

Leontief was not right after all

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Abstract We use recent international data on cost shares by industry to conduct the first robust test of Leontief’s hypothesis of factor-specific productivity differences. We strongly reject this hypothesis. Hence tests of the Heckscher–Ohlin–Vanek paradigm cannot be based upon simple modifications that define factors in efficiency units. We also discuss a theory of productivity differences that describes the factor content of trade well.

Keywords Factor-specific productivity differences · Leontief conjecture · Total factor productivity · Empirical studies of trade

JEL Classification D24 · F14

1 Introduction

Economists derive much of their intuition about general equilibrium theory from the Heckscher–Ohlin paradigm. It shows the clear link between goods prices and factor returns and forms the foundations of trade theory and parts of development economics. Heckscher (1919) had the deep intuition that differences in endowments are the source of comparative advantage, and Vanek (1968) emphasized that

trade in goods is really a veil for trade in the underlying factor services.

The empirical validity of this paradigm has met with mixed success. Leontief (1953) studied exports of the United States in 1947, when this country was the most capital-abundant on earth. He showed that American exports embodied considerably less capital and somewhat more labor than would be required for domestic production of competitive imports. These findings initiated a vast body of theoretical and empirical literature, and the Leontief paradox continues to spark new research. For example, Jones (2008) argued recently that the capital intensity of a country’s exports evolves in cycles, so that a capital-abundant country will at some time export relatively labor-intensive products. However, ten Raa (2008) responded that this resolution of the Leontief paradox inappropriately ignores the factor content of non-competitive imports. Also, ten Raa and Mohnen (2001) show that the Heckscher–Ohlin paradigm was substantiated by trade between Europe and Canada, after accounting for differences in technology and demand.

Leontief posited that correcting endowments for differences in “efficiency units” might explain this paradox, but he lacked data on other countries’ technologies to investigate further. Economists since Solow (1957) have used efficiency units to measure the quality of labor as it evolves across time, perhaps because of the accumulation of human capital. International economists recast efficiency units more broadly, applying them to all factors of production. The discipline of applied general equilibrium theory then imposes that factor-specific technological differences across countries are the dual of factor price differences among them. Relying again only on the United States’ input-output matrix to measure the factor content of trade globally, Trefler (1993) used an exactly identified model to compute productivity parameters for ten factors in many

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countries. He corroborated his computation with data on wages and prices of investment goods.

We examine consistent technology matrices from 33 countries to show that Leontief's idea is incorrect. In essence, we take advantage of factor uses in forty-eight industries to impose over-identifying restrictions that reject resoundingly factor-specific productivity differences.

The economic intuition behind our test is simple. If labor in France is half as productive as in the United States, then French wages will be half those in America. But a French firm *in any industry* will need twice as many workers as an American firm per unit of output. Hence the wage bill for any American firm and any French firm in the same industry will be identical. Since the factor called labor and the names America and France were arbitrary, the costs shares in any given industry should be identical in all the countries in the world. They are not.

Explaining international productivity differences correctly is not moot; standard undergraduate textbooks in international trade (Feenstra and Taylor 2012, p.102) routinely suggest that recasting Heckscher-Ohlin-Vanek theory simply in terms of "effective factors" goes a long way towards reconciling the theory and the data. We conclude by discussing a theoretical generalization that does work; factor-specific technical differences are a special case.

2 International productivity differences

There are n goods in the world economy and f factors in each country. A technology matrix for country c is an $n \times f$ array $A_c(w_c)$ that depends upon the $f \times 1$ vector of local factor prices w_c . Its ij th element is the direct and indirect local unit input requirement into good i of factor j . Since these input requirements minimize costs, the envelope theorem implies that every technology behaves locally as though it has fixed coefficients.

Fix local factor prices and consider two technology matrices:

$$A_c(w_c) = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix} \quad \text{and} \quad A_d(w_d) = \begin{bmatrix} 10 & 2 \\ 20 & 2 \\ 30 & 2 \end{bmatrix},$$

where the rows correspond to goods and the columns to capital and labor respectively. In this simple case, capital is ten times and labor is twice as efficient in Country c than in Country d . This example is germane because each matrix has full rank, and there are more goods than factors. The typical technology matrix in empirical work has often as many as 45 goods and between two and ten factors. Technical differences are *factor-specific* if and only if the columns of two technology matrices are collinear. Hicks-neutral technical differences are a special case, where the

international productivity differences for all factors are identical. The key insight is that if technical differences are factor-specific then all countries are competitive in all goods because local factor prices will reflect local productivity.

Another canonical case has differences in *total factor productivity by industry*:

$$A_c(w_c) = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix} \quad \text{and} \quad A_d(w_d) = \begin{bmatrix} 10 & 10 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}.$$

These differences define the chain of comparative advantage. Country c has absolute and comparative advantage in the first good. For example, if world goods prices are $p = (2, 3, 4)'$, then Country c is competitive in all three goods, but the other country produces only the last two. In this simple example, free trade equalizes factor prices, but Country d has a comparative disadvantage in the production of the first good. Again, Hicks-neutral technical differences are a special case, where the total factor productivity differences across all industries are identical. The central insight now is that countries typically produce a subset of all possible goods; the exact pattern of production will depend upon endowments and the local full-employment conditions. International economists refer to this pattern of specialization as "cones of diversification".

Heckscher-Ohlin-Vanek theory is the profound idea that trade in *final goods* is really trade in the *factor services* that produce them. An international economist sees an iPhone not simply as a commodity, but as the capital and labor that are used—directly and indirectly—in its manufacture. So a smart phone produced for final demand and recorded as a part of gross domestic product is really a bundle of different resources that constitute part of a country's endowment. National income accounting conventions ensure that the value of the final good is equal to the sum of income earned by the different factors that are embodied in it. The theory's central prediction is that a country tends to export those goods that use intensively those factors with which it is abundantly endowed.

Since Heckscher-Ohlin-Vanek theory is about trade in factor services, its simplest generalization assumes factor-specific productivity differences. But this extension of the theory has a very sharp implication that no one has ever put to the test. Let the reference country 0 have factor prices $w_0 = (w_{0,1}, \dots, w_{0,f})'$. Let $\lambda = (\lambda_1, \dots, \lambda_f)'$ be the factor-specific productivity parameters, and write $\Lambda = \text{diag}(\lambda)$.¹ Then factor prices in Country c are $w_c = (w_{0,1}/\lambda_1, \dots, w_{0,f}/\lambda_f)'$ by assumption. Unit input requirements in Country c are

¹ These parameters depend upon Country c and Country 0, but that dependence is suppressed for notational convenience.

$$A_c(w_c) = A_0(w_0)\Lambda.$$

The bill for factor j in industry i in Country c is

$$a_{c,ij}w_{c,j} = \lambda_j a_{0,ij}w_{0,j} / \lambda_j = a_{0,ij}w_{0,j},$$

where we have suppressed the dependence of $a_{c,ij}(\cdot)$ and $a_{0,ij}(\cdot)$ on factor prices for notational convenience. Since this is true for every factor j in industry i in any Country c , factor costs shares are identical in every country in the world

$$\theta(c)_{ij} \equiv \frac{a_{c,ij}w_{c,j}}{\sum_j a_{c,ij}w_{c,j}} = \frac{a_{0,ij}w_{0,j}}{\sum_j a_{0,ij}w_{0,j}} \equiv \theta(0)_{ij}.$$

This argument is much more general than assuming that every industry has a Cobb-Douglas production function.

Leontief was cognizant that his assumption of factor-specific productivity differences had strong implications for disaggregated data. Leontief (1953, p. 344) stated, “[T]he conventional argument must combine the foregoing observation with the implicit assumption that the relative productivity of capital and labor—if compared industry by industry—is the same here and abroad.” Of course, consistent international data on the direct and indirect factor requirements for a wide array of economic activities simply did not exist in 1953. But they do now.²

3 The data and a robust test

We use the OECD input-output tables benchmarked near 2000 for Australia, Austria, Belgium, Brazil, Canada, China, the Czech Republic, Denmark, Finland, France, Germany, Great Britain, Greece, Hungary, Indonesia, Ireland, Israel, Italy, Japan, Korea, the Netherlands, New Zealand, Norway, Poland, Portugal, Russia, Slovakia, Spain, Sweden, Switzerland, Turkey, Taiwan, and the United States.³

The raw data are in local currencies, but our technology matrices are unit-less factor shares. They are consistent in two ways. First, they are designed to be comparable across countries. Second, the factor shares for each industry are consistent with endowments; for example, the weighted average of capital’s shares across all local industries is

² Some data were available that might have undercut one’s belief in Leontief’s conjecture. Exploiting cross-country data on a wide sample of industries, Arrow et al. (1961) estimated elasticities of substitution that were typically quite different from unity. Since factor prices are not equalized, it could be inferred that an industry’s factor cost shares differed across countries. Also, scholars such as Rosefielde (1974) had long studied input-output matrices from other countries.

³ See OECD (2015). The URL http://www.oecd-ilibrary.org/industry-and-services/data/stan-input-output/input-output-database_data-00650-en?isPartOf=/content/datacollection/stan-in-out-data-en was retrieved on 12 October 2015.

equal to its share in macroeconomic accounts by construction.

We compute direct and indirect factor requirements in 48 industries for each of 33 countries. Our factors capital, labor, and social capital correspond to the three entries in national accounts for value added: gross operating surplus, compensation to employees, and indirect business taxes. It is slightly unconventional to define social capital as a factor. We do so for four reasons. First, indirect business taxes are completely analogous to payments to labor and capital in national accounts; so we define social capital as a factor for logical and statistical consistency. Second, different long-run patterns of indirect taxation by industry affect factor prices and thus local technologies. Third, our specification is consistent with the macroeconomic literature that measures after-tax rates of return to capital and labor. Fourth, social capital, interpreted as a firm’s access to a local market, is as much a fixed factor that is not traded as is labor or capital.⁴

Let

$$\theta(c)_{ij} - \theta(0)_{ij}$$

be the difference in industry $i \in \{1, \dots, 48\}$ between the share of factor $j \in \{K, L, G\}$ in country $c \in \{\text{Australia}, \dots, \text{Taiwan}\}$ and that in the United States.⁵ Figure 1 is a histogram of these factor share differences. There are 4050 observations, fewer than $47 \times 3 \times 32 = 4512$, because a few countries record no economic activity in some industries. The population mean is 0 by construction, and its median is -0.017 . Its standard deviation is 0.15, its skewness is 0.18, and its kurtosis is 8.0.

If Leontief’s conjecture were correct, then every difference would be identically zero. Figure 1 shows a Laplace distribution centered near the median -0.017 .⁶ This distribution is the difference between two independent and identically distributed exponential distributions; it would arise if factor shares by industry in the United States were independent of those in any other country. If this fact were true, then Leontief’s conjecture would be grossly incorrect. Since factor share differences are almost uniformly bounded between -1 and 1 , we are tempted to reject the theory simply by inspection. In fact, the

⁴ Our definition has a slight drawback. Factor shares always sum to unity, but there are a few subsidized industries where payments to social capital are negative. The most striking case is “Motor Vehicles, Trailers, and Semi-trailers” in Indonesia. Capital’s share is 1.6, labor’s is 0.8, and social capital’s is -1.4 . Some might consider it an advantage to identify rare cases of highly subsidized industries. These cases give the data fat tails.

⁵ We drop “Steam and Hot Water Supply” since that industry is not active in the United States.

⁶ See Everitt (1998, pp. 182–83) for a discussion of this distribution.

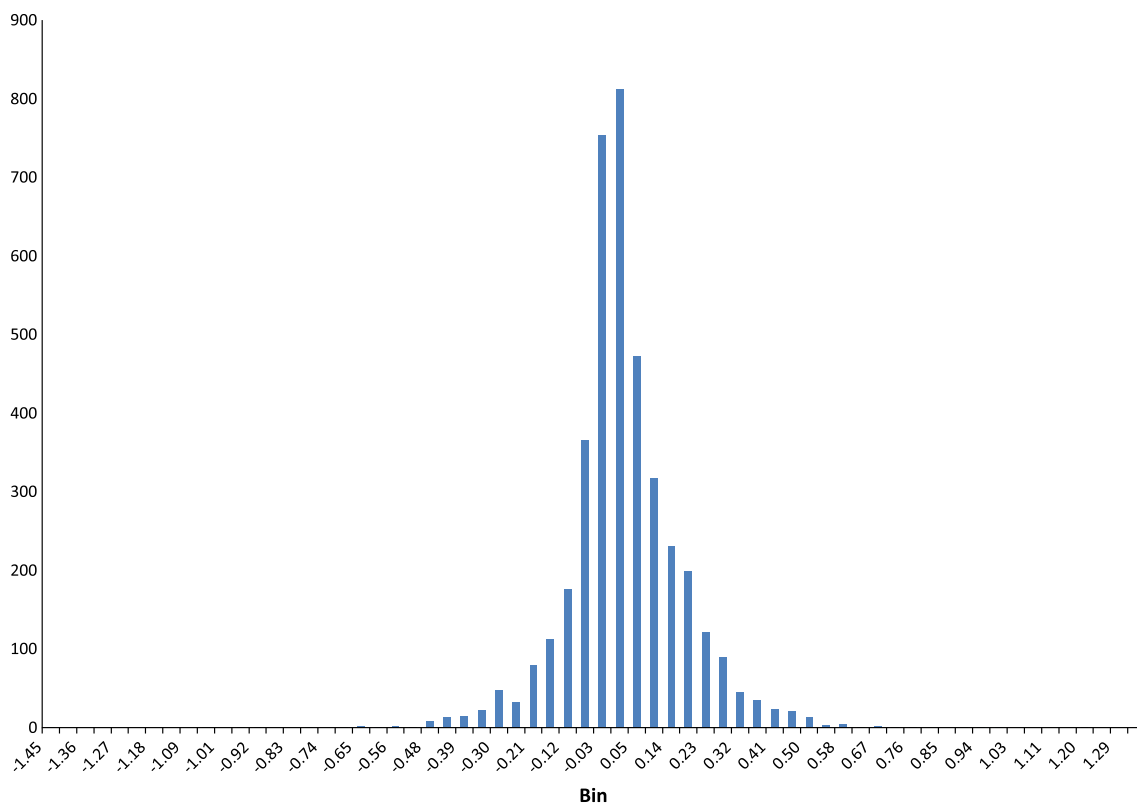


Fig. 1 Factor share differences

coefficients of variation (not reported) for each factor's share for a fixed industry across countries is just as large as its analog within a country across industries. That was why it was so easy to see that Leontief's conjecture was wrong.

Still, it is worth exploiting the wide variability of the cost shares. Hence we assume that the technology matrices are measured with error. Fix a factor j and a country c , and consider

$$\theta(c)_j - \theta(0)_j$$

where $\theta(c)_j$ is the 47×1 vector of factor shares in country c and $\theta(0)_j$ is its analog in the United States. Since factor prices and goods prices are fixed by assumption, these factor shares are measured with error perhaps because there is idiosyncratic local aggregation bias in each industry. For example, a different mix of firms might produce 'Rubber and Plastic Products' in Korea than in the United States. We assume only that measurement error is independent across industries. In essence, aggregation bias does not depend upon the name of the economic activity. We are making no parametric assumptions about any distribution.

We use the natural sign test based upon the null hypothesis that each element of this vector has an equal chance of being positive or negative. There are 32 country pairs. Since the factor shares in each industry in each

country are constrained to sum to unity, we have only 64 independent pairwise tests. We report those for capital and labor. The null hypothesis is: for a given factor and country pair, Leontief's description of factor-specific technical differences is correct. Fix your attention on capital for the moment. If the hypothesis is true, then about half of the local industries' cost shares for capital will be above those in the United States and about half will be below. One rejects the null hypothesis if the local uses of capital services deviate systematically from those across all the industries in the United States.

Almost all the p-values are near 0.⁷ Table 1 reports the fourteen cases that are large enough not to reject Leontief's specification for a test of size 5 %. Since there is a great deal of variability in these data, it is quite significant that we strongly reject the theory in 50 of 64 cases. France, Israel, Sweden, and Taiwan seem to use capital and labor in the same way as the United States, but the evidence against factor-specific productivity differences is overwhelming.

Why did Trefler (1993) find a strong correlation between real wages and his measures of labor productivity? Gabaix

⁷ The binomial distribution is discrete. Each marginal significance level is the two-sided probability of a more extreme value than that observed in the data.

Table 1 Marginal significance levels greater than 0.05

Country and factor	<i>p</i> value	N
Canada, labor	0.184	46
China, capital	0.073	37
France, capital	0.441	42
France, labor	0.441	42
Germany, labor	0.280	42
Israel, capital	0.878	42
Israel, labor	0.164	42
Japan, labor	0.243	47
Norway, labor	0.382	42
Spain, labor	0.542	43
Sweden, capital	0.441	42
Sweden, labor	0.280	42
Taiwan, capital	0.079	47
Taiwan, labor	0.560	47

(1997) gives a good answer.⁸ If the measured factor content of net exports in labor is near zero, then the imputed labor productivities are nearly equal to a country’s output per worker, and rich countries have high real wages. Treﬂer (1993) adduces three pieces of indirect evidence that corroborate Leontief’s conjecture: (1) almost all his computed productivity parameters are positive; (2) there is a high correlation between his labor productivity measures and real wages; and (3) there is a high correlation between his capital productivity measures and the prices of investment goods in the Penn World Table.

Gabaix’s algebra is powerful.⁹ As long the factor content of trade is near zero, then all the productivity parameters are simply output per unit of a factor. This fact explains why almost all computed productivity parameters were positive. We did not have Treﬂer’s data, but he used the perpetual inventory method for computing the capital stocks. If countries have the same depreciation rate and were in steady state in 1983, then one can compute a proxy for output per unit of capital from the Penn World

⁸ A standard graduate textbook (Feenstra 2004, p. 61) assigns an exercise that illustrates Gabaix’s algebra. It is unfortunate that Gabaix (1997) was never published and is not readily available. In essence, Gabaix shows that Treﬂer’s calculations are not identiﬁed. When the measured factor content of trade in labor services is zero, then Treﬂer (1993) computed either labor productivity parameters or GDP per capita.

⁹ Treﬂer’s (1993) productivity parameters are the solution to an invertible system of linear equations. Its kernel has the property that each country’s factor-speciﬁc productivity is simply national income per unit of that factor. For example, the productivity parameters for labor are just output per worker, and those for capital are the inverses of the capital-output ratios. Since any invertible linear mapping is continuous, the imputed productivity parameters are quite near output per factor when the system’s image is in a neighborhood of zero.

Table data. It is the inverse of the share of GDP spent on investment goods.¹⁰ The correlation between this measure and the price of capital in the Penn World Table is 0.10, not signiﬁcant but still positive. Using data that have nothing to do with the measured factor content of trade, one can corroborate a correlation between GDP per capita and the price of investment goods.

4 Which productivity adjustments actually work?

Leontief’s idea was elegant, but it does not survive a closer inspection of the data. We have a big advantage: we use the technology matrices themselves to show that Leontief’s conjecture is wrong. Still, we do not want to end on a nihilistic note. Is there a simple speciﬁcation of international productivity differences that works?

For several years, we have advocated using factor conversion matrices. A factor conversion matrix computes the local factor content of a foreign Rybczynski matrix. The logic of Heckscher–Ohlin theory is very exacting; since goods are produced at identical unit costs everywhere, the slightest impediment to trade—a negligible tariff or transport cost—pins down the location of production. At the two-digit level, each industry is an aggregate of many different products, but the ineluctable conclusion is that the local mix of ﬁrms adjusts so that every country is competitive on world markets in almost every good. *Local technologies adapt in the long run to local factor market conditions.*¹¹

4.1 Factor conversion matrices in theory

We describe the theory using physical technology matrices because the intuition is simpler in this case. In the next subsection, we will implement the theory using the observable data, which are cost shares by industry in every country.

Consider a world with only two economies, those of Country *c* and Country *d*. Fix the two $f \times 1$ vectors of local factor prices w_c and w_d , and let p be the $n \times 1$ vector of world goods prices. Let $A_c(\cdot)$ and $A_d(\cdot)$ be the

¹⁰ Let Y_c be the output of Country *c*. In steady state, investment is $I_c = \delta K_c$ where the variables have their usual meanings. The data report $\kappa_c = I_c/Y_c$, the share of investment in GDP. Hence $Y_c/K_c = \delta/\kappa_c$, and the depreciation rate is common across all countries. Thus the productivity parameters for capital should be correlated with the inverses of the investment shares of GDP.

¹¹ Schott (2003) contends that the rubrics in these data are too broad. He argues that countries produce highly disaggregated goods in different diversiﬁcation cones, depending upon their level of economic development. We must be agnostic about this claim, but we note that there are very few zeros in each country’s vector of imports and exports at this level of aggregation.

corresponding $n \times f$ technology matrices; we will henceforth omit the dependence of these matrices on factor prices for convenience. If both countries are competitive in all goods,

$$A_c w_c = p = A_d w_d.$$

Let A_c^+ be the Moore–Penrose pseudo-inverse of Country c 's technology matrix.¹² If it has full rank and $n \geq f$,

$$w_c = A_c^+ A_d w_d.$$

The $f \times f$ matrix $A_c^+ A_d$ translates the $f \times 1$ vector of factor prices in Country d into those in Country c . Its transpose shows the factor content in Country d of the Rybczynski effects in Country c .¹³ Since any empirically tractable technology matrix has rank f and there are at least as many goods as factors, the generalized inverse has a simple formula: $A_c^+ = (A_c' A_c)^{-1} A_c'$. Hence, the j th column of $A_c^+ A_d$ reports the coefficients from an OLS regression of the unit input requirements of factor j in Country d on all such factor requirements in Country c .

Computing the f^2 elements of $A_c^+ A_d$ consists of running a system of f seemingly unrelated regressions. Each omits an intercept and regresses the uses of a factor in Country d on all the factor uses in Country c . Leontief's assumption of factor-specific technical differences imposes that all but f of these regression coefficients are zero. Leontief restricted his attention to only the diagonal elements of $A_c^+ A_d$. Instead, factor conversion matrices in essence estimate $f^2 - f$ more coefficients from n pieces of data, the factor uses by industry.

Let's return to our first example. In this case,

$$A_c^+ A_d = \begin{bmatrix} -1/2 & 0 & 1/2 \\ 4/3 & 1/3 & -2/3 \end{bmatrix} \begin{bmatrix} 10 & 2 \\ 20 & 2 \\ 30 & 2 \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 0 & 2 \end{bmatrix}$$

where $(A_c^+)'$ is the Rybczynski matrix for Country c . In that country, capital is the enemy of the most labor-intensive good 1 and the friend of the most capital-intensive good 3; labor is a friend of goods 1 and 2 and an enemy of good 3. This calculation shows that rents in Country c are 10 times those in Country d , and wages in Country c are twice those in Country d . When factor-specific productivity adjustments are warranted, we compute them.

We continue with a much more realistic example that shows the generality and power of our approach. Assume

¹² Let $Ax = b$ be a system of n equations in f unknowns x . Then the set of all solutions is $x = A^+b + (I - A^+A)z$, where z is an arbitrary $f \times 1$ vector. If the rank of A is at least f , then $I - A^+A = 0$. In fact, the Moore–Penrose inverse gives a solution to an inconsistent system of equations $Ax \approx b$, a fact that will be useful below.

¹³ We are using the symmetry property of the Moore Penrose inverse: $(A')^+ = (A^+)'$.

that there has been labor-augmenting technical progress in industry 1 in Country c . Now the technology matrix is:

$$\tilde{A}_c = \begin{bmatrix} 1 & 0.99 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}$$

The technology difference between Country c and Country d is neither factor-specific nor described by a simple total factor productivity index by industry. Before computing $\tilde{A}_c^+ A_d$, it is worth reviewing a trade theorist's comparative statics.¹⁴ Since the first local industry has experienced technical progress, it becomes the world leader in that good. Under the assumption of constant returns to scale, all resources in that country flow into that industry. Since it is the most-labor intensive one, local wages are bid up, local rents fall, and the second industry is no longer competitive and must shut down. This result is actually quite general. Consider an economy with n industries, and assume world prices are fixed. If the most labor intensive industry in Country c experiences one iota of technological progress, only two local industries will survive: that one and the most capital-intensive one. A minuscule amount of technical progress will shut down all intermediate industries, and the most capital-intensive industry survives only because the Stolper-Samuleson effect lowers the rental rate enough so that it is still competitive on world markets. The data are crying out for an appropriate theoretical approximation that works.

What is the factor conversion matrix in this case? A small amount of technical progress—or measurement error—throws a huge monkey wrench into the link between world goods prices and local factor prices. But the Moore–Penrose generalized inverse has a very attractive property: it computes approximate solutions even when the system of equations is inconsistent. Let p_c and p_d be the $n \times 1$ vectors of unit costs. Since

$$\tilde{A}_c w_c = p_c \approx p_d = A_d w_d,$$

we may conclude $w_c \approx \tilde{A}_c^+ A_d w_d$. In fact this is the best linear mapping between the vector of factor prices in Country d and those in Country c . This mapping is best because

$$\tilde{p}_c = \tilde{A}_c \tilde{A}_c^+ p_d = \tilde{A}_c (\tilde{A}_c^+ A_d) w_d$$

are the actual unit costs in Country c that are closest to unit costs p_d . Indeed,

$$\tilde{A}_c^+ A_d = \begin{bmatrix} 10 & -0.0101 \\ 0 & 2.0269 \end{bmatrix}.$$

For example, if $w_d = (1, 1)'$, then unit costs in that country

¹⁴ Brecher and Choudhri (1982) give a nice diagrammatic exposition of an economy with more goods than factors.

are $p_d = (12, 22, 32)'$, Predicted factor prices in Country c are $w_c = (9.9899, 2.0269)'$. Labor-augmenting technical progress has created a magnification effect: local rents have decreased, and local wages have increased by slightly more than one percent. The predicted unit costs in Country c are:

$$\tilde{p}_c = (11.9966, 22.0067, 31.9966)'$$

If one is willing to assume measurement error, aggregation bias, or transport costs smaller than one part in ten thousand, then one can rationalize one percent labor-augmenting technical progress in one industry. Otherwise, a trade theorist would predict radical changes in industry outputs for one country at least.

4.2 Factor conversion matrices in practice

How do we implement these ideas? For country c , we observe an $n \times f$ technology matrix that records cost shares by industry $\Theta_c(w_c)$.¹⁵ By construction, each of its $n = 48$ rows sums to unity.¹⁶ Since we do not observe physical inputs, we cannot hope to capture efficiency units without an additional identifying assumption.

We use two identifying assumptions. The Stolper-Samuelson Theorem assumes that the unobserved local quantities of factors are completely inelastically supplied; since the supply of a factor is vertical, we can identify and describe factor-augmenting technical differences. The Rybczynski Theorem assumes that, at unobserved local factor prices, factors are completely elastically supplied; since the supply of a factor is horizontal, we can identify and describe efficiency units by translating each local factor into an amalgam of those in the reference country.

First, impose the identifying assumptions of the Stolper-Samuelson Theorem. Consider exogenous technical progress in a reference country. We model this phenomenon as a uniform reduction in each unit input requirement in each industry; hence, the corresponding unit-value isoquant shifts radially inward. Still, different industries can have different rates of technological progress. This kind of improvement is isomorphic to an increase in the price of an industry's output; it takes proportionally fewer units of each factor to produce a dollar's worth of output. We will exploit the equivalence between technological progress in an industry and an increase in factor rewards.

Let \hat{w}_0 denote element-by-element logarithmic differentiation of the f factor prices in the reference country. We make an important assumption: these disparate rates of

technical progress are such that every industry remains active at the new factor prices.¹⁷ Factor price changes in Country 0 satisfy these n equations:

$$\Theta_0 \hat{w}_0 = \hat{\pi}_0,$$

where $\hat{\pi}_0$ is an $n \times 1$ vector of the rates of industry-specific technical progress. We interpret the elements of the vector \hat{w}_0 as generalizing Leontief's idea because they show how rates of technical progress by sector are reflected exactly in factor-augmenting changes.¹⁸ What is the best prediction for how country c 's factor-specific productivities would respond to the same technological progress? Since the column space of Θ_c and Θ_0 may well be different, the following approximation is our best hope:

$$\Theta_c \hat{w}_c \approx \hat{\pi}_0.$$

These n equations are an overdetermined and perhaps inconsistent system in \hat{w}_c , the f rates of factor-augmenting technical progress in country c . These changes keep local unit costs in that country as competitive as possible. Hence,

$$\hat{w}_c = \Theta_c^+ \Theta_0 \hat{w}_0$$

gives the best linear mapping from \hat{w}_0 into \hat{w}_c . The $f \times f$ matrix $\Theta_c^+ \Theta_0$ is our factor conversion matrix.

Table 2 gives the factor conversion matrix for China, when the United States is the reference. K denotes private capital, L labor, and G social capital. The columns correspond to the factors in Country 0, and the rows show the effects in Country c . Each row sums to unity, illustrating an important homogeneity property of $\Theta_c^+ \Theta_0$. If all rates of factor-augmenting technical change in the reference country are equal because of Hicks-neutral technical progress, then Country c must experience the same uniform technological advancement.

When we assume that factor supplies are fixed, we focus on the columns of Table 2. Consider 1 % capital-augmenting change in the United States; thus $\hat{w}_0 = (0.01, 0, 0)'$. The first column of Table 2 shows that China must experience a combination of capital-augmenting technical change, labor-augmenting technical change, and social-capital-augmenting technical change in the amounts of 59.3, 22.5, and 18.8 basis points respectively. Likewise, the second column of that table shows that 1 % labor-augmenting technical change in the United States corresponds to 34.0, 81.0, and 58.4 basis point increases in

¹⁵ Again, for notational convenience, we will henceforth omit the dependence of $\Theta_c(\cdot)$ on local factor prices.

¹⁶ When a sector is not active, every element in that row is zero. Then the pseudo-inverse has a corresponding column that also has every element equal to zero.

¹⁷ Thus the rates of technological progress lie in the column space of $\Theta_0(w_0)$.

¹⁸ Because unit input requirements minimize costs, the envelope theorem implies that $\Theta_0(w_0)$ is unaffected for small changes \hat{w}_0 . Given fixed factor supplies, higher factor returns are equivalent to more productive factors.

Table 2 Factor conversion matrix for China

	K_0	L_0	G_0
K_c	0.593	0.340	0.067
L_c	0.225	0.810	-0.035
G_c	0.188	0.584	0.228

the efficiency of private capital, labor, and social capital in China.

Second, impose the alternative identifying assumptions of the Rybczynski Theorem. Since goods prices and thus factor returns are fixed in every country, one now measures quantities of factors at unobservable local factor prices. A dollar's increase in any factor's services will induce changes in quantities supplied in all local industries; some will expand and others contract. The sum of these supply responses respects national income accounts: an extra dollar of any factor's services will create on net another dollar of GDP.

The elements of the $f \times n$ matrix Θ_c^+ are interesting in their own right. Table 3 reports the largest and smallest elements in each of the three rows of Θ_c^+ for China. Consider a dollar's increase of real capital in China. Then output in the real estate industry will expand by \$0.70, and output in the finance industry will *contract* by \$0.55. A figurative way to describe this response is to say that real estate is the "strongest friend" of capital, and finance & insurance is the "strongest enemy" of capital! The strongest friend of labor is agriculture, and its worst enemy is refined petroleum. The strongest friend of social capital is finance & insurance, and its strongest enemy is real estate.

In analyzing Rybczynski effects across countries, we focus on the *rows* of Table 2. We are now translating changes in the supply of China's factors into equivalent changes into an amalgam of resources in the United States. Consider the second row in Table 2. An international dollar of labor services in China corresponds to \$0.23 of American capital, \$0.81 of American labor, and -\$0.04 of American social capital. Now an extra dollar of resources in Country c respects international macroeconomic conventions; it must translate into a dollar's increase in world GDP measured at the reference country's prices!

It is difficult to disentangle quantities and prices here, but we can use data from other sources to develop one's intuition. If wages in the United States are \$21.61 and those in China are \$1.79,¹⁹ a dollar of Chinese labor services represents $0.56 = 1/1.79$ h of work there. This amount of Chinese labor translates into renting \$0.23 of American capital,

¹⁹ These data are presented in Table 1 of Fisher (2015). We have assumed here that the work year consists of 2000 h.

hiring an American worker for $0.037 = 0.81/21.61$ h, and receiving a subsidy of \$0.04 of social capital.

Figure 2 considers these Rybczynski effects in the aggregate. China's endowment is labeled original data.²⁰ The converted data are calculated by multiplying the factor conversion matrix in Table 2 on the left by a row vector corresponding to China's endowment. If technology differences were indeed factor-specific, our factor conversion matrix would be the identity matrix, and the converted and original data would be identical. One hour of China's labor, for example, would correspond to about \$1.79/\$21.69 $\approx 1/12$ h of labor in the United States. The converted data in Fig. 2 show this adjustment alone is *not* sufficient to capture technology differences fully.

After controlling for factor price differences, we see that China's aggregate endowment of labor is worth somewhat more in the United States. In these data, China's GDP is \$3.5 trillion, and its payments to labor are \$1.9 trillion. The converted data show that these labor services would actually be worth \$2.2 trillion in the United States, about 16 % more than one would infer from the wage differential alone. We now see that that the simple conversion using factor prices only understates the value of Chinese labor by about one-sixth.

The striking implication of this factor conversion matrix is that China has a very inefficient system for the provision of social capital. Because a factor conversion matrix has a strict homogeneity property, not every factor's productivity can be adjusted upward. After controlling for the large factor price differences, Chinese labor and capital are both more efficient than those factors in America; something has to give. China's use of social capital is grossly less efficient. China is measured as paying \$0.51 trillion for social capital; in the United States, this would be worth only \$0.12 trillion.

Of course, we have examined only one factor conversion matrix in detail, even though we computed $1089 = 33 * 33$ of them.²¹ We chose the two largest economies in the world, and the productive structures of the United States and China are really quite different. These differences reflect underlying differences in both physical factor inputs, such as hours of labor, and factor earnings. Since the input-output tables and factor services are measured in local values, we have focused on payment shares in our empirical implementation. Factor inputs measured this way

²⁰ We constructed the dollar values of endowments from those in local currencies in the OECD data using purchasing power parity exchange rates from the World Bank's International Comparison Program.

²¹ Note that $\Theta_i^+ \Theta_j \neq \Theta_j^+ \Theta_i$, just as a regression of Y on X is different from a regression of X on Y . Also, $\Theta_j^+ \Theta_i = I_f$, so there are only 1056 such matrices in these data that are not trivial. Again, we will make all these data available to any interested researcher.

Table 3 China’s strongest Rybczynski effects

	Strongest positive effect	Strongest negative effect
ΔK_c	Real estate (0.70)	Finance and insurance (−0.55)
ΔL_c	Agriculture (0.46)	Refined petroleum products (−0.21)
ΔG_c	Finance and insurance (1.94)	Real estate (−1.14)

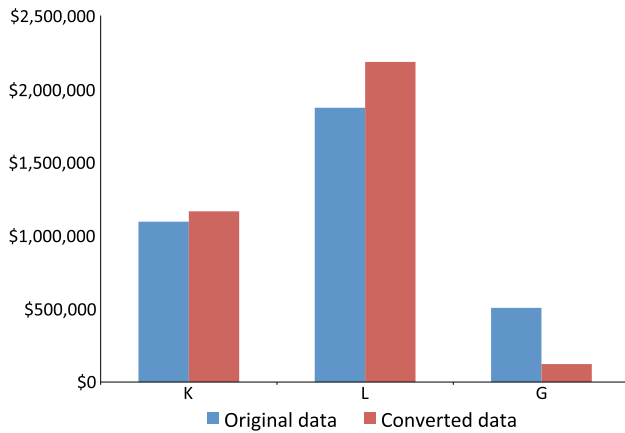


Fig. 2 Productivity adjustments for China, millions of international dollars

are consistent with national accounts, and our technique helps to address the difficulties in inherent measuring stocks of physical inputs among highly diverse countries. Nevertheless, our techniques can be readily applied to physical input matrices such as those provided in the World Input-Output Database, and we hope future research will lead to deeper insights into the nature of technological disparities and the large differences in wages and rents across countries.

These productivity adjustments were developed to corroborate classical international trade theory. They are not based upon estimates of technology matrices, as in Davis and Weinstein (2001). Their best empirical application is in Fisher and Marshall (2015). Using the Hessian of the national revenue function, Fisher and Marshall (2011) show the link between a country’s technology matrix and its Rybczynski matrix. Fisher (2011) sketches out the rudiments of Heckscher–Ohlin theory when countries have different technologies. Marshall (2012) links total factor productivity differences by industry with local factor prices.

5 Conclusion

Leontief (1953) set the agenda for half a century of scholarship that has sought to reconcile Heckscher–Ohlin theory with the data. Treffer (1993) is still one of the most economically meaningful attempts at this task. There was

nothing wrong with Treffer’s calculations or his corroborating evidence. He was simply hampered by using the American technology matrix only.

We had the big advantage of complete and consistent data on many countries’ technologies. Our insight about factor shares was overlooked by the literature in part because it is so easy. Also, most of that work concentrates on physical—not value—measures of direct factor uses; hence it neglects the important effects of local factor prices in measuring factor services properly. Once one computes a country’s technology matrix as factor shares by industry, it is simple to see that factor-specific productivity differences do not do full justice to the data.

Our important theoretical insight is that Leontief’s conjecture was a special form of a more general one. His idea reduces to testing whether the off-diagonal elements of a system of estimated equations are zero. Our factor conversion matrix can be computed simply by regressing a factor’s uses in one country on all factors’ uses in a trading partner. We derived this theory by applying the Moore–Penrose pseudo-inverse to the input-output table. Since input-output accounting was developed to describe the resources needed to produce a given vector of output, it should not be a surprise that the generalized inverse of the input-output matrix shows the goods that can be produced from a given vector of resources.

This observation relates to what Samuelson (1953) considered the most beautiful property of trade theory. The Stolper–Samuelson theorem examines how output price changes affect factor prices, when resources are in fixed supply. The Rybczynski theorem assumes that prices are fixed and then analyzes how changes in resources affect the mix of outputs produced. Perhaps the deepest insight in applied general equilibrium theory is that these two effects are identical. The symmetry properties of the Moore–Penrose pseudo-inverse imply that these price and quantity effects are intimately related, and it is at the crux of our factor conversion matrices. Indeed, our approach to international productivity adjustments exploits fully the duality between the Rybczynski and Stolper–Samuelson effects at the heart of international trade theory.

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