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Determining nugget:sill ratios of standardized variograms from aerial photographs to krige sparse soil data

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Abstract Maps of kriged soil properties for precision agriculture are often based on a variogram estimated from too few data because the costs of sampling and analysis are often prohibitive. If the variogram has been computed by the usual method of moments, it is likely to be unstable when there are fewer than 100 data. The scale of variation in soil properties should be investigated prior to sampling by computing a variogram from ancillary data, such as an aerial photograph of the bare soil. If the sampling interval suggested by this is large in relation to the size of the field there will be too few data to estimate a reliable variogram for kriging. Standardized variograms from aerial photographs can be used with standardized soil data that are sparse, provided the data are spatially structured and the nugget:sill ratio is similar to that of a reliable variogram of the property. The problem remains of how to set this ratio in the absence of an accurate variogram. Several methods of estimating the nugget:sill ratio for selected soil properties are proposed and evaluated. Standardized variograms with nugget:sill ratios set by these methods are more similar to those computed from intensive soil data than are variograms computed from sparse soil data. The results of cross-validation and mapping show that the standardized variograms provide more accurate estimates, and preserve the main patterns of variation better than those computed from sparse data.

Keywords Aerial photographs · Nugget:sill ratio · Kriging · Soil · Variogram

Introduction

To apply site-specific management in agriculture requires accurate information about the spatial variation in soil and crop properties. Viscarra-Rossel and McBratney (1998) said that grid sampling followed by kriging produces optimal predictions that can be used by

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farmers in a digital form or as contour maps for site-specific management. Accurate kriged estimates depend on a reliable variogram, however, which requires a minimum of about 100 data points if estimated by Matheron's usual method of moments (MoM) (Webster and Oliver 1992). This is usually too large a sample for most commercial soil and crop surveys of a field, which tend to have one sample per hectare at the most (Godwin and Miller 2003). The latter approach not only results in too small a sample to compute the variogram reliably, but also takes no account of the spatial scale of variation present. Without some knowledge of spatial scale there is no certainty that the sampling interval selected will provide data that are spatially dependent or correlated, which is essential for kriging and indeed for any kind of interpolation for mapping.

Kerry and Oliver (2003) showed that variograms from ancillary data, such as aerial photographs, satellite imagery, electromagnetic induction scans and digital elevation models, could be used to guide sampling and so take account of the scale of spatial variation. To ensure spatial dependence, the sampling interval should be less than half the range of spatial variation as a rule of thumb (Kerry and Oliver 2003, 2004). If the variation shows strong continuity, the variogram range might be large in relation to the size of the field and a sample size based on the variogram range would be too small to compute a reliable variogram. For example, Fig. 1a shows a multivariate variogram computed from three wavebands of an aerial photograph of a field; it has a range of 247 m which suggests a sampling interval of about 120 m would be suitable to resolve the spatial variation. The field is 43 ha and with such a sampling interval, there would be 23 sites. This sample size is too small to calculate an accurate variogram as we show later, but a sample size of 100 would waste sampling effort.

The difficulties described above indicate a need for alternative methods to compute reliable variograms when there are too few soil data. Variograms can be computed from intensive ancillary data, such as those described above, and they could provide a possible solution (Kerry and Oliver 2002). Such variograms can be standardized to a sill variance of 1 and then used to krige soil data standardized to zero mean and unit variance. The soil should be sampled to ensure that the data are spatially dependent, even though they might be sparse. Brooker (1986) noted that kriged predictions are most sensitive to poor estimates of the nugget variance. Kerry and Oliver (2002) showed that standardized variograms of

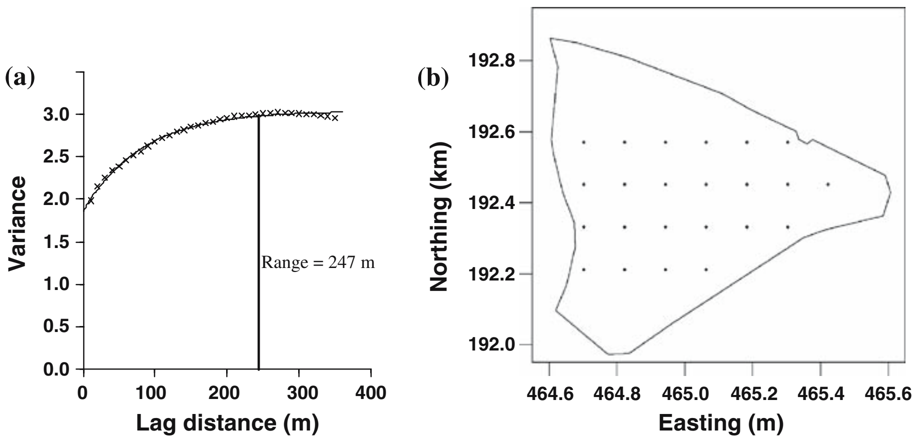

Fig. 1 (**a**) Multivariate variogram of three wavebands from an aerial photograph (1997) with a range of 247 m, and (**b**) map of a sampling scheme with a 120 m interval (23 points) at the Wallingford site

ancillary data can provide precise kriged estimates provided that they have similar nugget:sill ratios to those of variograms of the soil properties. Variograms of ancillary data, however, usually have a smaller nugget variance than those of soil properties because the data are more intensive than soil data. For example, if the sampling interval is 3–5 m for ancillary data and 30 m for soil data (this is intensive compared with that used for many farm surveys), more of the variation between 5 and 30 m is resolved by the ancillary data. Theoretically at zero lag, the variance should be zero. However, for most properties of the environment the variogram model has a positive intercept on the ordinate. This is the nugget variance, which comprises purely random variation, measurement error, but largely the variation at scales smaller than the sampling interval (Webster and Oliver 2007). Even when soil properties are measured at intervals of 5 m (Oliver and Webster 1987), they still tend to have a substantial nugget variance because of the local nature of soil variability. Therefore it is important that we reflect this source of uncertainty in prediction by setting nugget:sill ratios for ancillary variograms that are likely to reflect those of the soil properties to be estimated.

Our previous work (Kerry and Oliver 2002) showed a need to estimate the nugget:sill ratio of the ancillary variogram to correspond with that of the soil property of interest. The aim here is to examine and evaluate several ways of determining this ratio, such as from relatively small sets of data, from existing variograms (for example see McBratney and Pringle's (1999) suggestion for the use of average variograms), or by setting it somewhat arbitrarily to 0.25 for example. Kerry and Oliver (2005, 2007) and Lark (2000) have shown that a reliable variogram can be computed by residual maximum likelihood (REML) from about 50 data and method (a) will determine nugget:sill ratios of properties from variograms estimated by REML from 40 or fewer data. Method (b) will estimate the nugget:sill ratio from the average nugget:sill ratios of variograms from the same property at other sites, and method (c) from several properties measured previously at the same site. Method (d) sets the nugget:sill ratio to 0.25 based on our experience of the average value of this ratio for several soil properties. The nugget:sill ratios determined by these methods will be compared with those for several soil properties recorded in two arable fields. The methods will be evaluated by cross-validation with the set of nugget:sill ratios of standardized variograms from the ancillary data. These results will be compared with those based on variograms of the soil properties from the intensive and sub-sampled data. The accuracy of kriged predictions is always of importance in geostatistical analyses. However, from a practical viewpoint in precision agriculture, probably of greater importance is how well the spatial pattern of variation in the properties of interest in the fields can be represented.

Methods

Field sites, soil sampling and analysis

One field site was on Crowmarsh Battle Farms, Wallingford, Oxfordshire (OS reference SU 465000, 192000) where the soil has developed on the plateau gravel of the River Thames. The field has an undulating topography with dry valleys; two in the north-south direction and one east-west. The other field site was on the Yattendon Estate Yattendon, Berkshire (OS reference SU 456000, 181000) where the soil has developed on the Upper Chalk. The topography of the field comprises a plateau area and a north facing slope.

Soil samples were taken and observations made on a 30 m grid at Wallingford and Yattendon in 2000 (Figs. 2a and 3a, respectively). At each node of the sampling grid, six

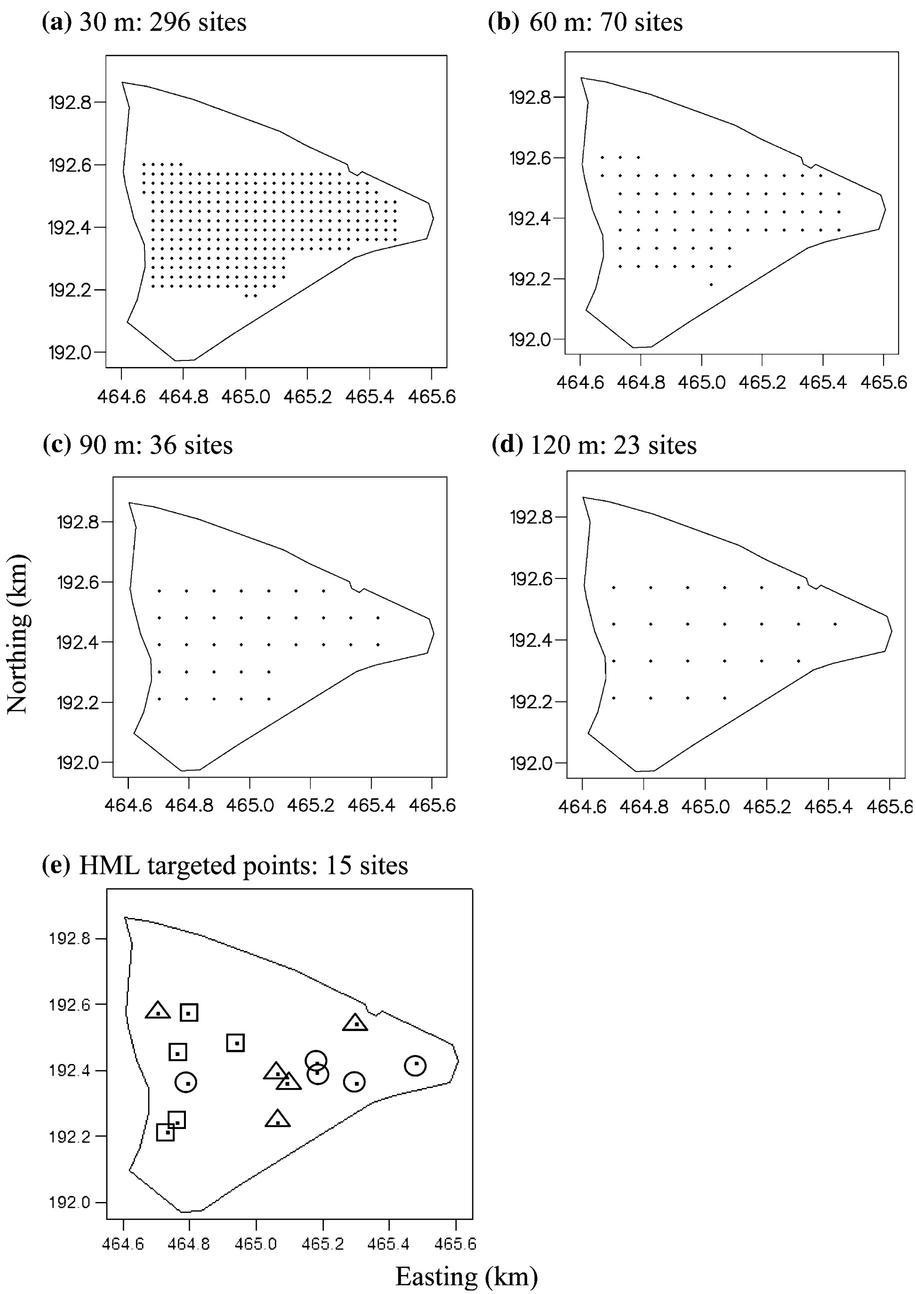

Fig. 2 Location of soil samples at Wallingford on grids of: (**a**) 30 m, (**b**) 60 m, (**c**) 90 m, (**d**) 120 m and (**e**) points targeted based on HML digital numbers—high (circles), medium (triangles) and low (squares)

cores of soil from a 1 m^2 support were bulked. Several standard laboratory analyses were done on the air-dry soil (<2 mm fraction); these are summarized together with the methods of field observation in Table 1. Replicates, controls and reference soils were measured for

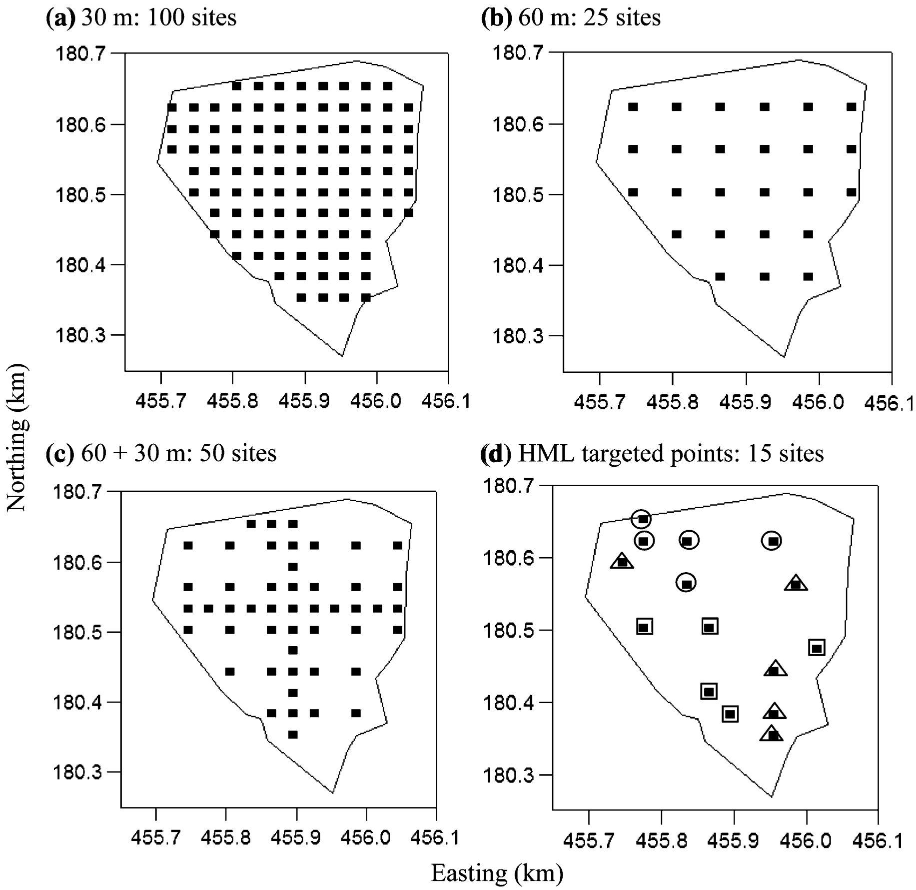

Fig. 3 Location of soil samples at Yattendon on grids of: (**a**) 30 m, (**b**) 60 m, (**c**) 60 + 30 m and (**d**) points targeted based on HML digital numbers—high (circles), medium (triangles) and low (squares)

Table 1 Field and laboratory methods

Soil property	Method
Depth (cm)	Auger and tape measure
Loss on ignition (LOI) (%)	10 g air dry <2 mm
Moisture correction factor (MCF) (%)	10 g air dry <2 mm
Munsell value	Air dry <2 mm
pH	2:1 water to soil ratio
Stoniness (%)	Standard charts
Texture (% sand, silt, clay)	Laser methods
Volumetric water content (VWC) (g kg^{-1})	Delta-T Theta probe calibrated for soil type

each set of laboratory analyses to ensure the accuracy of results. The soil properties observed are, in general, the more permanent ones that need to be recorded less frequently than the crop nutrients. The former are core properties for site-specific management because of their effects on many others, such as drainage and moisture status, bulk density, nutrient holding capacity, yield potential and many others. For example, soil texture and organic matter levels are important in determining the amounts of fertilizer and pesticide that should be applied.

Aerial photograph data and sub-sampling of soil data

Two aerial photographs of the bare soil were available for each field; a black and white one (1966) and a colour one (1997) for Wallingford and two colour ones for Yattendon (1986 and 1991). The photographs were obtained from standard surveys (www.aerofilms.com) for each field at a scale of 1:10 000 and were scanned at a resolution of 75 dpi resulting in a ground pixel size of 3.4 m. They were geo-corrected to UK Ordnance Survey coordinates, and digital numbers (DNs, 0–255) for the red, green and blue wavebands were extracted for each pixel with ERDAS Imagine (www.erdas.com) for the colour photographs.

An omni-directional variogram was computed from the DNs of the black and white photograph and modelled in GenStat (Payne 2006). Multivariate variograms (Bourgault and Marcotte 1991) were computed from the DNs of the red, green and blue wavebands of the colour aerial photographs and modelled. The ranges of these variograms were used to suggest an appropriate soil sampling interval; for Wallingford the ranges were 204 m (1966) and 247 m (1997) and for Yattendon they were 105 m (1986) and 80 m (1991). Sampling intervals for potential soil surveys were determined as approximately half the range of these variograms; for Wallingford, the interval indicated was 100–120 m and for Yattendon, it was 40–50 m. For the analyses that follow, the 30 m data at Wallingford were sub-sampled to grids of 60 m (70 points), 90 m (36 points) and 120 m (23 points). The sampling configurations are shown in Fig. 2b–d. The suggested sampling interval of 40–50 m for Yattendon would be too intensive for most soil surveys of a farm, therefore the 30 m data were sub-sampled to a 60 m grid (25 points) and to a 60 m grid supplemented by samples at 30 m (50 points) to provide an intermediate sampling intensity (Fig. 3b–c).

In addition to the above sub-samples, 15 data points were selected from the 30 m data that coincided with areas of high, medium and low (HML) digital numbers in the 1966 and

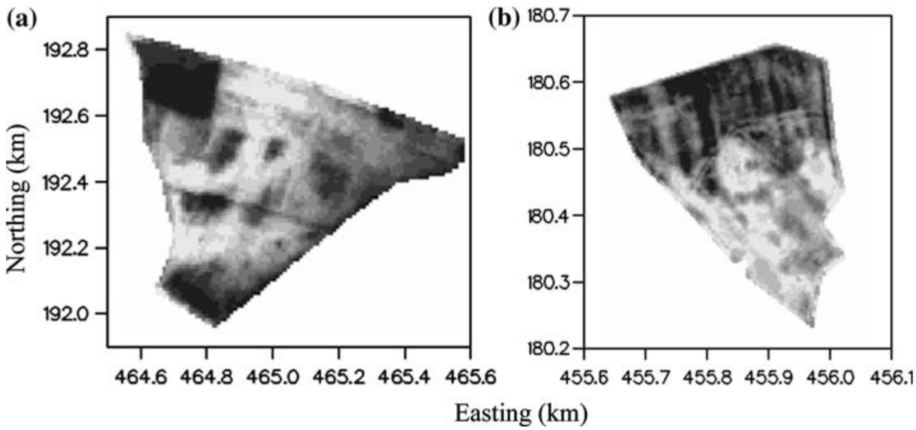

Fig. 4 Pixel maps of bare soil digital numbers at: (**a**) Wallingford (1966) and (**b**) Yattendon (1991). White areas have low digital numbers and black areas have high digital numbers

1991 aerial photographs for Wallingford and Yattendon, respectively (Fig. 4a, b). The high, medium and low areas corresponded with the lower, middle and upper 33% of the digital numbers. There were five sampling points in each of the three zones, and within each zone at least two of these were closely spaced (i.e. they were neighbouring points on the 30 m grid). The closely spaced points are important for improving the estimate of the nugget variance of the variogram. These targeted sampling schemes are shown in Figs. 2e and 3d, for Wallingford and Yattendon, respectively.

Standardized variograms of soil and aerial photograph data

Following exploratory data analyses, omni-directional variograms were computed for each of the soil properties from the original 30 m data (>100 sampling points) and the sub-sampled data at both sites using GenStat (Payne 2006). Transformed data were used where the coefficient of skewness was outside the bounds ± 1. The nugget:sill ratios were determined from the model parameters of the fitted variogram models and standardized to a sill variance of unity. Tables 2 and 3 give the model form, the range and nugget:sill ratio for each soil property at Wallingford and Yattendon, respectively.

The variogram of the DNs from the one channel, monochromatic photograph at Wallingford (1966) (Fig. 5a) and the multivariate variogram of red, green and blue DNs at Yattendon (1991) (Fig. 5b) were selected for further analysis because they appear to reflect the patterns of variation in the soil more clearly than the photographs of the other dates. In addition, the range of these variograms was close to the average range of those of the soil properties (Table 2). As for the soil data, the variograms from the aerial photographs were standardized, but in this case the nugget:sill ratios were determined according to the methods described below. Tables 2 and 3 give the model form and range of these variograms, and the nugget:sill ratios of each soil property for all methods for the two fields.

Estimating nugget:sill ratios of standardized variograms from aerial photographs

(a) Residual maximum likelihood variogram

A variogram was estimated for each variable at the Wallingford and Yattendon sites by REML using Pardo-Igúzquiza's (1997) MLREML program. Readers who want to know more about the method are recommended to read about it in his paper or in Webster and Oliver (2007). The REML variograms at Wallingford were computed from 38 data points comprising those from the 120 m sub-sample (Fig. 2d) and the 15 targeted data based on the HML zones in the aerial photograph (Fig. 2e). At Yattendon, the REML variograms were computed from 40 data points comprising those from the 60 m sub-sample (Fig. 3b) and the 15 targeted data (Figs. 3d).

(b) Property

The nugget:sill ratios were determined from variograms of a given soil property measured at the two sites described above and three additional sites investigated by Kerry (2004). The soil at the additional three sites was developed on the Oxford clay, the Lower Greensand and Upper Chalk. All five fields have soil developed on parent materials that are representative of the main areas of arable farming in southern England. The nugget:sill ratios of several variograms of soil depth, for example, from these sites were averaged to provide a ratio for the standardized variogram from the aerial photograph for this property

Table 2 Variogram parameters for 30 m soil data and nugget:sill ratios determined by methods (a)–(d), and differences between the former and latter at Wallingford

Soil property	Parameters for variograms of the 30 m soil data			Nugget:sill ratio determined by methods (a)–(d)				Difference from nugget:sill ratio of 30 m data			
	Model	$c_0{:}c_0 + c$	a ($3r$) m	Method (a) REML	Method (b) Property	Method (c) Site	Method (d) Fixed ratio	Method (a) REML	Method (b) Property	Method (c) Site	Method (d) Fixed ratio
Clay	Spher.	0.00	134.8	0	0.19	0.16	0.25	0.00	−0.19	−0.16	−0.25
Depth	Expo.	0.53	232.4	0.63	0.45	0.16	0.25	−0.10	0.08	0.37	0.28
LOI	Penta.	0.10	226.1	0	0.29	0.16	0.25	0.10	−0.19	−0.06	−0.15
M. Value	Penta.	0.24	218.9	0.20	0.47	0.16	0.25	0.24	−0.23	0.08	−0.01
MCF	Circ.	0.09	185.7	0	0.34	0.16	0.25	−0.11	−0.25	−0.07	−0.16
pH	Circ.	0.09	210.6	0	0.27	0.16	0.25	0.09	−0.18	−0.07	−0.16
Sand	Penta.	0.07	231.2	0	0.21	0.16	0.25	0.07	−0.14	−0.09	−0.18
Stones	Expo.	0.05	267.2	0	0.27	0.16	0.25	0.05	−0.22	−0.11	−0.20
VWC	Penta.	0.28	202.2	0.07	0.29	0.16	0.25	0.21	−0.01	0.12	0.03
		Average	212.1				RMSE of $c_0{:}c_0 + c$	0.13	0.18	0.16	0.18

Note: The standardized variogram from the aerial photograph was an exponential model with an effective range of 204.6 m. Circ., Circular; Expo., Exponential; Penta., Pentaspherical; Spher., Spherical; $c_0{:}c_0 + c$, the nugget:sill ratio; a and $3r$ the range parameter and the effective range of an exponential model, respectively; RMSE, the root mean squared error, REML, residual maximum likelihood

Table 3 Variogram parameters for 30 m soil data and nugget:sill ratios determined by methods (a)–(d), and differences between the former and latter at Yattendon

Soil property	Parameters for variograms of the 30 m soil data			Nugget:sill ratio determined by methods (a)–(d)				Difference from nugget:sill ratio of 30 m data			
	Model	c_0:$c_0 + c$	a, $3r$ (m)	Method (a) REML	Method (b) Property	Method (c) Site	Method (d) Fixed ratio	Method (a) REML	Method (b) Property	Method (c) Site	Method (d) Fixed ratio
Clay	Spher.	0.21	76.8	0.14	0.19	0.30	0.25	0.07	0.02	−0.09	−0.04
Depth	Expo.	0.24	79.3	0.10	0.45	0.30	0.25	0.14	−0.21	−0.06	−0.01
LOI	Spher.	0.33	80.1	0	0.29	0.30	0.25	0.33	0.04	0.03	0.08
M. Value	Circ.	0.40	93.4	0	0.47	0.30	0.25	0.40	−0.07	0.10	0.15
MCF	Circ.	0.72	111.2	0	0.34	0.30	0.25	0.72	0.38	0.42	0.47
pH	*	*	*	*	*	*	*	*	*	*	*
Sand	Spher.	0.10	95.2	0	0.21	0.30	0.25	0.10	−0.11	−0.20	−0.15
Stones	Penta.	0.37	111.0	0.20	0.27	0.30	0.25	0.17	0.10	0.07	0.12
VWC	Circ.	0.02	58.7	0	0.29	0.30	0.25	0.02	−0.27	−0.28	−0.23
			Average 88.2				RMSE of c_0:$c_0 + c$	0.32	0.20	0.20	0.20

Note: The standardized variogram from the aerial photograph was an exponential model with an effective range of 80 m. * Not measured at this site. Circ., Circular; Expo., Exponential; Penta., Pentaspherical; Spher., Spherical; c_0:$c_0 + c$, the nugget:sill ratio; a and $3r$ the range parameter and the effective range of an exponential model, respectively; RMSE, the root mean squared error; REML, Residual maximum likelihood

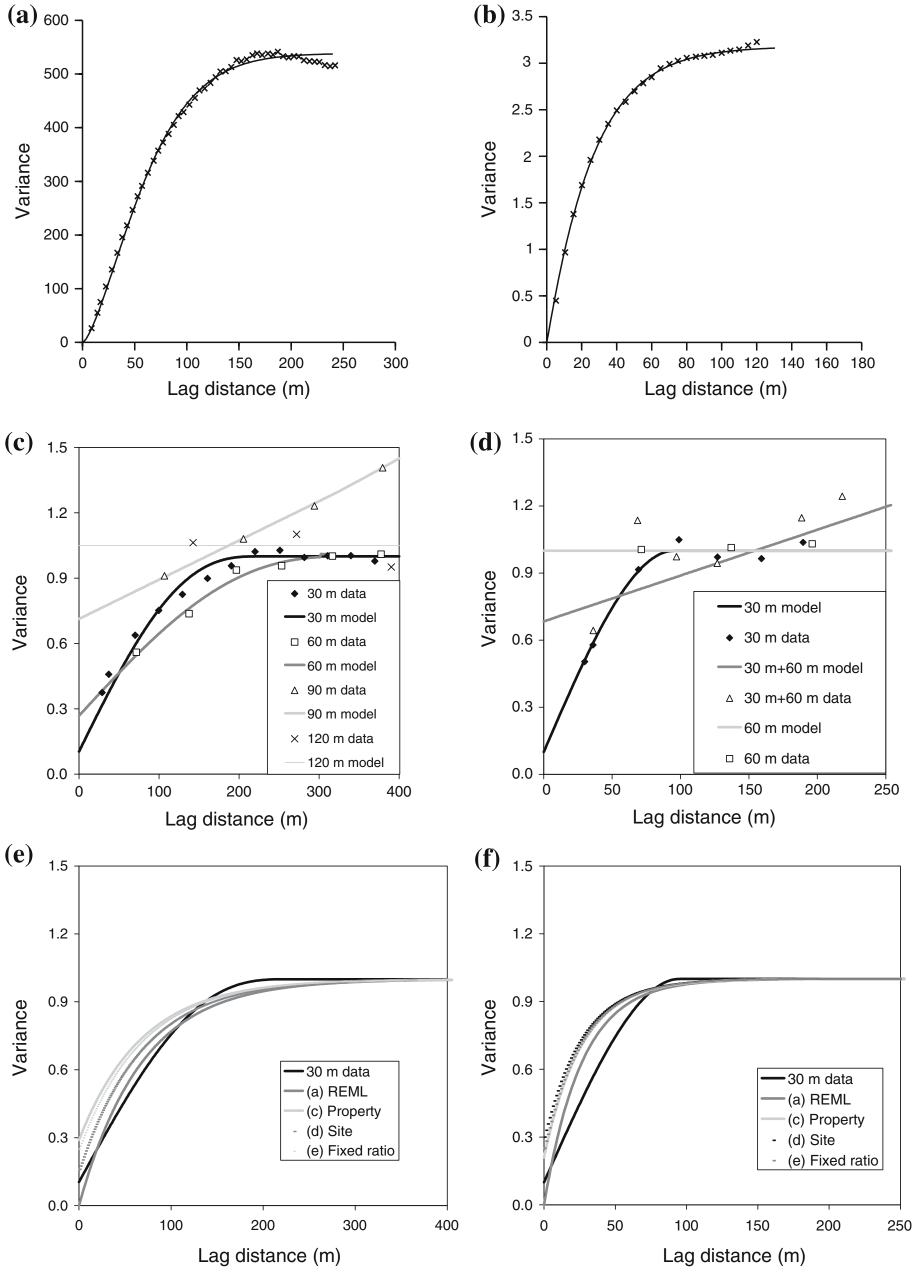

Fig. 5 Experimental variograms (symbols) and models (lines) for: (**a**) the 1966 monochromatic aerial photograph at Wallingford, (**b**) the multivariate variogram of digital numbers from the red, green and blue wavelengths at Yattendon from the 1991 aerial photograph, (**c**) 30 m and sub-sampled LOI data at Wallingford, and (**d**) 30 m and sub-sampled sand data at Yattendon, and standardized variogram models from ancillary data for (**e**) LOI at Wallingford and (**f**) sand at Yattendon

at the site of interest. The nugget:sill ratio for soil depth will be the same at all sites for this method; it requires no additional soil data at the field site of interest. However, it does need variogram models of the properties of interest from other studies.

(c) Site

This method used variograms of several soil properties measured at the same site in a previous study (Kerry 2004). For example, the nugget:sill ratios of variograms for soil depth, volumetric water content (VWC), clay, sand, loss on ignition (LOI) etc. at one site were averaged to provide a nugget:sill ratio for the standardized variogram from the aerial photograph of the site and for use with the standardized soil data for any variable measured at the same site. The nugget:sill ratio for all properties will be the same at a given site for this method. As above, this method requires no additional soil data at the field site, but depends on the availability of prior information.

(d) Fixed ratio

This method simply used a fixed nugget:sill ratio for the standardized variograms from aerial photographs for all soil properties at all sites. We chose a ratio of 0.25 which is based on our experience; it is a nugget:sill ratio that occurs for many variograms of soil properties from a variety of sites. As for methods (b) and (c), this method requires no additional sampling.

Cross-validation

The model parameters of the standardized variograms of the original and sub-sampled soil data were used with the original 30 m data standardized to zero mean and unit variance for cross-validation. Each point was removed in turn and was estimated using the surrounding data and appropriate model parameters. The variograms of the 120 m data for Wallingford and the 60 m data for Yattendon are pure nugget so linear models with a gradient of zero were used for cross-validation; this essentially amounts to simple interpolation by local averaging. We assumed that the variogram of the 30 m data of a given property is close to the underlying variogram. This is a reasonable assumption given the number of samples and the sampling interval from which they were computed. Therefore, the results based on the variograms of the 30 m data provide a basis for comparison when variograms from the sub-sampled data are used.

The nugget:sill ratios of the standardized variograms of ancillary data were determined by methods (a)–(d) and were used with the range and model type of the variograms from the aerial photograph data (Tables 2 and 3), which remained constant for a given field, for cross-validation with the original 30 m standardized data. As above, the results of cross-validation will be compared with those for the variograms of the original 30 m data for each soil property.

The following statistics provide the basis for comparing the results of cross-validation. The mean squared error (MSE) and mean squared deviation ratio (MSDR) were calculated for the full data, the sub-samples and all methods used to determine the nugget:sill ratio of the standardized ancillary variograms. The MSE is given by

$$\mathrm{MSE} = \frac{1}{N}\sum_{i=1}^{N}\{z(\mathbf{x}_i) - \hat{z}(\mathbf{x}_i)\}^2 \tag{1}$$

and the MSDR by

$$\mathrm{MSDR} = \frac{1}{N}\sum_{i=1}^{N}\frac{\{z(\mathbf{x}_i) - \hat{z}(\mathbf{x}_i)\}^2}{\hat{\sigma}^2(\mathbf{x}_i)} \tag{2}$$

where $\hat{\sigma}^2(\mathbf{x})$ are the kriging variances.

The MSDR is the average of the ratio of the squared error to the kriging variance at each prediction point; this should be close to 1 for the correct model. The larger the MSE values, the less accurate are the predictions.

Kriging

The original 30 m and sub-sampled standardized soil data were used for kriging with the associated standardized variograms. Where the variogram was pure nugget (120 m LOI data at Wallingford and 60 m sand data at Yattendon), all that could be done was a simple interpolation. Nevertheless, these data should be spatially dependent based on the variograms of the aerial photograph data. The most likely reason for the apparent lack of spatial structure in these variograms is that they were computed from too few data for the method of moments. Variograms computed from these data by REML showed some structure. For Wallingford, the 60, 90 and 120 m sub-sampled data were used with the standardized variograms of the aerial photographs for kriging; the nugget:sill ratios of these variograms were estimated by methods (a)–(d). For mapping, the kriged predictions were restored to the original scale of measurement by multiplying them by the standard deviation and adding the mean of the sub-sample used for kriging.

Correlation

Pearson's product moment correlations were calculated between kriged estimates from the 30 m data and the sub-sampled data with their associated variograms at both field sites. In addition, the correlations were calculated between the estimates from the 30 m data and those from the 120 and 60 m sub-samples for Wallingford and Yattendon, respectively, using standardized variograms with the nugget:sill ratios set by methods (a)–(d).

Results and discussion

Variogram analysis

Figure 5a, b shows the experimental variogram and fitted model of the digital numbers from the aerial photographs of Wallingford and Yattendon, respectively. They were both fitted with exponential functions; the former had an effective range of 204 m and the latter 80 m. Figure 5c, d shows the experimental variograms and fitted models of LOI for Wallingford and percentage sand content for Yattendon computed from the original 30 m data and for the range of sub-samples. These variograms have been standardized to a sill of one as for the ancillary variograms for ease of comparison. Figure 5c, d shows that as the sampling interval increases and the number of sampling points decreases, the form of the experimental variogram becomes increasingly unlike that of the original variogram. For Wallingford, the 60 m data result in a variogram that is not too dissimilar from the original one, but that of the 90 m sub-set has a much larger nugget variance and is fitted by a linear model and the 120 m model is pure nugget. The variogram of the 60 m sub-set at Yattendon is pure nugget (Fig. 5d). If the variogram is pure nugget, it does not necessarily mean that the data are spatially independent—it is more likely that the variogram is inaccurate because of the small size of the data set. The experimental variogram of the 60 + 30 m sub-set is erratic which made it difficult to model. Variograms of the sub-samples of other properties at each site are not shown as they show similar patterns.

Figure 5e, f shows the standardized ancillary variograms with the nugget:sill ratios determined by the four methods described above, together with the variogram models of LOI and sand from the 30 m data for Wallingford and Yattendon, respectively. For Wallingford, the standardized variograms are close to that of LOI, in spite of the variety of nugget:sill ratios, whereas for Yattendon they are more different from the variogram of sand, but the variation in nugget:sill ratios is a little less.

Tables 2 and 3 show that the variogram ranges for soil properties are on average similar to those of the aerial photographs at each site. For comparison at both sites, the nugget:sill ratio estimated by methods (a)–(d) was subtracted from the nugget:sill ratio of the variogram of the 30 m data (Tables 2 and 3). For Wallingford, the nugget:sill ratios estimated by the four methods are similar; they differ by between 0.13 and 0.18 on average from the variograms of the 30 m data (Table 2). The largest errors in the nugget:sill ratios are for soil depth when it was calculated by methods (c) and (d). Variograms of this soil property have a large nugget:sill ratio of about 0.5 at both Wallingford and Yattendon. For Yattendon, Table 3 shows that the methods give similar results on average for methods (b)–(d). There are no particular patterns for the best and worst estimates of the nugget:sill ratios at Yattendon, except that MCF and VWC are consistently under- and over-estimated, respectively, by the various methods. Method (a) is noticeably poor at estimating the nugget:sill ratios for LOI, MCF and Munsell value.

It is evident from Fig. 5c, d that the standardized ancillary variograms for Wallingford and Yattendon with nugget:sill ratios based on methods (a)–(d) are closer to the form of the variogram for the 30 m data than are those of any of the sub-samples, suggesting that they should give more accurate kriged estimates than those for sub-samples.

Cross-validation results

Table 4 gives the cross-validation results for LOI and soil depth at Wallingford based on the variograms of the full data and of the sub-samples. These show, in general, that as the sampling interval increases and the number of sampling points decreases, the MSEs increase and the MSDRs differ more from one. Table 4 also gives the cross-validation results for the ancillary variograms with the nugget:sill ratio determined by the four methods. For LOI, the MSEs are smaller and MSDRs are closer to 1 for most of the variograms from these methods than are those for the sub-samples; the MSDR for the 60 m sub-sample is the exception. For soil depth, the MSE is smaller and the MSDR more appropriate than the sub-sampled data for methods (a) and (b) only; these methods resulted in nugget:sill ratios similar to that for soil depth of the 30 m data (see Table 2). This suggests that for soil properties that consistently result in variograms with larger nugget:sill ratios, method (a), which has some soil data at small lags in addition to the 120 m grid, or

Table 4 Cross-validation results for LOI and soil depth at Wallingford using 30 m soil data and different models (a–d)

Model used	LOI (%)		Depth (cm)	
	MSE	MSDR	MSE	MSDR
30 m	0.477	1.504	0.674	0.902
60 m	0.495	1.191	0.686	0.890
90 m	0.543	0.661	0.677	1.211
120 m	0.574	0.521	0.695	1.225
(a) REML	0.472	1.367	0.675	0.810
(b) Property	0.483	0.817	0.674	0.953
(c) Site	0.475	0.976	0.693	1.434
(d) Fixed ratio	0.480	0.858	0.684	1.228

Table 5 Cross-validation results for percentage sand and stones at Yattendon using 30 m soil data and different models (a–d)

Model used	Sand (%)		Stones (%)	
	MSE	MSDR	MSE	MSDR
30 m	0.474	0.960	0.495	0.667
60 + 30 m	0.631	0.785	0.506	0.764
60 m	0.753	0.717	0.651	0.616
(a) REML	0.506	0.676	0.499	0.678
(b) Property	0.530	0.636	0.510	0.604
(c) Site	0.544	0.628	0.513	0.599
(d) Fixed ratio	0.536	0.632	0.509	0.607

method (b), which is property specific, should be used. Alternatively, a nugget:sill ratio of 0.5 should be used for properties that are known to vary in a locally erratic way or for those that have been measured on coarse measurement scales.

Table 5 gives the cross-validation results for percentage sand and stones at Yattendon. These show, in general, for both sand and stones that as the sampling interval increases and the number of sampling points decreases, the MSEs increase and the MSDR values differ more from one. For sand, the MSEs are smaller for variograms based on methods (a)–(d) than are those for the sub-sample variograms. The former are similar to those based on the model from the 60 + 30 m sub-sample. The MSDRs for methods (b)–(d) are less close to one, however, than are those of the sub-samples.

Maps of kriged estimates

The ‘proof of the pudding’ of this exercise from the precision farmer’s viewpoint is in the maps that result from kriging with variograms from the sub-sampled data and with the various standardized variograms from aerial photographs. Figure 6a–d shows the kriged maps of LOI for Wallingford based on kriging with the variograms and data from the 30 m grid and sub-samples of these data. The detail in the patterns of variation in the maps for the sub-samples becomes increasingly degraded as sampling intensity decreases compared

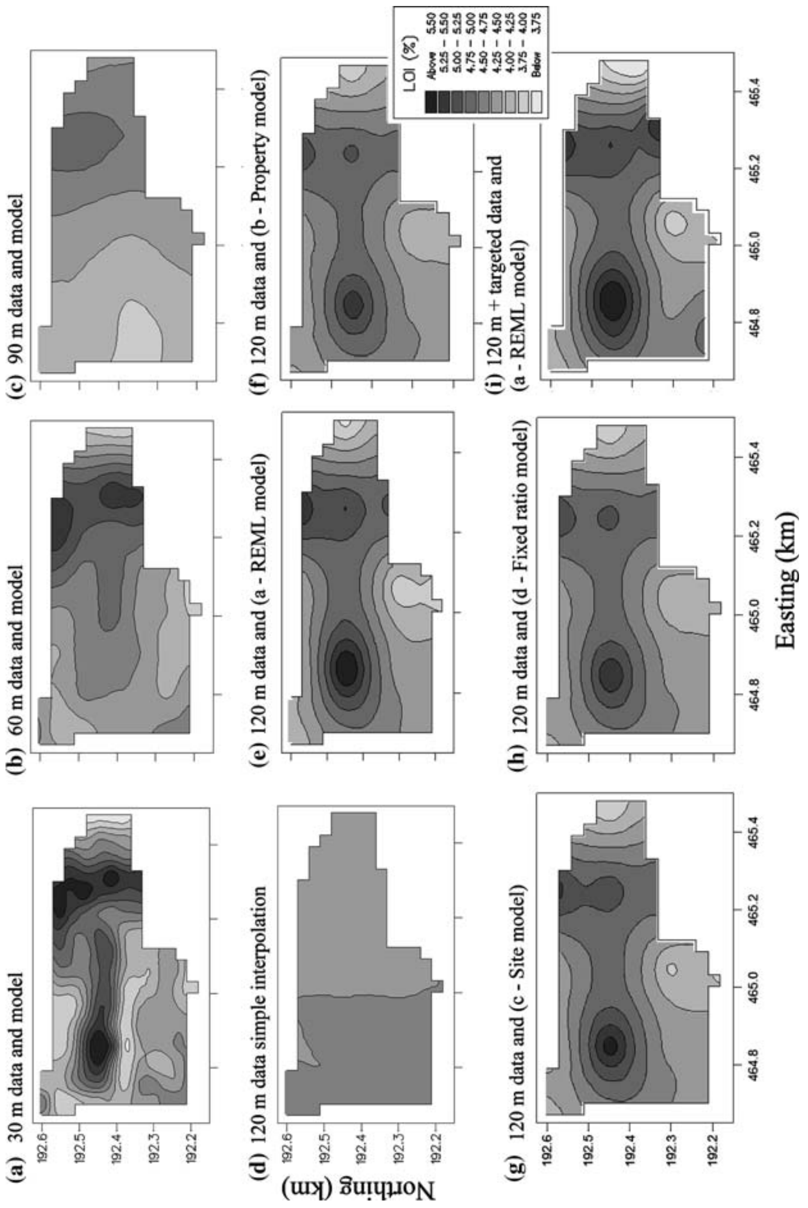

Fig. 6 Maps of loss on ignition (LOI) for Wallingford using: (**a**) 30 m data and model, (**b**) 60 m data and model, (**c**) 90 m data and model, (**d**) 120 m data and simple interpolation, (**e**) 120 m data and (a, REML model), (**f**) 120 m data and (b, Property model), (**g**) 120 m data and (c, Site model), (**h**) 120 m data and (d, Fixed ratio model) and (**i**) 120 m plus targeted data and (a, REML model)

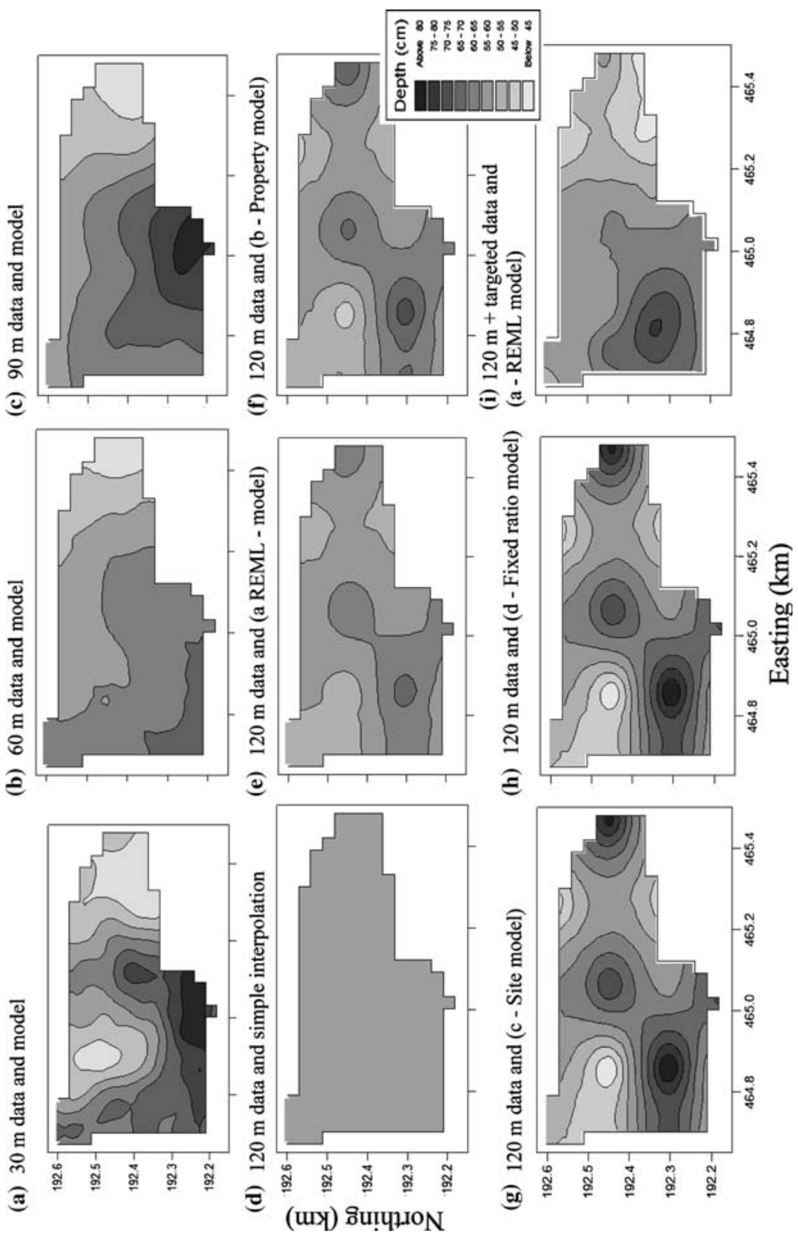

Fig. 7 Maps of soil depth for Wallingford using: (**a**) 30 m data and model, (**b**) 60 m data and model, (**c**) 90 m data and model, (**d**) 120 m data and simple interpolation, (**e**) 120 m data and (a, REML model), (**f**) 120 m data and (b, Property model), (**g**) 120 m data and (c, Site model), (**h**) 120 m data and (d, Fixed ratio model) and (**i**) 120 m plus targeted data and (a, REML model)

to that for the 30 m data. However, the results of kriging the standardized soil data on the 120 m grid with standardized variograms based on methods (a)–(d), Fig. 6e–h, show that the main features of the variation in Fig. 6a, are preserved. This is particularly so for methods (a) based on the REML variogram and (c) which was site specific. This supports the results in Table 4, but also shows the 'best case' scenarios where methods (a) and (c) estimate a similar nugget:sill ratio to that for the variogram of the 30 m data (Table 2). Although the nugget:sill ratios for methods (b) and (d) differ more (by up to 0.19, Table 2) from that of the variogram for the 30 m data, the main patterns of variation in the 30 m data (Fig. 6e–h) are preserved better than in Fig. 6d based on simple interpolation. Figure 6e shows the kriged map based on the REML variogram and the 120 m data. This variogram was computed from the 120 m data and the 15 targeted sampling points, and so it was also used to krige these data and the result is shown in Fig. 6i. This map shows that the additional data from the targeted samples have helped to resolve more of the variation in the south west corner of the field and it is more similar to the map from the 30 m data than are the others. Clearly, this part of the field was missed by the 120 m sampling interval.

Figure 7a–d shows the maps of soil depth for Wallingford based on kriging with the variograms and data from the 30 m grid and sub-samples of these data. As above, the detail in the patterns of variation in the maps from the sub-sampled data becomes increasingly degraded as sampling intensity decreases compared to that for the 30 m data. When the standardized variograms from methods (a)–(d) are used to krige with the 120 m data, the resulting maps (Fig. 7e–h) are considerably more detailed than those made with variograms of the sub-sampled data. There are some discrepancies, however, especially in the north-west of the field where methods (a)–(d) do not reproduce the pattern of variation. This problem relates to the configuration of the sparse sampling points as illustrated by Frogbrook and Oliver (2000) rather than a problem with the above methods. The large soil depths in the west of the field are associated with a dry valley running north-south that is less than 100 m across, and this was probably missed by the 120 m sampling interval. This example presents a 'worst case' scenario where the sampling has missed important features of the variation. Nevertheless, the maps based on variograms from methods (a) to (d) are far more plausible than the one made by simple interpolation of the 120 m data. Figure 7e shows the kriged map based on the REML variogram and the 120 m data. As above, this variogram was estimated from the 120 m data, and the 15 targeted data and the result of kriging with these additional data is shown in Fig. 7i. Some of the larger soil depths in the west of the field associated with the dry valley are detected when these additional data are used.

Table 6 gives the correlations between the estimates from the 30 m data and those from the sub-sampled data with their associated variograms at Wallingford. In addition, the correlations are also given between the estimates from the 30 m data and estimates from the 120 m sub-sample using standardized variograms with the nugget:sill ratios set by methods (a)–(d). For LOI, the correlations are strong for the 60 m data and weak for the 90 and 120 m data and they are all moderately strong for the methods (a)–(d). For soil depth, the correlations are strong for the 60 and 90 m data, whereas they are moderate for the 120 m data and for methods (a)–(d) using the 120 m data. Note that the correlation is slightly larger for method (a) when the 15 HML targeted points are added to the 120 m data for kriging (Table 6).

The maps of sand and stone contents at Yattendon (Figs. 8 and 9, respectively) illustrate an intermediate example between LOI and depth at Wallingford. The main features of the variation in the 30 m maps (Figs. 8a and 9a) are clearly degraded in the maps of the sub-samples (Figs. 8b, c and 9b, c). The maps based on the 60 m data and methods (a)–(d) for

Table 6 Correlations of predictions produced with various sub-samples and models with predictions produced with the 30 m data and model at Wallingford

Data and model used for kriging	Correlation with predictions from 30 m data and model	
	LOI	Depth
60 m data and model	0.82	0.81
90 m data and model	−0.14	0.85
120 m data and model	−0.04	0.43
120 m data and (a, REML model)	0.69	0.51
120 m data and (b, Property model)	0.83	0.39
120 m data and (c, Site model)	0.84	0.46
120 m data and (d, Fixed ratio model)	0.84	0.41
120 m + targeted data and (a, REML model)	0.73	0.69

estimating the nugget:sill ratios (Figs. 8d–g and 9d–g) are more similar to the map from the 30 m data than are those for the sub-samples. Moreover, the differences between the maps based on methods (a)–(d) are small for both properties. Although the main features of variation are reasonably preserved in the maps of the 60 + 30 m data for both properties (Figs. 8b and 9b), there is more detail in those based on the 60 m data and the variograms from methods (a)–(d) (Figs. 8d–g and 9d–g).

Table 7 gives the correlations between the estimates from the 30 m data and those from the sub-sampled data with their associated variograms at Yattendon. In addition, the correlations are also given between the former and estimates from the 60 m sub-sample using standardized variograms with the nugget:sill ratios set by methods (a)–(d). For sand content, the correlations are moderate for the 60 m data and strong for the 60 + 30 m data, and they are all strong for methods (a)–(d) of determining the nugget:sill ratio. For stones, the correlations are moderate for the 60 m data, whereas they are strong for the 60 + 30 m data and methods (a)–(d), although a little less strong for the latter.

Discussion

Differences between the nugget:sill ratios of variograms from the 30 m data and those estimated by the methods (a)–(d) show that no method gives consistently the best estimate of the ratio. This was also confirmed by our results for the three other sites mentioned above, but whose results have not been included here in detail. The use of additional data, targeted according to large, intermediate and small values of the digital numbers seems to have added little to the improvement of the overall results, except for soil depth at Wallingford where the 120 m sampling interval had missed some of the main patterns of variation. Farmers are constrained by costs as to the number of samples that they can take and have analyzed. Our aim here was to identify approaches that would produce acceptable results for site-specific management with a small, but spatially dependent sample. Results for other sites (not included) where variograms were computed by other authors (Frogbrook 2000; Oliver and Carroll 2004; Kerry 2004) showed that method (c, Site) can be unreliable when variograms from other surveys with a different suite of soil properties, and different sampling and bulking strategies are used. Method (c) should probably be avoided by farmers who do not have existing variograms from their own surveys. Although data

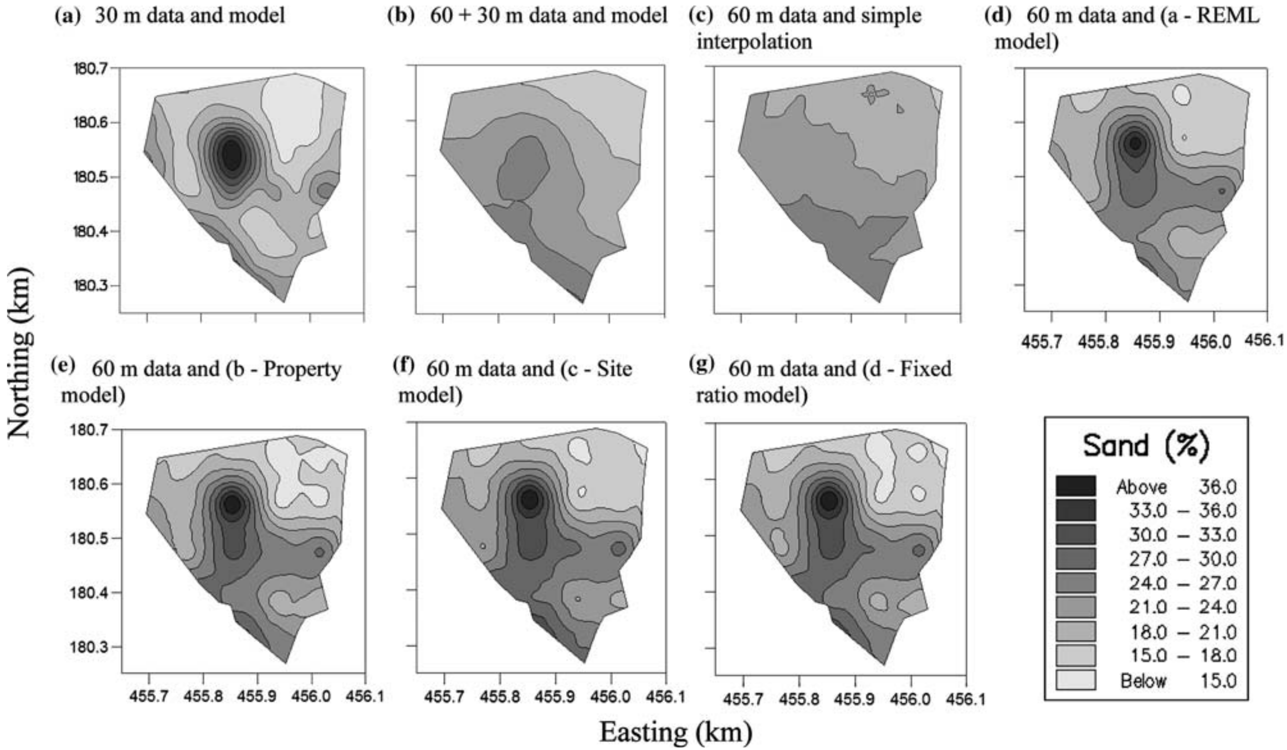

Fig. 8 Maps of percentage sand for Yattendon using: (**a**) 30 m data and model, (**b**) 60 + 30 m data and model, (**c**) 60 m data and simple interpolation, (**d**) 60 m data and (a, REML model), (**e**) 60 m data and (b, Property model), (**f**) 60 m data and (c, Site model) and (**g**) 60 m data and (d, Fixed ratio model)

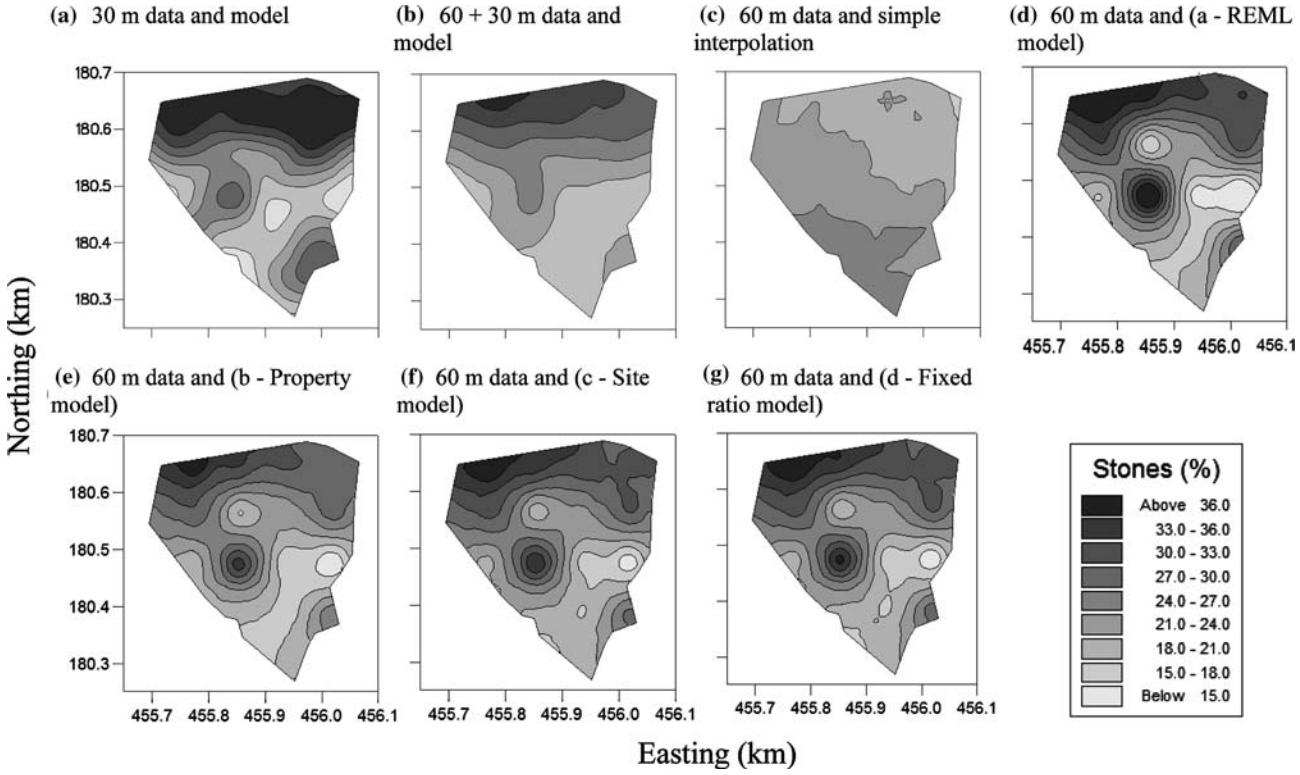

Fig. 9 Maps of percentage stones for Yattendon using: (**a**) 30 m data and model, (**b**) 60 + 30 m data and model, (**c**) 60 m data and simple interpolation, (**d**) 60 m data and (a, REML model), (**e**) 60 m data and (b, Property model), (**f**) 60 m data and (c, Site model) and (**g**) 60 m data and (d, Fixed ratio model)

Table 7 Correlations of predictions produced with various sub-samples and models with predictions produced with the 30 m data and model at Yattendon

Data and model used for kriging	Correlation with predictions from 30 m data and model	
	Sand	Stones
60 m data and model	0.59	0.64
60 + 30 m data and model	0.81	0.92
60 m data and (a, REML model)	0.85	0.83
60 m data and (b, Property model)	0.83	0.84
60 m data and (c, Site model)	0.84	0.84
60 m data and (d, Fixed ratio model)	0.84	0.84

from other studies were not used for method (b, Property), the effects of differences in the methods of data collection and analysis are likely to be similar to those of method (c). Bearing this in mind, it would be better to use a fixed nugget:sill ratio, such as 0.25 or 0.5. The latter would be especially useful for properties measured fairly crudely, such as soil depth, Munsell value and soil stoniness. This was supported by the results for these three variables from the three additional sites (results not included) because the nugget:sill ratios of the variograms for these properties tended to vary markedly from each other and between sites. For example, if there are few stones at a site, the variogram of stoniness will have a larger nugget:sill ratio than if the soil is more stony. It is interesting to note that in surveys of depth to bedrock others have found consistently that variograms had nugget:sill ratios of about 0.5 (Barry Rawlins, British Geological Survey, personal communication). The use of a nugget:sill ratio of 0.5 could also be adopted for properties that do not vary much at a given site.

Conclusions

The detailed results for Wallingford and Yattendon show that standardized variograms with nugget:sill ratios similar to those of the 30 m soil data produce more accurate predictions based on the results of cross-validation and preserve more of the main patterns of variation (Figs. 6–9) with sparse soil data than do variograms of the sub-samples. The results presented here for LOI and depth at Wallingford and for sand and stones at Yattendon are data with a near-normal distribution and no significant trend. The effects of the use of standardized variograms to krige skewed data and data with trend needs to be investigated in the future. Several scenarios were presented and the 'worst case' one for soil depth at Wallingford showed that even where sparse soil data are spatially dependent, important features of the variation may be missed because of the large sample spacing. However, even in this case, kriging with the 120 m data and the standardized variograms was better than simple interpolation of the 120 m data. The results for soil depth also emphasize the benefits of supplementing data on a coarse grid with a few additional samples at short intervals. Precision farming practitioners should not be tempted to krige with variograms computed from sparse data; otherwise the maps of soil properties used to determine variable fertilizer and pesticide rates will not reflect the main patterns of variation present. This will result in over- and under-application of agro-chemicals as in traditional farming where a single rate per field is used and this would partially defeat the object of practicing precision agriculture.

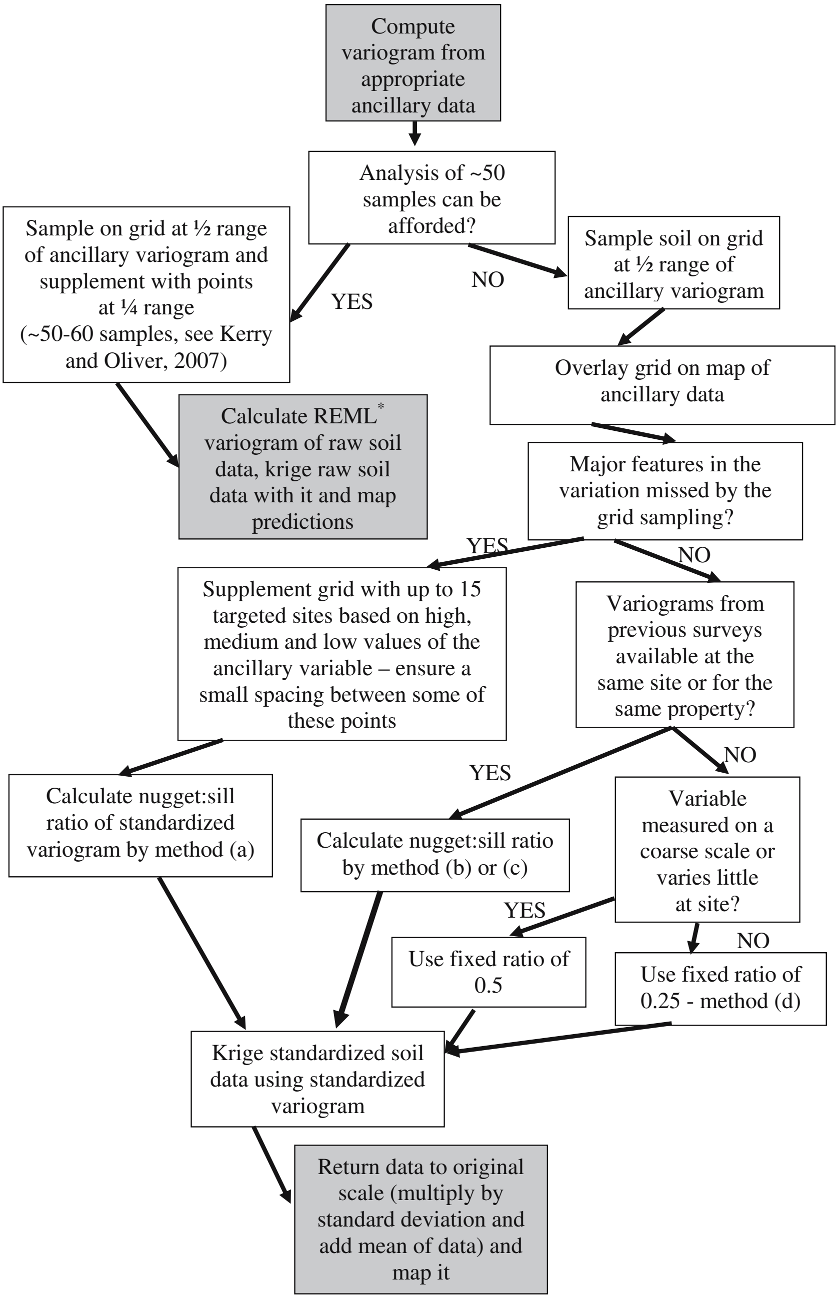

Fig. 10 Flow diagram summarizing recommended steps for mapping sparse soil data based on the present and previous research (Kerry and Oliver 2007). Grey boxes indicate the start and finish points

The methods of estimating the nugget:sill ratio performed similarly. Method (a, REML) tended to under-estimate the nugget variance more than the other methods. At Wallingford, for all properties, this was small and the differences between the REML nugget:sill variances and those for the 30 m data are less than for the other methods. For Yattendon, method (a) under-estimated the nugget:sill variance considerably more than the other methods. Nevertheless, the additional targeted data used for method (a) resolved the spatial variation better. Farmers might wish to use other ancillary data that are strongly related to soil properties, such as EC_a, elevation, yield etc. Further analysis is required to assess the benefits and limitations of standardized variograms computed from different types of ancillary data for estimating sparse, but spatially dependent soil properties. Standardized variograms computed from spectral data of a growing crop could be used to krige sparse, yet spatially structured data from N analysis of leaf samples. This could help to improve site-specific management of N.

Methods (b)–(d) could be applied to data on a coarse grid, but with additional targeted sampling points. This should help to reduce the risk of missing important features in the pattern of soil variation with a sparse sampling scheme and should also lead to a reasonable estimate of the nugget:sill ratio. The REML approach can produce reliable variograms with about 50 data, but this is at least 10 more sampling points than for the above approach. The results suggest that, with the use of standardized variograms from ancillary data, fewer variables are likely to be estimated poorly because the variogram is computed from a large data set. Practitioners have several methods to choose from, but in the absence of prior information about the spatial structure of the soil properties in a field, a fixed ratio provides acceptable results as we have shown.

Our recommendation at present is to adopt the approach set out in the flow diagram (Fig. 10). It requires practitioners to make informed decisions on how to choose a method to determine the nugget:sill ratio; this approach is used in the machine-learning community where expert knowledge provides an initial estimate of model parameters rather than the parameters being data driven.

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