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Elasticity measurement on multiple levels of DEA frontiers: an application to agriculture

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Abstract

Recently, the elasticity of response measures revealing the marginal characteristics of efficient frontiers have been developed and generalized for different types of DEA production technologies. In theory, the elasticity measures can be calculated for the units on the efficient frontier that satisfy a selective radial efficiency assumption. This corresponds to a subset of the evaluated units. In this research, we propose to extend the elasticity measurement to the entire production possibility set (technology) by stratifying the units to different levels of efficient frontiers. The stratification idea is inspired by the commonly known context-dependent DEA based on the exclusion of efficient units at each iteration and obtaining multiple levels of frontiers. We build the proposed methodology on the idea that a DEA technology theoretically consists of several frontiers and calculating elasticity measures on all frontiers may provide additional information on the returns-to-scale (RTS) characteristics of all the units whether they are on the first-level frontier or not. The proposed methodology is presented in an empirical application using the Farm Accountancy Data Network (FADN) data of the agricultural farms operating in the Aegean Region of Turkey. The results reveal that the proposed method enables us to obtain a wider perspective on the RTS characterizations of DEA production technologies.

Keywords Data envelopment analysis · Elasticity measures · Context-dependent DEA · Stratification · Agriculture

1 Introduction

The economic notion of returns to scale (RTS) has been widely studied within the framework of Data Envelopment Analysis (DEA) (see Banker et al. 2011 for a review). Following the early research focusing on qualitative identification of RTS characterization of the units on the frontier, recent efforts have been directed to the quantitative measures through the calculation of scale elasticity (Førsund and Hjalmarsson 2004). Since the DEA efficient frontiers are not defined in functional forms as in the classical economic theory, identifying the marginal characteristics of DEA frontiers requires special treatment and draws growing attention in DEA literature. Recently, the elasticity of response measures, which enable us to quantify the potential of efficient units in response to changes in any subset of input and/or outputs, have been developed and generalized for different types of DEA production technologies (see Hadjicostas and Soteriou 2006; Chambers and Färe 2008; Podinovski 2009; Podinovski and Førsund, 2010; Atici and Podinovski, 2012a; Podinovski et al., 2016).

In theory, the elasticity measures can be calculated for the Decision-Making Units (DMUs) on the efficient frontier that satisfy selective radial efficiency assumption (Podinovski and Førsund, 2010). Therefore, the developed measures provide information on a subset of the evaluated units corresponding to the units of relatively best-practice. On the other hand, DEA technology can be thought of as a set that consists of different levels of performance. Relying on the relative nature of DEA modeling, if the efficient units are removed from the technology, it is possible to obtain a new frontier, i.e. a new set of relatively best-practice. Previously, such an approach has been introduced by Seiford and Zhu (2003) as a part of context-dependent DEA modeling. In context-dependent DEA, the production possibility set is divided into different evaluation contexts that '*each evaluation context represents an efficient frontier composed by DMUs in a specific performance level*'. Context-

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dependent DEA is a practical approach in the sense that it enables one to rank the units within their contexts through attractiveness and progress scores, as well as to set efficient targets and reference sets for short-term and long-term improvements. Thus, it shows that DEA can be an effective tool for stratification of the DMUs without violating the nature of the production possibilities since each sub-technology is a member of the given production technology with hypothetical lower level frontiers that are obtained relative to a different subset of units. Here, we anticipate another advantage of stratification that if there are multiple frontiers (contexts) in a given technology, then it will be also possible to investigate marginal characteristics on those multiple levels of frontiers.

Relying on the above observation, in this research, we propose and show that the elasticity measurement can be expanded to the entire DEA technology through obtaining the elasticity of response measures at different levels of efficient frontiers. Inspired by the stratification approach of context-dependent DEA modeling by Seiford and Zhu (2003), we aim at an approach that unites two perspectives, resulting in the identification of marginal characteristics for a larger set of the evaluated units. The main idea of the research is based on obtaining levels of efficient frontiers through the exclusion of efficient units at each iteration, followed by the calculation of elasticity measures on every frontier based on the framework established by Podinovski and Førsund (2010) and Atici and Podinovski (2012a). With the proposed methodology, we aim to contribute to the economic notion of elasticity and its measurement in DEA production technologies by enriching its implications for the entire set of units. This also contributes to the returns-to-scale identification for inefficient units, which has been discussed from different angles in DEA literature.

Along with the proposed methodology, the research also promises an illustrative application of the proposed methodology and its implications in a case from a real-world agriculture data. The proposed methodology is applied to a sample of farms located in the Aegean Region of Turkey, which is one of the most active regions of the country in agriculture. The data is extracted from the Farm Accountancy Data Network (FADN) of Turkey for the set of farms in the given region accounting for the same farms between years 2015-2017. The response measures are calculated under different scenarios of changing and responding input and output factors, including the scale elasticity. The elasticity values are then interpreted in terms of RTS characterizations following the definitions presented by Podinovski et al. (2016). We suggest that such a methodological way of thinking would provide valuable additional information on the scale properties in a given DEA technology and generate wider policy implications for evaluating RTS. We also evaluate multiple years that enable us to assess the changes in RTS characterizations over the years.

The paper is organized as follows. In Section 2, we present the basics of elasticity measurement for output sets with some essential definitions and its model, explain the stratification framework of context-dependent DEA modeling and introduce the proposed methodology. In Section 3, we present the empirical application of the methodology in a sample of farms with a detailed description of data, model design, and findings. Finally, Section 4 concludes.

2 Elasticity measurement on multiple levels of DEA frontiers

A well-known measure for the marginal characteristics of DEA frontiers is the elasticity of response that reveals the magnitude of the response for a set of determined input factor(s) or output factor(s) to the changes in the determined sets of input and/or output factors at a given unit on the frontier. Such measures are calculated for the units located on the frontier and they provide insight into the Returns-to-Scale (RTS) characteristics of the frontier. Thus, they are available for a subset of units (efficient units) and their implications are limited to those units. On the other hand, DEA technology can be thought of as a set that consists of different levels of performance. This has been introduced to the DEA literature by Seiford and Zhu (2003) within the scope of Context-dependent DEA method. Context-dependent DEA is an effective approach that is based on stratification of the units to different performance levels, namely as contexts, and enables one to rank the units within their contexts through attractiveness and progress scores, as well as to set efficient targets and reference sets for short-term and long-term improvements.

We aim to contribute another use of stratification that is to expand the elasticity measurement to a larger set of units and enrich the practical implications on the RTS characterization of the DEA technologies. Because any unit in a given technology is a part of a frontier at a certain level, we propose that it is possible to obtain elasticity measures for the units based on their context. For this purpose, we integrate the stratification approach that is used within the context-dependent in DEA methodology to the elasticity measurement. Accordingly, our methodology consists of using two known DEA approaches together: elasticity measurement and the stratification in the context-dependent DEA. In this section, first, we present the basics of the elasticity measurement on DEA frontiers with its fundamental definitions. Second, we discuss the stratification approach of the context-dependent DEA. After defining both approaches, we introduce the integration framework.

2.1 Elasticity measures for output sets

The elasticity of response measures can be partially calculated for certain subsets of the inputs and output factors as well as for the whole set of inputs and outputs (scale elasticity). It is possible to calculate these measures both in Variable Returns-to-Scale (VRS) and Constant Returns-to Scale (CRS) production technologies. In modeling, we assume that all inputs and outputs can be divided into three disjoint sets: A, B, and C. The set A represents marginal change factors, set B expresses response factors with respect to the marginal changes of set A factors and set C shows factors that remain constant. Set B can be assumed to include either outputs or inputs. In this section, we present the case that the assumed technology is VRS, and set B consists of only outputs, which relevant to our empirical application. The sets A and C can include both input and output factors. Therefore, any unit $(X_0, Y_0) \in T_{VRS}$ can be defined as $(X_0, Y_0) = \left(X_0^A, X_0^C, Y_0^A, Y_0^B, Y_0^C\right)$, where the superscripts indicate the sub-vectors of X_0 and Y_0 corresponding to the sets A, B, and C. In the case of any empty sets, the corresponding sub-vectors are omitted.

The modeling for the elasticity of response measurement is built upon the proportional output response function $\overline{\beta}(\alpha)$ followed by the main assumption of selective radial efficiency pointing out that the given DMU (X_0, Y_0) is efficient in the production of its output vector Y_0^B (Podinovski and Førsund 2010).

2.1.1 Definition

Output response function. For the unit $(X_0, Y_0) \in T_{VRS}$, the output response function $\overline{\beta}(\alpha)$ is the following optimal value (as long as it exists, $\beta \in R^+$):

$$\overline{\beta}(\alpha) = max\left\{\beta | \alpha X_0^A, X_0^C, \alpha Y_0^A, \beta Y_0^B, Y_0^C\right\}$$

$\overline{\beta}(\alpha)$ function represents the maximum proportion of vector Y_0^B feasible in T_{VRS} if the vectors X_0^A and Y_0^A change in proportion α, and the vectors X_0^C and Y_0^C are kept constant.

For a unit to have a defined elasticity measure, it should satisfy the selective radial efficiency with respect to the output set B, stated as below in Atici and Podinovski (2012a):

2.1.2 Assumption

Selective radial efficiency with respect to the output set B. The function $\overline{\beta}(\alpha)$ is finite at $\alpha = 1$, and $\overline{\beta}(1) = 1$.

The elasticity measures are developed relying on the dual of the output response function and its differential characteristics. Two linear programming (LP) models are solved for the right-hand and left-hand elasticity measures for a unit, respectively. The right-hand elasticity measure corresponds to the response of the given output Y_0^B to the increases in the changing inputs and/or outputs. The left-hand elasticity is the response of the given output Y_0^B to the decreases in the changing inputs and/or outputs. For a unit that satisfies the selective radial efficiency assumption in technology T_{VRS}, Podinovski and Førsund (2010) present the LP model to solve for the right-hand elasticity measure $\varepsilon^+_{A,B}(X_0, Y_0)$ as below. The left-hand elasticity $\varepsilon^-_{A,B}(X_0, Y_0)$ can be calculated by changing the maximization problem to minimization subject to the same constraints.

$$\begin{aligned}
&\varepsilon^+_{A,B}(X_0, Y_0) = \min v^A X_0^A - \mu^A Y_0^A \\
&\textit{subject to} \\
&v^A X_0^A + v^C X_0^C - \mu^A Y_0^A - \mu^C Y_0^C + \mu_0 = 1 \\
&v^A \overline{X}^A + v^C \overline{X}^C - \mu^A \overline{Y}^A - \mu^B \overline{Y}^B - \mu^C \overline{Y}^C + \mu_0 \geq 0 \\
&\mu^B Y_0^B = 1 \\
&v^A, v^C, \mu^A, \mu^B, \mu^C \geq 0, \mu_0 \,\textit{sign free}
\end{aligned} \tag{1}$$

In Atici and Podinovski (2012a), it is clarified that when calculating one-sided elasticities via the LP given above, one does not need to check for the selective radial efficiency assumption to hold for a certain unit. The above programs produce three possible solutions. For a unit, if the above program has a finite optimal solution, then $\varepsilon^+_{A,B}(X_0, Y_0)$ exists and is equal to the optimal value. In this case, the selective radial efficiency assumption is also satisfied by the given unit. If the above program has an unbounded solution, then $\varepsilon^+_{A,B}(X_0, Y_0)$ is undefined although selective radial efficiency assumption is satisfied by the unit. Finally, if the above program is infeasible, DMU (X_0, Y_0) does not satisfy the selective radial efficiency assumption, therefore, $\varepsilon^+_{A,B}(X_0, Y_0)$ is also undefined. The same applies to the left-hand elasticities.

The RTS characterization of the units can be identified by observing the right-hand and left-hand elasticity measures obtained by given LPs. This is given in Definition 3 of Podinovski et al. (2016) as below.

The output radial efficient unit $(X_0, Y_0) \in T_{VRS}$ exhibits:

- increasing returns to scale (IRS) if $1 < \varepsilon^+_{I,O}(X_0, Y_0) \leq \varepsilon^-_{I,O}(X_0, Y_0)$
- decreasing returns to scale (DRS) if $\varepsilon^+_{I,O}(X_0, Y_0) \leq \varepsilon^-_{I,O}(X_0, Y_0) < 1$
- constant returns to scale (CRS) if $\varepsilon^+_{I,O}(X_0, Y_0) \leq 1 \leq \varepsilon^-_{I,O}(X_0, Y_0)$

2.2 Stratification in context-dependent DEA

The stratification approach in context-dependent DEA introduced by Seiford and Zhu (2003) can be explained as

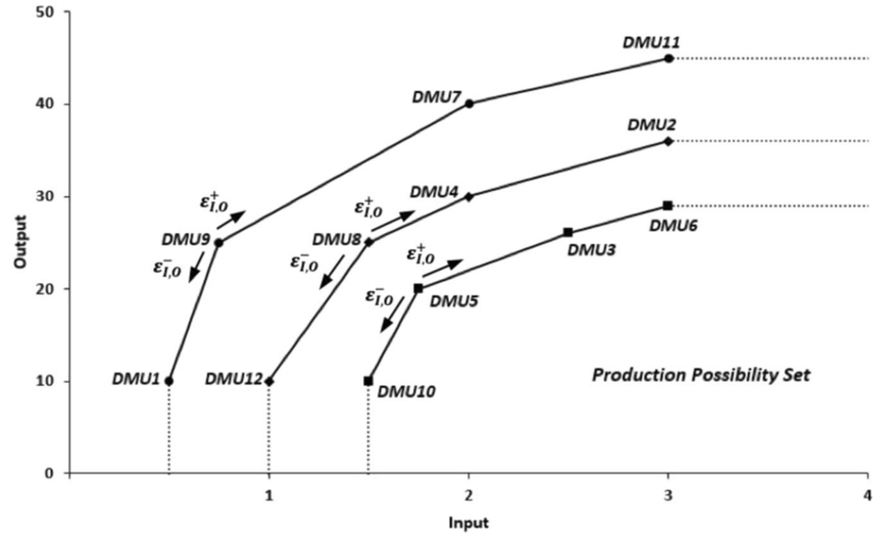

Fig. 1 Illustration for stratified units and their elasticity

follows. We define the set of all DMUs as N and set of efficient DMUs as E. N^1 defines all DMUs in the production technology and E^1 represents the set of units on the first-level efficient frontier and is a subset of N^1. Context-dependent DEA is based on defining different and successive contexts. This successive structure is provided from connections between N^1 and E^1, where l represents the level of the frontier, i.e. context. The arrangements of N^1 and E^1 are defined as $N^{l+1} = N^l - E^l$. Every l value creates a new evaluation context for determining the next level efficient frontier. The evaluation contexts are obtained by creating level-by-level efficient frontiers until only the last DMU is left (Zhu 2004). The output-oriented VRS efficiency score of DMU_o in context l is expressed as below.

$$\begin{aligned} &\phi^*(l,o) = \max_{\lambda_j,\phi(l,o)} \phi(l,o) \\ &subject\ to \\ &\sum_{j \in F(N^l)} \lambda_j y_j \geq \phi(l,o) y_o \\ &\sum_{j \in F(N^l)} \lambda_j x_j \leq x_o \\ &\sum_{j \in F(N^l)} \lambda_j = 1 \\ &\lambda_j \geq 0 \end{aligned} \tag{2}$$

In the above model, $F(.)$ represents the correspondence from a DMU set to the corresponding subscript index set. Therefore, $j \in F(N^l)$ implies $DMU_j \in N^l$. Note that $l = 1$ corresponds to the standard VRS technology. In practice, the stratification is based on solving DEA models by excluding the efficient units from the data set at each iteration. Below, we explain how stratification in context-dependent DEA can be used for elasticity measurement at different levels.

2.3 Integration of stratification to elasticity measurement

Relying on the observation that even the inefficient units in a production technology are thought to be on the frontier of consecutive contexts obtained when the efficient units are excluded, it is possible to compute elasticity measures for those units as well. This can be achieved by integrating the stratification approach of the context-dependent DEA into the elasticity measurement.

Figure 1 illustrates the multiple levels of frontiers for 12 hypothetical units. Originally, the right-hand ($\varepsilon^+_{A,B}$) and left-hand ($\varepsilon^-_{A,B}$) elasticity measures would be available only for the efficient units in the first-level (DMU1, DMU9, DMU7 and DMU11) as illustrated for DMU9 in Fig. 1 ($A = I$, $B = O$). With stratification, all DMUs are now on an efficient frontier of different contexts, which means that elasticity measures can apply to new DMUs. In the absence of the upper layer efficient units, the remaining feasible units form new efficient frontiers. Note that the units on these frontiers are inefficient units in the original technology; however, they are efficient in the sub-technologies constructed in line with production possibility set assumptions. Once the units are stratified into successive contexts, we can calculate the right-hand ($\varepsilon^+_{A,B}$) and left-hand ($\varepsilon^-_{A,B}$) elasticity measures for second-level efficient units (e.g. DMU8) or third-level efficient units (e.g. DMU5) that will be on a frontier in different contexts. Accordingly, we obtain the marginal characteristics for units that are originally out of the frontier and therefore, will be able to interpret the effects of changes for a wider set of units. Additionally, this opens up a new perspective for evaluating the RTS for inefficient units in the original technology as an alternative to projecting the units on the original frontier and evaluating the RTS for the projections (see Banker et al. (2004) for a discussion on RTS of inefficient units).

In any scenario of sets A and B for a given data set, the problem can be handled in three alternative methods summarized and explained below.

Method 1. Solve both models in consecutive order.

(i) solve program (2) for each $DMU \in N^l$
(ii) exclude efficient units at each iteration, define $N^{l+1} = N^l - E^l$
(iii) stratify the data set into $l = 1, 2, \ldots, n$ contexts until $N^{l+1} = \emptyset$
(iv) solve (1) for each unit in n samples.

Method 2. Solve only the elasticity model by excluding units that yield in optimal and unbounded solutions at each iteration.

(i) solve program (1) for each $DMU \in N^l$
(ii) exclude the units with an optimal and unbounded solution to (1)[1]
(iii) solve (1) for each unit in each N^{l+1} $(l = 1, 2, \ldots, n)$ until no infeasibility is observed.

Method 3. Solve only the elasticity model for the units exhibit infeasibility at each iteration.

(i) solve program (1) for each $DMU \in N^l$
(ii) segregate the units with an infeasible solution to (1)[2]
(iii) solve (1) for each unit in each N^{l+1} $(l = 1, 2, \ldots, n)$ until no infeasibility is observed.

The first method involves running two models in consecutive order. We can identify the data sets for each context using standard DEA modeling and then apply elasticity measures to stratified samples. It is also possible to rely on the elasticity measurement as indicated by Methods 2 and 3. The solution to program (1) can serve as an estimator if the unit is efficient or not in the given context. If a unit is satisfying the selective radial efficiency assumption in the given technology, then infeasibility will never be observed when the program (1) is solved (see Atici and Podinovski 2012a). The solution to program (1) will yield either an optimal or an unbounded solution. In this case, we can move to next iteration either by excluding the units with optimal and unbounded solutions and solving (1) for remaining units or segregating units with an infeasible solution and continue solving (1) for these units at each iteration. The advantages of pursuing Methods 2 and 3 would be early detection of the exceptional cases of units that might not be efficient but satisfy the selective radial efficiency in the given elasticity scenario (by given elasticity scenario, we refer to the selection of sets A and B).

In the following section, we put the proposed methodology in practice in a real-world case for a sample of agricultural farms.

[1] Note that this is analogical to excluding the efficient units and therefore it is basically defining $N^{l+1} = N^l - E^l$.

[2] Note that this is analogical to excluding the efficient units and the segregated set is identical with $N^{l+1} = N^l - E^l$.

3 Application

3.1 Data

To illustrate our proposed methodology in practice, we utilize a farm-level data set from the Farm Accountancy Data Network (FADN) of Turkey. FADN is a survey network that is carried out by the member and candidate states of the European Union within the scope of Common Agricultural Policy. It consists of standardized farm-level data and is representative of the commercial agricultural farms of the given state (European Commission, 2020). Our sample belongs to the farms operating in the Aegean Region of Turkey, which is one of the most active regions of the country in agriculture. The data set is obtained from the Turkish Ministry of Agriculture for the years 2015–2017. It consists of 146 commercial farms (same farms across years). The DMUs are labeled with the provinces that the farms are located and with farm-specific codes for privacy. The region consists of 8 provinces.

In the selection of input variables, out of the richer data set of FADN, we focus on common factors used in the farm efficiency research as land, labor, cost, and capital as well as livestock (see Atici and Podinovski 2012b for an extensive review). On the output side, we use the monetary values of crop and livestock production. Table 1 presents the input and output factors of the research followed by their definitions.

Table 1 Input and output factors

Input factors		
I1	Total land	Hectare
I2	# of total livestock	Number
I3	# of total machinery and equipment	Number
I4	Total labor	Annual Working Units
I5	Total cost	Turkish Lira
Output factors		
O1	Total crop production output	Turkish Lira
O2	Total livestock output	Turkish Lira

Table 2 Elasticity measurement scenarios

Scenario	Set A *(Changing)*	Set B *(Responding)*
Scenario 1 – *Scale elasticity*	All inputs (*I1*, *I2*, *I3*, *I4*, *I5*)	All outputs (*O1*, *O2*)
Scenario 2 – *Changing livestock vs. livestock output*	# of total livestock (*I2*)	Total livestock output (*O2*)
Scenario 3 – *Changing cost vs. outputs*	Total cost (*I5*)	Total crop production output (*O1*)
		Total livestock output (*O2*)
Scenario 4 – *Changing land vs. crop output*	Total land (*I1*)	Total crop production output (*O1*)

- *Total land* represents the total utilized agricultural area by the farm and it is expressed in hectares (1 hectare = 10,000 m^2).
- *The number of total livestock* refers to the livestock belongings of the farm. This factor is closely related to the total livestock output (revenues) through various livestock-based products.
- *The number of total machinery and equipment* is included to represent capital and involves tractors, trucks, panel vans, cars (not for personal use) together with the farm tools which worth more than 100 Euros.
- *Total labor* is expressed as an Annual Work Unit (AWU) that is the full-time employment equivalent. AWU is calculated with the total hours worked divided by the average annual hours worked in full-time jobs in the country (Eurostat Glossary, 2020).
- *Cost* is used as the sum of the following two costs and measured in Turkish Liras: total intermediate consumption and total purchases. It consists of total specific costs (including inputs for production) and overheads arising from production in the accounting year. Note that the cost of capital and livestock purchases are not inclusive to this item.
- *Total crop production output* is measured in Turkish Liras. It includes revenues by the sales of crops and crop products.
- *Total livestock output* consists of the revenues by the sales of livestock and their products like milk, egg, wool, etc. It is measured in Turkish Liras.

3.2 Model design

The proposed methodology is applied to the described data set of 146 commercial farms in two phases based on running two models as defined as Method 1 in Section 2.3. The data set covers three years (2015–2017) that enables to experiment with three sets independently and obtaining marginal characteristics in each year, as well as to observe the changes in marginal characteristic between years.

Phase I. We run standard DEA modeling for all years to observe the efficient and inefficient units. The first-level efficient frontiers are obtained for all years. This is the case where $l = 1$ in the program (2). This is the standard output-oriented VRS DEA model. VRS is selected for more variation in the elasticity measurement and output-oriented modeling is preferred because the elasticity of response measures for output sets is of interest. Then, the farms are stratified by excluding the efficient units at each iteration. We run the program (2) for $l > 1$ and identify the number of frontiers and efficient units on each frontier (i.e. in each context).

Phase II. After identifying the efficient units in each context, the data set is divided into subsets relying on the results of Phase II. By using the program (1), the elasticity measures are calculated in each sub-technology. We consider a variety of scenarios for changing and responding sets of A and B, respectively given in Table 2. The set C consists of the remaining factors for each scenario. The first scenario represents the scale elasticity, in which all outputs are responding to all inputs changing. The remaining scenarios consist of partial sets of inputs and outputs focusing on different aspects of the change. Scenario 2 represents the response of livestock output to changing livestock (number); whereas Scenario 4 is about the response of crop production to changing land. Scenario 3 focuses on the response of both outputs to changing costs. The elasticity values obtained in line with these 4 scenarios are interpreted to identify the RTS characterization of the units in each scenario. The changes over the years are observed.

As indicated in Section 2.3, it is possible to skip solving the standard DEA models to identify the efficient units in each context. Solving only elasticity models will also serve as an estimator if the unit is efficient or not. We also verify the findings of our above approach by applying Method 2 and observe that the same results for all scenarios are obtained.

3.3 Findings

In this section, we present the findings of two phases of the methodology described above. The evaluation period involves three years (2015–2017).

Table 3 Stratification results

Year	Context	# of efficient units	# of inefficient units	Average efficiency
2015	*First-level*	56	90	0.70
	Second-level	56	34	0.89
	Third-level	28	6	0.96
	Fourth-level	6	0	1.00
2016	*First-level*	50	96	0.69
	Second-level	50	46	0.85
	Third-level	33	13	0.91
	Fourth-level	12	1	0.96
2017	*First-level*	53	93	0.67
	Second-level	61	32	0.88
	Third-level	23	9	0.94
	Fourth-level	9	0	1.00

3.3.1 Findings of stratification with DEA

We begin with identifying the first-level context by applying the output-oriented VRS DEA model for three years (2015, 2016, and 2017). The average efficiency scores are obtained as 0.70, 0.69, and 0.67 for three years, respectively. Then, by excluding efficient units at each iteration, we obtain different levels of efficiency. The numbers of efficient and inefficient units with the average efficiency scores at each level are presented in Table 3. We observe that there are four contexts, therefore four levels of frontiers except for the year 2016. For 2016, we have a single farm remaining that can be considered as a fifth-level.

By stratification, all units are assigned to a frontier depending on their performance level. It is possible to calculate different efficient targets and identify levels of reference sets that may help to evaluate short-term and long-term improvement potential. In context-dependent DEA, the next step is to calculate the relative attractiveness and progress measures. Attractiveness and progress refer to the score of a unit when evaluated within the lower-level context and upper-level context, respectively. These measures enable us to differentiate between the units and therefore, obtain ranking at different contexts. With the proposed methodology of the current research, we suggest another advantage that it is also possible to obtain elasticity measures and therefore, RTS characterizations in each context. In our case, for all years, 34% to 38% of the units are found to be efficient in the first-level evaluations. In the standard setting, it would be possible to obtain elasticity measures of those units. However, with the stratification, we can now evaluate the marginal characteristics of the entire technology.

3.3.2 Findings of elasticity measurement

After obtaining levels of frontiers and stratifying the data set for each context, we calculate the right-hand and left-hand elasticity measures using the program (1) in every context for every year. We rely on four different scenarios of changing and responding sets of factors as given in Table 2 in Section 3.2. To be representative of our results, we present Table 4 for selected farms. The table summarizes the right-hand and left-hand elasticity measures of representative farms under four scenarios and three years. It includes representative farms from all provinces from the data set. The farm name indicates the province and the farm code within the province (the codes are unique and may include values greater than 146; they do not represent farm number). The single unit in 2016 at the fifth level is ignored in presenting the elasticity results and the results are standardized to four contexts. Also, there is a single unit that exhibits infeasibility in scenarios 2, 3, and 4 although it is reported as efficient on the third-level frontier in the previous phase. This farm does not satisfy the selective radial efficiency assumption for given scenarios. Below, we provide the implications that can be derived from the results presented. With the help of stratification, we now have the elasticity measures for units that are inefficient in the standard DEA model setting.

The elasticity measures can be seen as a tool for interpreting the sensitivity of the units to the percentage changes in factors at local level. Consider a scenario that changing set A includes an input factor and the responding set B includes an output. A right-hand elasticity (RHE) measure of 2 means that at the given unit, increasing the input by 1% will be responded by a 2% increase in the output. Left-hand elasticity (LHE) measure of 2 reveals the opposite case, in which decreasing the input by 1% will be associated with a 2% decrease in the output. Both one-sided elasticities can attain values greater than 1, which will imply increasing returns-to-scale (IRS) for the given unit (see Section 2.1 for the interpretation). For instance, the right-hand scale

Table 4 Elasticity measures for representative farms from different contexts

2015	Scenario 1		Scenario 2		Scenario 3		Scenario 4	
First-level	*RHE*	*LHE*	*RHE*	*LHE*	*RHE*	*LHE*	*RHE*	*LHE*
Aydın-75	1.22	1.28	15.16	*UD*	0.50	0.53	0	0.03
Uşak-1	0.87	1.70	0	2.03	0.14	1.27	0	0.33
Second-level								
Denizli-38	3.25	*UD*	0	*UD*	1.11	*UD*	0	*UD*
Aydın-140	0.85	3.29	0	*UD*	0	1.17	0.31	2.40
Third-level								
İzmir-146	1.85	11.94	0	0.51	0	2.10	0	*UD*
Aydın-68	0.47	1.06	0	*UD*	0	0.75	0	1.35
Fourth-level								
Denizli-3	0	2.97	0	*UD*	0	0.75	0	0.44
Denizli-23	0.84	*UD*	0	*UD*	0.23	*UD*	0	*UD*
2016								
First-Level	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**
Muğla-19	2.15	7.95	0	*UD*	1.02	1.97	1.39	4.05
Manisa-7	2.89	8.71	0	*UD*	1.49	3.29	0.39	3.97
Second-level								
Aydın-113	0.21	1.33	0.13	1.07	0	0.48	0	*UD*
Denizli-45	1.35	2.36	0.35	1.36	0	0.71	0	*UD*
Third-level								
Aydın-124	2.19	4.99	1.06	2.98	0	0.12	0	0.11
İzmir-87	0.19	2.11	0	*UD*	0	0.67	0.07	0.94
Fourth-level								
Denizli-95	0.20	2.97	0	*UD*	0	1.13	0	*UD*
Denizli-48	1.41	*UD*	0	*UD*	0	*UD*	0	*UD*
2017								
First-level	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**
Uşak-1	1.65	23.45	0.07	2.44	0.58	8.98	0	*UD*
Muğla-87	1.23	1.57	0	0.23	1.08	1.57	3.38	*UD*
Second-level								
Afyon-42	0.14	1.96	0	*UD*	0	0.65	0	0.99
Kütahya-130	0.44	1.16	0	1.20	0	0.07	0.19	0.72
Third-level								
İzmir-73	4.30	*UD*	0	*UD*	0	*UD*	4.10	*UD*
Aydın-149	0.34	1.64	0	*UD*	0	1.20	0	*UD*
Fourth-level								
İzmir-55	0	2.51	0	*UD*	0	0.70	0	*UD*
İzmir-99	5.90	UD	0	*UD*	0	*UD*	0	*UD*

RHE Right-hand Elasticity, *LHE* Left-hand Elasticity, *UD* Undefined

elasticity (Scenario 1) of the farm *Aydın-75* in 2015 is 1.22 implies that a 1% increase in all inputs will result in a 1.22% increase in the outputs. Similarly, the left-hand scale elasticity for the same farm implies a 1% decrease in all inputs will be responded with a 1.28% decrease in outputs. The values of right-hand and left-hand elasticities can be also less than 1 on either or both sides. For instance, both RHE and LHE of the farm *Aydın-75* in 2015 are less than 1 for Scenario 3 indicating a decreasing returns-to-scale (DRS) for this farm under Scenario 3. RHE and LHE values of *İzmir-87* in Scenario 1 of 2016 are 0.19 and 2.11, respectively. This is an indication of a constant returns-to-scale (CRS) for this unit under this scenario.

In measuring the response of output sets, when both sets include both outputs then the elasticity measure will be

Table 5 The farms with highest elasticity scores in the scale elasticity scenario (scenario 1)

	2015		2016		2017	
Context	Farm	RHE	Farm	RHE	Farm	RHE
First-level	*Aydın-108*	14.15	*Aydın-44*	10.01	*Aydın-41*	25.89
Second-level	*Aydın-164*	745.67	*İzmir-159*	17.80	*Aydın-82*	29.35
Third-level	*Denizli-78*	11.49	*İzmir-125*	52.82	*Aydın-11*	10.16
Fourth-level	*Aydın-26*	2.88	*Denizli-23*	4.73	*İzmir-99*	5.90
Context	**Farm**	**LHE**	**Farm**	**LHE**	**Farm**	**LHE**
First-level	*Aydın-86*	29.34	*Uşak-80*	89.34	*Aydın-23*	24.92
Second-level	*Aydın-11*	167.92	*Aydın-156*	24.47	*Manisa-130*	108.35
Third-level	*Afyon-180*	51.48	*Afyon-180*	24.13	*Aydın-124*	7.23
Fourth-level	*Denizli-3*	2.97	*Denizli-95*	2.97	*Denizli-23*	11.38

negative since increasing one output will require decreasing the other one. In our case, since the set A includes only inputs and set B includes only outputs, no negative values are observed.

As indicated in Section 2.1, the elasticity measures can be undefined due to the unbounded solutions to the program (1). This implies that the unit is on a part of the frontier that it is not feasible to increase or decrease its input and/or output vectors within the given technology. In Table 4, undefined elasticities are mostly observed in Scenarios 2, 3, and 4, which represent partial sets of inputs and outputs. Also, it is observed both in Table 4 and within the entire results that the unbounded elasticity measures are more in number four third and fourth contexts.

In addition, it is possible to observe zero values in both right and left elasticities, which imply that the change in factors in set A has no effect on the factors in set B for the given unit, corresponding to the horizontal bit of the frontier. For instance, observe the RHE of *İzmir-55* in all scenarios in the year 2017. It is also possible to notice several units with RHE as 0 and unbounded LHE such as *Denizli-38* and *Aydın-140* in 2015 for Scenario 2, which is also an indication of CRS.

In some cases, relatively higher values of elasticity can be observed. We can say that the outputs of these units are highly sensitive to changes in the inputs. To illustrate, Table 5 presents the units with the highest elasticity scores according to the scale elasticity (Scenario 1) measurements. The upper panel of Table 5 summarizes the highest right-hand elasticities (RHE) and the lower panel presents the highest left-hand elasticities (LHE) in each context and each year.

As can be seen in Table 5, some farms exhibit remarkably high elasticity values such as on the second-level frontier, the RHE of the farm *Aydın-164* in 2017, LHE of the farm *Aydın-11* in 2015 or LHE of the farm *Manisa-130* in 2017. Such high elasticity values are obtained due to the steepness of the frontier at the given unit in the given scenario. For such DMUs this means that the output values of the given DMUs are highly sensitive to the changes in the given inputs.

3.3.3 Returns-to-Scale (RTS) characterizations

The quantitative measures of elasticity of response can be converted into RTS classifications of the units. The protocol is given in Section 2.1 in line with Definition 3 of Podinovski et al. (2016). In this part of the analysis, we identify the RTS characterizations for all farms stratified in each context as Increasing Returns-to-Scale (IRS), Decreasing Returns-to-scale (DRS) and Constant Returns-to-scale (CRS). According to Table 6, in all scenarios, contexts, and years, CRS prevails for the farms in the data set. The scenario-specific evaluations for increasing different combinations of the inputs are provided below.

3.3.3.1 Scenario 1 (Scale elasticity) The farms exhibiting CRS are followed by IRS farms with noticeable percentages in every context. Increasing all the inputs with a certain percentage, which can be seen as relatively a long-run policy compared to other scenarios, will most likely result in an increase in outputs by the same rate or by a larger percentage for the farms of the region.

3.3.3.2 Scenario 2 (Changing livestock vs. livestock output) The farms exhibiting CRS are followed by DRS farms according to this scenario. The percentage of DRS and IRS farms is decreasing and CRS prevalence is increasing for the farms on the frontier in the third and fourth contexts. Increasing the livestock with a certain percentage will most likely be responded with an increase in the livestock outputs with the same rate; nevertheless a decreasing rate on the table with a noticeable possibility for the farms of the region.

3.3.3.3 Scenario 3 (Changing cost vs. outputs) The farms exhibiting CRS are also followed by DRS farms according to this scenario. DRS prevalence is more visible for all years in this scenario, especially regarding the first and second level units. Spending more on production by some percentage will most likely result in an increase in the crop and

Table 6 RTS characterizations (%) for the farms in all contexts, scenarios and years

		Scenario 1			Scenario 2			Scenario 3			Scenario 4		
Year	Context	IRS	DRS	CRS	IRS	DRS	CRS	IRS	DRS	CRS	IRS	DRS	CRS
2015	*First-level*	36%	5%	59%	4%	21%	75%	7%	41%	52%	14%	14%	71%
	Second-level	39%	4%	57%	5%	20%	75%	4%	43%	54%	9%	29%	63%
	Third-level	29%	4%	68%	4%	18%	79%	0%	36%	64%	7%	14%	79%
	Fourth-level	17%	0%	83%	0%	0%	100%	0%	17%	83%	0%	17%	83%
2016	*First-level*	30%	8%	62%	4%	20%	76%	8%	38%	54%	6%	22%	72%
	Second-level	36%	12%	52%	0%	20%	80%	4%	44%	52%	6%	32%	62%
	Third-level	28%	3%	69%	3%	6%	91%	0%	25%	75%	0%	19%	81%
	Fourth-level	17%	0%	83%	0%	0%	100%	0%	8%	92%	0%	0%	100%
2017	*First-level*	38%	17%	45%	6%	13%	81%	4%	45%	51%	8%	30%	62%
	Second-level	41%	7%	52%	2%	15%	84%	5%	46%	49%	13%	31%	56%
	Third-level	30%	0%	70%	0%	13%	87%	0%	22%	78%	4%	13%	83%
	Fourth-level	11%	11%	78%	11%	0%	89%	0%	33%	67%	0%	11%	89%

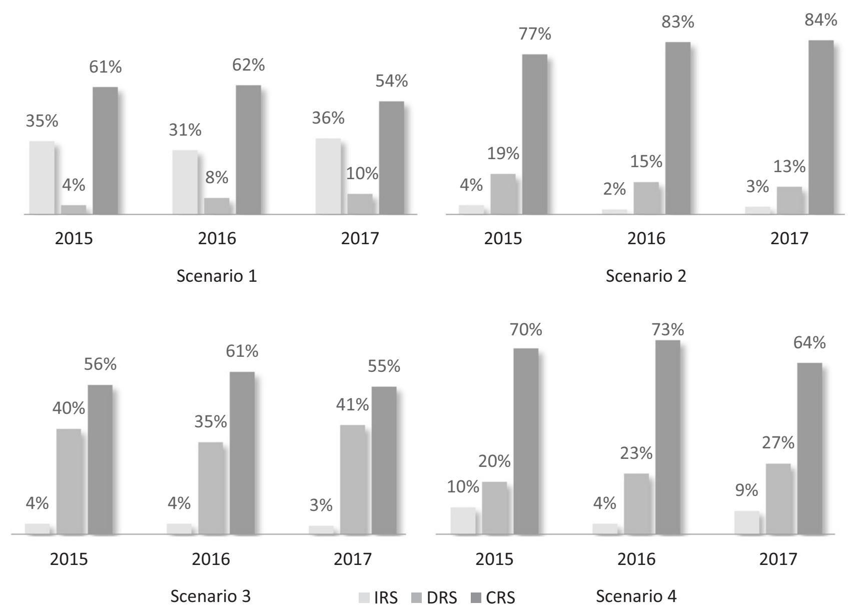

Fig. 2 Overall RTS characteristics of the farms

livestock outputs by the same rate or by a smaller percentage for the farms of the region.

3.3.3.4 Scenario 4 (Changing land vs. crop output) In this scenario, DRS farms are also subsequent to CRS farms in percentage. However, the percentages are less apparent. Therefore, CRS prevalence is observed to a high degree. Increasing land by some percentage will most likely result in an increase in crop output by the same rate for the farms of the region.

We summarize the RTS categorizations of the farms for the entire data set in Fig. 2. The prevalence of CRS is also visible for all scenarios and years. Moreover, IRS is subsequent to CRS in the scale elasticity scenario (Scenario 1) that focuses on changing all inputs. For the rest of the scenarios, which focus on changing different subsets of input factors, DRS is the successive characteristic after CRS. These scenarios focus on changing a single factor and thus can be seen as more straightforward alterations compared to changing all inputs.

Finally, we also examine the RTS classifications of the farms for status changes over three years. Figure 3 presents the farms preserve the same RTS classification for all years. Out of 146 farms in our sample, the number of farms that consistently possess the same RTS classification is almost half for Scenarios 1, 3, and 4. For scenario 2, the rate is higher (63%).

4 Conclusion

Investigation of the marginal characteristics of DEA frontiers is a notable stream in Data Envelopment Analysis (DEA)

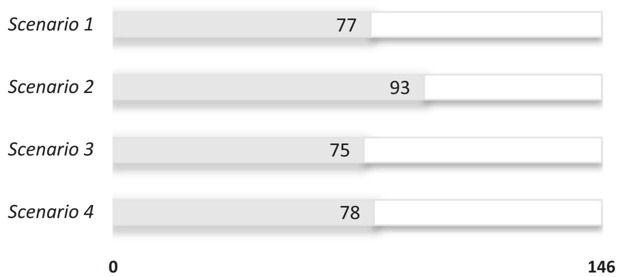

Fig. 3 # of farms that preserve the same RTS characteristics (2015–2017)

literature. A well-known measure is the elasticity of response that reveals the magnitude of the response of determined input factor(s) or output factor(s) to changes in the determined sets of input and/or output factors at a given unit on the frontier. With recent developments, we can calculate these measures on different types of DEA production technologies for either side (right-hand for proportional increases and left-hand for proportional decreases in a factor or a set of factors). Customarily, these measures are available for the units located on the frontiers of the production technology.

We propose a practical method for calculating the response measures at different levels of a given DEA technology based on stratification. The stratification was introduced to the DEA literature within the scope of context-dependent DEA methodology, in which the efficient units are excluded from the data set at each iteration to form several levels (contexts) of performance. Such an approach enables one to define short-term and long-term targets and reference sets for the units as well as to obtain a ranking at each level. With the current research, we attach the elasticity measurement to this scope. Because any unit in a given technology is a part of a frontier at a certain level, we propose and show that it is possible to obtain elasticity measures for the units based on their context.

We provide a procedure to implement the proposed methodology with three alternative paths to follow relying on the solutions to the linear programs to obtain elasticity measures: *(i)* the data can be stratified using DEA modeling and then the linear programs for elasticity measurement can be solved in stratified samples; *(ii)* the linear programs for elasticity measures can directly be solved by excluding the units with finite and unbounded elasticity measures at each iteration; *(iii)* the linear programs for elasticity measures can directly be applied by excluding the units with infeasible solutions at each iteration. The alternative methods differ in how they approach to the data. We focus on the elasticity of response measures for the output sets in our model expositions; however, in theory, the same procedure can be followed for the responses of inputs as well.

To see the proposed methodology in practice, we experiment with a farming data set retrieved from the Farmer Accountancy Data Network (FADN) of Turkey. The analyses conducted for the farms in the Aegean region of the country are presented to conclude both quantitative and qualitative marginal characteristics of the farming in the regions. Besides introducing a fresh methodological approach to the elasticity measurement on DEA frontiers, we also provide a set of discussions to provide insight into the interpretation of the obtained measures for consecutive years (2015–2017). In our experiments, we also focus on the responses of outputs to changing inputs in line with our model exposition. We design four scenarios of elasticity, in which each scenario includes different subsets of inputs and outputs including the scale elasticity.

Generally, when the programs for right-hand and left-hand elasticities are solved, it is possible to observe a variety of results from finite values, zeros, unbounded solutions to relatively high values of elasticity. We illustrate and interpret those results for the representative farms in our data set with an emphasis scale increase through proportionally increasing the given inputs. Following that, we convert the measures to Returns-to-Scale (RTS) classifications of the units within their context using the definitions of preceding research. With stratification, we obtain the elasticity of response and RTS classifications for farms at different layers of frontiers (i.e. contexts). For all contexts, scenarios, and all years, we present the RTS classification of all units in terms of percentages. The results reveal that constant Returns-to-Scale (CRS) is prevalent for the farms of the region followed by increasing Returns-to-Scale (IRS) or decreasing Returns-to-Scale (DRS) depending on the scenario. Interestingly, the share of the IRS is higher in scale elasticity scenario, in which the response of all outputs is measured to the change in all inputs. In the remaining scenarios that focus on different subsets such as changing livestock, changing cost, and changing land, the share of DRS is extremely visible compared to the IRS. Moreover, we examine the changes in the RTS status of the farms throughout the years and observe that only around half of the farms preserve the same RTS classification over the years of analysis.

Compliance with ethical standards

Conflict of interest The authors declare no competing interests.

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