

The value of conducting on‑farm feld trials using precision agriculture technology: a theory and simulations

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Abstract

The objectives of this article are to examine the practicality of on-farm precision experiments to sufficiently lower the costs of acquiring the information necessary to make sitespecifc nitrogen (N) fertilizer management proftable, and to examine the potential value of on-farm precision experiments in uniform rate N fertilizer management. After presenting a simple economic model as theoretical background, two hypotheses are tested. Hypothesis 1 is that if on-farm precision experiments are conducted over sufficiently many growing seasons on a "fat and black" central Illinois cornfeld, the information gained can be used to make site-specifc N application management more proftable than uniform rate N application management. Hypothesis 2 is that conducting on-farm precision experiments on that feld for only a few years will provide information that can increase profts for a farmer who otherwise would follow the N application rate recommendation of the Maximum Return to Nitrogen project. Monte Carlo simulations rejected Hypothesis 1, but failed to reject Hypothesis 2. On the modeled central Illinois feld, which was characterized by relatively little spatial heterogeneity, even ffteen years of on-farm precision experiments did not provide enough information to make using site-specifc N management proftable. But the information gleaned from just a few years of on-farm precision experiments provided very proftable information to improve spatially uniform N rate management.

Keywords On-farm precision experiments · Value of information · Nitrogen management

Site-specifc input application technology (SST) has been commercially available for two decades. Using truly space-age methods to customize input management to small parts of farm felds was then and still is exciting. However, actual adoption of SST remains far below the expectations expressed in the late 1990s by enthusiastic farmers, agricultural scientists, farm equipment dealers, and popular news media. Addressing this subject amidst the early excitement, Bullock et al. ([1998](#page-16-0), [2002](#page-16-1)), and Bullock and Bullock [\(2000\)](#page-16-2) discussed how the economic complementarity between SST and information about yield response made

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proftable use of SST unlikely without much more information about yield response than was then available.

The economic complementarity of SST equipment and yield response information has important research implications and, ultimately, important practical implications, which provide both bad and potentially good news about precision technology. The bad news: unless more information can be produced about yield response, the proftability of SST will remain limited. The potentially good news: using research methodologies now being called "on-farm precision experimentation" (OFPE), precision technology itself can be used to inexpensively run large-scale agronomic feld trials, and so increase the supply of the information needed to make SST proftable. The data thus generated could be used proftably in concert with other agricultural "Big Data," a topic generating much excitement in agricultural and popular media.

The objectives of this article are to examine the practicality of the idea that OFPEs can be used to sufficiently lower the costs of acquiring the information necessary to make sitespecifc nitrogen (N) fertilizer management proftable, and to examine the potential value of OFPEs in uniform rate N fertilizer management. In particular, two hypotheses are tested; these are necessarily narrow in scope, encompassing only two of dozens of important questions about OFPEs, and are examined using simulations with data from a single OFPE. It is hoped, however, that the reported research will encourage more applied and theoretical OFPE research. Hypothesis 1 is that if OFPEs are conducted over sufficiently many growing seasons on a "fat and black" central Illinois cornfeld, the information gained can be used to make site-specifc N application management more proftable than uniform rate N application management. Hypothesis 2 is that conducting OFPEs on that feld for only a few years will provide information that can increase profts for a farmer who otherwise would follow the N application rate recommendation of the Maximum Return to Nitrogen (MRTN) project (Sawyer et al. [2006;](#page-16-3) Iowa State University Extension and Outreach [2019](#page-16-4)).

In pursuit of these objectives, frst a simple economic theory is presented to argue that (a) current use of SST is low because many potential users possess too little information about crop yield response to use SST proftably; and (b) if the economic supply curve of that information can be shifted out, use of the technology will increase. Second, results of a Monte Carlo simulation are presented to gauge the practicality of this idea of using SST to increase the information needed to increase the demand for SST. Those simulations quantify (a) how the values of SST and uniform technology change as the number of years of on-farm experimentation increases, (b) how the value of SST [as compared to conventional, uniform rate technology (URT)] increases as the number of years of on-farm precision experimentation increases, and (c) the value of OFPE information to farmers who otherwise would follow university-based uniform rate management advice. The principal results reject Hypothesis 1, but fail to reject Hypothesis 2. At least on the modeled central Illinois feld, which was characterized by relatively little spatial heterogeneity, even ffteen years of OFPEs did not provide enough information to make using site-specifc N management proftable. But the information gleaned from just a few years of OFPEs provided very proftable information to improve spatially uniform N rate management.

A simple economic theory of SST adoption (and the lack thereof)

This section presents a simple economic theory to explain the current low adoption rates of SST, and discusses how equilibrium adoption rates would be increased if more information about yield response were available. Figure [1](#page-2-0) features a model of two markets. The left-hand panel shows a market for SST equipment. The curve labeled S_E' represents the supply of equipment in an initial period. The demand for SST equipment is initially $D_E(P_E, P_I^0)$, where P_I^0 is the initial price of yield-response information. (At this level of abstraction, no attempt is made to defne in exact quantitative terms what is meant by "information." For a more rigorous treatment of the subject, see Bullock et al. [\(2009\)](#page-16-5).) These initial curves have been drawn such that the vertical intercept of S_E' exceeds that of $D_E(P_{E_2}, P_I^0)$, implying that in the initial equilibrium SST equipment is neither bought nor sold $(Q_E⁰=0)$. This is meant to represent a state in which SST has not yet been invented—that is, is "so expensive" to produce that no buyers are willing to pay enough for any to be produced in equilibrium. In the initial state, even without SST, farmers do have some use for information about yield responses to inputs on their felds, because it can be helpful as they make URT management decisions. Therefore, as shown in the right-hand panel of Fig. [1](#page-2-0), in the initial equilibrium a positive but small amount of information, Q_I^0 , is bought and sold at a price per unit P_I^0 .

Figure [1](#page-2-0) illustrates that a subsequent equilibrium arises when SST equipment is brought to market ("invented"), which is modeled as an outward shift of the supply curve from S_F' to S_F'' . This shift results in the emergence of the market for SST equipment and an expansion of the information market. The subsequent equilibrium is shown with prices and quantities P_E^{-1} , P_I^1 , Q_E^{-1} , Q_I^{-1} . (In the subsequent equilibrium, demand for site-specific technology equipment has shifted in slightly from $D_E(P_E, P_I^0)$ to $D_E(P_E, P_I^1)$, because the price of information has risen.)

The purpose here is to consider whether the subsequent equilibrium illustrated in Fig. [1](#page-2-0) must be the long-run equilibrium. Figure [2](#page-3-0) illustrates why it may not have to be, because it is possible to use precision technology itself to lower the costs of generating yield-response information. Following early discussions (Bullock et al. [1998,](#page-16-0) [2002](#page-16-1); Bullock and Bullock

Fig. 1 Effects of the invention of site-specific technology equipment on the markets for site-specific technology equipment and yield-response information

Fig. 2 OFPE lowers the cost of generating information, so demand for precision technology equipment rises

[2000](#page-16-2)), here it is assumed that SST and yield-response information are economic complements; the technology is more valuable if the information is available, and the information is more valuable if the technology is available. The use of precision technology to cheaply generate yield-response information is represented by a shift down and out in the information supply curve, from S_I' to S_I'' . This drops the price of information, which in turn, because of the complementarity between information and precision equipment, shifts out the demand for precision equipment. In the new equilibrium, quantities have risen to Q_E^2 , and Q_I^2 , and prices are $P_E²$, and $P_I²$. Precision technology has been used to supply the information needed to increase the value of precision technology in crop production, and thereby create demand for itself.

Supply of information about site‑specifc yield response: how precision technology can be used to make precision technology proftable

Crop scientists and agricultural economists have been attempting for generations to estimate yield response functions with small amounts of data (e.g. Heady and Pesek [1954;](#page-16-6) Day [1965;](#page-16-7) Stauber et al. [1975;](#page-16-8) Cerrato and Blackmer [1990;](#page-16-9) Bullock and Bullock [1994;](#page-16-10) Tembo et al. [2008;](#page-17-0) Marenya and Barrett [2009](#page-16-11)). But that research has been insufficient to provide farmers needed detailed, site-specifc knowledge about yield response. The costs of conducting yield-response studies limited data generation. Trials were conducted using labor-intensive techniques; workers marked off plots of land using measuring tapes and orange fags, applied inputs by hand or with specialized equipment, and harvested without the beneft of large-scale farm machinery. This labor intensity meant that it was generally only fnancially feasible to run trials on very small areas of land, at few locations, usually only for a few years. Consequently, despite generations of research, little is known about optimal fertilizer management; university- and industry-provided fertilizer management recommendations still often rely on "rules-of-thumb" (e.g., Hoeft and Peck [2007\)](#page-16-12) that are, at best, based only loosely on data analysis and science (Rodriguez et al. [2019\)](#page-16-13).

At the end of the 1990s, a few small agronomic research projects (e.g., the development of Donald Bullock's and Ronald Milby's *Enhanced Farm Research Ana*lyst software (Rund [2000](#page-16-14)), and Cook et al. [\(1999](#page-16-15))) began demonstrating that precision technology could be used to conduct OFPEs on very large fields at very low costs. More recently, the Data-Intensive Farm Management (DIFM) project (Bullock et al. [2019](#page-16-16)) has expanded those early efforts, conducting hundreds of OFPEs in the US and South America). Figure [3](#page-4-0) depicts a corn OFPE, conducted by DIFM in 2018 on a 30-ha central Illinois commercial farm field. A participating farmer put the experiment "in the ground" by operating the farm machinery in his usual manner while rates of sidedressed N application were controlled by computer and Global Navigation Satellite Systems equipment according to the researchers' pre-programmed randomized field trial design, which included five N application rates.

Fig. 3 Nitrogen-rate trial design for a 30-ha feld in central Illinois, 2018, the farmer "putting the trail in the ground," and scatterplot of the (N, yield) data generated

A Monte Carlo simulation of the relationships between application rate technologies and information from on‑farm precision experimentation

To examine the potential value of conducting OFPEs such as the one depicted in Fig. [3](#page-4-0), a 1000-round Monte Carlo simulation of the physical and economic results of N fertilizer experimentation and management in corn production was conducted to examine the relationships between N application strategies and yield response information.

The simulated feld and simulated production

A farm feld with spatially autocorrelated production characteristics was simulated, then used in the simulation rounds. The simulated feld's size and grid design were typical of DIFM's (N, corn) OFPEs. As in Fig. [3,](#page-4-0) it was assumed that the trial design included a "buffer zone" inside the simulated field's perimeter, both for headlands to provide turnaround room, and also to not subject diferent parts of the experiment to diferent levels of wind exposure and self-shading of the crop. The bufer zone was not considered to be part of the experiment, just as it was not part of the 2018 OFPE illustrated in Fig. [3](#page-4-0). The "in-trial" part of the feld was partitioned into a rectangular grid of 160 plots, each 91.44 m (300 ft) long and 18.288 m (60 ft) wide. Following DIFM's usual practice, each plot was partitioned into five $18.288 \text{ m} \times 18.288 \text{ m}$ (60 ft×60 ft) subplots, as shown in Fig. [3,](#page-4-0) resulting in 800 subplots in total. Thus, the "in-trial" part of the feld was rectangular, 731.52 m (2400 ft) long, and 365.76 m (1200 ft) wide, making the trial 26.76 ha (66.12 ac) in size.

The simulated feld's characteristics maps

The simulated feld was assigned the two "characteristics maps," depicted in Fig. [4](#page-5-0). The characteristics modeled were the feld's stream power index (SPI) and its Illinois Soil Nitrate Test (ISNT) map. ISNT values are meant to indicate in-soil nitrate

Fig. 4 ISNT and SPI maps of the simulated feld

concentration, and SPI values describe topography. Rufo et al. ([2005](#page-16-17), [2006\)](#page-16-18) provided more detailed descriptions of the properties and agronomic implications of these feld characteristics. For the purposes of the simulations, the ISNT and SPI values were treated as exogenous and constant over time. The ISNT map was generated using the *geoR* package in *R* (Ribiero et al. 2007) to run a Gaussian random feld simulation, assuming a nugget of 300, a sill of 1617, and range of 418, which were parameters reported for the SY03 feld in Champaign County, Illinois in Rufo et al. [\(2005,](#page-16-17) [2006\)](#page-16-18). The nugget/sill ratio of the feld's semi-variogram is under 30%, indicating strong spatial autocorrelation, and therefore was useful for the purposes of the simulations. The calculated ISNT values were normally distributed, having a high *p*-value form the Shapiro–Wilk test, with mean 217.1 and standard deviation 26.9.

Creating SPI maps requires a topographical slope map, and a Specifc Catchment Area (SCA) map. A set of digital elevation data at a 1 m level of resolution were obtained for feld SY03 from U.S. Geological Survey [\(2017](#page-17-1)), and then slope values were calculated (Arundel et al. [2015](#page-16-19)). The SCA map data was then derived from the slope map data using the System for Automated Geoscientifc Analyses (SAGA) tool in QGIS (Conrad et al. [2015,](#page-16-20) QGIS Development Team [2019\)](#page-16-21), according to Eq. [\(1](#page-6-0)). The resultant SPI data had a mean of 1.76 and a standard deviation of 1.61.

$$
SPI = SCA \times slope \ (\%).
$$
 (1)

Trial design

In each experimental year, to each of the feld's 160 plots, one of the fve N fertilization target rates was randomly assigned: 125, 150, 175, 200, or 225 kg/ha. In each Monte Carlo round, each of the five rates was assigned to 32 plots, and all five subplots in a plot received the same N rate.

The "true" response function

It is assumed that the "true" response function is that reported in Bullock et al [\(2009](#page-16-5)), which was derived from regression analysis on data from an early central Illinois OFPE:

$$
f(N, M, S, I, u) \equiv \beta_0 + \beta_N N + \beta_D + \beta_I I + \beta_M M + \beta_S S + \beta_{NN} N^2 + \beta_{NI} NI + \beta_{NM} NM + \beta_{NS} NS + \beta_{NN} N^2 I + u,
$$
\n(2)

where the *f* function describes how a subplot's crop yield responds to the nitrogen fertilizer application rate *N*, May rainfall *M*, the subplot's ISNT and SPI values, *I* and *S*, and a random yield disturbance term, *u*.

Calling I_{bi} and S_{bi} the ISNT and SPI values in subplot *j* of plot *b*, each subplot's reducedform yield response function can be defned as,

$$
f_{bj}(N, M, u) \equiv f(N, M, S_{bj}, I_{bj}, u), \qquad (3)
$$

where f_{bi} denotes Mg yield on the 18.3 $m \times 18.3$ m subplot, and *N* is kg nitrogen fertilizer applied to that subplot. The coefficient values assumed are from Champaign County field *D* in Bullock, et al. ([2009\)](#page-16-5): $\beta_0 = 5524$, $\beta_N = 154.97$, $\beta_D = -2440$, $\beta_I = 52.33$, β_M = −108.5, β_S = −194.7, β_{NN} = −0.4461, β_{NI} = −0.4866, β_{NM} = 0.1624, β_{NS} = 0.7537, and β_{NNI} =0.001318. The subplots' error terms in year *t* are specified as u_t , an 800×1 vector, as presented in Kapoor et al. [\(2007](#page-16-22)). Mathematically,

$$
u_t = \lambda W u_t + v,\tag{4}
$$

$$
v = \mu + \varepsilon_t,\tag{5}
$$

$$
\mu \sim N\Big(0, \sigma_{\mu}^2 \cdot I_M\Big),\tag{6}
$$

$$
\varepsilon_t \sim N\big(0, \sigma_\varepsilon^2 \cdot I_M\big),\tag{7}
$$

where μ is 800×1 vector of subplot-level time invariant characteristics, ε_t is 800×1 subplot-level vector of error terms that vary over both plot and time, and *W* is a distance-based spatial weights matrix, where weights decrease with distance between subplots.

Simulated data generation

In one thousand rounds of Monte Carlo simulation, a 20-year series of agronomic experiments simulated yield in each of the feld's 800 subplots in every year. For every year of modeled experimentation, a value for the random variable May precipitation, in millimeters, was drawn from a normal distribution with mean of *M* =70 and standard deviation of $\sigma_M = 15$. These parameters were taken from Champaign County, Illinois rainfall data, 1895–2018. It was assumed that in a given year t , the same amount of rain, M_t , fell on every subplot in the feld. For each of the feld's 800 subplots, a random yield disturbance term was drawn from a normal distribution with mean 0 and standard deviation 200 kg/ ha. Thus, every observation in the resultant simulated data set had a value for the applied N fertilization rate, a value for May precipitation, a value for its random disturbance term, and a yield value. Since in each Monte Carlo round yield was simulated on 800 subplots for twenty years, there were 16,000 observations in each round's data set. Since one thousand rounds were run in the Monte Carlo process, one thousand "20-year" datasets were created. To simulate varying lengths of agronomic experimentation, for each Monte Carlo round data from years of its 20-year data set were simply omitted. For example, a simulation's 19-year data set contained the observations from experiment years 2 through 20. In this manner, data sets simulating experiments of 2, 3, 4, …, 19, and 20 years of length were generated.

Econometric methods

In each round, the spatial random efects (SPRE) model (Kapoor et al. [2007](#page-16-22)) of the *R* package *splm* (Millo and Piras [2012\)](#page-16-23) was used to estimate the yield response function. In the presence of spatial dependence in the error term, Ordinary Least Squares is not in general efficient. On the contrary, the SPRE is a Generalized Least Squares estimator that takes into account the spatial dependence of both the plot-level efects and idiosyncratic errors specifed above. In the context of the dynamic on-farm trial problems considered in this

article, statistical efficiency has economic value. The more accurate the estimate of yield response function with given amount of information obtained through on-farm experiments, the greater the potential beneft of site-specifc management in subsequent years.

Defnitions of values of information and technology

The term *full information* is used here to describe the situation in which the farmer knows the field characteristics maps, the distributions from which the yield disturbance ε and May precipitation *M* are drawn, and the "true" meta-response function in Eq. (2) (2) . A farmer is said to have *partial information* when (s)he does not know the true subplot-specifc response functions f_{b_i} , but has access to the OFPE data, with which the f_{b_i} functions can be estimated. The "amount of information" depends on the number of years of experimental data to which the farmer has access. (Following the microeconomic theory presented by Laffont [\(1989](#page-16-24)), Bullock et al. [\(2009](#page-16-5)) provide a formal mathematical treatment of "information" in the context OFPEs.)

Net revenues under SST and full information

For simplicity, it is assumed that farmers desire to maximize expected profts. (In reality, of course, farmers' objectives can be more complicated. Risk-averse farmers are willing to receive lower expected monetary returns if the variance of those can be diminished. Some farmers may simply want to tell their neighbors about high yields. Most farmers care about the environmental impacts of their production practices.) Let $B = 160$ be the number of plots in the OFPEs, and *j* be the index for subplots within a plot. In a representative year *t*, under full information, the farmer using SST for fertilization solves,

$$
\underset{N_1,\dots,N_B\geq 0}{Max} \left\{ \sum_{b=1}^{B} \sum_{j=1}^{5} \left[pf_{bj} \left(N_b, M_t, u_{bjt} \right) - w N_b \right] \right\},\tag{8}
$$

where p is the output price, and w is the price of N fertilizer, and E is the expectations operator. It is assumed that the delays in yield monitor and variable-rate applicator accuracy mean that producer can only choose N fertilizer rates by plot, not by subplot. That is, all subplots within a plot must be managed in the same way. The solution to (8) is the *vector of ex-ante optimal site-specifc N fertilization rates under full information*:

$$
\mathbf{N}^{ssfi}(p, w) \equiv \left(N_1^{ssfi}(p, w), ..., N_B^{ssfi}(p, w) \right) \equiv \underset{N_1, ..., N_B \ge 0}{argMax} E \left\{ \sum_{b=1}^B \sum_{j=1}^5 \left[p f_{bj} (N_b, M_t, u_{bjt}) - w N_b \right] \right\}.
$$
\n(9)

Ex-ante maximized profts under SST with full information are then,

$$
\pi^{ssf}(p, w) \equiv E \left\{ \sum_{b=1}^{B} \sum_{j=1}^{5} \left[p f_{bj} \left(N_b^{ssf}(p, w), M_t, u_{bjt} \right) - w N_b \right] \right\},
$$
(10)

which because *M* and *u* enter the response function linearly, can be rewritten,

$$
\pi^{ssf}(p, w) \equiv \sum_{b=1}^{B} \sum_{j=1}^{5} \left[pf_{bj} \left(N_b^{ssf}(p, w), \overline{M}, 0 \right) - w N_b^{ssf}(p, w) \right]. \tag{11}
$$

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Net revenues under uniform technology and full information

A farmer who will use uniform-rate technology under full information has the following N fertilizer demand function, which gives the optimal uniform application rate as a function of prices:

$$
N^{unfi}(p, w) \equiv argMax E \left\{ \underbrace{\sum_{N\geq 0}^{B} \sum_{j=1}^{5} p f_{bj}(N, M_t, u_{bjt})}_{f_{wf}(\mathbf{N}, M_t, \mathbf{u})} - w \sum_{b=1}^{B} \sum_{j=1}^{5} N \right\},\tag{12}
$$

where $f_{wf}(N, M, \mathbf{u})$ is the whole-field-response-to uniform- N function. Ex-ante maximized profts under uniform technology with full information are then,

$$
\pi^{unfi}(p, w) \equiv E \left\{ \sum_{b=1}^{B} \sum_{j=1}^{5} \left[p f_{bj} \left(N^{unfi}(p, w), M_t, u_{bjt} \right) - w N^{unfi}(p, w) \right] \right\}
$$
\n
$$
\equiv \sum_{b=1}^{B} \sum_{j=1}^{5} \left[p f_{bj} \left(N^{unfi}(p, w), \overline{M}, 0 \right) - w N^{unfi}(p, w) \right].
$$
\n(13)

Net revenues under SST and partial information

A farmer with partial information knows the estimated yield response functions. Since the estimates are generated from data from OFPEs, then in some sense the "amount" of information to which the farmer has access depends on the number of years that OFPEs have been run on the feld. Since the number of observations in the data set increases with the number of experiment-years, the trend will be for estimates of the *f* function to improve as OFPEs are run for additional years.

To model diferent information settings, the efects of conducting OFPEs for diferent numbers of years was simulated. The number of experiment-years is denoted by the variable *T*, which takes values between 2 and 20, depending on the simulation. Using "experiment *T*" to denote the experiment run for *T* years, call the estimated coefficients $\hat{\beta}_0^T$, $\hat{\beta}_N^T$, $\hat{\beta}_D^T$, $\hat{\beta}_I^T$, $\hat{\beta}_M^T$, $\hat{\beta}_S^T$, $\hat{\beta}_{NN}^T$, $\hat{\beta}_{NN}^T$, $\hat{\beta}_{NS}^T$, and $\hat{\beta}_{NNI}^T$. Then, for $b = 1, ..., 160, j = 1, ..., 5$, and $T = 2, ...$ 20, the estimated reduced-from site-specifc response function for subplot *j* of plot *b* after *T* years of experiments is,

$$
\hat{f}_{bj}^T(N,M) \equiv \hat{\beta}_0^T + \hat{\beta}_N^T N + \hat{\beta}_D^T + \hat{\beta}_I^T I_{bj} + \hat{\beta}_M^T M + \hat{\beta}_S^T S_{bj} + \hat{\beta}_{NN}^T N^2 + \hat{\beta}_M^T N I_{bj} + \hat{\beta}_{NM}^T N M + \hat{\beta}_{NS}^T N S_{bj} + \hat{\beta}_{NNN}^T N^2 I_{bj}.
$$
\n(14)

Letting *M* denote expected May precipitation, a farmer using uniform technology is assumed to maximize expected profts from the whole feld. If the only information that he/ she has is the estimated whole-feld response function, then his/her objective is,

$$
\max_{N_1,\dots,N_b \ge 0} E\left\{ \sum_{b=1}^B \sum_{j=1}^5 \left[p \hat{f}_{bj}^T (N_b, M_t) - w N_b \right] \right\}.
$$
 (15)

The solution to the problem above is the *vector of ex-ante optimal block-specifc N fertilization rates under information from an experiment of length T*:

$$
\mathbf{N}^{ssT}(p, w) \equiv \left(N_1^{ssT}(p, w), \dots, N_B^{ssT}(p, w)\right) \equiv \underset{N_1, \dots, N_B \ge 0}{\arg\,max} \sum_{b=1}^{B} \sum_{j=1}^{5} \left[p_{bj}^2\left(N_b, \overline{M}\right) - wN_b\right].
$$
\n(16)

Expected ex-ante maximized profts under SST with information from *T* years of OFPEs are then,

$$
\pi^{ssT}(p, w) \equiv E \left\{ \sum_{b=1}^{B} \sum_{j=1}^{5} \left[p f_{bj} \left(N_b^{ssT}(p, w), M_t, u_{bjt} \right) - w N_b^{ssT}(p, w) \right] \right\}
$$
\n
$$
\equiv \sum_{b=1}^{B} \sum_{j=1}^{5} \left[p f_{bj} \left(N_b^{ssT}(p, w), \overline{M}, 0 \right) - w N_b^{ssT}(p, w) \right].
$$
\n(17)

(Note that the true profts with incomplete information depend on the true reduced-form response functions f_{bi} , not the estimated response functions.)

Net revenues under uniform technology and partial information

The farmer operating with URT with information from an OFPE of length *T* years solves

$$
\max_{N\geq 0} E\left\{ \sum_{b=1}^{B} \sum_{j=1}^{5} \left[p \hat{f}_{bj}^{T}(N, M_{t}) - wN \right] \right\}.
$$
\n(18)

The solution to the problem above is the *ex-ante optimal whole-feld N fertilization rate under information from an experiment of length T*:

$$
N^{unT}(p, w) \equiv \arg_{N \ge 0} Max \sum_{b=1}^{B} \sum_{j=1}^{5} \left[p \hat{f}_{bj}^{T} \left(N, \overline{M} \right) - wN \right].
$$
 (19)

Expected ex-ante maximized profts under uniform technology with information from *T* years of experiments are then,

$$
\pi^{unT}(p, w) \equiv E \left\{ \sum_{b=1}^{B} \sum_{j=1}^{5} \left[p f_{bj} \left(N^{unT}(p, w), M_{t \cdot bj} u \right) - w N^{unT}(p, w) \right] \right\}
$$
\n
$$
\equiv \sum_{b=1}^{B} \sum_{j=1}^{5} \left[f_{bj} \left(N^{unT}(p, w), \overline{M}, 0 \right) - w N^{unT}(p, w) \right].
$$
\n(20)

The value of information under uniform and site‑specifc technologies

The per-ha *value of the information gap between uniform management with full information and uniform management with information from T years of experiments* is the diference in maximized expected net revenues between the two situations:

$$
v^{inun,T}(p,w) \equiv \left[\pi^{unf}(p,w) - \pi^{unT}(p,w)\right]c^{-1},\tag{21}
$$

where c = 0.0335 is the conversion rate from 18.3×18.3 m² to ha⁻¹. (That is, all values are measured per hectare, not per subplot.)

Similarly, the per-ha *value of the information gap between site-specifc management with full information and site-specifc management with information from T years of experiments* is,

$$
v^{inss,T}(p,w) \equiv \left[\pi^{ssft}(p,w) - \pi^{ssT}(p,w)\right]c^{-1}.\tag{22}
$$

Value of SST under full information

The maximization problem in (9) is less constrained than the one in (12). Therefore, under full information, if the cost of purchasing the technologies themselves is not considered, profts under site-specifc management cannot be lower, and in general will be higher than under uniform management. Call the diference between these the *ex-ante value of sitespecifc technology under full information*:

$$
v^{ssfi}(p, w) \equiv \left[\pi^{ssfi}(p, w) - \pi^{unfi}(p, w) \right] c^{-1} \ge 0.
$$
 (23)

Value of site‑specifc technology under partial information

Finally, the net revenues of the site-specifc farmer with some amount of information are compared to those of a uniform-rate farmer with that same information. This is the *ex-ante value of site-specifc technology under partial information*:

$$
v^{ssT}(p, w) \equiv \left[\pi^{ssT}(p, w) - \pi^{unT}(p, w) \right] c^{-1} \ge 0.
$$
 (24)

This value is the one most pertinent to farmers's technology decision. If the diference in the costs of the technologies is less than the diference in the net revenues defned in (23), then the farmer's best choice is to adopt site-specifc technology.

The value of *status quo* **information**

In the above, it is assumed that the farmer obtains information solely from OFPEs. The value of the information provided by OFPEs depends on the amount of information (s)he would possess if no experiments were run. In reality, farmers try to garner management information from many sources, and use that information to make decisions. Diferent farmers manage their felds diferently. Let *Nsq* denote the uniform N rate that the farmer would use in the *status quo* situation, that is, in the situation in which no OFPEs are run. How close N_{sa} is to the farmer's optimal uniform N rate will vary among farmers. For the purposes of illustration, in what follows it is supposed that the farmer's *status quo* rate is that provided by the Maximum

Returns to Nitrogen (MRTN) project's website (Iowa State University Agronomy Extension and Outreach [2019\)](#page-16-4). The MRTN system was developed and is supervised by researchers and extension personnel at seven land grant universities in the US Corn-Soy Belt. MRTN recommendations are based on a series of on-farm strip trials that those researchers have conducted since the early 2000s. MRTN recommendations are provided by geographic region and by prices. At the time the research here reported was conducted, and for a feld in central Illinois facing the N and corn prices assumed in the Monte Carlo simulation the MRTN recommended N rate was 264 kg ha⁻¹ (240 lbs ac⁻¹). Let this number be denoted *N^{mrtn}*, and the profits resultant from this status quo N rate in Monte Carlo simulation *m* be denoted π_m^{mrtn} . Let $\pi^{mrtn} = \left(\sum_{m=1}^{1000} \pi_m^{mrtn} \right)$ \int /1000 denote the mean of the status quo profits over the 1000 Monte Carlo simulations. Throughout the analysis, a corn output price of $p^0 = \$0.1535/kg$ (\$3.90/bu) and a N fertilizer price of $w^0 = 0.80$ /kg (\$0.36/pound) for N fertilizer were assumed.

Results

Let *m* index the Monte Carlo runs, so that $m=1, \ldots, 1000$. Let π_m^{ssT} be net revenues (crop revenues minus N fertilizer costs) measured in Monte Carlo round *m* from optimal sitespecific management, given that there have been T years of experiments (as in Eq. (17)). Let $\pi^{ssT} = \left(\sum_{m=1}^{1000} \pi_m^{ssT}\right)$ $\big)$ /1000 be the mean of this measurement over all the 1000 Monte Carlo rounds. Similarly, referring to Eq. [\(20\)](#page-10-1), let $\pi^{unT} = \left(\sum_{m=1}^{1000} \pi_m^{unT}\right)$ � ∕1000. Figure [5](#page-12-0) compares how net revenues depended on information (either status quo information, full information, or information from various numbers of experiment years), and the technology subsequently used. The result immediately apparent from Fig. [5](#page-12-0) is that expected status quo profts are far lower than the profts the farmer could use with after just a few years of experiments. Using information from just two years of experiments would allow the

Fig. 5 Means of the 1000 Monte Carlo simulations' expected maximized profts of a uniform and a sitespecifc farmer, when they have information from *T* years of on-farm precision experiments

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farmer, whether employing uniform rate technology or SST, to increase profts by nearly \$200 ha−1, from \$1578.88/ha to approximately \$1770/ha. It may seem surprising that the MRTN recommendation is quite so poor in this particular example. In fact, however, the developers of MRTN have reported that their regionally-recommended MRTN rates have frequently differed by over 50 kg ha⁻¹ from the estimated optimal uniform rates that they have measured for the same individual experimental fields. (See Fig. [1](#page-2-0) of Nafziger ([2018\)](#page-16-25).) The fact is, it may be very difficult for farmers who do not have adequate data from their felds to closely approximate their economically optimal N rates. The lesson taught by Fig. [5](#page-12-0) is that it may be possible to greatly ameliorate this situation with just a few years of OFPEs. A second lesson derived from Fig. [5](#page-12-0) is that after just a few years of trials, additional trials provide little additional useful information. Both curves are asymptotically approaching full-information profts as the number of trial years surpasses fve or six. That is, once a handful of experiments are run and analyzed, there is not much valuable information to be gleaned from further experimentation. Indeed, the information from the frst two years of field trials increased net revenues by almost \$200 ha^{-1} , but after about year 6, additional trial years increased net revenues by only a few dollars per ha.

Results from the one thousand Monte Carlo rounds show that the mean of the value of site-specific technology under full information, as defined in (23), was $v^{ssf}(p^0)$, *w*⁰) = \$0.216/ha. By using SST instead of URT, a farmer who knew every subplot's true response function, $f_{bi}(N, M)$ could expect to increase annual net revenues by only \$0.216/ ha. This does not include the costs of the precision agriculture equipment, nor account for the farmer possessing more information about the characteristics feld maps and the functional form of the "true" yield response function than is actually available. In short, the simulation results suggest that replacing uniform technology with site-specifc technology would not pay for itself on this "fat and black" central Illinois cornfeld, and so Hypothesis 1 is rejected.

Note from Fig. [5](#page-12-0) that for $T < 17$, $\Pi^{unT} > \Pi^{ssT}$. This implies that, unless the farmer has a great deal of information, s(he) can make more money by using URT than by using SST. The vertical distance between the, Π^{unT} and Π^{ssT} curves in Fig. [5](#page-12-0) represents $v^{ssT}(p^0,w^0)$ from [\(24](#page-11-0)), the Monte Carlo simulations' ex-ante values of site-specifc technology under various amounts of partial information. Note that for every experiment length from two to seventeen years, this value is negative. Until the farmer has information from seventeen years of feld trials, uniform management outperforms site-specifc management, even when the diferences in the costs of production under these technologies are not accounted for. The intuitive explanation for this result is consistent with the discussion about how information and SST are complements in Bullock et al. ([1998](#page-16-0)), Bullock and Bullock [\(2000\)](#page-16-2), and Bullock et al. [\(2002\)](#page-16-1), and later articles, which explain that site-specifc application management is information-intensive. Loosely speaking, managing a feld "on average" is a simpler task than managing it site-specifcally. Site-specifc management requires knowledge about how yield responds to management in many places on the feld. With enough information, eventually the farmer can take advantage of the ability to customize input application according to site characteristics. But while the same amount of information may not be valuable for managing small sites within a feld, that information may be more useful for describing the feld "on average," and therefore to managing the feld "on average." Of course, these results will depend on the heterogeneity of the spatial characteristics in the feld. The feld used in the model presented here simulated a central Illinois "fat and black" feld, and the spatial heterogeneities of the SPI and ISNT values were limited. This leads to similar site-specifc inputs rates among sites, and limits the value of the site-specifc technology. Obviously, extending the analysis to examine the values of OFPEs and technology on felds less spatially homogeneous is called for.

Limitations

The research reported here simulated an OFPE on a single feld. The size of the simulated feld, the "maps" used to describe the spatial distribution of the model's feld characteristics, and the design of the agronomic experiment were realistic, but the possibilities of running multiple similarly designed experiments on multiple felds, and using the data together to estimate yield response, were not examined. Therefore, the inference space of the research reported here is small, and no conclusion may be drawn about the value of running multiple trials on multiple felds in multiple yields. More OFPE trials are being and need to be run in new years and new locations. Research examining questions similar to those addressed in the current article, but using data from felds with highly heterogeneous characteristics maps would be especially interesting.

An additional limitation of the reported research is that the opportunity cost of running on-farm trials was not accounted for. However, in actual DIFM on-farm experiments many farmers have made money, because using the experimental target application rates ended up bringing in greater profts than those that would have been garnered had the producer used his *status quo* application rate. Therefore, the opportunity costs of the experiments depend highly on how close to optimal the farmer's *status quo* rate would have been in the frst place. More research is needed for reliable estimates of this type of opportunity cost.

The reported research ignored possibilities of dynamically optimizing feld trial design. This could involve centering the range of targeted input application rates around the actual optimal rate, which would lower the opportunity costs of the trials. Only one managed variable, N fertilizer, was included in the simulations. In reality, the DIFM project often conducts feld trials that vary both N and seed rates. Of course, all agronomic feld trials are limited because the mathematics of factorial design cause the number of treatments needed to increase rapidly as the number of variables in the trial increases.

OFPE methods are not applicable to all input management decisions. The methods have been proved to work well in examining N fertilizer application rates and seeding densities. But the nature of application machinery, for example of spinner spreaders, presents challenges to conducting phosphorous, potassium, or lime application OFPEs. Trying to conduct pesticide rate trials would also be difficult, as high rates in field trials would often violate government regulations. Given current commercially available planting technology, is not currently possible to implement gridded OFPEs to test hybrid varieties, and strip trials must be run instead.

An important caveat to OFPE is that data generated in a trial in one growing season may provide only limited insight into management decisions in later seasons. When new genetics are introduced, interactions among those genetics, feld characteristics, weather, and managed inputs may cause optimal management strategies to change over time. For farmers, the frequency of conducting OFPEs might be similar to the frequency of pulling and analyzing soil samples; after a few years, it simply has to be done again.

Finally, this study was limited because it only considered the efects of trial length; this is only one of a multitude factors in OFPE design that need to be considered. Plot length,

subplot length, bufer zone location, range of application rates, number of management variables considered, and number of application rates are among a multitude of factors that would need to be optimized to obtain "perfect" OFPEs. It is hoped that this article demonstrates methods that will be used to examine efficient strategies for improving the choices of these other trial design factors.

Conclusions

Precision agriculture technology has been available now in commercial markets for twenty years. When the technology initially appeared, it generated great enthusiasm in the agricultural press. Very gradually, a market for the technology has developed, with private consulting frms attempting to increase farm profts by ofering site-specifc management advice. The results of the simulations reported here suggest that how proftable this advice is for the farmers who purchase it is open to question. Managing inputs site-specifcally may require great amounts of information about how crop yields respond to managed inputs, and how those responses vary over time and space. Loosely speaking, it is simply harder to manage many sites within a feld diferently and "well" than it is to farm a feld well "on average." Purchases of precision agriculture equipment and consulting services continue to slowly increase in world markets, but a good deal of doubt about the efectiveness of commercial site-specifc N management prescription remains. Whether the precision agriculture equipment, software "decision tools," and consulting services being purchased are much more than "bells and whistles" remains to be seen.

On the other hand, the simulations reported here suggest that OFPEs, in which parts of felds, or even whole felds, are used for large-scale, randomized agronomic feld trials, may ofer great economic promise. In the simulations reported here, the promise of profts came not so much from using the experiments' data to improve site-specifc management. At least on the "flat and black" field modeled, site-specific management offered little advantage over uniform management. In fact, even when the costs of sitespecifc equipment and the opportunity costs of running on-farm trials were ignored, unless the producer possessed many years of feld trial information, the best obtainable uniform rate strategy was more proftable than the best obtainable optimal site-specifc strategy. The greatest part of the value in OFPE came from the information given by the frst few years of trials. That information helped the farmer develop fairly accurate estimates of the feld's optimal uniform application rate. For this particular feld, the MRTN project's recommended rate was far too high, and relying instead of information from two years of experiments showed potential to return to the farmer hundreds of dollars per hectare, providing strong evidence in support of Hypothesis 2.

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