uncorrelated.[14] While this result hinges on the assumption that all the supporting parties have bargaining power, we get a similar result if one of the supporting parties of the government does not have bargaining power and there is no pivotal party: Now, a minority government emerges if for the smallest party EV^i falls short of v^C. So in the case where the head of government is strong in the government formation stage, expenditures tend to be relatively low. The possibility of a strong formateur separates the n-party legislature from the three party legislature where the supporting party always has bargaining power.

5. Conclusion

This paper gives a systematic treatment of the relationship between budget outcomes and the majority status of the government. We derive our main result for the case of a three-party legislature acting under full information and in the absence of verifiable office rents or ideological benefits which are traded against expenditures. Here, we expect no systematic differences in spending between majority and minority governments. Suspending these assumptions results in deviations from neutrality in either direction. This suggests that neutrality is anything but an extreme case at the far end of the range of possible outcomes.

Under incomplete information, mid-term shocks can result in the failure of governments. Governments emerging as the result of political instability are predicted to show above average spending and come as minority governments with a positive probability. The availability of non expenditure benefits in the government would tend to lower cabinet expenditures. If the head of the government is relatively strong in the government formation stage, she chooses a minority government as a device to cut expenditures. This requires, however, that the supporting party is not pivotal in the government formation process.

The results presented here give a theoretical underpinning to the empirical papers which have found little support of the Roubini-Sachs indicator as a predictor of fiscal policies. At the same time our model is in accordance with a range of other findings of the empirical literature like the effects of fragmentation or political instability. Ideally, one would like to look not so much into the majority status of a government as into the circumstances in which it emerges.

Notes

1. Lupia and Strom (1995) derive sufficient conditions for government dissolution. These involve sufficiently high transaction costs of renegotiating the distribution within the governing coalition.
2. Other possible explanations are that in the case of ideologically coherent coalitions expenditures have strong public good properties across parties or that their members are more likely to behave cooperatively.
3. Huber, Kocher, and Sutter (2003) measure the bargaining power of a party using the Banzhaf index. They find that a more equal distribution of bargaining powers within a coalition government results in higher deficits.
4. Dividing the l.h.s. of (4) by $h_g(g^i)/(-h_{gg}(g^i))$ allows us to interpret it as a change in the marginal rate of substitution of expenditures on behalf of the other coalition members for expenditures on behalf of i when expenditures are varied proportionately (i.e., if g^k is varied at the rate λ^k). Expenditures increase if the marginal rate of substitution falls at a rate which is lower than the rate of transforming tax payments.
5. In the case where two parties are in the coalition (4) reduces to $h_g(g^j)^2/h_{gg}(g^j) > h_g(g^k)^2/h_{gg}(g^k)$ which for $g^k > g^j$ is fulfilled for all convex demand functions. It is easy to demonstrate that in a coalition of two equal sized parties with a demand function which is sufficiently steep and linear in the range (g^i, g^ω) the optimal allocation fulfills this requirement.
6. We assume that the pay off for any single party declines in the number of parties in the cabinet, but that this effect is not strong enough to give the formateur an incentive to include more than the minimum number of parties in his cabinet.
7. As discussed in Pech (2000), threats by the head of government to overspend in the initial budget proposal are ruled out if the restriction applies that the budget proposals have to be credible spending strategies when the budget is executed. The Nash equilibrium strategy is a credible strategy configuration because no-one has an incentive to individually defect ex post and spend less. Von Hagen (1992) finds that discretion in the execution of the budget is a cause for higher expenditures. This is in accordance with a two-stage budget game where discretion renders threats of the head of government incredible.
8. In claiming that there is the possibility of a government premium in a majority coalition government we have to discard the possibility of a vote of confidence, moved by the head of government. By such a vote, the head of parliament could theoretically force the other coalition members to their lower reservation value (see Huber, 1996). A vote of confidence, however, has been hardly used to discipline the cabinet. Furthermore, in all

states within the Roubini and Sachs sample were minority governments occur, restrictions apply to a vote of confidence.

9. Only some countries have a formal investiture vote. However, at least a majority of the parliament should be willing to accept that a minority government is run.
10. After rejection of its own offer party 1 is prepared to tolerate a minority government headed by 2 because the worst outcome for 1 is a minority government headed by 3 where expenditures have to pay off party 2. There are alternative procedures which support the equilibria of this section: One makes use of an exogenous restraint which cuts off the selection process after the first two parties have made an offer. This possibility is discussed in Baron (1991), see also note 11. Another is a procedure with an additional bargaining move for a majority of the chamber to agree on the dissolution of the parliament. Parties 1 and 2 would do so after 2's proposal has been defeated and if 2 were expecting a head to head race in the following elections.
11. If the bargaining game ends after two stages, 3 would randomize when receiving an offer by 1, i.e., a rejection occurs with positive probability. Such a procedure would account for the evidence provided in Merlo (1997) and Diermeier, Eraslan, and Merlo (2001) who show that in those cases where a minority government actually forms, negotiations tend to take longer than in those cases where a majority coalition governments forms.
12. Note that in the case of asymmetric information the head of government faces a similar problem when she runs a minority government. If she wants to offer $V^3|L > EV^3$ in order to account for the insecurity in the parliamentary stage an eventual government premium has vanished and proposition 2 holds in the government formation stage.
13. See the evidence in Strom (1990).
14. Empirically, minority coalition governments are do not play an important role outside of Norway.
15. 3 and 2 never propose the same policy with the same utility distribution when they change places. Let $x = EV^3$ and $y = EV^2$. Then, proposing the same policy would require that $F(x) = y$ and $F(y) = x$. Because both involve the same utility distribution it must also be that $F(x) + y = F(y) + x$ or $F(x) - F(y) = x - y$, which is only fulfilled for $x = y$. This violates lemma 2. But it is also impossible because in that case $x + y = F(x) + x$, so EV^2 and EV^3 would distribute the whole pay off at x which is only possible if 2 and 3 always impose their rule in the parliament. Therefore, 3 always proposes under a more restrictive constraint than 2 and, by lemma 1, has higher expenditures.

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Appendix

1. Proof of lemma 1

All allocations which are proposed in equilibrium are a solution to (2). Let λ^j be the Lagrange multiplier associated with j's restraint. In that case the f.o.c. require

$$h_g(g^\omega) = 1 + \sum\nolimits_{k\in C\backslash\omega} \lambda^k$$

$$h_g(g^j) = \frac{\pi^\omega + \sum_{k\in C\backslash\omega}\lambda^k}{\lambda^j}, j \in C\backslash\omega$$

Now consider an increase in the reservation pay off for i. Denominate the sum $\Gamma_{C\backslash\omega} = \sum_{k\in C\backslash\omega}\lambda^k$. Aggregate expenditures G can be expressed as a function of λ^i:

$$\begin{aligned} G(\lambda^i) &= g^\omega + g^i + \sum\nolimits_{j\in C\backslash\{i,\omega\}} g^j \\ &= h_g^{-1}(1+\Gamma_{C\backslash\omega}) + h_g^{-1}\left(\frac{1+\Gamma_{C\backslash\omega}}{\lambda^i}\right) + \sum\nolimits_{j\in C\backslash i,\omega} h_g^{-1}(\frac{1+\Gamma_{C\backslash\omega}}{\lambda^j}). \end{aligned}$$

Using $h^{-1\prime}(a\,\lambda) = \frac{a}{h_{gg}}$ one gets

$$\frac{\partial G}{\partial \lambda^i} = \frac{1}{h_{gg}(g^\omega)} + \sum\nolimits_{j\in C\backslash i,\omega} \frac{1}{\lambda^j h_{gg}(g^j)} - \frac{1+\sum_{j\in C\backslash\{\omega,i\}}\lambda^j}{(\lambda^i)^2 h_{gg}(g^i)}$$

Using $\lambda^j = \frac{h_g(g^\omega)}{h_g(g^j)}$ and the definition of μ^i, the condition in the lemma follows immediately for $\frac{\partial G}{\partial \lambda^i} > 0$ where we observe that $h_{gg} < 0$.

2. Proof of lemma 2

Because we know $V^i|S > V^{i+1}|S$ from the discussion in the text, it is sufficient to show that $V^i|L \geq V^{i+1}|L$ to proof that the lemma is true.

Consider the case $n = 3$. In that case the coalitions $(1, 3)$, $(2, 3)$ and $(1, 2)$ are all minimum winning. Suppose that $V^3|L > V^2|L$. In that case, $r^{12} = 1$ and $r^{32} = 1$. But then, because $\pi^2 > \pi^3$, it follows from (6) that $V^2|L > V^3|L$. Similarly, $V^2|L > V^1|L$ can be disproved.

For higher order cases, let the seat shares be $\pi^i > \pi^j > \pi^k$. With an arbitrary coalition S let $S \cup j \cup k \in M^\omega$ and $S \cup i \in M^\omega$ be minimum winning. If reservation prices coincide, it is always i which gets an offer from ω: If $h(g^{j\prime}) - G^{\omega\prime} = R$ and $h(g^{k\prime}) - G^{\omega\prime} = R$ then $h(g^{j\prime} + g^{k\prime}) - G^{\omega\prime} > R$. For a legislature with $S \cup j \cup k \in M^\omega$ and $S \cup i \notin M^\omega$, if i replaces j or k, the result is also a minimum winning coalition. But then $V^i|L < V^j|L$ cannot be true because the larger party, i, can always replace j in an offer once it is cheaper.

3. Proof of lemma 3

Initially, we construct the pay offs for the case where 3 gets an offer from every other party and makes an offer to 2 when selected to make a proposal. Let x be 3's reservation price. We have $\widehat{V}^2 = \widehat{\pi}^2 F(x) + \widehat{\pi}^1 z$ where $z < 0$ and $\widehat{\pi}^i = \pi^i/(1-\pi^3)$ and $\widehat{V}^3 = \pi^3 F(\widehat{V}^2) + (1-\pi^3)x$. Because x is the reservation price it must be that $x = \widehat{V}^3$ which implies that $F(\widehat{V}^2) = x$. We derive the lower boundary for x by setting $z = 0$ and define the function $T(x) = F(\widehat{\pi}^2 F(x))$. A fixed point of T satisfies $T(x^0) = x^0$ which is fulfilled at $x^0 = F(0)$. To see this, note that F(0) is the maximal value of the problem Max F^3 s.t. $U^2 \geq 0$ with $(\widehat{g}^3, \widehat{g}^2)$ as optimal choices. By duality, the problem Max F^2 s.t. $U^3 \geq F(0)$ yields $F^{2*} = 0$ and $(\widehat{g}^3, \widehat{g}^2)$. But then $F\left(\widehat{\pi}^2 F^{2*}(F(0))\right) = F(0)$. We now show that the fixed point is unique in a neighborhood of x^0 (it is straightforward but laborious to analyze the whole range of possible x). For a small variation of x^0, x we have

$$|T(x) - T(x^0)| \leq \widehat{\pi}^2 \mathrm{Max}[\lambda^{32}(0+\varepsilon), \lambda^{32}(0)]\mathrm{Max}[\lambda^{23}(x), \lambda^{23}(x^0)]|x - x^0|$$

where $\lambda^{ij} = \frac{h_g(g^i)}{h_g(g^j)}$. As x converges to x^0 and ε converges to 0, $\lambda^{32}(0+\varepsilon)$ converges to $\lambda^{32}(0)$ and $\lambda^{23}(x)$ converges to $\lambda^{23}(x^0)$. Because $\widehat{\pi}^2 << 1$, there is $\overline{\pi}$ with $\widehat{\pi}^2 \leq \overline{\pi} < 1$ such that $\overline{\pi}\lambda^{32}\lambda^{23} < 1$ where the inequality follows from our duality argument which implies that $\lambda^{32}(0)\lambda^{23}(x^0) = 1$. Then, T is a contraction mapping in a neighborhood of x^0. In this equilibrium where 3 always gets an offer its vote is so expensive that the proposer goes with a pay off of zero. But then, $\widehat{V}^3 \geq F(U(0,0))$ which exceeds $V^i|S = \pi^i F(U(0,0)) + (1-\pi^i)Z$ where we had $Z < 0$. The final step is to recognize that in a randomization equilibrium 3 realizes $V^i|L \in (V^i|S, x^0)$.

4. Proof of propositions 1 and 2

Consider the equilibrium strategies in an extensive form game for the responder: A party accepts an offer if it does at least as well as in the next offer which is accepted and there is no chain of subgames starting with the offer where it does at least as well in every subgame and strictly better in one. Therefore, an equilibrium strategy for a proposer is an offer which is acceptable to the responder which is worst off in the next offer which is accepted.

Define as party i's proposal a^i the pay off vector $\{U^1; U^2; U^3\}$ which is credibly associated with the government configuration which i proposes. We make the convention that i always keeps the position of head of government and that the head of a minority government cannot commit to offer anything to a party which is not the cheapest in the budgeting game. Let $\sigma^i|s$ be the move of party i in stage s of the government formation game where ω^s makes a proposal. $\sigma|s = (\sigma^1|s; \sigma^2|s; \sigma^3|s)$ accordingly describes a configuration of moves in stage s.

4.1. The case $v^C > EV^3$

The following description characterizes the moves which constitute a unique equilibrium for the game with a government premium:

- first stage: $a^1 = \{F(v^C); -G(v^C); v^C\}, \sigma|1 = (a^1$; reject; approve)
- second stage: $a^2 = \{-G(EV^3); F(EV^3); EV^3\}, \sigma|2 =$ (approve; a^2; reject)
- third stage: $a^3 = \{-G(EV^2); EV^2; F(EV^2)\}, \sigma|3 =$ (reject; approve; a^3)
- fourth stage: like first stage ...

where we have simply written "approve" in the first line for the (part of the) strategy of party 3 where it approves of an offer of a majority government (implying an offer v^C) if it is proposed when party 1 is the proposer. Note that $G(EV^2) > G(EV^3)$,[15]so a minority government headed by 3 and supported by 2 is the worst outcome for 1 and 1 tolerates a minority government headed by 2 once its proposal is rejected.

Were party 1 to deviate from its strategy by offering party 3 only a minority government involving EV^3, party 3 would reject because it is the minimum pay off it receives in all subgames following a rejection of a^1 and gets a better result in the subgame following a rejection of a^2.

4.2. The case $v^C \leq EV^3$

Here, an offer of a minority and a majority government imply the same pay off for party 3, the only credible supporter of a minority government in the budgeting stage. The equilibrium moves in the unique equilibrium of this game are:

- first stage: $a^{1\prime} = \{F(EV^3); -G(EV^3); EV^3\}, \sigma|1 = (a^{1\prime}$; reject; approve)
- second stage: $a^{2\prime} = \{v^C; F(v^C); -G(v^C)\}, \sigma|2 =$ (approve; $a^{2\prime}$; reject)
- third stage: $a^{3\prime} = \{-G(EV^2); EV^2; F(EV^2)\}, \sigma|3 =$ (reject; approve; $a^{3\prime}$)
- fourth stage: like first stage ...

It is easy to see that $a^{1\prime}$ cannot contain an acceptable offer to 2 because 2 in turn would be able to make an acceptable offer in the following stage to 3.

Note that the configuration of proposals (a^1, a^2, a^3) cannot be supported in an equilibrium. This can be seen by replacing v^C in a^1 by EV^3. As in section 5 party 2 relies on party 2 to inaugurate a minority government. Now, the strategies of party 2 and 3 can never form an equilibrium in the subgame starting with a^2: 2 accepts proposal a^3 only if 3 accepts proposal a^1. But if 2 accepts a^3, 3 does not want to accept proposal a^1. Observe that 2 has an incentive to switch from a^2 to $a^{2\prime}$, the unique equilibrium of the game.

5. Example for $n > 3$ with a pivotal party

For any party i the set of parties which support it in the budgeting stage, L_i, is well defined. A party $l \in L_i$ accepts an offer a^{il} by i unless there is a chain of proposers k which all propose $a^{kl} = a^{il}$ and one strictly better outcome which is realized at the end of the chain. A party $j \notin L_i$ accepts an offer by i only if the following offer a^{kj} which is accepted is worse for it.

Consider the following configuration: $\pi^1 = 0.41$, $\pi^2 = 0.29$, $\pi^3 = 0.2$, $\pi^4 = 0.06$, $\pi^5 = 0.04$. Then $L_1 = (4, 5)$, $L_2 = (3, 5)$, $L_3 = (2, 5)$, $L_4 = (2, 3)$, $L_5 = (2, 3)$. Note that minority governments incur increasing expenditures down the line and all majority coalition governments except (1, 2) and (1, 3) need support of two parties.

Now let $v^C(m = 1) > v^C(m = 2) > EV^4 > EV^5$. 4 has no bargaining power and would accept a minority government headed by 1. 5 has bargaining power if 2 finds acceptance for a minority government. 1 has to decide on whether to run a majority government where both parties get $v^C(2)$ or a minority government where 4 gets EV^4 and 5 gets $v^C(l = 1)$. This highlights the effect of the condition $L_1 \subseteq L_2$. But now, $L_1 \in \overline{M}_1$ is also violated: The cheapest solution for party 1 is to run a majority coalition government with party 3, paying $v^C(m = 1)$ as long as $G(Max(EV^3, v^C(m = 1))) < G(\{EV^4, EV^5\})$.

If $v^C(m = 1) > EV^3$ a government proposal where 1 offers 3 a cabinet post can be sustained as follows: If coalition (1, 3) emerges in the next stage, 5 cannot propose anythinġ: it can give at most $v^C(m = 2)$ to party 3. The same applies to 4. Party 3 finds acceptance for a minority government with parties 2 and 5. But then 1 helps 2 inaugurating a minority government and 3 has an incentive to accept the proposal of a majority coalition government (note that a minority government supported by 3 would not be credible).

If $EV^3 \geq v^C(m = 1)$ parties 4 or 5 can make a viable proposal to 3. A party, when indifferent says yes. So 2 accepts an offer of 3 and this induces 1 to inaugurate a minority government headed by 2. But then 1 cannot make a proposal to 3 because 3 can only improve by rejecting it. Therefore, 1 has to offer a minority government to 4 and 5.