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Exact decomposition of the Fisher ideal total factor productivity index

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Abstract Decompositions of total factor productivity (TFP) shed light on the driving factors behind productivity change. We develop the first exact decomposition of the Fisher ideal TFP index which contains no debatable mixed-period components or residuals. We systematically isolate five effects of (1) technical change, (2) technical efficiency, (3) scale efficiency, (4) allocative efficiency, and (5) price effect. The three efficiency components (2–4) represent the efficiency of achieving a given target point. Components (1) and (5) capture the changes of the target point. While the technical change component is well-established, changes in the relative input–output prices can have real effects on the scale and scope of the target. Such changes are captured by the new price effect component (5). The new decomposition is compared with existing decompositions both in theory and by means of an empirical application to a panel data of 459 Finnish farms in years 1992–2000.

Keywords Total factor productivity · Fisher ideal index · Malmquist index · Decompositions · Agriculture · Aggregation

JEL Classification C43 · D24

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1 Introduction

The notion of productivity is conceptually simple: it is the ratio of output to input. However, virtually all production activities make use of multiple inputs to produce one or more outputs. The fundamental challenge in the measurement of *Total Factor Productivity* (hereafter TFP) arises from the need to aggregate the various inputs and outputs. In the large and growing literature of TFP indices, two main approaches, referred to as the *axiomatic* and the *economic* approach, can be distinguished (e.g., Diewert 1992; Diewert and Nakamura 2003).

The *axiomatic* (or test) approach postulates a number of properties that any meaningful index number should satisfy, and then tries to construct an ideal index number formula to meet the axioms. For example, the Fisher ideal index and the Törnquist index have a number of desirable axiomatic properties (e.g., Diewert 1992). These indices are simple to calculate, and they assume very little about the economic objectives of the firms or their production technology.

By contrast, the *economic* (or exact index number) approach is based on the economic theory and its behavioral assumptions. In contrast to the axiomatic approach, the economic approach to TFP measurement requires estimation of some representation of the production technology (e.g., production, distance, or cost functions), which necessarily requires some assumptions to be made. A prime example of the economic approach is the Malmquist index (Caves et al. 1982; Nishimizu and Page 1982; Färe et al. 1994). While the Malmquist index requires estimation of the production frontier, it only requires quantity data of inputs and outputs. By contrast, the Fisher TFP index additionally requires price data for inputs and outputs.[1] Yet, under certain conditions, the axiomatic and economic approaches are equivalent (see e.g., Diewert 1992; Färe and Grosskopf 1992; Balk 1993, 1998).

One clear advantage of the economic approach to TFP measurement has been the availability of sound, economically meaningful decompositions that shed light on the underlying sources of productivity growth. In the economic approach, decompositions of productivity can be traced back to the seminal work of Farrell (1957), who expressed the overall economic efficiency as a product of the technical efficiency and allocative (price) efficiency components. Farrell restricted to a static framework, which left no room for technical progress. Nishimizu and Page (1982) proposed the first dynamic decomposition of TFP based on the work of Farrell (1957) and the Malmquist productivity index by Caves et al. (1982), which included technical change and technical efficiency components. Färe et al. (1994) (henceforth FGNZ) generalized and developed the decomposition further, introducing a scale efficiency component. The Malmquist approach has gained momentum after FGNZ, especially in the firm-level applications. Similar decompositions have been extended to variants of the Malmquist index such as the Hicks–Moorsten type productivity index (Bjurek 1996; Lovell 2003) and to the Malmquist–Luenberger type indexes (see Färe and Primont 2003).

The lack of a meaningful, operational decomposition is widely held as a disadvantage of the Fisher index (see e.g., FGNZ). Indeed, strict adherence to the axiomatic approach does not provide means to distinguish the frontier shifts from efficiency changes. However, if one is willing to cross over the methodological boundary between the axiomatic and economic approaches, the decomposition of the Fisher TFP index is possible along the lines of Nishimizu and Page (1982) and FGNZ. Such an amalgam of axiomatic and economic approaches to decompose the Fisher TFP index has been pursued by three earlier papers, to be reviewed next.[2]

Althin et al. (1996) were the first to link the Fisher index with the profitability change. They also decompose the profitability component as the product of technical efficiency change and scale elasticity change components along the lines of FGNZ. Ray and Mukherjee (1996) (henceforth RM) presented a more comprehensive decomposition that breaks the Fisher index into technical change, efficiency change, and scale components analogous to FGNZ, adding an allocative efficiency component (similar to Farrell 1957). Unfortunately, the RM decomposition restricts to the single output technology and does not immediately generalize to the multi-output settings. Moreover, the RM decomposition contains certain components that are difficult to interpret, as will be discussed in more detail below.

Recently, Zofio and Prieto (2006) (henceforth ZP) proposed a third decomposition of the Fisher TFP index based on a generalized graph distance function by Chavas and Cox (1999), which requires specification of a weighting parameter that determines the projection path to the frontier. The ZP decomposition consists of a technical component represented by the Malmquist index and an economic component consisting of an allocative efficiency component and a residual allocative term. The residual term contains components that are difficult to interpret, as shown below. Another debatable feature of the ZP decomposition is that the breakdown to different components crucially depends on the a priori specification of the weighting parameter. In conclusion, while all these three decompositions have their own merits, they all are *inexact* in the sense that they include a residual term that has no clear economic interpretation or they mix together prices and quantities from different time periods.

The purpose of this paper is to develop the first *exact* decomposition of the Fisher ideal TFP index (henceforth Fisher index). Our proposal differs from the earlier inexact decompositions in that we do not multiply the quantities of one period with the prices of another period, and we systematically abolish any residual terms that might emerge. We show that the Fisher index can be systematically broken down to five distinct components that represent changes in (1) production technology, (2) technical efficiency, (3) scale efficiency, (4) allocative efficiency, and (5) price effect. The three efficiency components (2–4) represent different aspects behind the efficiency of achieving a given target (i.e., the most profitable input–output vector). In contrast, components (1) and (5) capture the changes of this target. While the technical change component (1) is well-established, changes in the relative input–output prices can have real effects on the scale and scope of the optimal target, which have not been taken into account in the earlier decompositions of the Fisher index. Such price-induced changes of the target point are captured within the new price effect component (5), which indicates how much more (or less) profitable the most profitable target point has become as a result of the price change, as compared to the evaluated firm. Efficiency components (2–4) can be further split into separate sub-components for inputs and outputs, offering a detailed picture of the constituents of productivity change as measured by the Fisher index.

The proposed decomposition has many appealing features. The exactness of the decomposition is particularly important for methodological coherence when the

[1] However, if firms are assumed to operate with perfect allocative efficiency, then the quantity data suffices for estimating the Fisher index, as shown by Kuosmanen et al. (2004).

[2] In the axiomatic approach, decompositions of the Fisher ideal price and quantity indices have been developed (see Balk 2003, for a review).

components are estimated by means of deterministic methods (such as data envelopment analysis (DEA)) which leave no room for residuals. Yet, the components of the decomposition can be equally estimated by means of stochastic techniques (such as stochastic frontier analysis (SFA)). In addition, our decomposition provides a more detailed picture about the driving forces behind productivity growth (or decline) than any other TFP decomposition suggested before. For example, in our empirical application we find that the slow-down of the TFP growth in the Finnish farms in the aftermath of Finland's EU accession in 1995 is mainly due to the decline in the output prices. Another appealing feature of the new decomposition is that it is free of orientation to inputs or outputs, and (in contrast to ZP) it does not require any a priori specification of the measurement direction.

The remainder of this article is organized as follows. Section 2 introduces the standard notation and terminology used in production theory, and presents a formal duality theorem regarding profitability function. Section 3 introduces the FGNZ, RM, and ZP decompositions. Having discussed the pros and cons of the existing decompositions, we present our new decomposition and discuss its components in a step-by-step manner in Sects. 4 and 5. Section 6 presents an application to a large panel data of Finnish farms to illustrate the approach and to facilitate empirical comparison of the alternative indices and decompositions. Section 7 draws our concluding remarks. The formal proofs of the mathematical propositions are available in the working paper Kuosmanen and Sipiläinen (2004) and are hence omitted here.

2 Preliminaries

We focus on two-period comparisons and denote the base period as period 0 and the target period as period 1. Adopting the standard notation, $\mathbf{y}^t \in \mathbb{R}^s_+$ represents the output quantity vector of period $t \in \{0, 1\}$, and $\mathbf{p}^t \in \mathbb{R}^s_{++}$ the associated price vector. Similarly, $\mathbf{x}^t \in \mathbb{R}^r_+$ denotes the input quantity vector of period t and $\mathbf{w}^t \in \mathbb{R}^r_{++}$ the associated price vector. The production possibility set of period t is defined as $T^t = \{(\mathbf{x}, \mathbf{y}) \in \mathbb{R}^{r+s}_+ | \mathbf{x} \text{ can produce } \mathbf{y} \text{ in period } t\}$. These sets are assumed to be non-empty, closed and satisfy the scarcity and no-free-lunch assumptions (see Färe and Primont 1995 for details). Input–output vector (**x**, **y**) is technically efficient if it lies on the boundary of set T. The degree of inefficiency is traditionally measured using distance functions. Slightly deviating from Shephard's original definition, we define the input and output distance function as $D^t_x(\mathbf{x}, \mathbf{y}) \equiv \min\{\theta | (\theta\mathbf{x}, \mathbf{y}) \in T^t\}$ and $D^t_y(\mathbf{x}, \mathbf{y}) \equiv \min\{\theta | (\mathbf{x}, \mathbf{y}/\theta) \in T^t\}$. Distance functions can also be seen as representations of technology (Färe and Primont 1995).

The minimum cost of producing output **y** at given input prices **w** and the technology of period t is given by the cost function $C^t(\mathbf{w}, \mathbf{y}) \equiv \min_x\{\mathbf{w} \cdot \mathbf{x} | (\mathbf{x}, \mathbf{y}) \in T^t\}$. Analogously, the maximum revenue that can be obtained, given inputs **x**, output prices **p** and the technology of period t, is given by the revenue function $R^t(\mathbf{x}, \mathbf{p}) \equiv \max_y\{\mathbf{p} \cdot \mathbf{y} | (\mathbf{x}, \mathbf{y}) \in T^t\}$. By duality theory, cost and revenue functions are also representations of technology.

Profitability function $\rho^t(\mathbf{w}, \mathbf{p}) \equiv \max_{x,y}\left\{\frac{\mathbf{p}\cdot\mathbf{y}}{\mathbf{w}\cdot\mathbf{x}} \middle| (\mathbf{x}, \mathbf{y}) \in T^t\right\}$ indicates the maximum return to the dollar achievable with the given non-negative input–output prices (Althin et al. 1996; and Kuosmanen et al. 2004). Profitability function can also be used as a representation of technology under certain conditions:

Proposition 1 *If production possibility set* T^t *satisfies free disposability, convexity, and constant returns to scale, then profitability function* ρ^t *is a complete characterization of the technology. In particular,*

$$T^t = \left\{(\mathbf{x}, \mathbf{y}) \in \mathbb{R}^{r+s}_+ \middle| \frac{\mathbf{p}\cdot\mathbf{y}}{\mathbf{w}\cdot\mathbf{x}} \leq \rho^t(\mathbf{w}, \mathbf{p}) \forall (\mathbf{w}, \mathbf{p}) \in \mathbb{R}^{r+s}_+\right\}, \tag{1}$$

and

$$D^t_x(\mathbf{x}, \mathbf{y}) = \max_{(\mathbf{w},\mathbf{p}) \in \mathbb{R}^{r+s}_+}\left\{\frac{\mathbf{p}\cdot\mathbf{y}}{\mathbf{w}\cdot\mathbf{x}} \middle| \rho^t(\mathbf{w}, \mathbf{p}) \leq 1\right\}. \tag{2}$$

Properties of the profitability function ρ^t are similar to those of the standard profit function (cf., e.g., Färe and Primont 1995). The only notable difference is that while the standard profit function is homogenous of degree one in prices (**w**, **p**), the profitability function ρ^t is homogenous of degree zero in prices. For example, if all prices are doubled, the maximum profit doubles, too, but the maximum revenue-to-cost ratio does not change.

As a final remark, it should be noted that the representations of technology (production possibility sets, distance functions, cost and revenue functions, and profitability functions) are usually not known, and must be estimated from empirical data (see e.g., Coelli et al. 2005, for methods). This article abstracts from the estimation issues and focuses solely on the decomposition. No estimations are needed for calculating of the Fisher input, output or TFP indices, but the components of the decomposition must be estimated. In this sense, our approach is an amalgam of the axiomatic and economic approaches to index numbers.

3 Earlier decompositions

This section briefly introduces the three most closely related decompositions by FGNZ, RM, and ZP, pointing out their shortcomings and limitations that motivate our new decomposition.

3.1 FGNZ decomposition

FGNZ measure TFP by means of the Malmquist index, which they originally defined by means of the output distance functions as

$$M_y\left(\mathbf{x}^{0,1},\mathbf{y}^{0,1}\right) \equiv \left(\frac{D_y^{0,CRS}(\mathbf{x}^1,\mathbf{y}^1)}{D_y^{0,CRS}(\mathbf{x}^0,\mathbf{y}^0)}\cdot\frac{D_y^{1,CRS}(\mathbf{x}^1,\mathbf{y}^1)}{D_y^{1,CRS}(\mathbf{x}^0,\mathbf{y}^0)}\right)^{1/2}, \quad (3)$$

where the superscript *CRS* refers to the fact that FGNZ measure distance functions relative to the constant returns to scale (CRS) reference technology. When the CRS reference technology is not indicated by the superscript, the benchmark technology exhibits variable returns to scale (VRS). Following Nishimizu and Page (1982), FGNZ showed that the Malmquist index can be broken down to the components of technical change (*TECHCH*), pure technical efficiency change (*PEFFCH*) and scale efficiency change (*SCH*) as

$$M_y\left(\mathbf{x}^{0,1},\mathbf{y}^{0,1}\right) = TECHCH\cdot PEFFCH\cdot SCH, \quad (4)$$

where

$$TECHCH \equiv \left(\frac{D_y^{0,CRS}(\mathbf{x}^0,\mathbf{y}^0)}{D_y^{1,CRS}(\mathbf{x}^0,\mathbf{y}^0)}\cdot\frac{D_y^{0,CRS}(\mathbf{x}^1,\mathbf{y}^1)}{D_y^{1,CRS}(\mathbf{x}^1,\mathbf{y}^1)}\right)^{1/2}, \quad (5)$$

$$PEFFCH \equiv D_y^1\left(\mathbf{x}^1,\mathbf{y}^1\right)/D_y^0\left(\mathbf{x}^0,\mathbf{y}^0\right), \quad (6)$$

and

$$SCH \equiv \left(\frac{D_y^{1,CRS}(\mathbf{x}^1,\mathbf{y}^1)}{D_y^1(\mathbf{x}^1,\mathbf{y}^1)}\right)\Bigg/\left(\frac{D_y^{0,CRS}(\mathbf{x}^0,\mathbf{y}^0)}{D_y^0(\mathbf{x}^0,\mathbf{y}^0)}\right). \quad (7)$$

Component (5) gauges the shift of the CRS frontier at the fixed input–output vector; since there is no reason to prefer point $(\mathbf{x}^0,\mathbf{y}^0)$ over $(\mathbf{x}^1,\mathbf{y}^1)$, or vice versa, FGNZ use both and take the geometric mean of the resulting technical change measures. Component (6) measures the change in technical efficiency (as measured by the output distance functions). Finally, component (7) measures the change is scale efficiency by comparing the distance functions gauged relative to CRS and VRS reference technologies.

The FGNZ decomposition has at least two important limitations. First, the decomposition is oriented either to input or output efficiency. If one uses input distance functions instead of the output distance functions, the technical efficiency and scale efficiency components (6) and (7) are likely to yield different values. The Malmquist index (*M*) and its technical change component (5) are invariant to the choice of orientation because they are defined relative to the CRS benchmark technology. Second, the decomposition does not account for changes in allocative efficiency. This is because the Malmquist index does not account for price data. This is not a problem as such, but it is clearly a limitation of the method. If we assume that inputs and outputs are efficiently allocated, then the Malmquist index closely approximates the Fisher index (Balk 1993; Kuosmanen et al. 2004).

The FGNZ decomposition has also been criticized because it measures technical change by means of the CRS rather than VRS benchmark (Ray and Desli 1997). Indeed, the technical change component of the FGNZ decomposition may fail to identify technical changes that occur locally in some part of the frontier, but do not influence the CRS benchmark. However, use of the VRS benchmark technology to gauge technical change can lead to infeasible solutions. Moreover, the alternative approach proposed by Ray and Desli (1997) that uses VRS benchmark results as a scale efficiency measure that includes mixed period terms that are ambiguous and difficult to interpret (see Färe et al. 1997). Furthermore, recall that the purpose of the technical change component is to gauge the *impact* of technical change on TFP; not technical change as such. In this respect, achieving higher TFP levels requires that the global CRS frontier shifts outward. Local technical improvements that do not influence the CRS frontier cannot boost new TFP levels, because the same TFP growth could be achieved through improved scale efficiency. For these reasons, the distinction between the technical change and scale efficiency components is generally ambiguous and debatable.

3.2 RM decomposition

RM presented the first systematic decomposition of the Fisher index, which also accounts for changes in allocative efficiency. Consistent with the index number approach to TFP measurement (e.g., Diewert 1992; Diewert and Nakamura 2003), the Fisher TFP index is defined as the ratio of the output and input quantity indices:

$$F_{TFP}\left(\mathbf{p}^{0,1},\mathbf{w}^{0,1},\mathbf{y}^{0,1},\mathbf{x}^{0,1}\right) \equiv \frac{F_y(\mathbf{p}^{0,1},\mathbf{y}^{0,1})}{F_x(\mathbf{w}^{0,1},\mathbf{x}^{0,1})}, \quad (8)$$

where

$$F_y\left(\mathbf{p}^{0,1},\mathbf{y}^{0,1}\right) \equiv \left(\frac{\mathbf{p}^0\cdot\mathbf{y}^1}{\mathbf{p}^0\cdot\mathbf{y}^0}\cdot\frac{\mathbf{p}^1\cdot\mathbf{y}^1}{\mathbf{p}^1\cdot\mathbf{y}^0}\right)^{1/2} \quad (9)$$

and

$$F_x\left(\mathbf{w}^{0,1},\mathbf{x}^{0,1}\right) \equiv \left(\frac{\mathbf{w}^0\cdot\mathbf{x}^1}{\mathbf{w}^0\cdot\mathbf{x}^0}\cdot\frac{\mathbf{w}^1\cdot\mathbf{x}^1}{\mathbf{w}^1\cdot\mathbf{x}^0}\right)^{1/2} \quad (10)$$

are the Fisher ideal output and input quantity indices (i.e., geometric means of the Laspeyres and Paasche quantity indices), respectively. Restricting to the single-output case, RM express the Fisher TFP index as a product of technical efficiency index (TEI), allocative efficiency index (AEI), cost function index (CI) and average cost index (ACI):

$$F_{TFP}\left(\mathbf{p}^{0,1}, \mathbf{w}^{0,1}, \mathbf{y}^{0,1}, \mathbf{x}^{0,1}\right) = TEI \cdot AEI \cdot CI \cdot ACI, \quad (11)$$

where

$$TEI \equiv D_x^1\left(\mathbf{x}^1, y^1\right)/D_x^0\left(\mathbf{x}^0, y^0\right), \quad (12)$$

$$AEI \equiv \left(\frac{\left(\frac{C^1(\mathbf{w}^0, y^1)/\mathbf{w}^0\mathbf{x}^1}{D_x^1(\mathbf{x}^1, y^1)}\right)}{\left(\frac{C^0(\mathbf{w}^0, y^0)/\mathbf{w}^0\mathbf{x}^0}{D_x^0(\mathbf{x}^0, y^0)}\right)} \cdot \frac{\left(\frac{C^1(\mathbf{w}^1, y^1)/\mathbf{w}^1\mathbf{x}^1}{D_x^1(\mathbf{x}^1, y^1)}\right)}{\left(\frac{C^0(\mathbf{w}^1, y^0)/\mathbf{w}^1\mathbf{x}^0}{D_x^0(\mathbf{x}^0, y^0)}\right)} \right)^{1/2}, \quad (13)$$

$$CI \equiv \left(\frac{C^0(\mathbf{w}^0, y^0)}{C^1(\mathbf{w}^0, y^0)} \cdot \frac{C^0(\mathbf{w}^1, y^1)}{C^1(\mathbf{w}^1, y^1)} \right)^{1/2}, \quad (14)$$

and

$$ACI \equiv \left(\frac{C^0(\mathbf{w}^1, y^0)/y^0}{C^0(\mathbf{w}^1, y^1)/y^1} \cdot \frac{C^1(\mathbf{w}^0, y^0)/y^0}{C^1(\mathbf{w}^0, y^1)/y^1} \right)^{1/2}. \quad (15)$$

The components of the RM decomposition have the following interpretations. The technical efficiency index *TEI* is an input-oriented variant of the standard *PEFFCH* component by FGNZ. Allocative efficiency index *AEI* measures the change in allocative efficiency following Farrell's (1957) classic definition. Cost function index (*CI*) captures the effects of technical change: recall that cost function is a valid representation of technology. Finally, average cost index (*ACI*) captures the changes in scale efficiency by means of average costs: recall that the increasing (decreasing) returns to scale will cause the average costs to decrease (increase).

The RM decomposition has four major shortcomings. First, it restricts to the single-output technology, and does not easily extend to the general multi-output technologies. In particular, extending the average cost index *ACI* to the general multi-output setting proves problematic. Second, the decomposition is inherently input-oriented. In the single-input single-output case, replacing input distance functions by output distance functions and the cost functions by revenue functions would yield a consistent decomposition, with different results. Third, the allocative efficiency index *AEI* includes mixed-period measures that gauge whether inputs of period 0 have been allocatively efficient at prices of period 1, and vice versa (see (13)). However, there is no reason why inputs should be allocatively efficient at prices of some other period (compare with the sharp critique by Balk 1993). Fourth, the average cost index *ACI* is subject to a similar problem. Again, the average costs are measured counter-intuitively using the prices of period 0 and the technology of period 1, and vice versa (see (14)). In our view, the third and fourth problems are the most serious ones. Our new decomposition described in the next section avoids all these four problems.

3.3 ZP decomposition

ZP propose an alternative decomposition of the Fisher index that is based on the generalized distance function (introduced by Chavas and Cox 1999), defined as

$$D_G^t(\mathbf{x}, \mathbf{y}; \alpha) \equiv \min\left\{\theta \middle| \left(\mathbf{x}\theta^{1-\alpha}, \mathbf{y}/\theta^{\alpha}\right) \in T^t\right\}, \quad (16)$$

where $\alpha \in [0, 1]$ represents the relative weight of the input and output oriented projections to the frontier. In particular, ZP show that the Fisher index can be decomposed as

$$F_{TFP}\left(\mathbf{p}^{0,1}, \mathbf{w}^{0,1}, \mathbf{y}^{0,1}, \mathbf{x}^{0,1}\right) = M_G\left(\mathbf{y}^{0,1}, \mathbf{x}^{0,1}; \alpha\right) \quad (17)$$

$$\cdot \left[\frac{AE^1(\mathbf{p}^1, \mathbf{w}^1, \mathbf{y}^1, \mathbf{x}^1; \alpha)}{AE^0(\mathbf{p}^0, \mathbf{w}^0, \mathbf{y}^0, \mathbf{x}^0; \alpha)} \cdot \frac{A^0(\mathbf{p}^0, \mathbf{w}^0, \mathbf{y}^1, \mathbf{x}^1; \alpha)}{A^1(\mathbf{p}^1, \mathbf{w}^1, \mathbf{y}^0, \mathbf{x}^0; \alpha)} \right]^{1/2}, \quad (18)$$

where M_G is the Malmquist index (3) defined in terms of the generalized distance functions (16) (instead of the output distance functions),

$$AE^t(\mathbf{p}^t, \mathbf{w}^t, \mathbf{y}^t, \mathbf{x}^t; \alpha) = \frac{(\mathbf{p}^t\mathbf{y}^t/\mathbf{w}^t\mathbf{x}^t)/D_G^t(\mathbf{y}^t, \mathbf{x}^t; \alpha)}{\rho(\mathbf{p}^t, \mathbf{w}^t)}, \quad t \in \{0, 1\} \quad (19)$$

is the measure of allocative efficiency, and

$$A^t\left(\mathbf{p}^t, \mathbf{w}^t, \mathbf{y}^{1-t}, \mathbf{x}^{1-t}; \alpha\right) = \frac{\left(\mathbf{p}^t\mathbf{y}^t/\mathbf{w}^{1-t}\mathbf{x}^{1-t}\right)/D_G^t(\mathbf{y}^{1-t}, \mathbf{x}^{1-t}; \alpha)}{\rho(\mathbf{p}^t, \mathbf{w}^t)}, \quad t \in \{0, 1\} \quad (20)$$

is referred to as a "residual allocative term". The main merit of the ZP decomposition is that it further elaborates the connection between the Malmquist and the Fisher indices: the Malmquist index represents a technical component within the Fisher index, which can be further decomposed into technical change, scale efficiency change, and efficiency change components along the lines of FGNZ. On the other hand, the changes in allocative efficiency and the residual allocative terms represent the economic component of the Fisher index.

The ZP decomposition has two shortcomings. First, the residual allocative terms (20) include questionable mixed-period measures that gauge whether inputs of period 0 have been allocatively efficient at prices of period 1, and vice versa. In our terminology, this renders the decomposition inexact. Second, the magnitudes of different components depend on the choice of the parameter α that determines the projection path towards the frontier. In practice, the results can be sensitive to the specification of parameter α.

4 New decomposition

This section presents our new proposal for decomposing the Fisher ideal TFP index. We first present the decomposition, and then interpret and discuss each component and sub-component in more detail.

Proposition 2 *The Fisher ideal TFP index is the product of the technical efficiency* ($\Delta TEff$), *technical change* ($\Delta Tech$), *scale efficiency* ($\Delta SEff$), *allocative efficiency* ($\Delta AEff$), *and the price effect* (ΔPE) *components*:

$$F_{TFP} = \Delta TEff \cdot \Delta Tech \cdot \Delta SEff \cdot \Delta AEff \cdot \Delta PE, \tag{21}$$

where

$$\Delta TEff \equiv (\Delta ITEff \cdot \Delta OTEff)^{1/2} \tag{22}$$

$$\Delta ITEff \equiv D_x^1(\mathbf{x}^1, \mathbf{y}^1) / D_x^0(\mathbf{x}^0, \mathbf{y}^0) \tag{22a}$$

$$\Delta OTEff \equiv D_y^1(\mathbf{x}^1, \mathbf{y}^1) \Big/ D_y^0(\mathbf{x}^0, \mathbf{y}^0) \tag{22b}$$

$$\Delta Tech \equiv \left(\frac{\rho^1(\mathbf{w}^0, \mathbf{p}^0)}{\rho^0(\mathbf{w}^0, \mathbf{p}^0)} \cdot \frac{\rho^1(\mathbf{w}^1, \mathbf{p}^1)}{\rho^0(\mathbf{w}^1, \mathbf{p}^1)}\right)^{1/2} \tag{23}$$

$$\Delta SEff \equiv \frac{(ISEff^1 \cdot OSEff^1)^{1/2}}{(ISEff^0 \cdot OSEff^0)^{1/2}} \tag{24}$$

$$ISEff^t \equiv \left(\frac{\mathbf{p}^t \cdot \mathbf{y}^t}{C^t(\mathbf{w}^t, \mathbf{y}^t)}\right) \Big/ \rho^t(\mathbf{w}^t, \mathbf{p}^t) \tag{24a}$$

$$OSEff^t \equiv \left(\frac{R^t(\mathbf{x}^t, \mathbf{p}^t)}{\mathbf{w}^t \cdot \mathbf{x}^t}\right) \Big/ \rho^t(\mathbf{w}^t, \mathbf{p}^t). \tag{24b}$$

$$\Delta AEff \equiv \left(\frac{IAEff^1}{IAEff^0} \cdot \frac{OAEff^1}{OAEff^0}\right)^{1/2} \tag{25}$$

$$IAEff^t \equiv \frac{C^t(\mathbf{w}^t, \mathbf{y}^t)}{\mathbf{w}^t \cdot \left(D_x^t(\mathbf{x}^t, \mathbf{y}^t)\mathbf{x}^t\right)} \tag{25a}$$

$$OAEff^t \equiv \frac{\mathbf{p}^t \cdot \left(\mathbf{y}^t / D_y^t(\mathbf{x}^t, \mathbf{y}^t)\right)}{R^t(\mathbf{x}^t, \mathbf{p}^t)} \tag{25b}$$

$$\Delta PE \equiv \frac{\Delta PET}{\Delta PEA} \tag{26}$$

$$\Delta PET \equiv \left(\frac{\rho^0(\mathbf{w}^1, \mathbf{p}^1)}{\rho^0(\mathbf{w}^0, \mathbf{p}^0)} \cdot \frac{\rho^1(\mathbf{w}^1, \mathbf{p}^1)}{\rho^1(\mathbf{w}^0, \mathbf{p}^0)}\right)^{1/2} \tag{26a}$$

$$\Delta PEA \equiv \left[\left(\frac{\mathbf{p}^1 \cdot \mathbf{y}^0}{\mathbf{w}^1 \cdot \mathbf{x}^0} \Big/ \frac{\mathbf{p}^0 \cdot \mathbf{y}^0}{\mathbf{w}^0 \cdot \mathbf{x}^0}\right) \cdot \left(\frac{\mathbf{p}^1 \cdot \mathbf{y}^1}{\mathbf{w}^1 \cdot \mathbf{x}^1} \Big/ \frac{\mathbf{p}^0 \cdot \mathbf{y}^1}{\mathbf{w}^0 \cdot \mathbf{x}^1}\right)\right]^{1/2}. \tag{26b}$$

4.1 Technical efficiency

Change in technical efficiency can be measured in input oriented manner using (22a) (equivalent to the *TEI* component by RM) or in output oriented fashion using (22b) (equivalent to the *TECHCH* component by FGNZ). From the perspective of TFP measurement, both orientations are equally valid and relevant. Since we have no reason to prefer either orientation, in (22) we resolve the choice of orientation by taking the geometric mean of both measures. If the orientation makes a difference, our decomposition allows to report the sub-components of input and output efficiency as separate components within the overall TFP index.

4.2 Technical change

In measuring technical change, we follow RM and utilize a dual representation of technology. Whereas RM make use of the cost function, we resort to the profitability functions ρ^t. By Proposition 1, ρ^t is a valid representation of a benchmark technology that satisfies free disposability, convexity, and constant returns to scale; the standard properties in the context of TFP measurement. Thus, it is equally legitimate to measure technical change in terms of profitability functions ρ^t as with distance functions.

The ratio $\rho^1(\mathbf{w}^0, \mathbf{p}^0)/\rho^0(\mathbf{w}^0, \mathbf{p}^0)$ represents the change of maximum profitability from the base period to the target period, at the prices of the base period. As the same price vectors appear both in the nominator and the denominator, any change in profitability must inevitably be due to the change of technology. Technical progress would tend to increase profitability, and hence this ratio. Technical regress would decrease this ratio. However, we could similarly measure technical changes using the prices of the target period 1 as $\rho^1(\mathbf{w}^1, \mathbf{p}^1)/\rho^0(\mathbf{w}^1, \mathbf{p}^1)$. As we do not have any reason to prefer prices of period 0 or 1, in (23) we measure technical change as a geometric mean of the two ratios.

It is worth to note that the technical change component of FGNZ also measures technical change in terms of profitability; the dual formulation of the distance function relative to a CRS technology can be expressed as profitability measure (see (2)). Our technical change component differs from the *TECHCH* measure by FGNZ in that we measure profitability in terms of observed input–output prices, whereas FGNZ use the shadow prices. These two measures of technical change are equivalent if the assumption of allocative efficiency holds; the same assumption that guarantees equivalence of the Fisher and Malmquist TFP indices (see Färe and Grosskopf 1992, and Balk 1993, 1998).

4.3 Scale efficiency

Like RM, we measure scale efficiency in economic rather than technical terms.[3] In line with our treatment of technical change, we adopt the dual perspective and characterize the optimal scale size in terms of the profitability function ρ^t. Thus, the most natural measure of scale efficiency is the ratio of the maximum profitability at the current scale size to the maximum (global) profitability. In this definition, scale inefficiency represents firm's inability to adjust its scale size in the short run.

The current scale size can be measured either in terms of inputs or outputs. If we fix the output level to $\mathbf{y}^t$, the maximum profitability (or return on the dollar) in period t is given by the ratio $\mathbf{p}^t \cdot \mathbf{y}^t / C^t(\mathbf{w}^t, \mathbf{y}^t)$. Thus, the input oriented profitability based scale efficiency measure is the ratio of the size-constrained maximum profitability to the unconstrained maximum profitability (see (24a)). Similarly, if we fix the inputs to $\mathbf{x}^t$, the maximum profitability is $R^t(\mathbf{x}^t, \mathbf{p}^t)/\mathbf{w}^t \cdot \mathbf{x}^t$. Thus, the output oriented profitability based scale efficiency measure is given by (24b). To avoid the dependence on the arbitrary choice of orientation, in (24) we again take the geometric mean of the input oriented and output oriented scale efficiency measures.

It is well known that input and output oriented scale measures can give very different, even conflicting results. In contrast to FGNZ and RM, our scale efficiency measure accounts for both input and output oriented measures of scale efficiency. In stark contrast to RM, our scale efficiency measure avoids mixing prices and technologies from different time periods.

4.4 Allocative efficiency

The concept of allocative efficiency dates back to Farrell's (1957) static efficiency decomposition. RM were the first to introduce changes in allocative efficiency into the dynamic TFP decompositions. Allocative efficiency can be measured in terms of inputs or outputs (costs or revenues). Input allocative efficiency is given by (25a), directly analogous to RM. Output allocative efficiency is defined in terms of revenues rather than costs as in (25b). The overall allocative efficiency measure should take into account the changes in both input and output sides. As before, in (25) we measure the overall allocative efficiency change as the geometric mean of the input and output oriented measures. This is in line with the fact that changes in input and output allocative efficiency contribute to profitability with equal weight and in multiplicative form.

Our allocative efficiency component differs from that of RM in two important respects. First, our measure accounts for changes in allocative efficiency of the output mix (since RM operate in the single-output framework, there is no allocative inefficiency in outputs). Second, our measures of allocative efficiency consistently apply the prices and the technology of the same period, whereas RM also measure allocative inefficiency of input quantities observed in period 0 (1) with respect to input prices observed in period 1 (0). However, there is no reason to expect input quantities to be allocatively efficient with respect to past or future prices (compare with Balk 1993).

4.5 Price effect

Price effect components analogous to (26) have earlier appeared in the parametric TFP decomposition by Bauer (1990) and the decomposition of profit by Grifell-Tatjé and Lovell (1999), but the specific formulation of the price effect in (26) is a new contribution. To our knowledge, such price effect component has not been introduced to an exact decomposition of index number before.

Although TFP index is a ratio of two quantity indices, changes in relative prices lead to reallocation of input and output mixes, which may have *real* effects on the most profitable scale and scope of the firms. While the allocative and scale efficiency components capture the rate and efficiency with which the firm adjusts to the new most profitable target point, these efficiency measures do not account for the fact that the new target point may be more or less profitable than the earlier target. Our price effect captures such a change of target.

The overall price effect ΔPE consists of two parts: the price effect on the target point ΔPET and the price effect on the actual observation ΔPEA. The overall price effect is the ratio of these two subcomponents. Both subcomponents measure the effect of price change on profitability, but the input–output vectors used as quantity weights differ. The nominator ΔPET represents the effect of price changes on the optimal target. More specifically, consider the ratio $\rho^0(\mathbf{w}^1, \mathbf{p}^1)/\rho^0(\mathbf{w}^0, \mathbf{p}^0)$. This ratio captures the change in the maximum profitability due to the change in prices, given the technology of period 0 (note that the input–output vectors serving as the index weights in this ratio can be different for the nominator and the denominator). Instead of using period 0 technology, one could equally well take the period 1 technology as the benchmark and use profitability functions ρ^1. Since we have no reason to prefer

[3] There is a good reason for resorting to the economic (dual) measures of scale efficiency. The conventional distance function based scale measure would be problematic here, because the decomposition would then depend on the order in which its components are calculated, as aptly pointed out by McDonald (1996). The measure of allocative efficiency tends to give different results depending on whether we define it relative to the VRS or CRS benchmark. The dual measures of scale efficiency circumvent this problem.

either period, we measure the price effect on the target point by taking the geometric mean.

The price effect on the actual observation ΔPEA differs from ΔPET in that it uses the observed input–output vectors of the evaluated firm as the fixed index weights. Interestingly, we may interpret ΔPEA as the price deflator: it can be rephrased as the ratio of the Fisher output price index to the Fisher input price index

$$\Delta PEA = F_p\left(\mathbf{p}^{0,1}, \mathbf{y}^{0,1}\right)/F_w\left(\mathbf{w}^{0,1}, \mathbf{x}^{0,1}\right), \tag{27}$$

where the input and output vectors of the actual observation are used as the index weights. This deflator essentially measures the change of output prices relative to the change of input prices. If average output prices increase more (less) than the input prices, then the value of the deflator is greater (smaller) than one. If output and input prices change by the same factor, then this deflator is equal to one.

Combining these two terms, the overall price effect ΔPE indicates how much more (or less) profitable the optimal target point has become as a result of the price change, as compared to the observed input–output vector. Like Bauer's (1990) price effect, our overall price effect component (26) does not change if all prices are multiplied by some positive scalar $a > 0$. Moreover, the overall price effect ΔPE does not change if output prices are multiplied by a scalar $a > 0$ and the input prices by a scalar $b > 0$, $a \neq b$, because subcomponents ΔPET and ΔPEA will change by the same factor (a/b).

It is worth to emphasize that the technical change $\Delta Tech$ and the price effect on target ΔPET both differ from the efficiency components ($\Delta TEff$, $\Delta SEff$, and $\Delta AEff$) in an important respect. While the efficiency components capture the change of efficiency relative to some fixed target, both $\Delta Tech$ and ΔPET measure the impact of a change in the target point. Both $\Delta Tech$ and ΔPET measure this impact in terms of the change in the maximum profitability. Whereas the $\Delta Tech$ component represents the change of target due to technical progress, the price effect component captures the change of target resulting from price changes.

In contrast to the subcomponent ΔPET, the overall price effect ΔPE can be interpreted as an efficiency measure. Define the *profitability efficiency* (analogous to Nerlove's profit efficiency) as the ratio of the observed and the maximum profitability:

$$\rho Eff^t(\mathbf{w}^t, \mathbf{p}^t; \mathbf{x}^t, \mathbf{y}^t) = \frac{\mathbf{p}^t \cdot \mathbf{y}^t / \mathbf{w}^t \cdot \mathbf{x}^t}{\rho^t(\mathbf{w}^t, \mathbf{p}^t)}, \quad t = 0, 1. \tag{28}$$

By rearranging the elements of ΔPET and ΔPEA, we find that the overall price effect ΔPE can be stated as

$$\Delta PE = \left(\frac{\rho Eff^0(\mathbf{w}^1, \mathbf{p}^1; \mathbf{x}^0, \mathbf{y}^0)}{\rho Eff^0(\mathbf{w}^0, \mathbf{p}^0; \mathbf{x}^0, \mathbf{y}^0)} \cdot \frac{\rho Eff^1(\mathbf{w}^1, \mathbf{p}^1; \mathbf{x}^1, \mathbf{y}^1)}{\rho Eff^1(\mathbf{w}^0, \mathbf{p}^0; \mathbf{x}^1, \mathbf{y}^1)}\right)^{1/2}. \tag{29}$$

In this expression, the first ratio $\rho Eff^0(\mathbf{w}^1, \mathbf{p}^1; \mathbf{x}^0, \mathbf{y}^0)/\rho Eff^0(\mathbf{w}^0, \mathbf{p}^0; \mathbf{x}^0, \mathbf{y}^0)$ can be interpreted as the change in profitability efficiency due to price changes, given the technology and the observed input–output vector of period 0. The second ratio uses analogously the technology and the observed input–output vector of period 1. The overall price effect ΔPE is the geometric mean of these two price effects on profitability efficiency.

As a final remark, we note that prices can change due to different reasons: price changes can be exogenous shocks to the business environment (e.g.,changes in trade barriers) or endogenous choices by imperfectly competitive firms. We must emphasize that our decomposition is built upon the neoclassical assumption of price taking firms; the notions of cost, revenue, and profitability functions fail if the evaluated firms set prices endogenously. Thus, the price changes must be exogenous to the firms under evaluation. Yet, this does not mean that all input and output markets where the firms operate must be perfectly competitive. Indeed, there may be other firms that exhibit market power on the relevant input–output markets, either on the supply or demand side. In this sense, the price effect may capture effects of imperfect competition in the input and/or output markets, even though the evaluated firms themselves are assumed to take prices as given.

5 Decomposing profitability

Productivity and profitability are closely related notions (cf. Grifell-Tatjé and Lovell 1999; and Lawrence et al. 2006). In this section we shed some new light on this connection from the perspective of the decomposition proposed in Sect. 4.

To begin with, it is worth to observe that multiplying the Fisher input (output) quantity index by the corresponding price index yields the change in the total cost (revenue) (Diewert 1992). Thus, making use of Eq. 27 and moving the price effect ΔPEA to the left-hand side of the identity (21), we obtain the following alternative decomposition:[4]

$$\frac{\mathbf{p}^1 \cdot \mathbf{y}^1 / \mathbf{w}^1 \cdot \mathbf{x}^1}{\mathbf{p}^0 \cdot \mathbf{y}^0 / \mathbf{w}^0 \cdot \mathbf{x}^0} = \Delta TEff \cdot \Delta Tech \cdot \Delta SEff \cdot \Delta AEff \cdot \Delta PET. \tag{30}$$

The expression on the left hand side of (30) can be interpreted as the change in the observed profitability. On the right hand side, we have the same five components as in the Fisher TFP index, except the price effect now reduces to the price effect on the target point ΔPET only. Identities (21) and (30) provide further insight to the intimate relation

[4] We are grateful to an anonymous reviewer of this journal for suggesting this alternative interpretation.

between profitability and the Fisher TFP index: the latter can be seen as the price deflated variant of the former, using the price effect ΔPEA as an intuitive deflator. This connection can be exploited for decomposing the observed changes in profitability in analogous manner to the decomposition developed in Sect. 4.

Another interesting route to interpret identity (30) is to express it in terms of profitability efficiency, as defined in (28) above. The change in profitability efficiency is denoted by

$$\Delta\rho Eff = \rho Eff^{1}(\mathbf{w}^{1}, \mathbf{p}^{1}; \mathbf{x}^{1}, \mathbf{y}^{1}) / \rho Eff^{0}(\mathbf{w}^{0}, \mathbf{p}^{0}; \mathbf{x}^{0}, \mathbf{y}^{0}). \tag{31}$$

Multiplying both sides of Eq. 30 by $\rho^{0}(\mathbf{w}^{0}, \mathbf{p}^{0})/\rho^{1}(\mathbf{w}^{1}, \mathbf{p}^{1})$, we find that the remaining price effect and the $\Delta Tech$ components cancel out on the right-hand side, and the identity (30) reduces to

$$\Delta\rho Eff = \Delta TEff \cdot \Delta SEff \cdot \Delta AEff. \tag{32}$$

Interestingly, identity (32) reveals that the change in profitability efficiency is invariant to both technical change and changes in the price effect: only the three efficiency components remain in (32). This is due to the fact that the profitability function ρ, which captures the changes in the target point due to technical changes and price changes, is now placed on the left-hand side of (32). Therefore, any changes in the target point are already explicitly accounted for on the left-hand side, so any deviations from the target are attributable to inefficiency (which can be technical, allocative, or scale inefficiency in its nature). In contrast, identities (21) and (30) do not contain the profitability function ρ on the left-hand side, so the three efficiency components are not enough; it is necessary to account for the changes in the target point, too. We believe the developments in this paper provide new insight to this important distinction between the changes in the target point on one hand, and the efficiency of achieving the target on the other.

6 Application

This section applies the proposed TFP decomposition to the panel data of 459 Finnish bookkeeping farms in years 1992–2000 (see Kuosmanen and Sipiläinen 2004, for details). The main purpose of the application is to illustrate the new insights gained by the new decomposition and compare the results with those obtained by FGNZ and RM methods.[5] This empirical case is very useful for that purpose. Firstly, Finnish agriculture is subject to difficult, volatile weather conditions that frequently cause random, exogenous technology shocks. As a result, technical progress is not smooth and gradual; strong fluctuations and temporary regress are common. Hence, it is interesting to assess how the observed technology shocks show up in the alternative decompositions. Secondly, Finland's accession to the European Union (EU) in 1995 resulted as drastic changes in the output prices. The prices of meat and other animal products and crop products decreased more than 50%, but the price of milk decreased only by 15%. Input prices did not fall as dramatically. These price changes have led farms to adjust their scale size as well as their input–output mix, the effects of which are expected to show up in the technical, allocative, and scale efficiency measures. Thirdly, the data set is rich in detailed, disaggregated information about most relevant inputs and outputs as well as their prices, and the sample size is sufficiently large for estimation purposes.

Our sample includes different types of farms varying from specialized animal production and crop farms to more conventional farms engaged both in animal husbandry and crop farming. The data set itemizes three outputs and nine inputs. The outputs are crops, milk, and other animal products. The inputs are: labor, animal units, land, machinery, buildings, fertilizers, energy, purchased feed, and materials. Since we are mainly interested in the effect of the alternative index numbers and decompositions at the aggregate level of the entire sample, we focus on the productivity of the "representative farm", characterized by the arithmetic average of the input and output vectors of the sample farms.[6]

The Fisher ideal TFP index can be directly calculated from the price and quantity data, but its components depend on technology representations that must be estimated empirically. In the present type of a multi-output setting, the nonparametric Data Envelopment Analysis (DEA) approach is usually employed. The values of distance functions as well as cost and revenue functions relative to a convex, variable returns-to-scale DEA technology are obtained as optimal solution to specific linear programming problems (see e.g., Coelli et al. 2005). The profitability function was calculated by simply enumerating through all observed combinations of price and quantity vectors to calculate the full profitability distribution at all observed prices.[7]

[5] The results of the ZP decomposition depend on the weight assigned on the input and the output projections, and are thus not directly comparable.

[6] Kuosmanen and Sipiläinen (2004) report sample averages based on farm-specific productivity indices. For the Fisher TFP index and its decomposition, these sample averages are very similar to those of the representative farm discussed here.

[7] To make this critical profitability measure more robust to data errors and outliers, the 95 percentile of the profitability distribution was used as the empirical estimator for the profitability function. The clipping of the profitability distribution did not have a notable influence on the results to be presented.

Table 1 Results of the proposed decomposition (cumulative)

	1992	1993	1994	1995	1996	1997	1998	1999	2000
Fisher index (F_{TFP})	1	0.929	0.955	0.949	0.963	0.967	0.949	0.977	1.014
Output quantity index (F_o)	1	0.992	1.031	1.063	1.083	1.117	1.155	1.244	1.338
Input quantity index (F_i)	1	1.067	1.079	1.120	1.124	1.155	1.216	1.272	1.319
Technical efficiency ($\Delta TEff$)	1	0.993	1.005	1.061	0.989	0.911	0.864	0.943	0.946
Output oriented (*OTEff*)	1	1.003	1.020	1.084	1.015	0.938	0.869	0.957	0.960
Input oriented (*ITEff*)	1	0.983	0.991	1.039	0.964	0.885	0.859	0.930	0.932
Technical change ($\Delta Tech$)	1	0.875	0.937	1.001	1.038	1.051	1.074	1.037	1.060
Scale efficiency ($\Delta SEff$)	1	1.084	1.058	0.978	1.136	1.156	1.244	1.142	1.179
Output oriented (*OSEff*),	1	1.087	1.092	0.953	1.072	1.119	1.237	1.107	1.133
Input oriented (*ISEff*)	1	1.081	1.024	1.002	1.205	1.194	1.252	1.177	1.228
Allocative efficiency ($\Delta AEff$)	1	1.009	0.963	0.971	1.037	1.109	1.028	1.094	1.071
Output oriented (*OAEff*)	1	0.996	0.919	0.975	1.071	1.120	1.028	1.112	1.098
Input oriented (*IAEff*)	1	1.021	1.009	0.967	1.004	1.097	1.028	1.076	1.044
Price effect (ΔPE)	1	0.977	0.996	0.941	0.796	0.788	0.800	0.800	0.802
Target quantities (ΔPET)	1	1.108	1.029	0.754	0.767	0.986	0.830	0.799	0.793
Actual quantities (ΔPEA)	1	1.134	1.034	0.801	0.963	1.261	1.038	0.999	0.989

The cumulative Fisher TFP index and its components are reported for the representative farm in Table 1. Recall that the Fisher TFP index is obtained as the ratio of the output quantity index (F_o) to the input quantity index (F_i), reported in the first three rows of Table 1. Both aggregate input and output grew fast during the study period, implying a growth in the average farm scale size. However, input consumption grew faster than output production, which implies productivity decline (TFP levels <1). Only in year 2000, the cumulative output growth finally exceeded the input growth, and the Fisher TFP index recovers the initial levels of 1992.

Table 1 also reports the cumulative indices of each component of our proposed decomposition. The results suggest that scale efficiency of the sample farms has improved as a result of increased farm size. Ceteris paribus, the improvement in scale efficiency would imply nearly 18% TFP growth during the study period. Both input and output oriented scale efficiency measures indicate efficiency improvement, but the input oriented measure shows even higher growth than the output oriented variant. Other important growth factors were improved allocative efficiency and technical change, which implied about 7 and 6% TFP growth during the study period, ceteris paribus. However, these three growth factors were almost completely offset by the declining technical efficiency, and especially the vast 20% decrease in the price effect. In the present application, the price effect component captures the exogenous price shock due to Finland's EU membership in 1995, which resulted as a drastic fall in the output prices while the input prices decreased only little. We note that this drastic price change decreases profitability of both the most profitable farm and the average farm in years 1995–1996, but the most profitable farm is hit harder. By year 2000 the cumulative price effect on the average farm is negligible, but the price effect on the most productive farm is over 20% decline in profitability. This explains the significant drop in the overall price effect. This example clearly indicates that the price effect and its subcomponents can provide insight to the driving factors behind TFP.[8]

Parallel results of the RM decomposition are reported in Table 2. Obviously, the overall TFP index reported on the first row of Table 2 is identical to our decomposition; only the partition to the components is different. The original RM decomposition measures change in scale efficiency (*SCA*) by means of average costs per unit of output in the single-output setting, but it is unclear how to adapt this measure to the present multi-output setting. In Table 2 we approximate the scale efficiency term by the "residual term" obtained by dividing the Fisher index by the product of all other components.

Also Table 2 suggests improved allocative and scale efficiency as the main growth factors with nearly 6 and 5% cumulative growth. However, both these components indicate lower growth rates than our new decomposition. The difference is particularly large in the scale efficiency component. The RM decomposition also recognizes decline in technical efficiency (after all, $TEI = ITEff$), but the cost function index that represents technical change suggests technical regress. It seems unrealistic to claim that farmers have experienced negative technology shocks in

[8] The price effect index tracks very closely the observed patterns in the output prices (cf. Fig. 6 in Kuosmanen and Sipiläinen 2004).

Table 2 Results of the RM decomposition (cumulative)

	1992	1993	1994	1995	1996	1997	1998	1999	2000
Fisher index (F_{TFP})	1	0.929	0.955	0.949	0.963	0.967	0.949	0.977	1.014
Technical efficiency (*TEI*)	1	0.983	0.991	1.039	0.964	0.885	0.859	0.930	0.932
Cost function index (*CI*)	1	0.925	0.951	0.927	0.978	0.969	1.044	0.930	0.974
Scale residual[a]	1	0.994	0.995	1.001	1.001	1.007	1.015	1.035	1.051
Allocative Eff. index (*AEI*)	1	1.028	1.019	0.985	1.021	1.120	1.043	1.092	1.063

[a] Average cost index (*ACI*) cannot be computed in the present multi-output setting, so we have approximated the scale efficiency component by the residual $F_{TFP}/(TEI \cdot CI \cdot AEI)$

almost all years of the study period; it appears that this *CI* component also captures the decline in output prices which we have attributed to the price effect.

Finally, the results of the FGNZ decomposition are reported in Table 3. For better comparability, we report the results of the input oriented variant instead of the output oriented decomposition originally presented by FGNZ (and discussed above). The FGNZ decomposition is based on the Malmquist index, which shows very similar pattern to the Fisher TFP index in the first half of the study period but diverges somewhat especially in the last 2 years. Comparing the first rows of Tables 1 and 3, we see that the Malmquist index falls behind the Fisher index by 5.3% points by year 2000.

The FGNZ decomposition identifies only three components. In line with Tables 1 and 2, the results of Table 3 suggest improved scale efficiency as the main growth factor with the impact of 4% growth in TFP, ceteris paribus. This is offset by the decline in technical efficiency; the same input oriented technical efficiency component is included in all three decompositions, but our decomposition also accounts for the output oriented technical efficiency. The technical change component of FGNZ shows initially decline, but almost catches up the initial level by year 2000.

It is worth to note some interesting observations regarding technical change and scale efficiency. Regarding technical change, all three indicators suggest negative technology shocks in years 1993 and 1999, and all indices reach their highest value in year 1998. Our $\Delta Tech$ index shows initially the strongest decline but then shows relatively quick and steady progress throughout the sample period, ending up with 6% cumulative growth. By contrast, the two alternative indices both suggest technical decline throughout the study period, except for year 1998. Observe that *CI* and *TECHCH* indicators reach their lowest point in 1995, the year of Finland's EU accession. This suggests that these two indices capture under the label of technical change also some effects of the drastic price changes and the resulting reallocations of input–output mixes.

The improvement in scale efficiency ($\Delta SEff$) proved the most important growth factor in our decomposition. We already noted that the scale components of FGNZ and RM decompositions indicated more modest improvement in scale efficiency. In reality, the sizes of the sample farms grew in this period; both input and output quantity indices show over 30% point cumulative growth during the study period (see Table 1). The distance function based FGNZ component measures the proportionate change in the output levels resulting from an *equiproportionate* scaling of inputs. In the present data, the growth of farms changed the input mix dramatically: while machinery and buildings grew by 86 and 63% during the study period, the labor input grew only by 9%, and the energy use actually decreased by 9%. The Malmquist approach views these changes as movements along the input isoquant, and not as increase in the scale size. By contrast, our scale efficiency index ($\Delta SEff$) and RM's *ACI* component measure the scale size in terms of cost and revenue aggregates. In this respect, the rapid increase in economies of scale suggested by our $\Delta SEff$ index is supported by the parallel growth in the input and output quantity indices. The fact that the residual component of the RM decomposition fails to capture the considerable increase in the average farm size

Table 3 Results of the FGNZ decomposition (cumulative)

	1992	1993	1994	1995	1996	1997	1998	1999	2000
Malmquist index (*M*)	1	0.935	0.951	0.950	0.962	0.963	0.946	0.952	0.961
Technical efficiency (*PEFFCH*)[a]	1	0.983	0.991	1.039	0.964	0.885	0.859	0.930	0.932
Technical change (*TECHCH*)	1	0.925	0.922	0.870	0.949	1.017	1.095	0.996	0.991
Scale efficiency (*SCH*)[a]	1	1.027	1.041	1.051	1.053	1.070	1.005	1.027	1.041

[a] For better comparability, we report the input-oriented variants of the *PEFFCH* and *SCH* components

Table 4 Summary statistics of the distribution of the technological change index ratio $TC = TECHCH/\Delta Tech$ at the farm level

	TC9293	TC9394	TC9495	TC9596	TC9697	TC9798	TC9899	TC9900
Mean	1.193	0.984	0.974	1.058	0.964	1.045	0.964	1.086
Median	1.156	0.975	0.932	1.027	0.955	0.996	0.982	1.066
Std	0.452	0.260	0.424	0.398	0.252	0.293	0.213	0.351
5% low	0.986	0.812	0.739	0.877	0.794	0.783	0.637	0.851
95% high	1.383	1.209	1.154	1.226	1.167	1.500	1.212	1.368

TECHCH and $\Delta Tech$ are the components representing technical change in the Malmquist and Fisher indices, respectively

Table 5 Summary statistics of the distribution of the technical efficiency ratio $TEC = PEFFCH/\Delta TEff$ at the farm level

	TEC9293	TEC9394	TEC9495	TEC9596	TEC9697	TEC9798	TEC9899	TEC9900
Mean	0.995	1.006	1.001	1.000	1.000	1.016	1.027	0.987
Median	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Std	0.034	0.054	0.063	0.042	0.050	0.325	0.379	0.093
5% low	0.950	0.964	0.957	0.954	0.948	0.572	0.926	0.830
95% high	1.036	1.061	1.036	1.048	1.044	1.489	1.138	1.065

PEFFCH and $\Delta TEff$ are the technical efficiency components of the Malmquist and Fisher indices, respectively

Table 6 Summary statistics of the distribution of the scale efficiency ratio $SEC = SCH/\Delta SEff$ at the farm level

	SEC9293	SEC9394	SEC9495	SEC9596	SEC9697	SEC9798	SEC9899	SEC9900
Mean	0.979	1.004	1.109	0.910	1.095	0.878	1.121	0.898
Median	0.960	0.994	1.063	0.874	1.061	0.860	1.044	0.865
Std	0.173	0.151	0.252	0.190	0.253	0.248	0.336	0.410
5% low	0.739	0.753	0.815	0.681	0.862	0.554	0.789	0.557
95% high	1.273	1.255	1.612	1.196	1.419	1.179	1.672	1.177

SCH and $\Delta SEff$ are the scale efficiency components of the Malmquist and Fisher indices, respectively

seems to suggest that the interpretation of the residual as scale efficiency index may not be appropriate.

It is worth to stress that the differences between alternative indices and decompositions tend to be much larger at the firm level than at the aggregate level. Therefore, considering just the representative farm may conceal interesting patterns emerging at the farm level. To illustrate the distributions of the differences between the components of the Malmquist index (FGNZ decomposition) and the Fisher index (the proposed decomposition), Tables 4, 5 and 6 report some summary statistics of the technical change, technical efficiency change and scale efficiency change ratios of the corresponding Malmquist and Fisher index decompositions. These ratios have been calculated by dividing the technical change (technical efficiency change, scale efficiency change) component of the Malmquist index by the corresponding component of the Fisher index. When the ratio is equal to one, both components yield the same value. If the ratio is greater than one, the component of the Malmquist index is larger than the corresponding component of the proposed Fisher index decomposition. The situation is reversed when the ratio is less than one.

Table 4 compares the technical change components. The results show that farm specific and even average values may differ considerably between years: in half of the years the mean values of technical change are larger for the Malmquist index the median being larger only in three of 8 years. The results also show that the mean values are not necessarily consistent with technical change based on the representative or average farm. On the other hand, the technical efficiency change components (Table 5) of the two indices do not differ considerably, as could be expected. The median values of the ratios are all close to one in every year. The variation in the ratio is the largest from 1997 to 1998 and from 1998 to 1999 when poor weather conditions increased the variation of yields. Finally, in the scale efficiency change (Table 6), the differences between approaches are fairly large. Recall that the FGNZ decomposition defines scale efficiency as the ratio of technical efficiencies with respect to VRS and CRS technologies, our proposed scale efficiency component is defined in economic terms. Table 6 aptly illustrates that farm specific differences may be considerably larger than the differences at the level of the representative farm.

7 Concluding remarks

We have proposed a new *exact* decomposition of the Fisher ideal TFP index that which leaves out no questionable mixed-period components or residual terms. By combining insights from both axiomatic and economic approaches to index theory, the proposed decomposition further enhances our understanding about productivity. By distinguishing between input and output oriented sub-components, and by introducing allocative efficiency and the new price effect component, the present decomposition can provide a more detailed picture about the driving forces behind productivity changes than any other decomposition suggested before. Our decomposition also recognizes the price-based allocative efficiency and the price effect as important elements of productivity change. In particular, the price effect captures the change of the target point (relative to which the efficiency components are defined) due to the changes in the relative prices of inputs and outputs, which has not been taken into account in earlier decompositions of the Fisher index.

The exactness of the decomposition is particularly important for methodological coherence when the components are estimated by means of deterministic methods (such as data envelopment analysis, DEA) which leave no room for residuals. Yet, the components of the decomposition can be equally estimated by means of stochastic techniques (such as stochastic frontier analysis, SFA).

While our decomposition has a number of appealing characteristics, it obviously has some debatable features. One of them is the fact that our technical change component is based on the CRS rather than VRS benchmark, similar to FGNZ. Thus, the critique by Ray and Desli (1997) applies to our approach as well (see also Lovell 2003, for recent discussion). On the other hand, the approach of Ray and Desli has its own problems. In particular, the VRS route spoils the scale efficiency component by introducing mixed-period terms that are difficult to interpret: we see no reason to expect the observation of period t be scale efficient relative to the most productive scale size of periods $t-1$ and $t+1$. As yet, there is no generally accepted distinction between the technical change and scale efficiency. We have presented arguments to support our preference of the FGNZ interpretation, but this question remains subject to debate.

In conclusion, we hope this study might also enhance the position of the Fisher index as a useful index number formula for productivity analysis, and inspire further debate about the relative merits of different approaches. We believe there is no single superior index number for all empirical studies, but different index formulae may be appropriate depending on the purposes of the analysis and the interpretation of productivity. Further research could consider other important indices such as the widely used Törnqvist productivity index.

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