

Flat earth economics and site-specific crop management: how flat is flat?

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Abstract The practical significance of achieving optimality in on-farm decision making has been debated in the agricultural economics literature since the 1950s, with some arguing that optimal input application is less critical if farmers are faced with a flat pay-off function. This issue has considerable implications for the adoption of site-specific crop management (SSCM), where the optimal management of inputs across space is emphasised. This paper contributes to this debate by addressing some previously unresolved issues. Firstly, a new metric is proposed, termed 'relative curvature' (RC), that is used for more accurate and versatile quantification of the flatness of pay-off functions. Secondly, this metric is used to compare the difference in profitability between management classes within the same field, where SSCM is practiced. Thirdly, the RC metric is used to examine the effect of considering environmental damage costs from non-optimal input application on the flatness of pay-off functions. The key findings of this paper are that there exists a high degree of variability in relative curvature of pay-off functions derived for different management classes within the same field. The RC procedure can be used to identify fields which are most suitable to variable-rate management intervention. Also, RC increases substantially when environmental costs are accounted for, implying that optimality of input use may be more important than previously thought.

Keywords Economics · Flatness · Payoff functions · Relative curvature · Site-specific crop management

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Introduction

The practical importance of achieving optimality in on-farm decision making has been debated in the agricultural economics literature since the 1950s. While the optimal pay-off is often emphasised as best practice, it has been argued that achieving optimal input application is less critical if farmers are faced with a flat pay-off function (Hutton and Thorne 1955; Jardine 1975; Pannell 2006).

This issue was first highlighted in the literature by Hutton and Thorne (1955), in which they offered criticism of a profitability framework for nitrogen and phosphorus application constructed by Heady and Pesek (1954). Hutton and Thorne (1955) argued that the framework is of "trivial economic importance" due to the large range of input values for which profit changed only slightly. Jardine (1975) echoed this sentiment in the context of agricultural risk, suggesting that the flat shape of the pay-off function allows the farmer to choose a sub-optimal level of input application without suffering a damaging loss in the face of uncertainty.

Dillon (1977) attributed the flat shape of pay-off functions to the shape of the input response function, which is often characterised by small changes of the marginal physical product as input quantity changes, reflecting the biological behaviour of the system. Dillon (1977) states that as a result, "marginal profit is necessarily close to zero in the region of best operating conditions". However, Anderson (1975) used cost functions and whole farm models as examples to argue that this issue does not only affect input response functions, but is a "general phenomenon which pervades all optimisation processes and models". This stance has considerable implications for the adoption of site-specific crop management (SSCM).

Most recently, Pannell (2006) identified some "far reaching consequences" for economic decision making, particularly with respect to SSCM. He suggested that the existence of flat pay-off functions effectively devalues information used for SSCM, as improvements in efficiency of input use often lead to a minimal increase in profit. However, proponents of precision agriculture (PA) argue that optimal input use under SSCM provides a range of economic and environmental benefits, such as reduced potential for excess inputs from the cropping system finding their way into the environment (Cassman 1999; Zhang et al. 2002; Bongiovanni and Lowenberg-Deboer 2004).

Mixed results from a number of SSCM profitability studies have served to further polarize the debate. While some studies, such as Babcock and Pautsch (1998), have found that variable-rate input application can be more profitable and sustainable than uniform rate application, Bullock et al. (1998) argue that SSCM only pays off when the cost of acquiring information and technology decreases substantially.

As this debate is yet to be resolved, there are a number of key issues that this study aims to address. Firstly, there does not appear to be a definitive and accurate measure of flatness for pay-off functions reported in the literature. Pannell (2006) provided a number of examples where the degree of flatness is defined as a range of input levels over which profit is within a certain percentage of the optimum. However, this approach is not interpretable in a straight forward way and does not allow for a direct comparison of relative curvature among multiple functions under different circumstances. Therefore, one of the specific objectives of this study is to define a metric for quantifying the relative curvature of pay-off functions, in order to make them directly comparable between each other.

Secondly, the examples used in Pannell (2006) assumed that whole fields are spatially homogeneous production units. However, as is well documented, the scale of within-field

variation can be substantial, particularly with respect to soil properties and crop yield (e.g. Beckett and Webster 1971; Robert 1993; Cook and Bramley 1998; Boydell and McBratney 2002). Taylor et al. (2007) identified a range of data sources for delineating unique, within-field management classes, such as yield maps, soil surveys, proximal soil sensors and their combinations. Whelan and Taylor (2005) and Whelan et al. (2012) used such data to estimate crop yield response across management classes, finding considerable variability in yield response to applied nitrogen and phosphorus between the classes. Given that within-field spatial variation in crop yield response has been shown to exist, the present study hypothesizes that within-field profitability also varies. Therefore, the second specific objective of this study is to examine the degree of variability in the flatness of pay-off functions derived for management classes within a field.

Thirdly, environmental damage costs from excess input application were not accounted for in examples presented in Pannell (2006). Indeed, the implications of environmental damage costs for the shape of pay-off functions have been largely overlooked in the literature. A uniform application of inputs to a field which exhibits a degree of spatial variability in crop response is likely to lead to excess application in some areas and underapplication in others (Wang et al. 2003). Excess application of fertiliser, irrigation or pesticide can cause varying degrees of environmental damage, which imposes an external cost (McBratney et al. 2005). This cost, while not directly pertaining to a farmer's pay-off, is likely to have significant external implications that are often not factored in on-farm decision making. Consequently, the present study hypothesises that the inclusion of environmental damage costs in the calculation of the pay-off is likely to reduce the flatness of the pay-off function. Therefore, the third specific objective of this study is to quantify the change in flatness of the pay-off function under a range of likely environmental damage costs.

The paper proceeds by presenting a theoretical framework to outline the derivation of a metric for quantifying the flatness of payoff functions. This metric is then used to examine the degree of variation in flatness of pay-off functions calculated using crop yield response data for fourteen fields that are partitioned into management classes. Finally, the environmental costs associated with excess nutrient application are included in the calculation

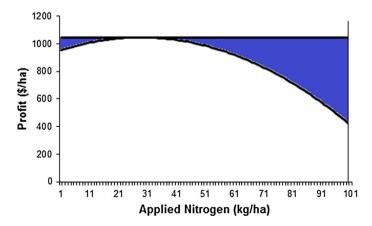


Fig. 1 Geometric approach to measuring flatness of profit functions. The size of the *blue area* provides a measure of curvature (Color figur online)

of the pay-off, to arrive at a quasi-social net benefit function (SNBFs), the flatness of which is compared to a purely private payoff function.

Materials and methods

Theoretical framework

A new metric for measuring the flatness of pay-off functions is proposed. Termed relative curvature (RC), the metric is obtained by calculating the area lying between the graph of the pay-off function and a horizontal line that is tangent to this graph at the point of maximum pay-off (profit) over a given range of input values (Fig. 1).

This area is simply determined by subtracting the definite integral of the pay-off function from the area of the rectangle formed by the tangent line, the horizontal axis, the vertical axis, and a line parallel to the vertical axis that crosses the horizontal axis at the point of maximum input value. The resulting difference is then divided by the total area of the rectangle, leaving a dimensionless metric of flatness between 0 and 1. Small values of RC imply a relatively flat profit function, while larger values imply a relatively high curvature.

This method can be represented algebraically. Consider a concave pay-off function, $\pi(x)$, with a maximum π_{max} ; further consider an interval of input values $[0, \hat{x}]$ over which one wishes to calculate RC.¹ The formula for RC is then:

$$RC(\pi, \hat{x}) = \frac{\pi_{max} * \hat{x} - \int_0^x \pi(x) dx}{\pi_{max} * \hat{x}}$$
(1)

Note that the term, $\pi_{max} * \hat{x}$ represents the area of the rectangle formed by the line tangent to $\pi(x)$ at π_{max} .

A problem can arise in cases where $\pi(x) < 0$ over the interval $[0, \hat{x}]$ (i.e. where there may be negative profits as a function of inputs) because the formula expressed in Eq. 1 implies summing the area that lies above the horizontal axis with the negatively signed area that lies below the horizontal axis, which understates the measure of the relative curvature of the function (Fig. 2). The figure shows an example where $\hat{x} = 100$. The definite integral of $\pi(x)$ over the interval [0, 100] is equal to the area (A-C). When this is subtracted from the rectangle (A + B) as proposed in Eq. 1, the relative curvature of $\pi(x)$ is understated.

One way (out of several possibilities) to rectify this problem, is to use a scalar to shift the whole pay-off function upwards such that $\pi(x) \ge 0$ everywhere over the interval $[0, \hat{x}]$. The value of the scalar is determined by the vertical distance between the point of intersection, labelled D in Fig. 2, and the horizontal axis. In this case, the modified formula for relative curvature can be written as follows:

$$RC(\pi, \hat{x}) = \frac{(\pi_{max} + a) * \hat{x} - \int_0^{\hat{x}} [\pi(x) + a] dx}{(\pi_{max} + a) * \hat{x}}$$
(2)

where a is defined as a scalar such that $\pi(x) + a \ge 0$ everywhere over the interval $[0, \hat{x}]$.

As the magnitude of RC depends on the selection of \hat{x} , the choice of the value of \hat{x} has to be consistent for all pay-off functions that are compared within a given exercise. For instance, when comparing the RC of pay-off functions with respect to nitrogen fertilizer

¹ More generally, the interval of input values can be defined as $[z,\hat{x}]$, such that $z \ge 0$.

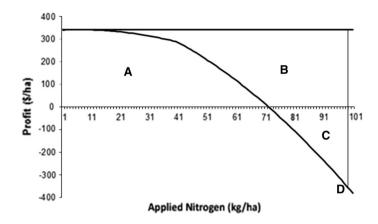


Fig. 2 A pay-off function for applied nitrogen, where part of the pay-off is negative. The calculation of the definite integral overstates RC

input, the value of maximum input has to be the same for all fields for which the comparison is undertaken.² If \hat{x} remains unchanged, RC will remain unchanged regardless of the scalar used to shift the intercept.

The RC metric described here could be applied in a variety of production contexts, but its main purpose is to serve in studying optimisation of input decisions in agricultural production and it is applied here to an empirical case study of SSCM. The RC metric has its advantages and disadvantages. For instance, it is a superior metric of 'flatness' compared to the one used by Pannell (2006), as it is precisely formulated and can be used to compare across very different payoff functions, under different circumstances. The main disadvantage is that for the comparisons to be meaningful, the interval of input values $(0, \hat{x})$ has to be fixed across the fields being compared (i.e. \hat{x} has to have the same value), which limits its generality.

Data

Yield response data for this analysis were obtained from field trials reported in Whelan et al. (2012). The trials, conducted at various sites throughout south eastern Australia, involved partitioning fields into management classes based on interpolated crop yield, apparent soil electrical conductivity (ECa) and elevation data following the method of Taylor et al. (2007). In the study by Whelan et al. (2012), crop yield response functions were calculated for each management class, and used to examine the differences in magnitude and form of within-field crop yield response under a range of Australian conditions. The yield response functions for nine fields covering 14 crop-years from Whelan et al. (2012) were used to estimate the pay-off functions in this study.

An example of the experimental design and data for one of these fields (designated Field 4 in this study) is shown in Fig. 3. The delineated management classes and the location of fertiliser trial plots are shown in Fig. 3a. Three treatment rates (0, 30, 45 kg N/ha), in two

² Obviously, if comparison of other fields with respect to the same input, or another input are made, the value of \hat{x} will change. All that is required is consistency of this value within a given comparison.

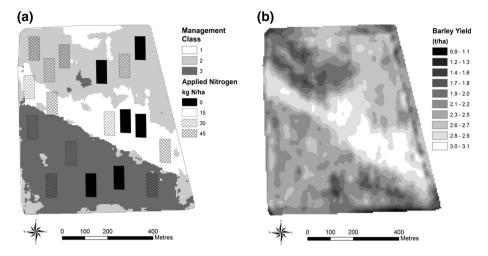


Fig. 3 Management class delineation and location of fertiliser treatment plots for Field 4 (a), and barley yield response for the whole field (b)

replicates, were applied in each of the three management classes. The subsequent barley yield response data is shown in Fig. 3b.

Calculating pay-off functions

Using the approach described in the theoretical framework section, pay-off functions for the response to either applied nitrogen (N) or phosphorus (P) fertiliser in each management

Field	Year	Crop	Nutrient applied	Price per unit input (\$A/kg nutrient)	Price per unit output (\$A/kg grain)
1	2003	Wheat	Nitrogen	0.87	0.197
2	2003	Canola	Nitrogen	0.87	0.397
3	2004	Canola	Nitrogen	0.77	0.38
4	2004	Barley	Nitrogen	0.87	0.13
5	2004	Wheat	Nitrogen	0.87	0.15
6	2003	Wheat	Phosphorus	2.50	0.176
6	2004	F-Peas	Phosphorus	2.50	0.23
6	2005	Wheat	Phosphorus	2.50	0.23
6	2006	Barley	Phosphorus	2.27	0.20
7	2006	Wheat	Phosphorus	1.95	0.25
7	2007	Canola	Phosphorus	1.95	0.65
7	2008	Wheat	Phosphorus	2.72	0.27
8	2005	Wheat	Phosphorus	2.30	0.195
9	2007	Wheat	Phosphorus	1.95	0.33

 Table 1
 Summary of input and output costs for the field years

class were calculated for each crop-year. The pay-offs were calculated by deducting costs from revenue, using output and input prices summarised in Table 1. This calculation took the following form:

$$\pi_i = PQ_i - wX_i \tag{3}$$

where π_i is pay-off (profit) from the *i*th management class (\$/ha), *P* is the exogenous output price (\$/t), Q_i is the estimated yield for the *i* th management class (t/ha), *w* is the price of N or P application (\$/kg), and X_i is the amount of applied N or P to the *i* th management class (kg/ha). A pay-off function was also calculated for the entire field using the average yield.

Linear or quadratic yield-response functions to N and P input in each crop-year were fitted dependent on the best fit for each management class and for the entire field. These functions were then used to calculate the definite integral over a fixed input interval, with \hat{x} set at 100 kg/ha. The maximum value of input was set at 100 kg/ha because that was consistent with the range of input data from the sites that were used in the investigation. For other studies, where fertility ranges differ, this maximum value will have to be set according to the respective input use data. The relative curvature (RC) was then calculated for each management class using Eq. 2, and compared to the RC for the average pay-off over the entire field to highlight any effect of spatial heterogeneity in crop yield response on flatness.

Determining the quantity of nutrients potentially available for loss to the environment

To illustrate the effect of the potential for the unused nutrients to be lost from the field into the environment, the study focused on potential N losses. For this purpose, a basic soil N budget was constructed for each crop-year. This was achieved by calculating the N requirement (NR) of the crop based on the average yield achieved, and then subtracting the amount of applied nitrogen (X). This provided a theoretical measure of 'surplus' N (SN) in the soil derived as follows:

$$NR(Q, PC)_{(kg/ha)} = QxPCxF \tag{4}$$

where Q is yield (t/ha), PC is protein percentage (%), and F is a N factor relevant to the proportion of nitrogen in the protein of specific grains. Consequently the surplus N, available to escape in the environment is given by:

$$SN(X)_{(kg/ha)} = NR_{(kg/ha)} - X_{(kg/ha)}$$
(5)

It is assumed in this study that wheat crops achieved a target protein content of 12.5 % with a nitrogen factor of 1.75, while the barley crop achieved a target of 10 % protein with a nitrogen factor of 1.60. Using the law of mass balance, it is assumed that any *SN* is available to enter the environment (e.g. through leaching into ground water, run-off into waterways or gaseous loss to the atmosphere) and cause environmental damage (Ayres and Kneese 1969). Therefore, SN(X) can be interpreted as an environmental damage function as it may be seen as imposing an environmental damage cost (*Z*), such that:

$$Z_{(\$A/ha)} = SN(X)_{(kg/ha)} * C_{(\$A/kg)}$$
(6)

where C is the marginal damage cost (\$/kg) of SN entering the environment, which is assumed to be exogenous. Once calculated in this way, the damage costs (Z) were then

Field	Location	Year	Crop	Input	Management Class	Size (ha)	Relative curvature
1	Albury, NSW	2003	Wheat	Nitrogen	1 2 3 Whole field	12 20 15 47	0.12 0.10 0.15 0.12
2	Yarrawonga, VIC	2003	Canola	Nitrogen	1 2 3 Whole field	18 53 59 130	0.05 0.03 0.01 0.01
3	Stockport, SA	2004	Canola	Nitrogen	1 2 3 Whole field	3 5 14 22	0.03 0.30 0.21 0.24
4	Crystal Brook, SA	2004	Barley	Nitrogen	1 2 3 Whole field	12 19 19 50	0.13 0.42 0.43 0.31
5	Moree, NSW	2004	Wheat	Nitrogen	1 2 Whole field	50 29 79	0.09 0.03 0.05
6	Crystal Brook, SA	2003	Wheat	Phosphorus	1 2 Whole field	84 26 110	0.08 0.34 0.28
6	Crystal Brook, SA	2004	F-Peas	Phosphorus	1 2 Whole field	84 26 110	0.68 0.34 0.25
6	Crystal Brook, SA	2005	Wheat	Phosphorus	1 2 Whole field	84 26 110	0.32 0.02 0.08
6	Crystal Brook, SA	2006	Barley	Phosphorus	1 2 Whole field	84 26 110	0.39 0.34 0.44
7	Yarrawonga, VIC	2006	Wheat	Phosphorus	1 2 Whole field	29 29 55	0.42 0.30 0.34
7	Yarrawonga, VIC	2007	Canola	Phosphorus	1 2 Whole field	29 26 55	0.15 0.27 0.28
7	Yarrawonga, VIC	2008	Wheat	Phosphorus	1 2 Whole field	29 26 55	0.11 0.32 0.22
8	Stockport, SA	2005	Wheat	Phosphorus	1 2 3 Whole field	13 20 7 40	0.05 0.04 0.02 0.02
9	Yarrawonga, SA	2007	Wheat	Phosphorus	1 2 3 Whole field	32 37 22 91	0.08 0.25 0.49 0.27

Table 2 Relative curvature values for within-field management classes and whole fields

incorporated into the pay-off function (Eq. 3), yielding a 'social' net benefit function (*SNBF*):

$$SNBF = PQ - wX - Z \tag{7}$$

The 'social' net benefit function (SNBF) was calculated for each crop-year using the average pay-off functions. These functions were also calculated for different values of marginal damage costs, C, which were chosen arbitrarily to highlight the possible difference in their magnitude i.e. from a relatively low marginal damage cost, to a relatively high marginal damage cost. These values were 10c/kg, \$1/kg and \$3/kg of surplus N. Using the measure of relative curvature, the shape of the SNBF for each value of C was compared to the shape of the purely private pay-off function (Eq. 3).

Results and discussion

Spatial variation in flatness

All fields exhibited a degree of spatial variation in profitability as evidenced by the variability in RC values between management classes in Table 2. As a consequence, the RC of the whole-field pay-off function also differs from those derived for the management classes. The extent of the difference between the RC of the management class and the whole-field pay-off functions varies between fields and crop-years as expected.

As an example, Fig. 4 shows the payoff functions for Field 4 and the goodness of fit (Adj R^2) for the associated yield response functions. The uniform whole-field pay-off function has an RC of 0.31, while class 1 has a considerably flatter pay-off function, with an RC of 0.13. Pay-off functions for classes 2 and 3 have much higher RCs than the average (0.42 and 0.43 respectively). In this case, these areas make up close to 80 % of the

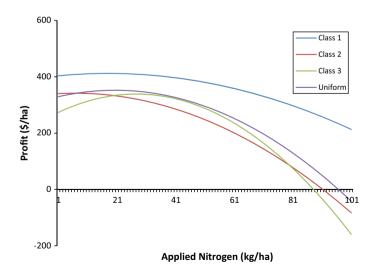


Fig. 4 Profit functions calculated for each management class for Field 4, showing relative degrees of flatness. Adjusted R^2 values for associated yield response functions: Class 1 = 0.96, Class 2 = 0.90, Class 3 = 0.76, Whole field = 0.82

field. There is clear evidence here that the shape of pay-off functions can be influenced by inherent spatial differences in field characteristics.

The results also show that whole-field pay-off functions can indeed be relatively flat as Pannell (2006) has stated. Fields 2 and 8 display low RC values for the whole field and also similarly low RC values for the component management classes. However, the results also show that while the whole-field average payoff function may appear flat, an assessment of the payoff functions using within-field management classes can identify substantial differences in flatness. Field 6 in 2005 is a prime example where the whole field RC (0.08) suggests a flat pay-off for P across the 110 ha field, but in the 82 ha of management class 2, the RC (0.32) indicates a substantially more sensitive pay-off response.

The results do not imply that pay-off functions derived from management classes will always have a higher RC than those derived from uniform management. Rather, these results show that while pay-off functions for whole fields may be relatively flat as Pannell (2006) has stated, the presence of spatial heterogeneity in soil characteristics has the potential to exhibit functions that are far less flat than if only considered under uniform management.

Such findings provide further justification for SSCM. The results imply that targeting input use optimally at the level of management classes can be important for profitability, even in cases when a whole field profitability analysis would suggest that optimality of input use is not important. This, in particular, has relevance for inputs that have relatively higher share in the overall cost structure of the production process. Such inputs are likely to have higher RC scores than other, lower cost inputs, and consequently their optimal management is going to be more significant for profitability.

The RC will also be sensitive to changes in the cost of inputs and the output price. The greater the 'terms of trade' (the output price vs. the cost of input) the less important it is to apply inputs optimally. So an increase in the output price would reduce the RC, and a general reduction in cost would have the same effect. A decrease in output price, and/or a general increase in costs of production would make the RC greater, i.e. it would become more important to optimally apply inputs.

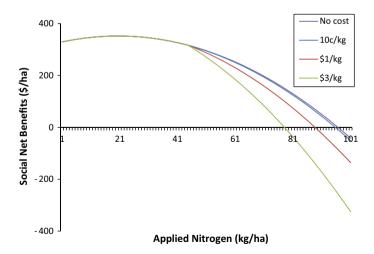


Fig. 5 Social net benefit functions calculated for the uniform application pay-off function in Field 4 using different marginal damage costs

The impact of changes in prices of individual inputs on RC will depend on how significant expenditures on a particular input are within the overall cost of production. The more significant a contribution of an input, the greater the RC increases when the price of that input goes up, and vice versa.

Environmental damage costs

Field 4 is again used as the example for estimating the impact of damage costs of the flatness of the pay-off functions. While a marginal damage cost of 10c/kg leads to a relatively small increase in RC for the uniform application pay-off function, this changes substantially when surplus N is priced at \$1 and \$3/kg. At a marginal damage cost of \$1/kg, RC increases by 25 %; while at \$3/kg, RC increases by 75 % (Fig. 5; Table 3).

These results indicate that the inclusion of environmental damage costs could lead to substantial changes in the shape of the pay-off functions, depending on how the damage is valued. As marginal damage costs increase; the SNBF becomes substantially less flat.

This highlights the importance of taking into account potential environmental damage costs. By doing so, the perceived flatness of the pay-off functions is reduced, justifying the significance of achieving optimal input application through SSCM.

This study is not without limitations. It is important to note here that the method used to quantify the costs of environmental damage is not precise, as it was used predominantly for illustration purposes. The assumption that all surplus N may enter the environment and pose a threat to the environment, while convenient for the present analysis, could overstate the implications for RC, as portions of any surplus N may become immobilised in the soil (Barrett and Burke 2000). Any surplus is however unused in the season/crop of application, which is detrimental to annual crop budgets and may risk becoming a loss to the overall enterprise economics.

The method does not take into account the behaviour of different forms of soil N, which may cause different levels of environmental damage, depending on the way in which they enter the environment. For example, the associated environmental costs of ammonia emissions may substantially differ from those associated with nitrate run-off which contributes to eutrophication and groundwater pollution (Von Blottnitz et al. 2006). Such a divergence in costs implies that environmental costs are likely to vary between sites, depending on a number of site-specific factors, including the proximity of a field to rivers and aquifers, as well as site-specific soil N dynamics (Ryther and Dunstan 1971).

To accurately determine the costs of environmental damage, soil N dynamics need to be modelled to account for transfers in soil N (Bradbury et al. 1993). These transfers may include the amount of N being leached, being lost to volatilisation and being lost to run off. Unfortunately, the data required to build such a model are scarce in Australia, and were simply not available for the present study. If a soil N model could be incorporated into the

Table 3 Relative curvature values for Field 4, using different	Cost/kg surplus N (\$A)	Relative curvature	
marginal damage costs	no cost	0.31	
	0.10	0.32	
	1.00	0.41	
	3.00	0.49	

above analysis, the environmental damage costs could be more accurately estimated. This study also does not account for the effect of carry-over N from previous seasons.

This study used only examples of environmental damage caused by excess soil N, whereas phosphorus (P) application may also contribute to nutrient loading in the environment (Tomaso 1995; Iho and Laukkanen 2012). While movement of excess phosphorus from the soil is generally regarded as slower than nitrogen, it can be envisaged that including the cost of excess P (and other macro nutrients) in the environmental damage costs would increase the curvature of the pay-off, making them less flat, and further emphasising the significance of SSCM.

Conclusions

This study has sought to address the debate over the importance of achieving optimality in on-farm decision making about input use. A general approach to quantifying the flatness of pay-off functions in agriculture has been formulated, and applied to pay-off functions which accounted for the presence of spatial soil variation and environmental damage costs.

This study has found that the shape of pay-off functions may differ substantially when spatial heterogeneity in soil characteristics is considered. While functions calculated for a whole field may be relatively flat in general, this flatness can vary substantially when the field is partitioned into management classes. This study also found that flatness of pay-off functions could differ substantially when taking into account the environmental damage costs stemming from the sub-optimal input use. In some cases, flatness decreased by up to 75 % given marginal damage costs. Findings of such substantial differences associated with the inclusion of environmental damage costs suggests that achieving optimal input application in agriculture is more important than some would argue. This is particularly valid in the context of site-specific crop management, where optimal input application within management classes of heterogeneous fields is economically justified.

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